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The Solution of the Inverse Problems on the Basis of the Analitical Continuation of the Transient Electromagnetic Field in the Reverse Time

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The method of the transient electromagnetic field migration based on the "reverse" continuation of the field within the conducting Earth is developed. The properties of the migrated field and the possibilities of electromagnetic migration method application for the geoelectric inverse problems solution are studied in detail.

1. Introduction

During the last decade in geoelectrics the interest in the problem of electromagnetic fields continuation through the conducting Earth and to the solution of the inverse geoelectric problems by means of the continued fields has been significantly increased.

This interest is stimulated by the progress of potential field continuation theory, of optical holography in physics and technics and also by the effective application of seismoholography principles in seismic data interpretation.

The theory of analytical continuation of the transient electromagnetic fields into the conducting Earth was developed in the following works: ZHDANOV (1981), ZHDANOV and FRENKEL (1982). The main principles of so called "reverse" continuation of the field were also formulated there. The technique pointed presents the generalization on the case of variable electromagnetic fields of the well known methods of the reverse continuation of wave fields (so called seismic migration), widely applied in seismic prospecting (TIMOSHIN, 1972; PETRASHEN and NAHAMKIN, 1973; BERKHOUT, 1980). So it is natural to regard the suggested method of electromagnetic field transformation as the "electromagnetic migration".

In this work we would like to study the main features of electromagnetic migration and to analyse the possibilities of using this method for solving the geoelectric inverse problems.

2. The Stratton-Chu Integral Formulas for the Transient Field.

The electromagnetic migration is founded on the Stratton-Chu integral formulas for the transient field.

We will present here the direct derivation of the Stratton-Chu formulas for the transient case, based on the integration of the field equations (Zhdanov, 1981). Let's consider the following electrodynamic problem: to determine the quasistationary electromagnetic field E, H within some domain V by means of the values of E, H on the surface Γ, confining V. It is supposed that the conductivity σ inside V is constant, the magnetic permeability is everywhere equall to μ_0- the permeability of vacuum, and the electromagnetic field sources are outside V. The electromagnetic field inside V and on Γ satisfies the equations:

$$rot\, H = \sigma E;\ rot\, E = -\mu_0 \frac{\partial H}{\partial t}.$$

Let's turn to the Ostrogradsky—Gauss' formula:

$$\iiint_V (\nabla \cdot F)\mathrm{d}V = \iint_\Gamma (F \cdot \mathrm{d}S)$$

and set

$$F = (c \cdot H)\nabla U + [[c \times H] \times \nabla U] - [c \times E]\sigma U,$$

where c is the arbitrary constant vector, and $U = U(r, t)$ is the function, twice continuously differentiable and integrable in modulus on $-\infty < t < \infty$ time axis.

A simple vector manipulation yields:

$$(\nabla \cdot F) = \left(c \cdot \left[H\Delta U - \mu_0 \sigma \frac{\partial H}{\partial t} \cdot U\right]\right).$$

$$(F \cdot \mathrm{d}S) = (c \cdot \{(\mathrm{d}S \cdot H)\nabla U + [[\mathrm{d}S \times H] \times \nabla U] + [\mathrm{d}S \times E]\sigma U\}).$$

Hence,

$$\left(c \cdot \iiint_V \left[H\Delta U - \mu_0 \sigma \frac{\partial H}{\partial t} U\right]\mathrm{d}V\right) =$$

$$\left(c \cdot \iint_\Gamma \{(n \cdot H)\nabla U + [[n \times H] \times \nabla U] + [n \times E]\sigma U\}\mathrm{d}S\right). \qquad (1)$$

where n—a unit vector of the outer normal to Γ.

Integrating (1) for t over infinite interval and taking into consideration that c is an arbitrary vector, we can write finally:

$$\int_{-\infty}^{\infty}\iiint_{V} H\left(\Delta U + \mu_0\sigma\frac{\partial U}{\partial t}\right)\mathrm{d}V\mathrm{d}t =$$

$$\int_{-\infty}^{\infty}\iint_{\Gamma} \{(\boldsymbol{n}\cdot H)\nabla U + [[\boldsymbol{n}\times H]\times\nabla U] + [\boldsymbol{n}\times E]\sigma U\}\,\mathrm{d}S\,\mathrm{d}t. \qquad (2)$$

Let's choose as "U" the function

$$\tilde{G} = \tilde{G}(\boldsymbol{r},t|\boldsymbol{r}',t') = \frac{(\mu_0\sigma)^{1/2}}{2\pi^{1/2}(t'-t)^{3/2}}\exp\left(-\frac{\mu_0\sigma|\boldsymbol{r}-\boldsymbol{r}'|^2}{4(t'-t)}\right)æ(t'-t),$$

where

$$æ(t'-t) = \begin{cases}1, & t < t'\\ 0, & t > t'\end{cases}$$

satisfies the equation

$$\Delta\tilde{G} + \mu_0\sigma\frac{\partial\tilde{G}}{\partial t} = -4\pi\delta(\boldsymbol{r}-\boldsymbol{r}')\delta(t-t')$$

(here δ is the Dirac delta-function and the laplacian acts on the variable $\boldsymbol{r}$). So $\tilde{G}$ is the function conjugate to the fundamental Green's function G for the diffusion equation (MORSE and FESHBACH, 1953):

$$G(\boldsymbol{r},t|\boldsymbol{r}',t') = \tilde{G}(\boldsymbol{r},-t|\boldsymbol{r}',-t'). \qquad (3)$$

Substituting $U=\tilde{G}$ into (2) and considering the field sources to be "switched on" at the moment $t = 0$ ($H \equiv 0$, $E \equiv 0$, if $t \leqslant 0$), we obtain the Stratton-Chu integral formula for the transient magnetic field:

$$-\frac{1}{4\pi}\int_{0}^{t'}\iint_{\Gamma}\{(\boldsymbol{n}\cdot H)\nabla\tilde{G} + [[\boldsymbol{n}\times H]\times\nabla\tilde{G}] + [\boldsymbol{n}\times E]\sigma\tilde{G}\}\,\mathrm{d}S\,\mathrm{d}t =$$

$$\begin{cases}H(\boldsymbol{r}',t'), & \boldsymbol{r}'\in V\\ 0, & \boldsymbol{r}'\notin \bar{V}\end{cases} \qquad (4)$$

where E and H values refer to the internal side of the surface Γ, ∇ operates with respect to $\boldsymbol{r}$, $\bar{V}$ is the domain V with it's boundary.

The Stratton-Chu integral formula for the transient electric field is derived by analogy. We introduce only the final expression for this formula:

$$-\frac{1}{4\pi}\int_{0}^{t'}\iint_{\Gamma}\left\{(\boldsymbol{n}\cdot E)\nabla\tilde{G} + [[\boldsymbol{n}\times E]\times\nabla\tilde{G}] + [\boldsymbol{n}\times H]\mu_0\frac{\partial\tilde{G}}{\partial t}\right\}\mathrm{d}S\,\mathrm{d}t =$$

$$\begin{cases}E(\boldsymbol{r}',t'), & \boldsymbol{r}\in V\\ 0, & \boldsymbol{r}'\notin\bar{V}\end{cases} \qquad (5)$$

3. The Stratton-Chu type integrals for the transient field.

By the analogy with the integrals of Stratton-Chu type for the fields harmonically varying in time (ZHDANOV, 1980a; BERDICHEVSKY and ZHDANOV, 1980) we can introduce the integrals of Stratton-Chu type for the transient fields.

Let some vector fields E and H be given on the internal side of the smooth closed surface Γ, confining the domain V_i. These vectors satisfy the following conditions:

a) they have the continuously differentiable tangential components E_{tg}, H_{tg}; b) their normal components E_n, H_n are related with E_{tg}, H_{tg} by the following equations:

$$E_n = -\frac{1}{\sigma}\operatorname{div}^{\Gamma}[n \times H_{tg}]; \qquad \frac{\partial H_n}{\partial t} = \frac{1}{\mu_0}\operatorname{div}^{\Gamma}[n \times E_{tg}]. \tag{6}$$

where σ -- a constant, $\operatorname{div}^{\Gamma}$ -- a symbol of the surface divergence (the same symbol, which was used in our paper (ZHDANOV, 1980b)).

As the relation (6) is the two-dimensional analog of the Maxwell's equations, the fields E, H refer to the electromagnetic field class.

Consider the expressions:

$$K^{E}(r', t') = -\frac{1}{4\pi}\int_0^{t'}\iint_{\Gamma}\left\{(n \cdot E)\nabla\tilde{G} + [[n \times E] \times \nabla\tilde{G}] + [n \times H]\mu_0\frac{\partial\tilde{G}}{\partial t}\right\}\mathrm{d}S\,\mathrm{d}t \tag{7}$$

$$K^{H}(r', t') = -\frac{1}{4\pi}\int_0^{t'}\iint_{\Gamma}\{(n \cdot H)\nabla\tilde{G} + [[n \times H] \times \nabla\tilde{G}] + [n \times E]\sigma\tilde{G}\}\,\mathrm{d}S\,\mathrm{d}t. \tag{8}$$

These expressions are identical with the integrals in Stratton-Chu formulas (4), (5) and may be called the integrals of Stratton-Chu type. The functions E and H are the "electric" and "magnetic" densities of the Stratton-Chu type integrals for the transient field. The Stratton-Chu type integrals (7), (8) have some features (analogous to those of their harmonical analog), the main of which is the following: the integrals of Stratton-Chu type outside the surface Γ satisfy the Maxwell's equations, and the vector function K^{E}, K^{H} -- the diffusion equation:

$$\Delta K^{E,H} - \mu_0\sigma\frac{\partial K^{E,H}}{\partial t'} = 0.$$

The Stratton-Chu type integrals (7), (8) have a simple physical interpretation. Let the electric and magnetic currents and charges with the surface densities I^{E}, I^{H} and q_S^{E}, q_S^{H} be distributed on the closed surface Γ:

$$I^{E} = [n \times H]; \quad I^{H} = -[n \times E];$$

$$q_S^{E} = \sigma(n \cdot E); \quad q_S^{H} = \mu_0(n \cdot H).$$

Then the electromagnetic field excited by these sources outside Γ is equal to the field $\boldsymbol{K}^{\mathrm{E}}$, $\boldsymbol{K}^{\mathrm{H}}$.

4. The "Reverse" Continuation of the Transient Electromagnetic Field

Let's formulate the following problem. The electromagnetic field E^0, H^0 excited by the transient source J^Q, located either on the external side of the surface Γ_0 or raised above the Earth and switched on at the moment $t = 0$, is given on the Earth's surface. The Earth is characterised by the constant conductivity σ, except some domain D (finitely or infinitely extended), where the conductivity σ_D may vary arbitrarily.

The problem consists in finding the domain with anomalous conductivity through the fields $E^0\ (r, t)$, $H^0(r, t)$, $(0 < t < T)$ measured on the Earth's surface. For solving this problems let's consider the following transformation of the field. From the ordinary time t we turn to the reverse time $\tau = T - t$ and assume auxiliary fields $\boldsymbol{F}\ (\boldsymbol{r}, \tau)$, $\boldsymbol{R}\ (\boldsymbol{r}, \tau)$ on the surface of the Earth Γ_0 using the following formulas.

$$\begin{aligned} F_{tg}(\boldsymbol{r}, \tau) &= E^0_{tg}(\boldsymbol{r}, T - \tau); \quad F_n(\boldsymbol{r}, \tau) = E^0_n(\boldsymbol{r}, T - \tau); \\ R_{tg}(\boldsymbol{r}, \tau) &= H^0_{tg}(\boldsymbol{r}, T - \tau); \quad R_n(\boldsymbol{r}, \tau) = -H^0_n(\boldsymbol{r}, T - \tau). \end{aligned} \tag{9}$$

We can see, that in such a definition the function F, R satisfy the condition (6) (if the time t is substituted by the reverse time τ):

$$F_n = -\frac{1}{\sigma}\operatorname{div}^{\prime}[\boldsymbol{n} \times \boldsymbol{R}_{tg}]; \quad \frac{\partial R_n}{\partial \tau} = \frac{1}{\mu_0}\operatorname{div}^{\prime}[\boldsymbol{n} \times \boldsymbol{F}_{tg}]. \tag{10}$$

Thus, these functions may be used as the densities of the Stratton-Chu type integrals, written for the reverse time τ:

$$E^*(r', \tau') = -\frac{1}{4\pi}\int_0^{\tau'}\iint_{\Gamma_0} \left\{(\boldsymbol{n} \cdot F)\nabla\tilde{G} + [[\boldsymbol{n} \times F] \times \nabla\tilde{G}] + [\boldsymbol{n} \times R]\mu_0\frac{\partial\tilde{G}}{\partial\tau}\right\} \mathrm{d}S\,\mathrm{d}\tau \tag{11}$$

$$H^*(r', \tau') = -\frac{1}{4\pi}\int_0^{\tau'}\iint_{\Gamma_0} \{(\boldsymbol{n} \cdot R)\nabla\tilde{G} + [[\boldsymbol{n} \times R] \times \nabla\tilde{G}] + [\boldsymbol{n} \times F]\sigma\tilde{G}\}\,\mathrm{d}S\,\mathrm{d}\tau. \tag{12}$$

In correspondance with the main features of the Stratton-Chu type integrals the fields E^*, H^* outside Γ_0 satisfy the Maxwell's equations:

$$\operatorname{rot} H^* = \sigma E^*; \operatorname{rot} E^* = -\mu_0\frac{\partial H^*}{\partial\tau'} \tag{13}$$

According to Zhdanov (1981), the field $E^*\ (r', \tau')$, $H^*\ (r', \tau')$ will be called the "reverse" continuation of the electromagnetic field $E^0(r, t)$, $H^0(r, t)$ or simply the "reverse" field.

We can give a simple physical interpretation for the "reverse" field E^*, H^*: the "reverse" field is the electromagnetic field, excited in the homogeneous conductive Earth by the system of fictitious currents and charges, defined through the observed fields E^0, H^0, but in the reverse time (that is these sources are switched on at the final moment $t = T$ of registration of the field working in the reverse time τ and switched off at the moment $t = 0$). Substituting (9) into (11) and (12) and replacing the variable τ in the expressions (11), (12) by $(T - t)$, we find:

$$E^*(r', T - t') = -\frac{1}{4\pi}\int_{t'}^{T}\iint_{\Gamma_0}\Big\{(n \cdot E^0)\nabla G + [[n \times E^0] \times \nabla G] + [n \times H^0]\mu_0\frac{\partial G}{\partial t}\Big\}\,\mathrm{d}S\,\mathrm{d}t \tag{14}$$

$$H^*(r', T - t') = -\frac{1}{4\pi}\int_{t'}^{T}\iint_{\Gamma_0}\{-(n \cdot H^0)\nabla G + [[n \times H^0] \times \nabla G] + [n \times E^0]\sigma G\}\,\mathrm{d}S\,\mathrm{d}t, \tag{15}$$

where $G = G(r, t \mid r', t')$ — is the fundamental Green's function for the diffusion equation

$$G = \frac{(\mu_0\sigma)^{1/2}}{2\pi^{1/2}(t - t')^{3/2}}\exp\left(-\frac{\mu_0\sigma|r - r'|^2}{4(t - t')}\right)æ(t - t')$$

connected with the function $\tilde{G}$ by the formula (3).

Note, that in practical application of the "reverse" continuation formulas (14), (15) to real electromagnetic data it is worth (as it is done in seismic survey) taking for σ the apparent conductivity of the Earth $\sigma_a(\tau)$, obtained using the corresponding formulas of the electromagnetic sounding and averaged in time in the interval $(0, \tau')$:

$$\sigma = \frac{1}{\tau'}\int_0^{\tau'}\sigma_a(\tau)\,\mathrm{d}\tau. \tag{16}$$

In such a case the real inhomogeneous section of the Earth is substituted by some homogeneous model (unique for each time τ').

5. The Electromagnetic Field Migration

The way of introducing the auxiliary fields F and R on the Earth's surface

Γ_0 suggested in the previous is not unique.

Really, let's define F and R as the followings:

$$F_{\tau a}(r,\tau) = E^{0}_{\tau a}(r, T-\tau); \quad F_n(r,\tau) = \frac{1}{c}\dot{E}^{0}_{n}(r, T-\tau);$$
$$H^{0}_{\tau a}(r,\tau) = H^{0}_{\tau a}(r, T-\tau); \quad R_n(r,\tau) = -H^{0}_{n}(r, T-\tau). \tag{17}$$

where $c = \mathrm{const} > 0$.

The direct test shows that the fields F and R defined by (17) will satisfy the conditions (10) if σ is substituted by $c\sigma$. I.e. for determining the Stratton-Chu type integrals by means of F and R functions, we substitute the real Earth by a half-space with some fictitious conduction

$$\sigma^{m} = c\sigma. \tag{18}$$

In such an introduction of the F and R fields the formulas (14) and (15) transform to the following form:

$$E^{m}(r', T-t') = -\frac{1}{4\pi}\int_{t'}^{T}\iint_{\Gamma_0}\left\{\frac{(n\cdot E^{0})}{c}\nabla G^{m} + [[n\times E^{0}]\times\nabla G^{m}] + [n\times H^{0}]\mu_0\frac{\partial \dot{G}^{m}}{\partial t}\right\}\mathrm{d}S\,\mathrm{d}t \tag{19}$$

$$H^{m}(r', T-t') = -\frac{1}{4\pi}\int_{t'}^{T}\iint_{\Gamma_0}\{-(n\cdot H^{0})\nabla G^{m} + [[n\times H^{0}]\times\nabla G^{m}] + [n\times E^{0}]\sigma^{m}G^{m}\}\,\mathrm{d}S\,\mathrm{d}t \tag{20}$$

where

$$G^{m} = \frac{(\mu_0\sigma^{m})^{1/2}}{2\pi^{1/2}(t-t')^{3/2}}\exp\left(-\frac{\mu_0\sigma^{m}|r-r'|^2}{4(t-t')}\right)æ(t-t').$$

Transformation of the field E^{0}, H^{0} observed on Γ_0, carried out by the formulas (19), (20) we will call electromagnetic field migration, and the field E^{m}, H^{m} itself — the migrated electromagnetic field. Obviously the "reverse" field is a particular case of the field obtained as a result of the migration (when $c = 1$, $\sigma^{m} = \sigma$). But as it will be shown below the migrated fields E^{m}, H^{m} has the properties more convenient for the electromagnetic data interpretation than the ordinary "reverse" field E^{*}, H^{*}. In conclusion of this section we should note, that the conception of the electromagnetic migration is introduced analogous to the seismic migration in order to emphasize the principle resemblance of these methods.

As a matter of fact, the migration procedure, as we will see later, allows us to "focus" the electromagnetic field on the geoelectric inhomogeneity. It should be underlined that the field transformation described by the formulas (19) and (20) is a stable procedure because the operators in the right part of these formulas are limited in L_2 space. Therefore the computer implementation of the electromagnetic field migration procedure is reduced to the simple quadratures.

6. The "Pseudomigrated" Field

Let's call "pseudomigrated" the field E^p, H^p, obtained as a result of the formal substitution in formulas (4), (5) the ordinary Green's function G instead of $\tilde{G}$, conjugate to the fundamental Green's function (with the corresponding change of the limits of integration over time t):

$$E^p(r', T - t') = -\frac{1}{4\pi}\int_{t'}^{T}\iint_{\Gamma_0}\Big\{(n \cdot E^0)\nabla G + [[n \times E^0] \times \nabla G] + [n \times H^0]\mu_0\frac{\partial G}{\partial t}\Big\}\,\mathrm{d}S\,\mathrm{d}t \tag{21}$$

$$H^p(r', T - t') = -\frac{1}{4\pi}\int_{t'}^{T}\iint_{\Gamma_0}\{(n \cdot H^0)\nabla G + [[n \times H^0] \times \nabla G] + [n \times E^0]\sigma G\}\,\mathrm{d}S\,\mathrm{d}t. \tag{22}$$

Evidently, the fields E^p, H^p, determined by the equations (21), (22) don't satisfy the Maxwell's equations (therefore, they shold be examined independently). It is easy to see that each of them separately satisfies the diffusion equation in reverse time $\tau = T - t$, for example:

$$\Delta H^p - \mu_0\sigma\frac{\partial H^p}{\partial \tau'} = 0.$$

The "pseudomigrated" field E^p, H^p is a result of a formal (without the corresponding coordination with the boundary conditions on Γ_0) transformation of the E^0, H^0 field observed on Γ_0, but nevertheless this field (more precisely the "magnetic" component of the "pseudomigrated" field H^p) has the remarkable properties which are very suitable for practical research. We will show in the next Section that the field H^p makes possible the exact localization of the anomalous objects, therefore it may be also used in solving the inverse problems of EM-induction in the inhomogeneous medium for the accuracy estimation of the anomalous field sources location.

7. The Properties of Electromagnetic Migration

In order to make the features of the "reverse" continuated field E^*, H^* more visible let's turn again from the reverse time τ to the ordinary direct time t:

$$\hat{E}(r',t') = E^*(r', T - t'); \quad \hat{H}(r',t') = H^*(r', T - t').$$

Acording to (13) the fields $\hat{E}$, $\hat{H}$ in direct time satisfy the equations:

$$\operatorname{rot} \hat{H} = \sigma \hat{E}; \quad \operatorname{rot} \hat{E} = \mu_0 \frac{\partial \hat{H}}{\partial t'}$$

and, therefore, satisfy the following equations:

$$\Delta \hat{E} + \mu_0 \sigma \frac{\partial \hat{E}}{\partial t'} = 0; \quad \Delta \hat{H} + \mu_0 \sigma \frac{\partial \hat{H}}{\partial t'} = 0. \tag{23}$$

If the ordinary diffusion equation describes the process of the electromagnetic field propagation in time increasing from the sources to the observer, then the Eq. (23) describes the same process in the inversed order from the final distribution of the field E^0, H^0 on the Earth's surface to its initial distribution in the sources. Therefore, the field transformation suggested (14), (15) describes the transformation of the electromagnetic fields, diverged in the real medium and diffracted on the geoelectric inhomogeneities, into the fields converged to the corresponding points, lines and surfaces of diffraction. In this case at the moment $\tau' = T$ (or $t' = 0$) as it takes place in seismoholography, the fields $E^*(r', T) = \hat{E}(r', 0)$ and $H^*(r', T) = \hat{H}(r', 0)$ form images of the field sources, connected with geoelectric inhomogeneities.

The process described is essentially equivalent to the field transformation carried out in the ordinary optical holography. Really, if we go in formulas (14), (15) to the time spectra, we get in their right-hand parts the formulas analogous to the ordinary Frenel's formulas (only written for the diffusion process) and the transition to the reverse time is reduced to the sign inversion of the electromagnetic field phase and correspondingly the replacement of the diverged waves superposition by the superposition of the converged waves.

We will illustrate this statement with some theoretical and practical examples, which show the resolving power of the method, and with this base determine some principles of the transient electromagnetic fields interpretation.

Let's consider the three-dimensional situation in which the electromagnetic field is excited by the horizontal electric dipole, immersed in the infinitely homogeneous medium with the conductivity σ. Dipole is fixed in r_0 (x_0, y_0, z_0) point of the medium and moreover $z_0 > 0$. The current in the source is varied according to the law

$$J(t) = j\delta(t), \tag{24}$$

where $\delta(t)$ — the Dirac delta function; j — a constant.

Denote as E^0, H^0 the electromagnetic field registrated at the time interval $(0, T)$ on the horizontal surface Γ_0, coinsident with $X0Y$ plane, and investigate the features of the field E^*, H^*, obtained as the result of the "reverse" continuation of the field E^0, H^0 into the low half space. The horizontal components of the "reverse" field H_x^*, E_y^* according to (14), (15) are the following:

$$H_x^* = \frac{1}{4\pi}\int_{t}^{T}\int\int (H_x^0 \cdot G_{,z} - H_z^0 \cdot G_{,x} - E_y^0 \sigma G)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}t$$
(25)
$$E_y^* = \frac{1}{4\pi}\int_{t}^{T}\int\int (H_x^0 \mu_0 G_{,t} + E_y^0 \cdot G_{,z})\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}t.$$

In formulas (25):

$$G_{,\varphi} = \frac{\partial G}{\partial \varphi}, \quad \varphi = x, y, z, t,$$

where

$$H_x^0 = -\frac{j}{4\pi} G_{,z}; \quad H_z^0 = \frac{j}{4\pi} G_{,x} \tag{26}$$

$$E_y^0 = \frac{j}{4\pi}\left(\frac{1}{\sigma} G_{,yy} - \mu_0 G_{,t}\right).$$

In this case we can show that the extremum points of the "reverse" field (of H_x^* and E_y^* components) are to be searched for in the half-space $z > 0$ on the vertical axis, passed through the dipole centre. This property is true also for the analogous components of the migrated and "pseudomigrated" fields, that essentialy simplifies the research of the extremum properties of these fields.

So, let $x' = x_0$; $y' = y_0$ (without restricting of generality we may take $x_0 = y_0 = 0$) and $t' = 0$, then the expressions (25) may be written:

$$\begin{aligned} H_x^* &\sim \int_0^T \int\int \left(z_0^2 - z_0 z' - \frac{4t}{\mu_0\sigma}\right) t^{-5} \exp\left(-\frac{\mu_0\sigma}{4t}(2x^2 + 2y^2 + z'^2 + z_0^2)\right)\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}t \\ &\sim \int_0^T \left(z_0^2 - z_0 z' - \frac{4t}{\mu_0\sigma}\right) t^{-4} \exp\left(-\frac{\mu_0\sigma}{4t}(z'^2 + z_0^2)\right)\mathrm{d}t \\ &\sim (z_0^2 + z'^2)^{-3}(z'^2 + 2z_0 z' - z_0^2) \end{aligned} \tag{27}$$

$$\begin{aligned} E_y^* &\sim \int_0^T \int\int \left\{\frac{3}{2} z_0 + z' - \frac{\mu_0\sigma}{4t}[(z' + z_0)z' z_0 + z_0(x^2 + y^2) + z' x^2]\right\} \\ &\quad \cdot \exp\left(-\frac{\mu_0\sigma}{4t}(2x^2 + 2y^2 + z'^2 + z_0^2)\right)\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}t \\ &\sim (z_0^2 + z'^2)^{-4}(3z'^3 - 8z'^2 z_0 - 9z' z_0^2 + 4z_0^3) \end{aligned} \tag{28}$$

(Here and further the symbol " ~ " means, that the expressions presented are accurate up to the constant coefficients).

The necessary conditions of the local extremum for the H_x^* and E_y^* functions lead correspondingly to the algebraic equations of three- and four-order.

$$2\alpha^3 + 5\alpha^2 - 4\alpha - 1 = 0 \tag{29}$$

$$\frac{5}{8}\alpha^4 - 2\alpha^3 - 3\alpha^2 + 2\alpha + \frac{3}{8} = 0, \tag{30}$$

where

$$\alpha = \frac{z'}{z_0}$$

We are interested only in positive roots of the equations (29), (30). The first equation has only one positive root:

$$\alpha_{x1} \cong 0.8$$

and the second - two:

$$\alpha_{y1} \cong 0.65; \; \alpha_{y2} \cong 4.2.$$

Note that

$$\left.\frac{\partial^2 H_x^*}{\partial z'^2}\right|_{\alpha_{x1}} \neq 0; \qquad \left.\frac{\partial^2 E_y^*}{\partial z'^2}\right|_{\alpha_{y1};\,\alpha_{y2}} \neq 0.$$

So both necessary and sufficient conditions of local extremum are satisfied for the "reverse" field components H_x^*; E_y^* in the α_{x1}, α_{y1} and α_{y2} points.

Write the coordinates of these points through the depth of cable bedding z_0:

$$z^e_{H1} \cong 0.8\, z_0; \quad z^e_{E1} \cong 0.65\, z_0; \quad z^e_{E2} \cong 4.2\, z_0. \tag{31}$$

The conditions (31) show that the dipole location in the medium is uniquely difined by the location of the "reverse" field local extremum at the moment $t' = 0$.

The change of z-axis scale for analytical field continuation in reverse time according to (31) makes possible the precise location of the anomalous field sources. However, the electric component E_y^* has "false" extremum z^e_{E2}, that can make the interpretation more complicated, for example, in the case, when Z_0 has a small value. The excluding of this "reverse" field extremum from consideration is possible when the electric and magnetic components H_x^*; E_y^* are interpretated jointly.

More accurate determanation of the "reverse" continuation singularities corresponding to the real anomalous field sources is possible if the interpretation is done by means of the electromagnetic field migration procedure.

In this case the "reverse" continuation is carried out into the medium with some fictitious conductivity $\sigma^m = c\sigma$. Let's estimate a value of the positive constant "c" from the condition that the location of the extremum of the migrated field magnetic component $H_x{}^m$ coincides with the source (dipole) location.

Carrying out the transformation analogous to (27) and (28) we obtain the following expressions for $H_x{}^m$ and $E_y{}^m$:

$$H_x^m \sim (z_0^2 + cz'^2)^{-3}(cz'^2 + 2z_0z' - z_0^2) \tag{32}$$

$$E_y^m \sim (z_0^2 + cz'^2)^{-4}[z'^3(2c^3 + c^2) - z'^2z_0(5c^2 + 3c) - z'z_0^2(4c^2 + 5c) + z_0^3(c + 3)]. \tag{33}$$

Obviously, when $c = 1$ formulas (32), (33) coincide with (27), (28). Further,

$$\left.\frac{\partial H_x^m}{\partial z'}\right|_{\substack{z'=z_0\\ c=0.5}} = 0; \quad \left.\frac{\partial E_y^m}{\partial z'}\right|_{\substack{z'=1.07z_0\\ c=0.5}} \cong 0; \quad \left.\frac{\partial E_y^m}{\partial z'}\right|_{\substack{z'=z_0\\ c=0.55}} \cong 0.$$

So the value of $c = 0.5$ may be taken as optimum for the procedure of the three-dimensional field migration.

The "pseudomigrated" field has the same properties. Really, following (21) and (26), we can write:

$$H_x^p - \int_{t'}^{T}\iint_{-\infty}^{\infty} (G_{,x0}G_{,x} - G_{,z0}G_{,z} - G_{,yy0}G + \mu_0\sigma G_{,t0}G)\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}t,$$

where index "o" of the Green's function denotes the dipole field.

$$\left.\frac{\partial H_x^p}{\partial z'}\right|_{\substack{r'=r_0\\ t'=0}} \sim \int_0^T\iint_{-\infty}^{\infty} (G_{,x}G_{,xz} - G_{,z}G_{,zz} - G_{,yy}G_{,z} + \mu_0\sigma G_{,t}G_{,z})\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}t$$

$$\sim \int_0^T\iint_{-\infty}^{\infty} G_{,z}(G_{,xx} + G_{,yy} + G_{,zz} - \mu_0\sigma G_{,t})\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}t \equiv 0.$$

We can show, that $\partial^2 H_x{}^p / \partial z'^2 \,|_{r'=r_0,\, t'=0} \neq 0$ and the point r_0 (x_0, y_0, z_0) is the only extremum point $H_x{}^p$ in the low half-space.

It should be pointed out that the "reverse", migrated, "pseudomigrated" fields' properties obtained above are naturally (by means of reflection method) generalized on the case when the field sources are immersed into the conductive homogeneous half-space, that corresponds to the real geoelectric situations. The analogous property takes place in two-dimensional case when the field is excited by the infinitely long cable.

8. The Migration of the Model Electromagnetic Field

Let's show the theoretical regularities found with some numerical experiments. For example determine the anomalous fields excited by: 1) infinitely long cable or horizontal electric dipole, immersed into the Earth in which the current varies according to Eq. (24) (or a system of infinitely long cables, located at different depths); 2) conducting bodies of simple geometric form immersed into a homogeneous half-space of poor conductivity. The source of the primary field is located on the surface of the Earth and works in the impulse-regime (the switch on impulse - regime is used).

Consider the first series of models with the elementary sources of the anomaly field.

In Figs. 1a, b there are shown the isoline maps of the "pseudomigrated" field H_x^p of one cable (Fig. 1a), and two cables, located at different depths (Fig. 1b). It is distinctly seen, that the location of the sources is well fixed by the extremum points of the field H_x^p.

A good agreement of the theory with practical results was also demonstrated with a three-dimensional case. The maps of isolines H_x^m $(c = 0.5)$, constructed for the moment $t' = 0$ in two mutually perpendicular planes are given in Figs.2a,b: a) in the vertical plane being perpendicular to the dipole axis and passing through its centre (Fig. 2a); b) in the horizontal plane in which the dipole is located (Fig. 2b). We see from these figures that the location of the source in the three-dimensional medium is confidently defined by extremum points of the migrated field.

Let's describe now the second series of models in which the initial anomalous

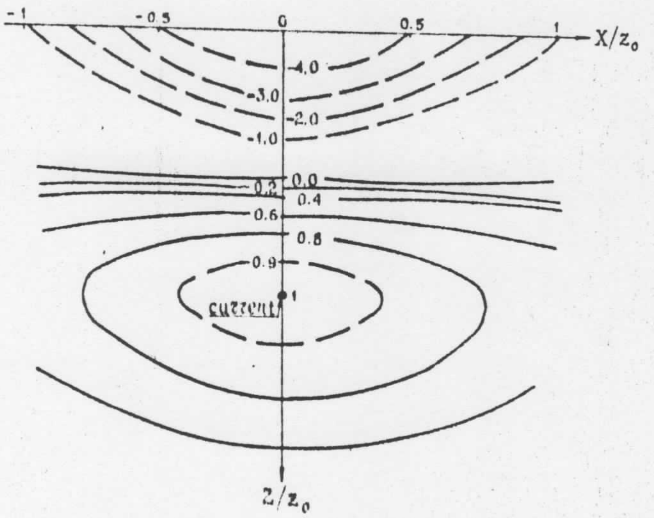
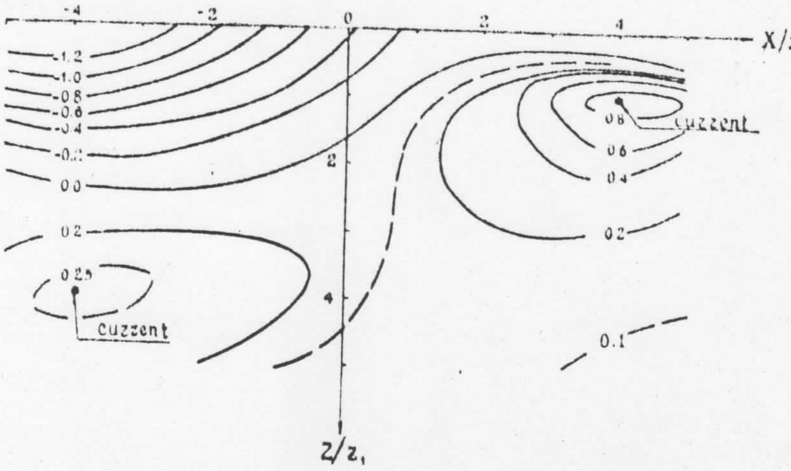


Fig. 1 (a)



(b)

Fig. 1.

field on the horizontal inhomogeneous Earth's surface was calculated by the finite-difference method (ZHDANOV *et al.*, 1982). The simple geoelectric situations were chosen for the modelling (Fig. 3), and the external field was given as a plane wave with amplitude being changed according to the switching on impulse law. This allowed calculation of the electromagnetic fields at several periods by the net program rather quickly. As the field spectrum is located in the narrow frequency range and appears to be a smooth frequency function, the Fourier transform of the field spectrum may be carried out by the symple computational scheme of Filon's method.

The different methods of the electromagnetic field migration which take into consideration the specificity of the concrete model and the properties of the "migrated" field were tested on medium models which contained local geoelectric inhomogeneities.

Model 1. A thin, well conducting horizontal strip (Fig. 3a). At the zero-moment of time ($t' = 0$) the anomaly field sources concentrated along the strip on the horizontal plane are "switched on". Therefore, the location and the form of the geoelectric inhomogeneity can be defined by the isolines of the "reverse" field*) (Fig. 4).

Model 2. The conductive square inset (Fig.3b). The electromagnetic migra-

*Here and in later examples the isoline maps of the "reverse" field electric component E_y^* is adduced. We concluded by experience with practical calculations that this component of the "reverse" field in the case of E-polarization provides more information about the anomalous object than the magnetic components H_x^* and H_z^*.

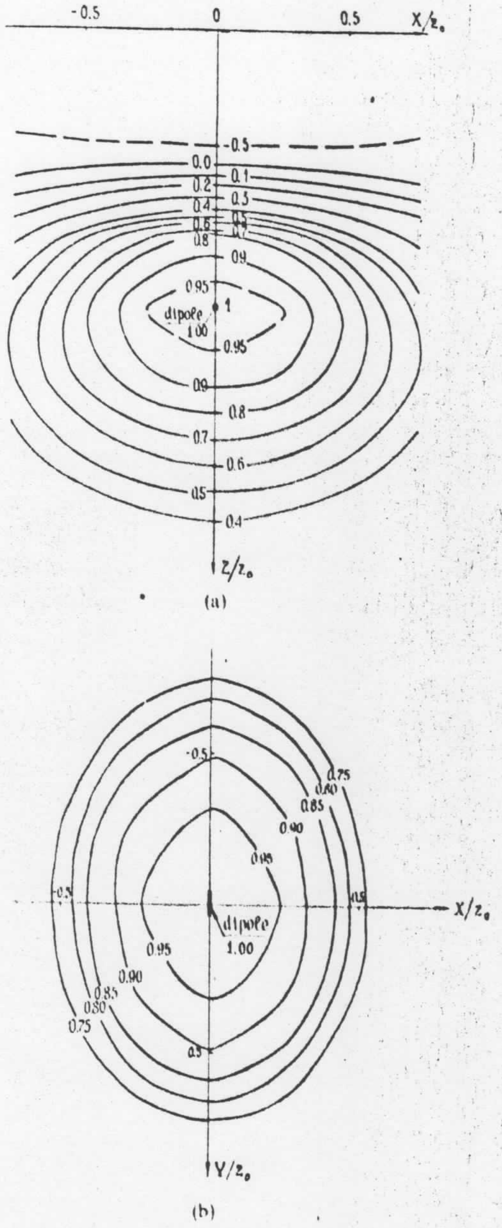
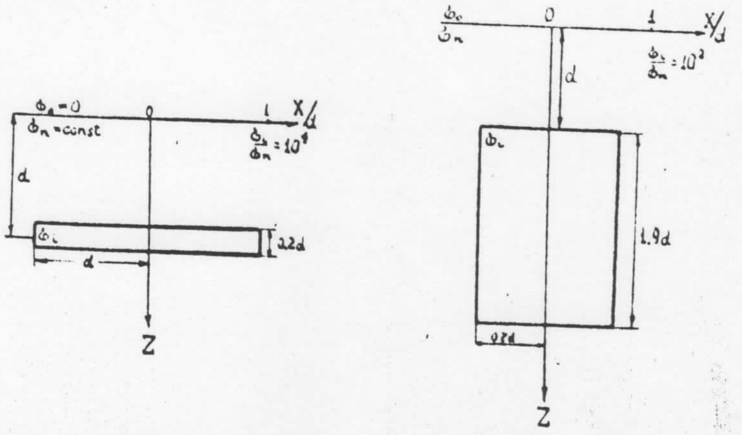

-0.5
0
0.5
X/z₀
-0.5
0.0
0.1
0.2
0.3
0.4
0.8
0.9
0.95
dipole
1
1.00
0.95
0.9
0.8
0.7
0.6
0.5
0.4
Z/z₀
(a)
-0.5
0.75
0.80
0.85
0.90
0.95
0.5
X/z₀
dipole
1.00
0.95
0.90
0.85
0.80
0.75
0.5
Y/z₀
(b)

Fig. 2.



(a) (b)

Fig. 3.

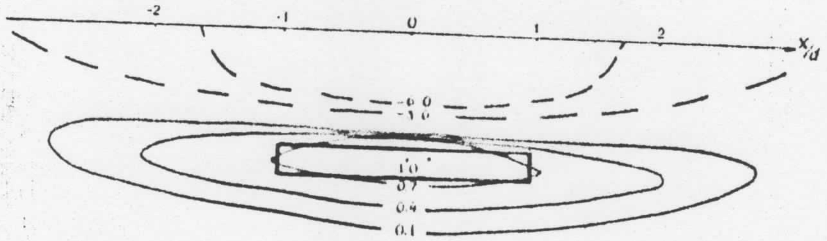


Fig. 4.

tion carried out according to the formulas of the "reverse" continuation allows us to distinguish at each moment of time the most active layer of geoelectric section, therefore, the restoration of the spatial distribution of the "reverse" field has to be carried out layer by layer: the field at each layer is restored at the moment of switching on the excess currents (the "sources" of the field in the layer) corresponding to the layer. The time of switching on is defined by the expression for the skin-depth of the transient field (MAKAGONOV, 1977):

$$h(t) = a\sqrt{2\pi t/\mu_0\sigma},$$

where

a - some constant, independent upon t,
σ - apparent conductivity, calculated by (16).

The result of such a restoration for the model 2 is presented in Fig. 5. A local extremum, which may be connected with maximum intensity of induction currents is distinctly shown in the figure. Evidently, the restoration of general spatial distribution of the "reverse" field at the moment corresponding to the local extremum is the most informative. Really, as it is seen in Fig. 6, the $E_y{}^*$ isolines distinctly reflect the location and form of the geoelectric inhomogeneities. The picture of the "reverse" field drawn at the other moments of time is not so expressive and can lead to errors in the definition of the location and form of the anomalous body.

9. Conclusion

In conclusion we will indicate the main features of the method suggested:

1) the simplicity of the numerical realization of the electromagnetic field migration procedure both in two-and three-dimensional situations;

2) the method is true to the transient fields of both artificial and natural origin;

3) for localization of deep inhomogeneity and definition of it's form we need the information only about the electromagnetic field observed at the surface of the Earth and the conductivity of the normal section.

It seems to us, that these features, in addition to the theoretical to and experimental investigation made above, show that the migration of the electromagnetic field may be used as an effective method of solving the inverse geoelectric problem, to determine the "geoelectric image" of the medium.

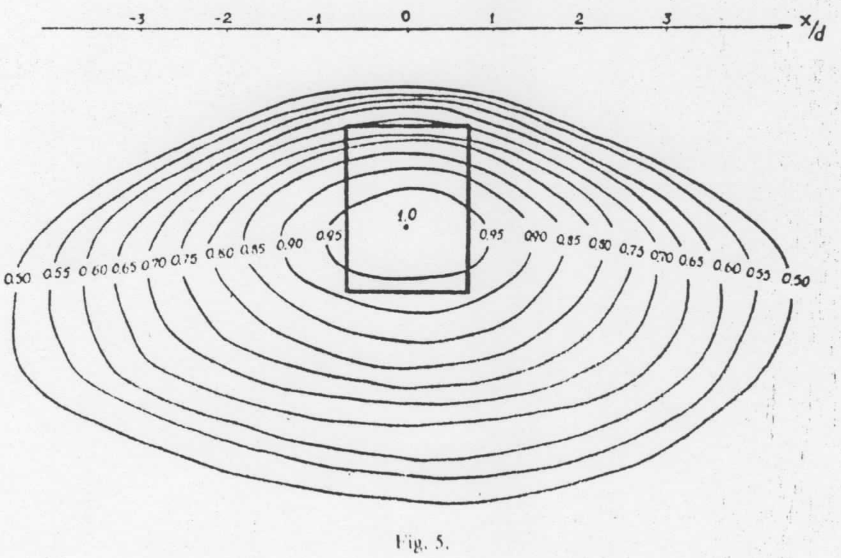


Fig. 5.

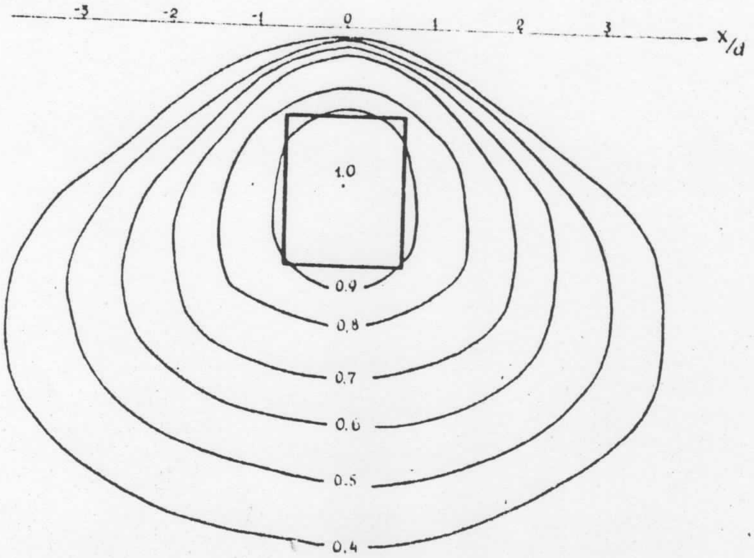


Fig. 6.

The authors hope that electromagnetic migration will become in future as powerful a method of solving the inverse geoelectric problems as seismic migration is in seismoprospecting.

We would like to thank Dr. J. R. Booker who suggested call our method of the continuation of the transient EM-field in the reverse time "electromagnetic migration".

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