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Interpretation of local two-dimensional electromagnetic anomalies by formalized trial procedure

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Summary. An iterative algorithm is presented to be used in the search for the shape of a 2-D local deep geoelectric inhomogeneity lying in a layered medium; an anomaly having been identified in the usual way by observing an alternating time-harmonic electromagnetic field along the surface of the Earth. The normal section parameters (conductivities and thicknesses) and the excess electrical conductivity (inside inhomogeneity) are assumed to be known. The shape of the inhomogeneity is determined by means of a misfit functional minimization technique. A gradient minimization algorithm is constructed and Tikhonov's regularization scheme is applied to achieve stability of the solution. The effectiveness of such an approach is demonstrated by model calculations and by the interpretation of the Carpathian geomagnetic anomaly. Finally, a brief discussion of the problems of the practical application of this formalized trial procedure is presented. Because of the lack of reliable estimates of the excess conductivity, it is proposed to consider a family of models selected for the set of probable values of model parameters. This family can be treated as a generalized solution of the interpretation problem.

1 Introduction

The study of variable electromagnetic field anomalies caused by the horizontal heterogeneities of geoelectric sections is the real challenge of today's geoelectric methods. Known anomalies can be subdivided into two large groups according to the superficial or deep nature of their sources (Rikitaki 1966; Schmucker 1970; Berdichevsky & Zhdanov 1981). The first group includes the anomalies related to the inhomogeneity of the Earth's crustal layer adjacent to the surface; the second group consists of the anomalies caused by the deep inhomogeneities in the Earth's crust and upper mantle. The geomagnetic fields observed on the surface of the Earth are, in general, defined by both the superficial and deep factors; therefore the first problem in the study of geomagnetic anomalies is to separate them into superficial and deep parts. A number of papers refer to this problem (Schmucker 1970, 1971; Berdichevsky & Zhdanov 1975; Zhdanov 1975, 1980).

The problem of the geological interpretation of the superficial and deep anomalies must then be solved. The interpretation of superficial anomalies for the majority of cases can be reduced to the determination of the total longitudinal conductivity of the heterogeneous layer adjacent to the surface. This can be solved by means of the integral transforms of the observed field (Schmucker 1970, 1971; Zhdanov 1975; Zhdanov, Berdichevsky & Zhdanova 1975; Rokityansky 1975; Weidelt 1975).

The problem of the interpretation of deep geomagnetic anomalies is far more complicated. Zhdanov (1975, 1980) presented a method to solve this problem based on analytical continuation of a field into the lower half-space. Analysis of the continued electromagnetic field vector lines makes it possible to determine in a number of practically important cases the location and form of deep inhomogeneities (Zhdanov, Varentsov & Golubev 1978; Berdichevsky & Zhdanov 1981; Varentsov 1981).

To determine the form of deep geoelectric inhomogeneities in detail it is useful to employ the method of successive approximations, widely used in various geophysical investigations. In this method the form of a region with anomalous electrical conductivity is corrected by comparing the theoretically calculated fields with observations.

Such an approach was developed for the solution of the 2-D inverse problem by Weidelt (1975, 1978), Jupp & Vosoff (1977), and Oristaglio & Worthington (1980). In these algorithms rather complicated geoelectric models were considered and strict modelling techniques employed. However, all the schemes mentioned above contain a great number of independent, discrete parameters that cause unreliable convergence in the approximation process, and instability with respect to initial data errors.

So it is very important to improve the parameterization of optimized models and thus achieve better convergence and stability in the approximation process. In the present paper this idea is extended in a rather simplified form, and an effective trial algorithm is suggested for the inversion of local conductivity structure.

Statement of the problem

Consider a 2-D model of geoelectric section which consists of the conducting horizontally layered earth contacting at $Z=0$ with the homogeneous non-conducting atmosphere. Specific electrical conductivities (σ_n, $n=1, N$) and thicknesses (h_n, $n=1, N-1$, $h_N=\infty$) of layers in the model are assumed to be known. Let the geoelectric inhomogeneity Q in the lth layer be such that the electrical conductivity in this layer is defined as:

$$\sigma(\bar{r}_q) = \begin{cases} \sigma_l, \bar{r}_q \notin Q \\ \sigma_l + \Delta\sigma(\bar{r}_q), \bar{r}_q \in Q \end{cases}$$

where $\bar{r}_q$ is the radius vector of the observation point and $\Delta\sigma(\bar{r}_q)$ is an arbitrary function describing the anomalous (excessive) electrical conductivity.

A field in the model is generated by extrinsic electrical currents distributed through the region P of the atmosphere. The time dependence of the field is expressed using the multiplier $\exp(-i\omega t)$. Magnetic permeability is constant in the whole space and equals $\mu_0 = 4\pi \cdot 10^{-7}\,\mathrm{H\,m^{-1}}$. Displacement currents are negligible. We assume that the field and medium are uniform along the y-axis; i.e. we can solve the problem with a 2-D formulation. Thus we restrict ourselves to the consideration of the most interesting case of E-polarization.

We assume that we know the electromagnetic field along a certain profile X on the Earth's surface in the range of frequencies Ω, the parameters of the normal geoelectric section $\{\sigma_n, h_n\}$, and the functional dependence of the anomalous electrical conductivity $\Delta\sigma$ on the coordinates of the point of observation. The problem is to determine a boundary for the region Q, characterized by the anomalous electrical conductivity.

When formulating this problem we naturally face the question of the uniqueness of its solution. In the theory of potential fields there is Novikov's theorem (Novikov 1938) which guarantees the uniqueness of the solution of the inverse problem for stellar bodies with known distribution of excessive density. We have no such fundamental theorem in the theory of electromagnetic fields; therefore, we must search for one of many possible solutions. At the same time, if electromagnetic fields are accurately measured along the rather prolonged profile at the Earth's surface, and in a wide range of periods, we have every justification to suppose that this information would be enough to determine uniquely the deep inhomogeneities. Theoretical proof of this statement would need a special study, transgressing the bounds of this paper (Weidelt 1978).

In order to solve the formulated problem with the approximation method it is necessary to compare the observed fields with the results of numerical calculations.

An accurate solution of the direct problem for a region of inhomogeneity of a rather arbitrary shape requires the consideration of complicated boundary problems, or integral equations (Berdichevsky & Zhdanov 1981). Therefore, in developing effective trial algorithms, some simple approximative methods may be used to solve the direct problem if the required accuracy of calculations is stated.

It has been shown by Kaufman (1974) that under certain conditions the solution to the direct problem in the introduced model may be obtained easily and exactly, if the anomalous currents $\bar{j}_{ex}$ induced in the region of inhomogeneity Q are assumed to be proportional to the normal electrical field. In particular under E-polarization

$$\bar{j}_{ex} = (0, j_{ex}, 0)$$

and the scalar function j_{ex} can be approximately determined by the formula:

$$j_{ex} = \Delta\sigma E_y^n \tag{1}$$

where E_y^n is the normal electrical field.

The calculations performed by Kaufman show that with the approximation of a plane wave external field, and with the assumption that the excessive electrical conductivity $\Delta\sigma$ is constant, the conditions for validity of relationship (1) are reduced to:

$$\frac{D}{\lambda_i} < 0,1 \quad \text{at} \quad 1 < \frac{\sigma_l + \Delta\sigma}{\sigma_l} < 10^3 \tag{2}$$

$$\frac{D}{\lambda_l} < 0,2 \quad \text{at} \quad \frac{\sigma_l + \Delta\sigma}{\sigma_l} < 1$$

where D is the maximum linear dimension in the section of cylindrical inhomogeneity, λ_i is the wavelength within it, and λ_l is the wavelength in the lth layer outside the heterogeneity. In other words, (1) gives a good approximation, if the dimensions of the inhomogeneity are small enough compared to the length of the electromagnetic wave within it and in the surrounding medium.

The distribution of the normal electrical field E_y^n in the layered earth is defined as the continuation of the normal component of the electrical field selected from the observed field at the surface of the Earth (Berdichevsky & Zhdanov 1975, 1981; Zhdanov 1980). Thus, the inverse problem of the analysis of the deep anomalies can be reduced to a search for a boundary of the region Q, which is characterized by the known distribution of excessive currents j_{ex}. In other words, the problem is to determine the form of the region Q, which is filled with the extrinsic currents of known density.

3 Determining of the boundary of local geoelectric inhomogeneity (tightening surface method)

Let a deep electromagnetic anomaly $U(U = H_x, H_z$ or $E_y)$ be known along a certain profile X at the surface of the Earth in the frequency range Ω. Moreover, let the distribution of the density of 'extrinsic' anomalous currents j_{ex} also be known.

From now on we shall assume that the unknown region Q is concentrated around a certain intrinsic point O, the coordinates of which are assigned. We then place the origin of the polar coordinate (ρ, ϕ) system at the point O and describe the boundary of the region Q in these coordinates by the equation:

$$\rho = \hat{f}(\phi)$$

It is necessary to find the boundary of the anomalous region Q, i.e. to determine the unknown function $\hat{f}(\phi)$ by means of the observed electromagnetic anomaly $U(x, \omega)$ $(x \in X, \omega \in \Omega)$.

We shall restrict ourselves to a search for the function $\hat{f}$ in the class of continuous functions F:

$$F = \{f(\phi): f(\phi) > 0, f(\phi + 2\pi) = f(\phi), -\pi \leqslant \phi \leqslant \pi\}.$$

Each function f from F describes in the polar coordinates the boundary δQ_f of a certain region Q_f.

The electromagnetic anomaly U_f generated by the currents j_{ex} in the region Q_f is defined as follows:

$$U_f(x, \omega) = \int_{-\pi}^{\pi} \int_0^{f(\phi)} j_{ex}(\bar{\tau}^M)\, G^U(\bar{\tau}, \bar{\tau}^M)\, \rho d\, \rho d\phi, \tag{3}$$

where

$$G^U(\bar{\tau}, \bar{\tau}^M) = \begin{cases} G_n, & U = E_y \\ -\dfrac{1}{i\omega\mu_0}\dfrac{\partial G_n}{\partial z}, & U = H_x \\ \dfrac{1}{i\omega\mu_0}\dfrac{\partial G_n}{\partial x}, & U = H_z \end{cases}$$

G_n is Green's function of a layered (normal) section defined by the equation:

$$\Delta G_n(\bar{\tau}, \bar{\tau}^M) + i\omega\mu_0\sigma_n G_n(\bar{\tau}, \bar{\tau}^M) = -i\omega\mu_0\,\delta(\bar{\tau} - \bar{\tau}^M),$$

where $\bar{\tau}$ is the radius vector of the observation point $(x, 0)$ and $\bar{\tau}^M$ is the radius vector for the infinitesimal point of integration $M \in Q_f$ with polar coordinates (ρ, ϕ).

In the trial process a function $\bar{f} \in F$ should be found for which the corresponding direct problem solution $U_{\bar{f}}$ is close enough to the observed field U. As the measure of closeness it is advisable to use the metric of complex space $\mathscr{L}_2[x, \omega]$:

$$\|U\| = \sqrt{\int_\Omega \int_X |U|^2 dx d\omega} = \sqrt{\int_\Omega \int_X U \cdot U^* dx d\omega} \tag{4}$$

where the asterisk denotes the complex conjugate value, and Ω and X are the frequency range and the profile of observation, respectively, where the function U is known.

With this metric the trial process can be used to determine the function $\tilde{f} \in F$ for which

$$\| U_{\tilde{f}} - U \| \leqslant \epsilon_0 \| U \| \tag{5}$$

in which ϵ_0 is the required relative trial error, which dependes on the accuracy δ of the observed data. If the square error functional $I[f]$ is defined as

$$I[f] = \| U_f - U \|^2 \tag{6}$$

then condition (5) may be rewritten in the form:

$$I[\tilde{f}] \leqslant \epsilon_0^2 \| U \|^2. \tag{7}$$

So the inverse problem is reduced to a minimization of the error function I.

It is necessary to note here that this problem is ill-posed, as are most geoelectric inverse problems. Therefore a direct minimization of the functional I, which contains the errors from the observations and the calculations, can lead to unstable, geologically useless results. To obtain a stable solution the methods of regularization of ill-posed problems must be employed. So, in order to construct a regularized solution of the trial problem for the shape of the deep geoelectric inhomogeneity, the Tikhonov parametric functional (Tikhonov & Arsenin 1979) is introduced:

$$M_p[f] = I[f] + pS[f] \tag{8}$$

with the stabilizing functional $S[f]$ used in the form proposed by Glasko & Starostenko (1976):

$$S[f] = \| f - f_0 \|^2 = \int_{-\pi}^{\pi} (f - f_0)^2 \, d\phi \tag{9}$$

where $f_0 \in F$ is an initial approximation based on *a priori* geological and geophysical information.

The parametric functional is minimized in a two-cycle procedure (Glasko & Starostenko 1976). In the external cycle the optimal value of the regularization parameter p is selected from the discrete sequence $\{p_K\}$ convergent to zero:

$$p_{K+1} = \nu p_K, \quad K = 1, 2, \ldots; \quad 0 < \nu < 1.$$

In the internal cycle the iterative process is constructed to minimize the function M_p for the fixed value of parameter p (Zhdanov & Varentsov 1980; Varentsov 1981). The common term of the minimizing sequence · $\{f_i\}$ is formulated as:

$$f_{i+1}(\phi) = f_i(\phi) + t_i \Delta f_i(\phi), \; i = 0, 1, 2, \ldots, \tag{10}$$

where $\Delta f_i(\phi)$ are the increment functions defined in the class of periodic continuous functions with the uniform metric, and t_i is a certain set of positive constants defining the value of minimization steps. If the constants t_i are sufficiently small, then the functions f_i, as well as f_0, belong to the class F.

Next is calculated the functional M_p of the function f_{i+1}. Because of the differentiability of this function the following representation is valid:

$$M_p[f_{i+1}] = M_p[f_i] + t_i M_p^{(1)}[f_i, \Delta f_i] + t_i^2 \bar{O}(\| \Delta f \|) \tag{11}$$

where $M_p^{(1)}[f_i, \Delta f_i]$ is the first variation of the functional M_p defined by the expression

$$M_p^{(1)}[f_i, \Delta f_i] = \lim_{t_i \to 0} \frac{1}{t_i} \{M_P[f_i + t_i \Delta f_i] - M_P[f_i]\}$$

$$= 2 \int_{-\pi}^{\pi} \Delta f_i \left\{ f_i \int_\Omega \mathrm{Re} \left[j_{\mathrm{ex}}(\bar{\tau}_i) \int_X (U_{f_i} - U)^* G^U(\bar{\tau}, \bar{\tau}_i)\, dx \right] d\omega + p(f_i - f_0) \right\} d\phi \quad (12)$$

and $\bar{\tau}_i$ is the radius vector of a point with the coordinates $(f_i(\phi), \phi)$. Equation (11) shows that for rather small t_i the condition of minimization

$$M_P[f_{i+1}] < M_P[f_i] \quad (13)$$

holds true as soon as

$$M_P^{(1)}[f_i, \Delta f_i] < 0. \quad (14)$$

To fulfil the latter condition it is expedient to preset the increment functions Δf_i in the form:

$$\Delta f_i = |q_i|^\beta \operatorname{sign} q_i \quad (15)$$

where

$$q_i(\phi) = -f_i \int_\Omega \mathrm{Re} \left[j_{\mathrm{ex}}(\bar{\tau}_i) \int_X (U_{f_i} - U)^* G^U(\bar{\tau}, \bar{\tau}_i)\, dx \right] d\omega - p(f_i - f_0), \quad (16)$$

and β is a positive constant. In fact, for such a definition of the function Δf_i the first variation of the functional M_p is negative:

$$M_P^{(1)}[f_i, \Delta f_i] = -2 \int_{-\pi}^{\pi} |q_i|^{1+\beta}\, d\phi < 0. \quad (17)$$

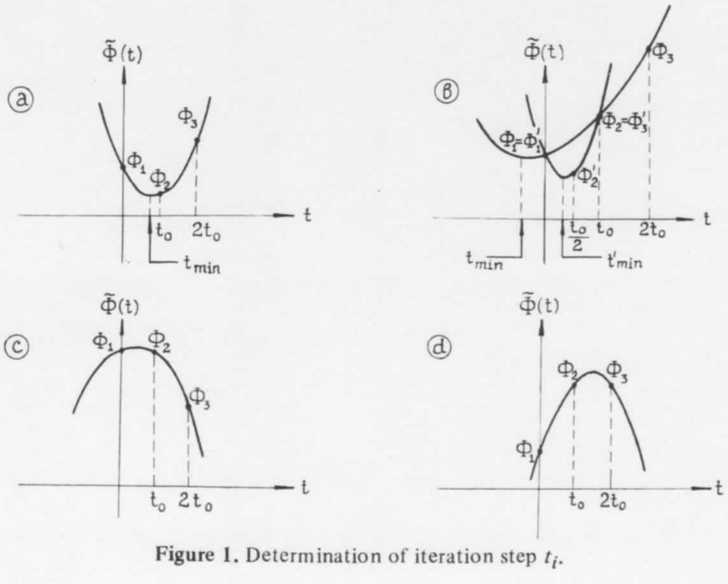

Figure 1. Determination of iteration step t_i.

To determine the value of minimization step t_i the scalar minimization problem should be considered:

$$\Phi(t) = M_P[f_i + t\Delta f_i] = \min_t .$$

Let us approximate the function $\Phi(t)$ by the parabola $\tilde{\Phi}(t)$ (Fig. 1) passing via the points $\Phi_1 = [0, \Phi(0)]$, $\Phi_2 = [t_0, \Phi(t_0)]$ and $\Phi_3 = [2t_0, \Phi(2t_0)]$ where $t_0 = t_{i-1}$. This parabola is described by the equation:

$$\tilde{\Phi}(t) = \frac{t^2}{2t_0^2}[\Phi(0) - 2\Phi(t_0) + \Phi(2t_0)] - \frac{t}{2t_0}[3\Phi(0) - 4\Phi(t_0) + \Phi(2t_0)] + \Phi(0).$$

When the inequality

$$\Phi(0) - 2\Phi(t_0) + \Phi(2t_0) > 0 \tag{18}$$

holds true the parabola has a minimum at the point with abscissa

$$t_{\min} = \frac{t_0}{2} \cdot \frac{3\Phi(0) - 4\Phi(t_0) + \Phi(2t_0)}{\Phi(0) - 2\Phi(t_0) + \Phi(2t_0)}.$$

If $t_{\min} > 0$ (Fig. 1a), we assign

$$t_i = t_{\min}. \tag{19}$$

The situation $t_{\min} \leqslant 0$ is possible when the step t_0 is too large. In this case we put

$$t_0 = t_0/2 \tag{20}$$

and repeat the process described with a new value of t_0 (Fig. 1b).

The inequality (18) is not fulfilled if the parabola $\tilde{\Phi}(t)$ branches are directed downward, or if it has degenerated into a straight line. In the case $\Phi(2t_0) < \Phi(0)$ (Fig. 1c) we assign

$$t_i = 2t_0. \tag{21}$$

Instead (Fig. 1d) we decrease the step t_0 according to (20) and repeat the whole process with a new t_0 as shown in Fig. 1(b).

Therefore, (10)–(21) allow the construction of the sequence of the functions $\{f_i\}$ that minimizes the functional M_p, starting from a certain initial approximation $f_0 \in F$ and initial step t_0. These functions describe *cylindrical surfaces tightening to the surface of the anomalous region Q*. So the technique is called the *tightening surfaces method*.

The iterative process in the internal cycle comes to an end according to the condition of stabilization:

$$\| f_i - f_{i-1} \| \leqslant \epsilon_1 \| f_i \| \tag{22}$$

where $\epsilon_1 > 0$ is a certain prescribed value. The minimization can also be finished naturally when the inequality (5) is fulfilled, i.e. when

$$I[f_i] \leqslant \epsilon_0^2 \| U \|^2. \tag{23}$$

For each value of the regularization parameter $p \in \{p_K\}$ the minimization of the functional $M_{p_K}[f]$ is carried out according to the scheme described. The result obtained in the internal cycle – the function f_{p_K} – is taken as an initial approximation for the minimization of the functional $M_{p_{K+1}}[f]$ (the initial approximation for the functional $M_{p_1}[f]$ minimization is the function f_0).

To complete the algorithm for the stable minimization of the parametric functional, a criterion for the search of the quasi-optimal parameter of regularization $\tilde{p} \in p_K$ should be formulated. We use the approach developed by Tikhonov & Arsenin (1979), Glasko & Starostenko (1976). In the absence of errors in the initial data the following expansion holds true

$$f_P = \hat{f} + p\,\frac{\partial f_P}{\partial p} + \dots\,. \tag{24}$$

If an error δ exists in the original data, it is possible to use as a criterion for selecting $\tilde{p}$ the condition

$$\left\| p\,\frac{\partial f_P}{\partial p} \right\| p = \min_P. \tag{25}$$

For the discrete sequence $\{p_K\}$ the latter criterion may be rewritten in approximate form:

$$\| f_{P_{K+1}} - f_{P_K} \| = \min_K \tag{26}$$

This criterion is especially useful when we do not known exactly the accuracy δ of the initial data (Glasko & Starostenko 1976). The function $\bar{f} = f_{\tilde{p}}$ corresponding to the quasi-optimal parameter $\tilde{p}$ found from (26) is considered to be a regularized solution of the trial problem.

4 Spectral modification of the tightening surfaces method

Let us consider the algorithm derived in the previous section in the spatial frequency domain. The spatial spectrum $u(\alpha, \omega)$ of the electromagnetic anomaly $U(x, \omega)$ is defined as:

$$u(\alpha, \omega) = F_x[U(x, \omega)] = \int_{-\infty}^{\infty} U(x, \omega) \exp(i\alpha x)\, dx.$$

The direct solution corresponding with (3) in the frequency domain takes the form:

$$u_f(\alpha, \omega) = F_x[U_f(x, \omega)] = \int_{-\pi}^{\pi} \int_0^{f(\phi)} j_{\text{ex}}(\bar{\tau}^M)\, g^U(\alpha, \bar{\tau}^M)\, \rho\, d\rho\, d\phi, \tag{27}$$

where

$$g^U(\alpha, \bar{\tau}^M) = F_x[G^U(\bar{\tau}, \bar{\tau}^M)|_{z=0}].$$

Equation (27) requires significantly less computational expense than the initial expression (3), when there are simple analytical representations for the function g^U (Weidelt 1975, 1978; Varentsov 1981)..Therefore, when solving the trial problem it is advisable to compare the spatial spectra of observed and modelled fields.

So the spectral analogue of the error functional (6) may be introduced:

$$J[f] = \| u_f - u \|^2 \tag{28}$$

where the metric of complex space $L_2[\alpha, \omega]$ is used:

$$\|u\| = \sqrt{\int_\Omega \int_A |u|^2 \, d\alpha d\omega} = \sqrt{\int_\Omega \int_A uu^* d\alpha d\omega}$$

and A is the spatial frequency range for which the spectrum is known. The structure of the functionals $I[f]$ and $J[f]$ is quite similar; moreover they are equal when the observed profile X and the frequency range A are expanded to infinity. Therefore in the spatial frequency domain it is natural to employ the regularized minimization procedure described above for the inverse problem solution. In this case we have only to substitute in equations (8)–(16) and (23) $I[f]$ for $J[f]$, $U(x, \omega)$ for $u(\alpha, \omega)$, $x \in X$ for $\alpha \in A$ and G^U for g^U.

5 Localization of deep geoelectric inhomogeneity

When formulating the inverse problem and elaborating the algorithm for its solution we assumed the centre of the polar coordinate system (ρ, ϕ) to be situated inside the conductivity inhomogeneity Q. Now let us assume that the pole O_0 is originally outside the region Q (at a point with cartesian coordinates x_0 and z_0). In this case the formal use of the tightening surfaces method generates the sequence of surfaces $\{f_i\}$ which do not converge to the boundary of the region Q. Nevertheless these surfaces are stretching towards the real position of the inhomogeneity, and the centres of regions Q_{f_i} are displacing in the same direction. This effect can be detected by the analysis of the eccentricity coefficient:

$$\theta[f_i] = \max_{-\pi \leq \phi \leq \pi} \frac{f_i(\phi)}{f_i(\phi - \pi)}. \tag{29}$$

When the eccentricity of the region Q_{f_i} is significant, i.e. when

$$\theta[f_i] \geq \theta_0,$$

with θ_0 equal to 1.5 or 2 say, then a correction should be made and the pole removed from the point O_1 to the point O_0 with coordinates:

$$x_1 = x_0 + [f_i(\phi_0) - f_i(\phi_0 - \pi)] \cos \phi_0$$

$$z_1 = z_0 + [f_i(\phi_0) - f_i(\phi - \pi)] \sin \phi_0$$

where ϕ_0 is the extreme value of the angle ϕ in equation (29). Then the function f_1 is recalculated in the new coordinate system, and the trial process is continued. After several corrections the polar coordinate system centre is moved sufficiently close to the centre of the region Q.

Thus by using the tightening surfaces method it is possible to find the location of the deep geoelectric inhomogeneity and then to determine the form of its boundary.

6 Model experiments

The spectral modification of the tightening surfaces method is realized in a FORTRAN-IV program (Zhdanov, Varentsov & Baglaenko 1980). The effectiveness of this program can be demonstrated clearly on theoretical models.

The first example shows the effectiveness of the regularization technique. The model consists of a horizontal circular cylinder (hatched region in Fig. 2b) submerged in the homogeneous earth (the conductivity of inhomogeneity being 100 times greater than that

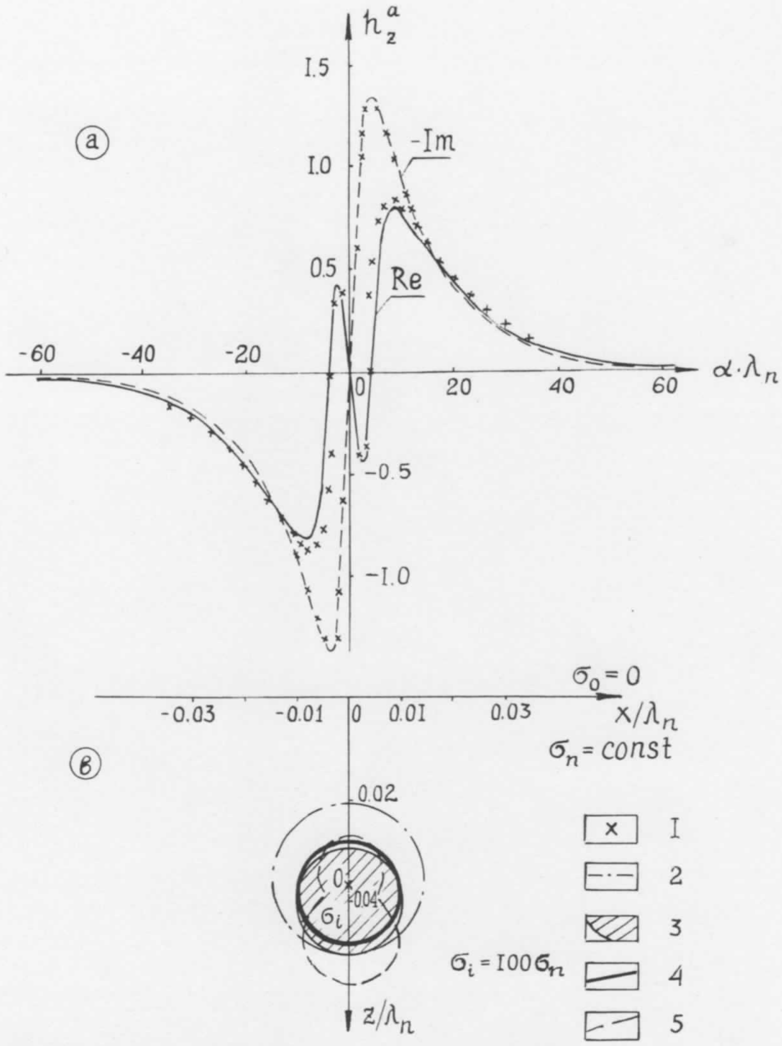

Figure 2. Inverse problem solution in model with conductive cylinder: (a) spatial spectra of observed magnetic field (solid and dotted lines) and field, calculated for the final model (crosses); (b) shape of inhomogeneity models. (1) spectrum h_z^a for model, (2) initial approximation Q_{f_0}, (3) real location of inhomogeneity, (4) model selected with regularization, (5) solution without regularization.

of the surrounding space). The solution of the inverse problem obtained without regularization (Fig. 2b – dotted line) differs radically from the true shape of the inhomogeneity; however, the deviation between the observed and modelled data (spatial spectra of anomalous vertical magnetic field – Fig. 2a) is less than 2 per cent. So that proves the high degree of instability of the trial problem. But the use of the regularized algorithm leads us to a quite accurate and stable solution (Fig. 2b – solid line). The error of approximation in this case is also less than 2 per cent (Fig. 2a).

The next example illustrates the process of the polar coordinate centre correction procedure. In the model containing a horizontal semicircular cylinder in a homogeneous half-space (Fig. 3) the initial approximation Q_{f_0} (dashed line) is taken far away from the real

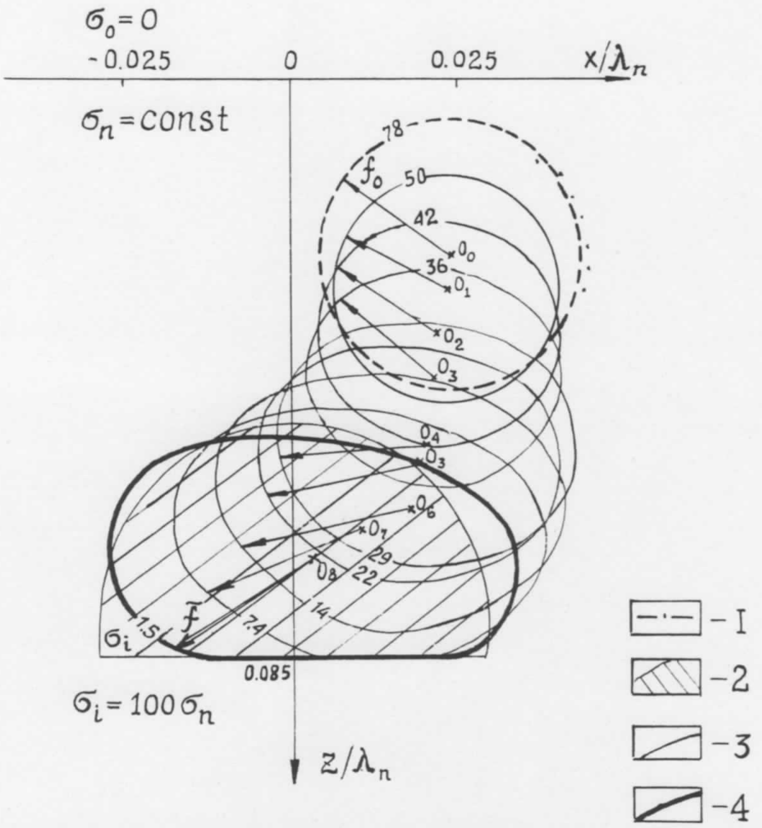

Figure 3. Determination of shape of semicircular inhomogeneity with correction of polar coordinate system centre position. (1) initial approximation, (2) real location of inhomogeneity, (3) intermediate solutions of trial process, (4) final solution.

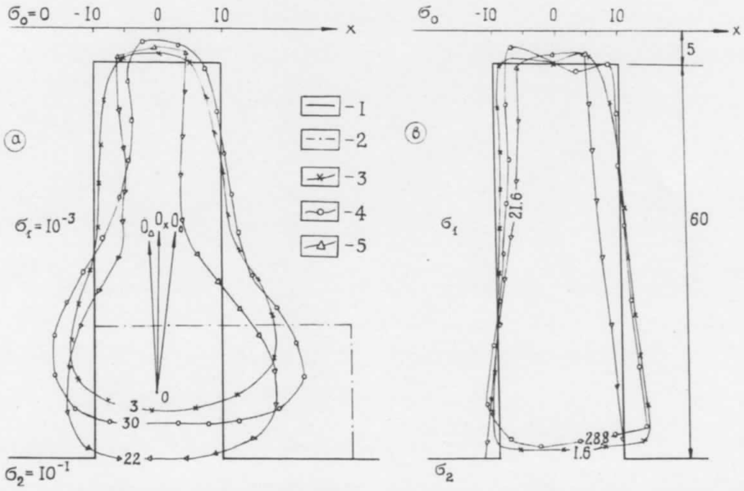

Figure 4. Inverse problem solution in model with a horst structure: (a) localization of inhomogeneity, (b) improving the accuracy of its boundary, (1) real location of horst structure, (2) initial approximation, (3) inhomogeneity contours determined through the approximate modelling data, (4) solutions for the same data complicated by 20 per cent random noise, (5) results obtained using finite-difference modelling data.

position of the inhomogeneity (hatched region). Nevertheless, after eight corrections the pole is successfully moved close enough to its centre. The shape of the inhomogeneity determined after 96 iterations with a trial error of 1.5 per cent is shown in Fig. 3 by a solid line.

A two-layered model with a narrow horst structure of the layers' interface (Fig. 4) was used to study the stability of the tightening surfaces method to errors in the initial approximation of the inhomogeneity and to errors in the determination of the spectra of the observed and modelled fields. The region Q_{f_0} was chosen in the form of an isometric rectangle placed at a certain distance from the horst (a dotted line in Fig. 4a). Three variants of the initial data were considered:

the spectrum of the anomalous magnetic field h_z^a as defined by the approximate equation (27);

the same spectrum complicated by 20 per cent randon noise;

the spectrum of the strict finite-difference solution of the direct problem.

The values of the magnetic fields H_z^a, corresponding to these spectra, are shown in Fig. 5.

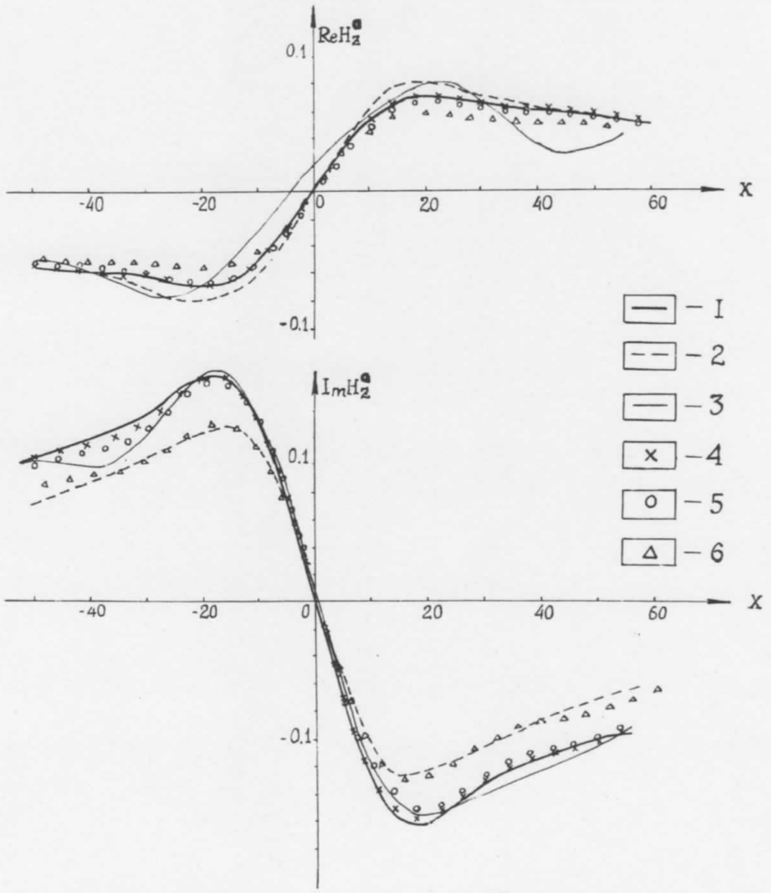

Figure 5. Magnetic anomaly H_z^a for horst structure and its models selected in trial process. (1) field calculated using approximate method (Section 3), (2) the same field with 20 per cent random noise, (3) finite-difference modelling results, (4, 5, 6) approximate direct problem solution for models determined through initial data (1, 2, 3) accordingly.

The inverse problem in the model described was solved in two stages. In the first stage the location of the geoelectric inhomogeneity was accurately determined and its shape outlined (Fig. 4a). As the *a priori* model of inhomogeneity was chosen rather roughly, the trial process at this stage was conducted without regularization. Then the results of the trial were corrected. This inhomogeneity was delineated approximately by horst-like structures, so that its boundaries, the lower part of which were distorted previously because of the inaccuracy of the initial guess, were straightened and prolonged to the layers' interface.

Starting from these corrected models the trial process at the second stage of inverse problem solutions was continued with regularization. The final results depicted in Fig. 4(b) describe the shape of the horst structure quite accurately. The corresponding magnetic fields H_z^a are shown in Fig. 5. It is important to point out that the presence of 20 per cent random noise in the inverted data affected only weakly the solution obtained.

We now consider a variant of inverse problem solution in which the input data are obtained from accurate finite-difference modelling. As shown in Fig. 5 the approximate solution of the direct problem used in the trial procedure differs systematically (by 20–30 per cent) from the finite-difference modelling data. This is caused by the relatively large dimensions of the inhomogeneity – the cut-off value for the radio D/λ_i in the criterion (2) is twice exceeded. Nevertheless the trial results (curves with triangles in Fig. 4a, b) define the location of the horst structure rather well, especially the depth to its upper edge. Though the square of the determined inhomogeneous region is diminished in proportion to the accuracy of the approximate modelling technique, still the shape of the structure is recognized with a proper resolution. Therefore the trial procedure technique could be applied in a wider frequency range than is described by the criterion (2).

Finally, we can conclude on the basis of the numerical experiments described above that the tightening surfaces method is very stable to mistakes in the initial guess (location and shape) of the local geoelectric inhomogeneity, and provides sufficient resolution of its structure even in the presence of significant (20–30 per cent) random or systematic errors in the initial data (electromagnetic fields and their spatial spectra).

7 Practical results – interpretation of Carpathian geomagnetic anomaly

The Carpathian geomagnetic anomaly can be traced beneath the whole extension of the Carpathians and is considered to be one of the most evident and well-investigated regional geomagnetic anomalies. It has a well-defined 2-D structure typical of the anomalies caused by local deep conductivity inhomogeneities.

A great number of geomagnetic observations have been held in the Soviet Carpathians. Many deep MT soundings have been done here (Rokityansky *et al.* 1976), and synchronous magnetic and electric measurements at periods of geomagnetic bays have been carried out along several profiles (Bondarenko & Bilinsky 1976).

Recent studies of the Carpathian anomaly in this region (Rokitynasky 1975; Rokityansky *et al.* 1976; Adam 1980; Berdichevsky & Zhdanov 1981) show that it originates from the mutual effect of the subsurface geoelectric inhomogeneities (thick conducting sediments in the troughs at the boundaries of the Folded Carpathians) and a deep crustal conductivity anomaly located in the narrow contact zone between the Folded Carpathians and the Transcarpathian trough.

This geoelectric situation gave us every justification to investigate the deep conductivity structure by the tightening surfaces method. Electromagnetic data measured simultaneously along the III International DSS profile (in the interval Beregovo-Korets) were selected for the interpretation. Using the integral transformations of observed fields (Berdichevsky &

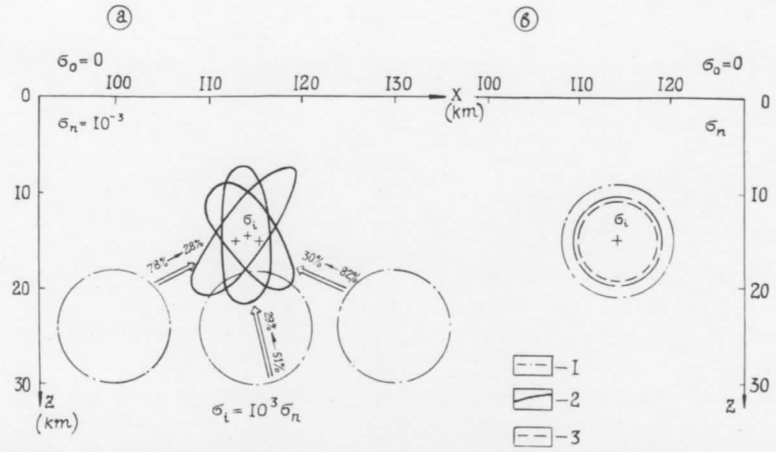

Figure 6. Investigation of deep conductivity anomaly in the Soviet Carpathians: (a) localization of inhomogeneity, (b) further determination of its shape. (1) initial approximations, (2) models selected through H_z^D field, (3) model selected through H_x^D field.

Zhdanov 1981), the deep geomagnetic anomaly $H_{x,z}^D$, free of subsurface distortions, was determined for a 1 hr period (solid curves in Fig. 7). The initial approximation of the inhomogeneity was a circular cylinder with a radius of 6 km and with the centre at a depth of 24 km. The cylinder was placed under the point of zero value of H_z^D anomaly. Two other initial approximations were made to control the stability of the trial process – the cylinder described above was first moved 15 km to the right, and then moved to the left from the anomaly axis (dash-dot curves in Fig. 6a).

The inverse problem was solved as earlier (Section 6) in two stages. In the first stage the inhomogeneity was located through the H_z^D anomaly and its centre defined at the depth 15 km (Fig. 6a). Then a new initial approximation was constructed (Fig. 6b), and the trial process was continued with regularization, using both the vertical and the horizontal

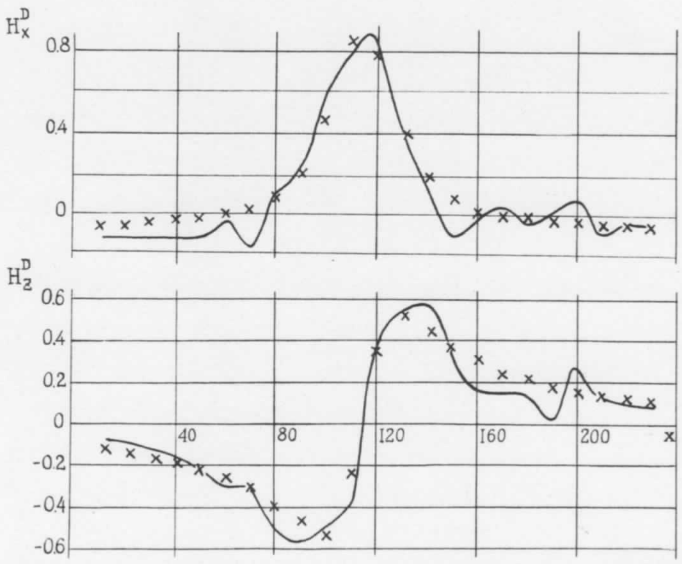

Figure 7. Deep magnetic anomalies calculated for models in Fig. 6(b) (crosses) and separated from the observed field (solid lines).

component of the deep geomagnetic anomaly. The solutions obtained are shown in Fig. 7(b). They are quite similar and differ slightly from the initial approximation.

If we compare the models selected at the first and second stages of the trial procedure, we notice that though the value of the trial error is almost the same (near 30 per cent), the shape of the inhomogeneity depends significantly on the choice of the initial approximation. The detected instability can be explained in the first instance by the high level of the field observation errors (20–30 per cent for the amplitudes and still more for the phases). At the same time the position of the centre of the inhomogeneity, its area (70 km^2) and integral conductivity

$$\sum_i = \iint_Q \sigma_i \cdot dQ = 0.7 \times 10^8 \text{ S m}^{-1}$$

are well-defined. These data agree with the modern generalized ideas of the deep geoelectric structure of the Carpathian region (Rokityansky 1975; Adam 1980).

8 Discussion

In this paper we have derived a theory for the new formalized 2-D inversion technique (tightening surfaces method) and have demonstrated its effectiveness in simple applications. Now we should like to mention some problems that arise in making use of this technique.

First, it is necessary to find the anomaly in synchronously measured electromagnetic fields on a rather long profile (in order to determine spatial Fourier harmonics if one is to use the Fourier domain modification of the method). Also, the parameters of the normal geoelectric section should be found.

Next, we need to determine a starting model for the structure of the geoelectrical inhomogeneity explored, especially the excess conductivity value. Sometimes this parameter can be estimated on the basis of *a priori* geophysical data.

Anyway, it is desirable to obtain several inverse problem solutions for the set of probable conductivity contrasts. The final solution can be chosen from these, depending on its fit to observed electromagnetic anomalies. However, we think it would be more useful to present this parametric family of models as a general solution of our geophysical problem. At a later stage of geological interpretation the suitable solution can be chosen from this family, taking into account geological ideas and results of other geophysical methods. This procedure is not very expensive, because it is rather fast and, moreover, we have good starting models for the second and all the following solutions.

The next problem will be to develop the application of the tightening surfaces method for the interpretation of multi-frequency profiling data. A natural first step would be to solve (with middle range accuracy) a set of single-frequency problems to determine an average model of inhomogeneity, as well as the frequencies with the best resolution. Then the multi-frequency modification of the algorithm should be used to delineate further the geoelectrical structure.

All of these problems are of great practical significance and we hope to study them in more detail in a special paper.

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References

Adam, A., 1980. The change of electrical structure between an orogenic and ancient tectonic area (Carpathians and Russian platform), *Geomagn. Geoelect.*, **32,** 1–46.

Berdichevsky, M. N. & Zhdanov, M. S., 1975. Analysis of the anomalies of variable geomagnetic field at the surface of multilayered horizontal heterogeneous medium, *AN SSSR, Geomagn. aeron.*, **15,** 325–330 (in Russian).

Berdichevsky, M. N. & Zhdanov, M. S., 1981. *Interpretation of the Anomalies of Variable Electromagnetic Field of the Earth,* Nedra, Moscow, 327 pp. (in Russian).

Bondarenko, A. P. & Bilinsky, A. I., 1976. Anomaly of geomagnetic bays in the East Carpathians, in *Geoelectric and Geothermal Studies,* pp. 589–599, ed. Adam, A., Akademiai Kiado, Budapest.

Glasko, V. B. & Starostenko, V. I., 1976. Regularizing algorithm for solving the system of non-linear equations in the inverse problems of geophysics, *Izv. AN SSSR, Fizika Zemly, No. 3,* pp. 44–53 (in Russian).

Jupp, D. L. B. & Vosoff, K., 1977. Two-dimensional magnetotelluric inversion, *Geophys. J. R. astr. Soc.*, **50,** 333–352.

Kaufman, A. A., 1974. *Fundamentals of the Theory of Inductive Ore Electrical Reconnaissance,* Nauka, Novosibirsk, 352 pp. (in Russian).

Novikov, P. S., 1938. Uniqueness of the solution of inverse problem in potential theory, *DAN SSSR,* **18,** 165–168 (in Russian).

Oristaglio, M. L. & Worthington, M. H., 1980. Inversion of surface and borehole electromagnetic data for two-dimensional electric conductivity models, *Geophys. Prospect.*, **28,** 633–657.

Rikitake, T., 1966. *Electromagnetism and the Earth's Interior,* Elsevier, Amsterdam.

Rokityansky, I. I., 1975. *Investigation of Electroconductivity Anomalies by the Method of Magnetovariational Profiling,* Naukova dumka, Kiev, 280 pp. (in Russian).

Rokityansky, I. I., Amirov, V. K., Kulik, S. N., Loginov, I. M. & Shuman, V. N., 1976. The electric conductivity anomaly in the Carpathians, in *Geoelectric and Geothermal Studies,* pp. 604–612, ed. Adam, A., Akademiai Kiado, Budapest.

Schmucker, U., 1970. *Anomalies of Geomagnetic Variations in the Southwestern United States,* University of California Press, Berkeley.

Schmucker, U., 1971. Interpretation of induction anomalies above non-uniform surface layers, *Geophysics,* **36,** 156–165.

Tikhonov, A. N. & Arsenin, V. Ya., 1979. *Methods of Solving Incorrect Problems,* Nauka, Moscow, 386 pp. (in Russian).

Varentsov, Iv. M., 1981. Development of the methods for magnetovariational profiling data interpretation in the class of two-dimensional heterogeneous models, *Cand. thesis,* IZMIR AN SSSR, Moscow (in Russian).

Weidelt, P., 1975. Inversion of two-dimensional conductivity structures, *Phys. Earth planet. Int.*, **10,** 282–291.

Weidelt, P., 1978. *Entwicklung und Erprobung eines Verfahrens zur Inversion Zweidimensionaler Leitfähigkeitsstrukturen in E-polarisation,* George August Universität, Gottingen.

Zhdanov, M. S., 1975. Problems in the theory of interpretation of deep geomagnetic anomalies, *Izv. AN SSSR, Fizika Zemly, No. 9,* pp. 59–73 (in Russian).

Zhdanov, M. S., 1980. Caushy integral analogues for the separation and continuation of electromagnetic fields within conducting matter, *Geophys. Surveys,* **4,** 115–136.

Zhdanov, M. S., Berdichevsky, M. N. & Zhdanova, O. N., 1975. On the surface anomalies of the variable electromagnetic field of the Earth, *AN SSSR, Geomagnetism i aeronomia,* **15,** 532–536 (in Russian).

Zhdanov, M. S. & Varentsov, Iv. M., 1980. Interpretation of local geomagnetic anomalies by the method of tightening surfaces, *AN SSSR, Geologiya i geofizika, No. 12,* pp. 106–117 (in Russian).

Zhdanov, M. S., Varentsov, Iv. M. & Baglaenko, N. V., 1980. Program of the solution of magnetovariational profiling inverse problem, in *Algoritmy i programmy analiza geomagnitnogo polya,* pp. 177–201, eds Fainberg, E. B. & Zhdanov, M. S., IZMIR AN SSSR, Moscow (in Russian).

Zhdanov, M. S., Varentsov, Iv. M. & Golubev, N. G., 1978. Determination of the geoelectric inhomogeneity location by the method of analytic continuation of variable geomagnetic fields, *AN SSSR, Geologiya i geofizika, No. 7,* pp. 54–63 (in Russian).