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MIGRATION OF ELECTROMAGNETIC FIELDS IN THE SOLUTION OF INVERSE GEOELECTRIC PROBLEMS[1]

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1. A basic problem in geoelectrics today is to develop methods for interpreting the data from electromagnetic studies carried out in regions with complex, horizontally inhomogeneous geoelectric profiles. This problem is particularly acute in the analysis of the results of electromagnetic sounding with high-power artificial current sources (MHD generators), since the fields generated by these sources propagate over a broad territory, through regions with very different geological structures. The solution of this problem requires the development of some fundamentally new interpretation methods based on an analysis of the entire space-time pattern of the electromagnetic field detected at the earth's surface. In the present paper we describe one such method, "electromagnetic migration" (named by analogy with seismic migration [1]).

This method is based on the "inverted" extension of the field into the conducting earth [2-4] and is a generalization, to the case of variable electromagnetic fields, of existing methods of inverse extension of wave fields. These other methods are used widely in seismic exploration [5-7].

In this paper we analyze the properties of the fields obtained as a result of migration, and we analyze the possibilities of using this method to solve inverse problems of geoelectrics.

2. The problem is formulated as follows. At the earth's surface[2], Γ, an electromagnetic field $\mathbf{E}^v$, $\mathbf{H}^v$ is specified for the time interval from 0 to T. This field is excited by a time-varying source which is either on the outer side of the surface Γ or is raised slightly above the earth and turned on at the time $t = 0$. The earth is assumed to have a constant electrical conductivity σ except in a certain deep zone D (of either finite or infinite size), in which the conductivity (σ_D) can vary in any arbitrary way. We are to find the region with an anomalous conductivity from measurements of the time-varying fields $\mathbf{E}^v(\mathbf{r},t)$, $\mathbf{H}^v(\mathbf{r},t)$ at the earth's surface.

To solve this problem we construct the following field transformation. We transform from the ordinary time t to the inverted time $\tau = T - t$, and we specify auxiliary fields $\mathbf{F}(\mathbf{r},\tau)$ and $\mathbf{R}(\mathbf{r},\tau)$ at the earth's surface Γ in the following way:

$$F_{tg}(\mathbf{r},\tau) = E^v_{tg}(\mathbf{r}, T-\tau), \quad F_n(\mathbf{r},\tau) = \frac{1}{c} E^v_n(\mathbf{r}, T-\tau);$$
$$R_{tg}(\mathbf{r},\tau) = H^v_{tg}(\mathbf{r}, T-\tau), \quad R_n(\mathbf{r},\tau) = -H^v_n(\mathbf{r}, T-\tau), \tag{1}$$

where the subscripts "tg" and "n" specify the tangential and normal components of the vectors $\mathbf{F}$ and $\mathbf{R}$.

It is not difficult to see that the fields $\mathbf{F}$ and $\mathbf{R}$ satisfy on the surface Γ the two-dimensional analogs of Maxwell's equations in the inverted time τ (since the real fields $\mathbf{E}^v$ and $\mathbf{H}^v$ satisfy on Γ the two-dimensional analogs of Maxwell's equations in the ordinary time t)[3], under the condition that the conductivity (σ^m) of the region adjacent to Γ satisfies $\sigma^m = c\sigma$:

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[1]Translated from: Migratsia elektromagnitnykh poley pri reshenii obratnykh zadach geoelektriki. Doklady Akademii Nauk SSSR, 1983, Vol. 271, No. 3, pp. 589-594.

[2]It is assumed that Γ is a piecewise-smooth surface, either closed or passing through an infinitesimally remote point.

[3]As usual in geoelectrics, we are dealing with a quasisteady model of the electromagnetic field. In other words, we are ignoring the displacement currents in Maxwell's equations.

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$$F_n = -\frac{1}{\sigma^m} \operatorname{div}^\Gamma [n \times R_{tg}], \quad \frac{\partial R_n}{\partial \tau} = \frac{1}{\mu_0} \operatorname{div}^\Gamma [n \times F_{tg}], \tag{2}$$

where $\operatorname{div}^\Gamma$ is a surface divergence, μ_0 is the permeability of free space, and n is the vector of the external unit normal to Γ.

We can now write the following integral relations which, as in [8], we call "Stratton-Chew integrals for a time-varying field":

$$E^m(r', \tau') = -\frac{1}{4\pi} \int_0^{\tau'} \iint_\Gamma \Big\{ (n \cdot F) \nabla \bar{G} + [[n \times F] \times \nabla \bar{G}] + + [n \times R] \mu_0 \frac{\partial \bar{G}}{\partial \tau} \Big\} dS\, d\tau, \tag{3}$$

$$H^m(r', \tau') = -\frac{1}{4\pi} \int_0^{\tau'} \iint_\Gamma \{ (n \cdot R) \nabla \bar{G} + [[n \times R] \times \nabla \bar{G}] + + [n \times F] \sigma^m \bar{G} \} dS\, d\tau. \tag{4}$$

The function $\bar{G} = \bar{G}(r, \tau | r' \tau')$ in expressions (3) and (4) is the adjoint of the fundamental Green's function for the diffusion equation [9]:

$$\bar{G}(r, \tau | r' \tau') = \frac{(\mu_0 \sigma^m)^{1/2}}{2\pi^{1/2} (\tau' - \tau)^{3/2}} \exp\left(-\frac{\mu_0 \sigma^m |r - r'|^2}{4(\tau' - \tau)} \right) \kappa(\tau' - \tau),$$

where

$$\kappa(\tau' - \tau) = \begin{cases} 1, & \tau < \tau', \\ 0, & \tau > \tau'. \end{cases}$$

The integrals in (3) and (4) have several properties which are analogous to the properties of their harmonic analogs [8]. The most important of these properties is that the Stratton-Chew integrals satisfy Maxwell's equations away from surface Γ, and the vector functions E^m and H^m themselves satisfy a diffusion equation,

$$\Delta(E^m; H^m) - \mu_0 \sigma^m \frac{\partial (E^m; H^m)}{\partial \tau'} = 0. \tag{5}$$

<u>Definition.</u> The transformation by Eqs. (1), (3), and (4) of the E^v, H^v field observed at Γ is called the "migration of the electromagnetic field," and the field E^m, H^m itself is the "migrated electromagnetic field."

The migrated field E^m, H^m can be given a simple physical meaning: This is the electromagnetic field which is excited in a homogeneous conducting half-space with conductivity $\sigma^m = c\sigma$ by a system of certain virtual currents and charges distributed on the surface Γ. These operate in inverted time τ [i.e., they begin to operate (are turned on") at the end of the field-detection interval, at $t = T$, and cease to operate at $t = 0$].

Substituting (1) into (3) and (4), and replacing the variable τ in (3) and (4) by $T - t$, we find

$$E^m(r', T - t') = -\frac{1}{4\pi} \int_{t'}^{T} \iint_\Gamma \Big\{ \frac{(n \cdot E^v)}{c} \nabla G + [[n \times E^v] \times \nabla G] + + [n \times H^v] \mu_0 \frac{\partial G}{\partial t} \Big\} dS\, dt, \tag{6}$$

$$H^m(r', T - t') = -\frac{1}{4\pi} \int_{t'}^{T} \iint_\Gamma \{ -(n \cdot H^v) \nabla G + [[n \times H^v] \times \nabla G] + + [n \times E^v] \sigma^m G \} dS\, dt, \tag{7}$$

where $G = G(r, t | r', t')$ is the fundamental Green's function for the diffusion equation, which is related to the function $\bar{G}$ by the reciprocity condition

$$G(r, t | r', t') = \bar{G}(r, -t | r', -t').$$

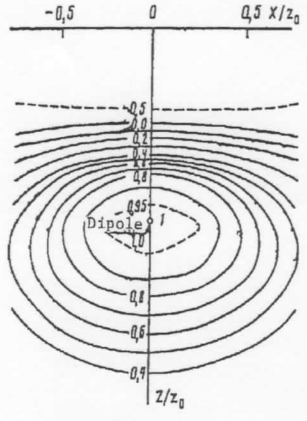

Fig. 1

Fig. 1. Contour map of H_x^m. The values of H_x^m are divided by the value of the field at the local extremum (a similar normalization has been used in plotting the contour maps of the migrated field in Figs. 2-4). Here z_0 is the depth of the dipole.

Fig. 2. Contour map of E_y^m in a model with a thin band. Here d is the distance from the observation surface to the center of the band.

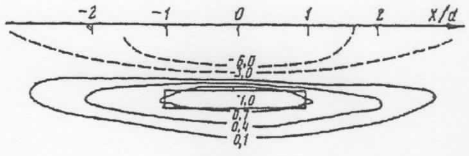

Fig. 2

Equations (6) and (7) implement the procedure of electromagnetic migration. When these equations are applied to actual data, we should follow the lead of seismic-exploration geophysicists and adopt as σ the apparent specific electrical conductivity of the earth, $\sigma_{\text{app}}(\tau)$, averaged over the time in the interval $(0,\tau')$:

$$\sigma = \frac{1}{\tau'} \int_0^{\tau'} \sigma_{\text{app}}(\tau) d\tau. \qquad (8)$$

The actual inhomogeneous profile of the earth in this case is, in a sense, replaced by some homogeneous model (which depends on the time τ').

The field transformation described by Eqs. (6)-(8) is a stable procedure because the operators on the right side of these expressions are bounded in the space L_2.

3. For a more graphic representation of the properties of the migrated field $\mathbf{E}^m$, $\mathbf{H}^m$, we again transform from the inverted time τ to the ordinary direct time t, and for simplicity we set $c = 1$:

$$\mathbf{E}'(\mathbf{r}',t') = \mathbf{E}^m(\mathbf{r}', T-t'), \quad \mathbf{H}'(\mathbf{r}',t') = \mathbf{H}^m(\mathbf{r}', T-t').$$

According to (5), the field $\mathbf{E}'$, $\mathbf{H}'$ satisfies in direct time the equations

$$\Delta \mathbf{E}' + \mu_0 \sigma \frac{\partial \mathbf{E}'}{\partial t'} = 0, \quad \Delta \mathbf{H}' + \mu_0 \sigma \frac{\partial H'}{\partial t'} = 0. \qquad (9)$$

Whereas the ordinary diffusion equation describes the evolution with time of the propagation of the electromagnetic field from the source to the observer, Eqs. (9) describe the same process in the opposite order: from the final distribution of the field $\mathbf{E}^v$, $\mathbf{H}^v$ on the earth's surface to its initial distribution at the sources. Our field transformation (6)-(8) therefore describes a transformation of electromagnetic fields which are diverging in the real medium and are being diffracted by geoelectric inhomogeneities into fields which are converging into the corresponding diffraction surfaces, lines, and points. At the $\tau' = T$ (or $t' = 0$), as in seismic holography, the fields $\mathbf{E}^m(\mathbf{r}', T) = \mathbf{E}'(\mathbf{r}', 0)$ and $\mathbf{H}^m(\mathbf{r}', T) = \mathbf{H}'(\mathbf{r}', 0)$ form an image of virtual field-exciting sources associated with the geoelectric inhomogeneities.

Let us illustrate the situation with some theoretical and model examples.

4. We consider a first series of models with elementary sources of an anomalous field. Direct analysis shows that the positions of the elementary sources (a horizontal electric dipole or an infinitely long cable) coincide with the positions of local extrema of the migrated field at the time at which the current is turned on in these sources (i.e., at $t' = 0$ or $\tau' = T$). For example, for an E-polarized primary field this property holds for the horizontal magnetic field component H_x^m in the case of field migration into a medium with a conductivity $\sigma^m = 0{,}5\,\sigma$, and it holds for the horizontal electric field component E_y^m in the case of migration into a medium with $\sigma^m \simeq 0{,}55\,\sigma$.

The migrated field retains the same properties in the three-dimensional situation, when an anomalous field is excited by a horizontal electric dipole. The values of the migration constant $c = 0.5$ (for the magnetic component H_x^m) and $c \simeq 0.55$ (for the electric component E_y^m) can therefore be taken to be the optimum values for the migration procedure for both two-dimensional and three-dimensional fields.

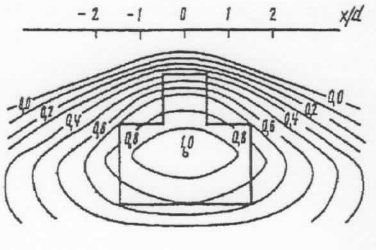

Fig. 3

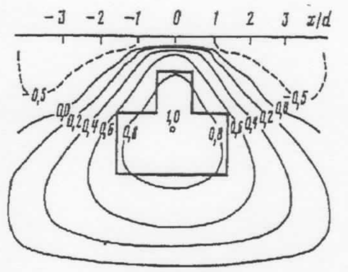

Fig. 4

Fig. 3. Contour map of E_y^m for a layer-by-layer reconstruction of the migrated field in a model with a composite (complex) inclusion. Here d is the distance from the observation surface to the upper edge of the inclusion.

Fig. 4. Contour map of E_y^m in a model with an inclusion. This map is plotted for the instant corresponding to the local extremem in Fig. 3.

Let us see how the properties of the migrated field are manifested in numerical simulations. Figure 1 shows an H_x^m contour map in a vertical plane perpendicular to the axis of the dipole and passing through its center ($t' = 0$ and $c = 0.5$). The surface Γ, on which the observed field is specified, coincides with the XOY surface. We see that the spatial position of the dipole can be determined accurately from the position of the local extremum of the migrated field (an analogous property is observed in the horizontal plane parallel to the dipole axis). We wish to emphasize that the migration procedure makes it possible to determine the spatial positions of anomalous-field sources at various depths.

5. In a second series of models we consider local two-dimensional geoelectric inhomogeneities of a simple geometric form, immersed in a homogeneous conducting half-space of lower conductivity (σ). The field in this model is excited by a plane electromagnetic wave whose amplitude varies in accordance with the triggering pulse. The anomalous fields at the earth's surface are calculated by finite-difference simulation [10].

In model 1 the geoelectric inhomogeneity is a narrow, highly conducting band (outlined by the heavy lines in Fig. 2); the ratio of the conductivities of the perturbing medium and the band is $\sigma_l/\sigma = 10^4$.

At the initial time, $t' = 0$, in the case of electromagnetic migration of the anomalous field specified at the X axis, there arise some sources of anomalous field, concentrated along the band. From the contour lines of the migrated field we can determine the position and shape of the object which forms the anomaly (Fig. 2).

In model 2 the inhomogeneity is formed by two connected rectangular inclusions ($\sigma_l/\sigma = 10^2$). The migration of the anomalous electromagnetic field by means of Eqs. (6) and (7) makes it possible to see at each instant the regions of the geoelectric profile with the greatest excess currents. Accordingly, the spatial distribution of the migrated field in this model is reconstructed in a layer-by-layer fashion: The field at each level is constructed at the time at which the excess currents (the field "sources" in the layer) corresponding to this level are "turned on." The "turn-on" time is determined from the expression for the depth of the skin layer of the time-varying field [11]. Figure 3 shows the result of this reconstruction of the horizontal electric component of the migrated field E_y^m ($c = 1$). We can clearly see a local extremum, which can be ascribed to maximum excess currents. The reconstruction of the overall spatial distribution of the migrated field at the time corresponding to the position of this extremum obviously gives the most information. As can be seen from Fig. 4, the E_y^m contour lines give a clear pattern of the position and shape of the geoelectric inhomogeneity.

6. These theoretical and experimental studies show that the procedure of electromagnetic-field migration can be used as an extremely effective method for solving the inverse problem of reconstruction of a "geoelectric image" of a medium.

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TIDAL EVOLUTION OF THE EARTH-MOON SYSTEM UPON RESONANT EXCITATION OF TIDES IN THE EARTH'S OCEANS[1]

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Theoreticians of the tidal evolution of the earth-moon system have assumed until recently that the basic sink for tidal energy is its dissipation in the solid earth. This assumption contradicts certain calculations (see [1-3], for example) which show instead that the oceans are the primary dissipators of tidal energy, and the solid earth is responsible for only a few percent of the total amount of the dissipated tidal energy. These calculations were carried out for the modern epoch, assuming resonant excitation of tides in the earth's oceans characteristic of this epoch [4, 5]. Were the same mechanism for the excitation of ocean tides also dominant in the past,[2] then the conclusion that the oceans are dominant in the dissipation of tidal energy would remain in force throughout the geological history of the earth. This would force a revision of the theory for the tidal evolution of the earth-moon system. Such a revision is the subject of the present paper.

We consider the simplest case, in which the moon revolves around the earth in a circular equatorial orbit. We can then write the equation for angular-momentum conservation in the earth-moon system and Kepler's third law, which define the distance (c) between the earth and the moon, the sidereal rotation velocity of the earth (ω), and the average velocity ($n_☾$) of the orbital motion of the moon, as follows:

$$\frac{1}{\kappa}\frac{d}{d\tau}\frac{\omega}{\omega_0} = -\frac{d}{d\tau}\left(\frac{c}{c_0}\right)^{1/2} = \frac{1}{13}\frac{L}{L_0}. \tag{1}$$

$$\left(\frac{n_☾}{n_{☾0}}\right)^2 = \left(\frac{c}{c_0}\right)^{-3}. \tag{2}$$

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[1]Translated from: Prilivnaya evolyutsiya sistemy Zemlya-Luna pri rezonansnom vozbuzhdenii prilivov v Mirovom okeane. Doklady Akademii Nauk SSSR, 1983, Vol. 271, No. 3, pp. 594-498.

[2]This assumption might be based on the results calculated in [6] for the natural mode spectrum of a hemispherical ocean in different positions of this ocean on the earth's surface.

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