Single-borehole and cross-borehole resistivity anomalies of thin ellipsoids and spheroids

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ABSTRACT

Hole-to-hole resistivity measurement is a useful method of detecting deeply buried fractures and ore deposits in the subsurface. With drilling costs continually rising, there is a growing need for developing methods of borehole geophysics such as this. In this study, we present theoretical results relating to detection of thin oblate spheroids and ellipsoids with arbitrary attitude.

If we assume that individual fractures within a fracture zone are connected to each other and are of finite lateral and vertical extent, then we can model the fracture zone as a thin conductive oblate ellipsoid or spheroid with arbitrary orientation of the major axis. Detection of such deeply buried fracture zones is the object of this study. Here the effects of the surface of the Earth are neglected and the body is assumed to be enclosed within an infinite homogeneous mass. The surface of the body is divided into a series of subsurfaces, and a numerical solution of the Fredholm integral equation is applied. Once a solution for the surface charge distribution is determined, the potential can be specified anywhere by means of Coulomb's law. The theoretical model results indicate that cross-borehole resistivity measurements are a more effective technique than single-borehole measurements for delineating resistivity anomalies in the vicinity of a borehole. In some cases, the depth to the center of the body and the dip and strike of the major axes of the body can be estimated.

INTRODUCTION

The methods of single-borehole, cross-borehole, and borehole-to-surface resistivity measurements for detecting anomalies in the vicinity of a borehole have been discussed by several authors. An analytical solution for a spherical body in a half-space, with a buried current source, was presented in Daniels (1977). Recently, theoretical solutions and discussions of the apparent resistivity anomalies of a sphere and a horizontal oblate or prolate spheroid in a whole space for cross-borehole and single-borehole methods were presented in Dobeci (1980) and Lyle (1982). However, computation of the potential about a thin ellipsoid with an arbitrary attitude is a practical problem which has not received sufficient attention. We can calculate this potential directly using the finite-element method. However, the following discussion demonstrates the advantage in solving this problem by means of the surface-charge integral equation technique.

The use of surface-charge integration to solve electric potential problems is a classical method in physics. Al'pin (1964) showed the theoretical formula for the computation of a complicated horizontally layered model involving the drilling fluid and the invaded zone for resistivity logging with this technique. Harrington (1968) described its applications to electrostatic and electromagnetic fields. This approach was first applied to solve three-dimensional (3-D) problems in resistivity and induced electrical polarization by Dieter et al. (1969). Barnett (1972, 1976) and Daniels (1977) applied it to solve the potential problems for 3-D bodies of spherical and arbitrary shape.

In conventional resistivity surveys, borehole measurements are limited in their ability to locate deeply buried targets such as fracture zones or tabular orebodies, from which only small responses are measured using conventional surface electrode configurations. This paper investigates the possibility of detecting a thin, anomalous body by borehole measurements. Several body attitudes and three different buried electrode configurations are considered. Theoretical modeling for a thin oblate body not only can be used to model fracture zones occurring in geothermal environments, but can also be used to model a thin resistive or thin conductive orebody in mining exploration. We use the term "spheroid" to describe a thin oblate ellipsoid possessing two axes of equal dimensions. (ellipsoid of revolu-
THEORY

Consider the ellipsoidal model for single-borehole and cross-borehole techniques depicted in Figure 1, in which \( \rho_1 \) is the resistivity of the whole space and \( \rho_2 \) is the resistivity of the body. Assume that the depth to the anomalous body is much greater than its size so that the effects of the surface of the earth can be neglected. The current and measuring electrodes A and N are placed at infinity on the surface; the electrodes B and M are thus the only electrodes downhole. In this model the center of the ellipsoid is the zero of the coordinate system and the ellipsoid can take any attitude. If the effects of the borehole are neglected, then the potential \( U_M \) at point M is given by (Al'pin, 1964)

\[
U_M = \frac{p_1 I}{4\pi R_{BM}} + \frac{1}{4\pi} \int_{S} \frac{\sigma(P)}{R_{PM}} ds,
\]

where \( S \) is the series of subsurfaces used to approximate the ellipsoid, \( P \) is a point on the surface of the ellipsoid, \( R_{BM} \) and \( R_{PM} \) are the distances from the current source B to the measuring electrode M and from point P to the measuring electrode M, respectively (Figure 1). The quantity \( \sigma(P) \) is the surface density of charge at the point P while \( \varepsilon \) is the dielectric permittivity. From boundary conditions and using a simple transformation of \( \sigma'(P) = \sigma(P)\varepsilon_p \), the density of surface charge at any point Q on the body surface should satisfy the following integral equation (Al'pin, 1964)

\[
\sigma(Q) = K \int_{S'} \frac{\partial \sigma'(P)}{\partial n} \left( \frac{1}{R_{PQ}} \right) ds,
\]

where \( K = \frac{(\rho_2 - \rho_1)}{(\rho_2 + \rho_1)} \) is the reflection coefficient associated with the surface of the body. \( S' \) is all of the surface except that occupied by the point P.

We approximate \( \sigma' \) by an N-term expansion (Harrington, 1968),

\[
\sigma'(P) \approx \sum_{j=1}^{N} f_j \sigma_j,
\]

where the expansion functions are given by

\[
f_j = \begin{cases} 1 & \text{on the jth subsurface } S_j, \\ 0 & \text{on all other subsurfaces} \end{cases}
\]

Substituting relation (3) into equation (2) gives

\[
\frac{2\pi}{K} \sigma_i - \sum_{j=1}^{N} \sigma_j \int_{S_j} \frac{\partial \sigma_j}{\partial n} \left( \frac{1}{R_{PQ}} \right) ds = \frac{\varepsilon}{\varepsilon_p} \left( \frac{1}{R_{PQ}} \right); \quad j \neq i.
\]

Writing equation (5) for each of the N values of \( i \) results in the system of linear equations

\[
\sum_{j=1}^{N} A_{ij} \sigma_j = B_i; \quad i = 1, 2, \ldots, N.
\]

By solving this system of linear equations and utilizing the resulting values of \( \sigma_j \), we can calculate the potential and apparent resistivity at any point for different electrode configurations. Barnett (1972, 1976) discussed the singularity in the integral of equation (5) and also discussed the method of computation of the integral over a triangular facet. Hence, we do not discuss these matters here.

Now the question is how to divide the body surface into a series of subsurfaces, suitable for any orientation of the ellipsoid, and satisfy the desired computational accuracy. The answer depends on one hand on the fact that the distribution of surface charge relates to the relative positions of the current source and the body and, on the other hand, our desire to use...
Fig. 2. The discretization of the surface of the oblate body: (a) division of contours normal to the z-axis, (b) division of triangles for a spheroid, and (c) division of triangles for an ellipsoid. The semiaxes are $a$, $b$, and $c$ in the $x$, $y$, and $z$ or $x'$, $y'$, $z'$ directions, respectively.
the minimum number of subsurfaces to attain the desired accuracy and automatically perform division of the body surface into subsurfaces. In our programming, we first divided the ellipsoid into 8 slices along the minor axis by 7 contours (Figure 2a), and then created a series of triangles on the body surface. For the spheroid, we used a symmetric division of equal arc length and created 136 triangles on the body surface (Figure 2b). It is quite evident that most of the triangles are isosceles and the differences in their areas are small. For an ellipsoid, the surface of the body is divided into 140 triangles as shown in Figure 2c. Thus in this discretization, the 3-D body to be modeled is represented by a polyhedron bounded by a series of triangular facets. Of course, for different body attitudes we can adjust the discretization contour positions. For instance, if a spheroid used in the following examples (body size $a = 2$, $b = 2$, $c = 0.2$) assumes a horizontal attitude, the radii of the circular contours (Figure 2b) are 0.3a, 0.6a, and 0.8a; if it assumes a vertical attitude the radii are 0.17a, 0.33a, and 0.67a, respectively.

In the computation procedure, a convenient method for specifying the size and dip of the body and assembling the subsurfaces of a polyhedron for a spheroid in an arbitrary attitude is as follows: (1) input the body size (semimajor axes $a$, $b$, and $c$); (2) assemble the coordinates of the apex for each triangle as a horizontal spheroid; and (3) rotate the spheroid and transform the coordinates for each apex. We use $x$, $y$, $z$ to represent the coordinates of the apex for each triangle in the original coordinate system $X$, $Y$, $Z$ (Figure 3) for a horizontal oblate body. The new coordinates $(x_{new}$, $y_{new}$, $z_{new})$ of each apex, after rotating successively about three semimajor axes by the angles $\alpha$, $\beta$, $\gamma$, are given by the following transformation:

$$
\begin{bmatrix}
    x_{new} \\
    y_{new} \\
    z_{new}
\end{bmatrix} =
\begin{bmatrix}
    \cos \alpha \cos \beta & \cos \alpha \sin \beta \sin \gamma - \sin \alpha \cos \gamma & \cos \alpha \sin \beta \cos \gamma + \sin \alpha \sin \gamma \\
    \sin \alpha \cos \beta & \sin \alpha \sin \beta \sin \gamma + \cos \alpha \cos \gamma & \sin \alpha \sin \beta \cos \gamma - \cos \alpha \sin \gamma \\
    -\sin \beta & \cos \beta \sin \gamma & \cos \beta \cos \gamma
\end{bmatrix} \begin{bmatrix}
    x \\
    y \\
    z
\end{bmatrix}
$$

Of course, we also can solve this potential problem by transforming the coordinates of the current and potential electrodes. An important point to note is that by keeping the coordinates of the body in a horizontal attitude and transforming the coordinates of the positions of the current and potential electrodes, we can save computation time, provided we do not change the body size but only the body attitude. It is obvious that the integration in equation (5) is not relative to the positions of the current and potential electrodes but only to the size and shape of the body. If we keep the size and shape of the body constant and only change its attitude, it is not necessary to recompute equation (5). Therefore the integral equation method is more efficient than the finite-element method in studying the responses of simple bodies.

In all the following figures, $a$, $b$, and $c$ represent the semimajor axes while $\alpha$, $\beta$, and $\gamma$ represent the rotation angles shown in Figure 3, and SBX, SBY, and SBZ and EMX, EMY, and EMZ represent the coordinates of the current and potential electrodes, respectively. All computations were made on a Hewlett-Packard model 9826 desktop computer (64 bit word length) in noncompile BASIC. The total computer time for one profile is 90 minutes. A large portion of the computer time is spent calculating the integration [equation (5)] and solving the system of linear equations [equation (6)]. If the size and shape of the body are kept constant and only the position of the measuring borehole or the electrode configuration is changed, the computer time for one profile is 15 minutes.

MODEL RESULTS

Dimensionless units are used throughout the study: the size of the body and all distances in the following model examples are expressed in terms of normalized distance units, and the anomalous response is expressed by the normalized apparent resistivity $\rho_a/\rho_i$. For cross-borehole and single-borehole measurements, three different electrode configurations are considered. The simplest possible electrode configuration for cross-borehole measurements is the fixed, single current electrode and moving single potential electrode. The other two arrays used are a widely spaced normal array (e.g., Figure 6) and a moving-bipole source with a moving-bipole receiver (e.g., Figure 22) for single- and cross-borehole measurements, respectively. Three cases, i.e., horizontal, dipping, and vertical oblate spheroids, are discussed separately. Finally, two representative results for an ellipsoid are presented.
Horizontal spheroid

The spatial variation of the normalized apparent resistivity \( \left( \frac{\rho_a}{\rho_1} \right) \) in the \( x-z \) plane is shown in Figure 4 for a conductive spheroid with a resistivity contrast of \( \rho_2/\rho_1 = 0.10 \). This result is very similar to the apparent resistivity distribution from a spherical body (Lytle, 1982). As seen from Figure 4, since the current lines concentrate into the body and flow out divergently from it, the normalized apparent resistivity measured on the current source side of the spheroid typically is less than one. However, on the side of the spheroid opposite the current source it typically is greater than one. Thus a conductive body located between boreholes always produces \( \rho_a/\rho_1 > 1.00 \) for the cross-borehole method. We can consider that \( \rho_a/\rho_1 > 1.00 \) and \( \rho_a/\rho_1 < 1.00 \) represent positive and negative anomalies, respectively. However, since our results are in terms of the normalized apparent resistivity, the sign and shape of the anomalies do not depend upon the sign of the current source. The resistivity anomaly in a vertical borehole containing the source is much smaller than in any borehole to the right of the source in Figure 4. This suggests that a larger anomaly can be obtained by using cross-borehole probing than by using single-borehole probing. Note, however, that only a small anomaly would be present for a measuring borehole passing through \( EMX = 0 \). Figure 4 also indicates that the cross-borehole anomalies in normalized apparent resistivity \( \rho_a/\rho_1 \) are markedly dependent upon the location of the measuring borehole.

**Figure 4.** The variation of normalized apparent resistivity \( \left( \frac{\rho_a}{\rho_1} \right) \) in the \( x-z \) plane for a horizontal spheroid with a resistivity contrast of \( \rho_2/\rho_1 = 0.10 \). The body size is \( a = b = 2, c = 0.2 \). The coordinates of the current source are \( SBX = -3.0, SBY = SBZ = 0 \).

**Figure 5.** Cross-borehole resistivity profiles for a thin horizontal spheroid as functions of the position of the borehole in which potential measurements are made. The source electrode is fixed at \( B \) in the first borehole while the measuring electrode \( M \) is moved down the second borehole. All parameters are indicated in the figure.
For example, if the measuring borehole were located at $EMX = -2$, the anomaly of $\rho_a/\rho_1$ would be negative, whereas if the measuring borehole were located at $EMX = +2$, the anomaly of $\rho_a/\rho_1$ would be positive. If the $L$ spacing $(MN)$ were 1 unit, the anomalies would be reasonably resolved regardless of the location of the measuring borehole. On the other hand, if the $L$ spacing were 4 units, the anomaly would not be resolved and its amplitude would be smoothed and decreased. Figure 5 illustrates this observation clearly.

Model results for single-borehole measurement with a normal array of different $L$ spacings are shown in Figure 6. The plotting point of the measurement is midway between the source electrode $B$ and the current electrode $M$. The anomaly for single-borehole measurement in Figure 6 is quite small compared with the cross-borehole measurement in Figure 5.

**Dipping spheroid**

If we rotate the oblate spheroid about the $y$-axis, an asymmetrical spatial variation in normalized apparent resistivity will occur, along with an asymmetrical distribution of the induced surface charge on the boundary between the body and the surrounding homogeneous medium. For a dipping conductive model of $\beta = 45$ degrees with the same size and resistivity contrast as in the horizontal case presented in Figure 4, the spatial distribution of the apparent resistivity in the $x$-$z$ plane is depicted in Figure 7. It is quite evident that the anomaly is asymmetric about the body. The apparent resistivity perturbation is usually larger for cross-borehole than for single-borehole as before.

Figures 8 and 9 illustrate the anomalies caused by a conductive spheroid dipping at $\beta = 45$ degrees for resistivity contrasts of 0.10 and 0.01, respectively, the distances from the center of the body to the borehole in which the potential is measured are varied. The configuration used involves a fixed source and a moving potential electrode. As shown in these two figures, the asymmetry of the curves is related to the dip of the body. For both resistivity contrasts, the positions of peak amplitude of apparent resistivity not only are almost the same, but are very close to the upper edge of the body. Also note in these two figures that the maxima of the anomalies decrease rapidly as the potential measuring borehole is placed farther from the center of the body. When the distance $EMX$ is three times greater than the semimajor axis ($a = 2$) of the spheroid, the anomaly produced is less than 2 percent. Of course, if the borehole containing the current source is located closer to the dipping spheroid, the anomaly will be more readily detected.

Next we observe the change in shape and amplitude of the profile of apparent resistivity as a function of dip (rotation of the body about the $y$-axis). The plane of the body is perpendicular to the plane of the two boreholes used in a cross-borehole survey. The model results for various dips are given in Figures 10 and 11. The resistivity contrast is 0.10 for the data of both

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**Fig. 6** Single-borehole resistivity profiles for a thin horizontal spheroid as functions of the electrode spacing $L$ of a normal array. All parameters are indicated in the figure. The plotting point of the measurement is midway between the source and the potential electrode $M$.
figures. Figure 10 illustrates the results for dips $\beta$ of 0, 15, 30, and 45 degrees, while Figure 11 illustrates the results for dips $\beta$ of 45, 60, 75, and 90 degrees. As would be expected, the anomalous amplitude decreases with the dip $\beta$ from 0 to 90 degrees. The positions of the peaks of the curves are always approximately opposite the upper edge of the body. It appears, therefore, that we can obtain a good idea of both the location and the dip of the body by using cross-borehole probing.

A comparison of the normalized apparent resistivity curves for various resistivity contrasts is shown in Figure 12. In this figure we keep the model parameters and electrode configuration constant, and allow only the resistivity contrast to change. The anomalous amplitude produced by a resistive spheroid is slightly higher than that of a conductive spheroid when comparing curves for reciprocal resistivity contrast. This result is similar to that obtained by Lytle (1982) for a spherical model with infinite and zero contrasts. However, for a dipping conductive spheroid (Figure 12), the position of the peak of the anomaly shifts to the depth of the upper edge of the body, as the resistivity contrast is increased, but the shifts are quite slight for a dipping resistive spheroid. The variation of the peak amplitude of $\rho_\alpha/\rho_1$ versus resistivity contrast $\log \rho_2/\rho_1$ for the dipping spheroid at $\beta = 45$ degrees is given by Figure 13.

The variation of $\rho_\alpha/\rho_1$ versus the size of the semiminor axis $c$, the spatial distribution of the normalized apparent resistivity caused by a dipping spheroid ($\beta = 45$ degrees, $a = 2$, $b = 2$, $c = 0.2$) in the $x$-$z$ plane. The current source is located at $SBX = -2.5, SBY = SBZ = 0$. The resistivity contrast $\rho_2/\rho_1$ is 0.10. A hypothetical borehole, containing the source, is shown in order to facilitate later discussion of potentials measured at $M_1$ and $M_2$ with respect to single-borehole measurements.

### Fig. 7

The spatial distribution of the normalized apparent resistivity caused by a dipping spheroid ($\beta = 45$ degrees, $a = 2$, $b = 2$, $c = 0.2$) in the $x$-$z$ plane. The current source is located at $SBX = -2.5, SBY = SBZ = 0$. The resistivity contrast $\rho_2/\rho_1$ is 0.10. A hypothetical borehole, containing the source, is shown in order to facilitate later discussion of potentials measured at $M_1$ and $M_2$ with respect to single-borehole measurements.

### Fig. 8

Cross-borehole apparent resistivity profiles, for a conductive spheroid dipping at $\beta = 45$ degrees, versus the position of the borehole in which the potential measurements are made. The source electrode is fixed at $B$ in the first borehole while the measuring electrode $M$ is moved down the second borehole. All parameters are indicated in the figure. The resistivity contrast $\rho_2/\rho_1$ is 0.10. These profiles should be compared with those of Figure 5.
FIG. 9. Cross-borehole apparent resistivity profiles, for a conductive spheroid dipping at $\beta = 45$ degrees, versus the position of the borehole in which potential measurements are made. The only change from Figure 8 is that the resistivity contrast $\rho_2/\rho_1$ is now 0.01. These profiles should be compared with Figure 8.

FIG. 10. Fixed source, moving potential configuration: effect of dip on cross-borehole measurements for a gently dipping conductive spheroid.
**Borehole Resistivity Anomalies**

**Fig. 11.** Effect of dip on cross-borehole measurements, for a steeply dipping conductive spheroid. All parameters are indicated in the figure for this fixed-source, moving-potential configuration of electrodes.

**Fig. 12.** Effect of resistivity contrast on cross-borehole measurements, for a dipping spheroid. All parameters are indicated in the figure for this fixed-source, moving-potential configuration of electrodes.
for a thin conductive spheroid dipping at $\beta = 45$ degrees with a resistivity contrast $P_2/P_1 = 0.10$, is presented in Figure 14. The peak position of the curves gradually shifts to the depth of the edge of the body as the spheroid becomes thinner, but the amplitude of the anomaly rapidly decreases (Figure 14).

Figure 15 shows the results of offsetting the borehole relative to the center of the dipping spheroid ($\beta = 45$ degrees). The shapes of the anomalies vary slightly; the amplitude decreases very rapidly as the boreholes are offset. If the offsetting distance is greater than the semimajor axes of the spheroid, the anomaly virtually disappears. If the offset is in the negative direction of the $y$-axis by the same distances used for the data shown in Figure 15, the shape and the amplitude of the anomalies will appear the same as in Figure 15. Thus, it is impossible to determine on which side of the plane defined by the two boreholes the body lies.

Figure 16 shows the response for a dipping spheroid with the attitude of $\gamma = 45$ degrees (rotation about the $x$-axis). The effects of offsetting the boreholes in the positive and negative $y$-direction by the same distances applied in the previous example are different from the last case: the anomalies in $P_2/P_1$ are always greater than $1$. If the offsetting distance $EMY$ is zero, the anomaly appears as a symmetric curve. Comparing this curve with the curve ($\beta = 0$ degrees) depicted by Figure 10, we see that the shape and amplitude of these two curves are almost the same; the maximum of the amplitude is exactly the same. Thus it is impossible to recognize the true dip of the spheroid from the curves, when the dip of the body is in a direction normal to the plane of the boreholes. For instance, as the spheroid takes the attitude of $\gamma = 45$ degrees or $\gamma = -45$ degrees, the anomalies from these two situations are exactly the same. Even if the boreholes are offset in the positive or negative $y$ direction, as shown in Figure 16, it is impossible to determine the real dip of the spheroid, even though some small shifts occur in the peak position of the anomalies. In this situation, the only way is to search for help from a third borehole.

**Vertical spheroid**

If the current source is placed on the axis of rotation of a vertical conductive spheroid and close to the body, a dramatic variation in the spatial distribution of normalized apparent resistivity occurs as seen in Figure 17. The normalized apparent resistivity near the outer edges of the body is high. However, on the axis and in the vicinity of the axis on the side of the body opposite the current source, there is a low of normalized apparent resistivity. The spatial distribution of the induced surface charge which causes this shielding effect is the result of a particular combination of eccentricity of the body, resistivity contrast, and distance from the source to the center of the body. For the spheroid model depicted in Figure 17, the induced negative charges are concentrated in the central area on the surface of the body facing the current source. However, most of the induced positive charges are repulsed and distributed on the outer edges of the body opposite the current source. The current flows out from the body in a loudspeaker shape. The current density near the axis of rotation probably is lower for the spheroid than for the sphere because of the surface charge distribution peculiar to the spheroid. Only at infinity does the current density recover to normal and the anomaly disappear.

This conclusion is similar to the result described in Seigel (1952) for a special case in which the electrode array is just passing through the center of the spheroid along its axis of rotation.

Figure 18 shows the normalized apparent resistivity responses versus the different positions of the borehole used for measurement of potential, with the configuration of a fixed current source and a moving potential electrode. Figure 18 illustrates that the three peaks of each anomaly, two positive and one negative, exactly indicate the positions of the upper edge, lower edge, and center of the body, respectively. The amplitude of the anomaly decreases rapidly as the borehole used for potential measurements is located farther from the vertical spheroid.

The model results with a widely spaced single-borehole normal array are given in Figure 19. Note that the anomalous shape and amplitude depend upon the array spacing. Referring again to Figure 7, assume that the borehole (dashed line) is parallel with the plane defined by the semimajor axes $a$ and $b$ and assume a normal array moving along the borehole. It is clear from Figure 7 that for the spacing $L_1 = BM_1$, which is less than $a$ or $b$, then $M_1$ is located in a region of $P_2/P_1 < 1.00$. On the other hand, for the spacing $L_2 = BM_2$, which is greater than $a$ or $b$, then $M_2$ is located in a region of $P_2/P_1 > 1.00$. As shown in Figure 7, if the spacing $L_1$ is less than the semimajor axis, then $P_2/P_1 < 1.00$. If the spacing $L$ is equal to the semimajor axis, the curve displays $P_2/P_1 = 1.00$ peaks at the upper and lower extents of the body with $P_2/P_1 \approx 1.00$ adjacent to the center of the body. Because the spacing $L$ is greater than the semimajor axis, the curve displays $P_2/P_1 < 1.00$ peaks above and below the body with $P_2/P_1 > 1.00$ adjacent to the center of the body and the $P_2/P_1 = 1.00$ crossovers approximately define the upper and lower extents of the body. The undulation of the curve $L = 1.00$ is caused by the nonuniformity of the spatial distribution of the apparent resistivity (see Figure 7). It must be noted that the conditions for occurrence of this undulation not
**Fig. 14.** Effect of aspect ratio $c/a$ on cross-borehole measurements, for a dipping conductive spheroid. All parameters are indicated in the figure for this fixed-source, moving-potential configuration of electrodes.

**Fig. 15.** Effect of offset-distance for a spheroid whose dip vector is parallel to the plane of the two boreholes used in cross-borehole measurements. All parameters are indicated in the figure for this fixed-source, moving-potential, configuration of electrodes.
FIG. 16. Effect of offset-distance for a spheroid whose dip vector is perpendicular to the plane of the two boreholes used in cross-borehole measurements. All parameters are indicated in the figure for this fixed-source, moving-potential, configuration of electrodes. The variation of normalized apparent resistivity \((P_a/P_i)\) for a vertical spheroid in the x-z plane, with a resistivity contrast \(P_a/P_i = 0.10\) the body size is \(a = b = 2, c = 0.2\). The coordinates of the current source are \(SBX = -1.0, SBY = SBZ = 0\). Notice the dramatic change in pattern compared with Figure 4.

**Vertical ellipsoid**

Here we present two typical profiles of apparent resistivity near a vertical ellipsoid using a fixed current source and moving potential electrode to establish that the algorithm can also handle this geometry. The normalized apparent resistivity profile for an ellipsoid of \(a = 6, b = 2, c = 0.2\) for which \(\beta = 90\) degrees is shown in Figure 20; the long axis of the body is vertical. The shapes of the anomalies are almost the same as for the case of a vertical spheroid (Figure 18), but the amplitude is higher due to the larger size of the body. (Note the difference in scale between Figures 18 and 20.) From these symmetric curves, the center of the body and the size of the major axis can be recognized from the locations of the negative and positive peaks. When the potential-measuring borehole is offset from opposite the center of the spheroid, i.e., moved into or out of the page in Figure 20, there is a change in amplitude of the anomaly, but there is no reversal in sign of the anomaly (not shown).

If the strike of the major axis of the body is horizontal (parallel to the \(x\)-axis, Figure 21, note here \(a = 6\)) and the boreholes lie opposite the center of the body (zero offset distance), the anomaly appears as \(P_a < P_i\) and the two shoulders
CROSS - BOREHOLE

**Electrode Configuration** | **Body Size** | **Angles** | **Resistivity Contrast**
---|---|---|---
Fixed Source | $a = 2 \ (z)$ | $a = 0^\circ$ | $\frac{P_2}{P_1} = 0.10$
Moving Electrode | $b = 2 \ (y)$ | $\beta = 90^\circ$ | $\gamma = 0^\circ$
$SBX = -10 \ \text{SBY + SBZ} = 0$ | $c = 0.2 \ (x)$
$EMX = 0$

**Fig. 18.** Cross-borehole resistivity profiles for a thin vertical spheroid as function of position of the borehole in which potential measurements are made. The source electrode is fixed at $B$ in the first borehole while the measuring electrode $M$ is moved down the second borehole. All parameters are indicated in the figure.

SINGLE - BOREHOLE

**Electrode Configuration** | **Body Size** | **Angles** | **Resistivity Contrast**
---|---|---|---
Normal Array | $a = 2 \ (z)$ | $a = 0^\circ$ | $\frac{P_2}{P_1} = 0.10$
$SBY = 0$
$b = 2 \ (y)$ | $\beta = 90^\circ$
$c = 0.2 \ (x)$ | $\gamma = 0^\circ$

**Fig. 19.** Effect of $L$-spacing for a vertical spheroid in single-borehole, widely spaced, reversed, normal array measurements. All parameters are indicated in the figure. The plotting point of the measurement is midway between the source and potential electrode.
Fig. 20. Cross-borehole apparent resistivity profiles, for a vertical conductive ellipsoid, versus the position of the borehole in which the potential measurements are made. All parameters are shown in the figure. Compare with Figure 18 but note change of scales. The long axis of the ellipsoid is vertical.

<table>
<thead>
<tr>
<th>Electrode Configuration</th>
<th>Body Size</th>
<th>Angles</th>
<th>Resistivity Contrast</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed Source</td>
<td>(\alpha = 6 , (z))</td>
<td>(\alpha = 0^\circ)</td>
<td>(\frac{P_a}{P_i} = 0.10)</td>
</tr>
<tr>
<td>Moving Electrode</td>
<td>(b = 2 , (y))</td>
<td>(\beta = 90^\circ)</td>
<td>(\gamma = 0^\circ)</td>
</tr>
<tr>
<td>SBX = -1.0 SBY = SBZ = 0</td>
<td>(c = 0.2 , (x))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EMY = 0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 21. Cross-borehole apparent resistivity profiles, for a vertical conductive ellipsoid, versus offset of the position of the borehole in which potential measurements are made. The long axis of the ellipsoid is horizontal. All parameters are shown in the figure.

<table>
<thead>
<tr>
<th>Electrode Configuration</th>
<th>Body Size</th>
<th>Angles</th>
<th>Resistivity Contrast</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed Source</td>
<td>(\alpha = 2 , (z))</td>
<td>(\alpha = 0^\circ)</td>
<td>(\frac{P_a}{P_i} = 0.10)</td>
</tr>
<tr>
<td>Moving Electrode</td>
<td>(b = 6 , (y))</td>
<td>(\beta = 90^\circ)</td>
<td>(\gamma = 0^\circ)</td>
</tr>
<tr>
<td>SBX = -1.0 SBZ = 0</td>
<td>(c = 0.2 , (x))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EMX = 1.0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
for $\rho_s > \rho_t$ (as in Figure 20) disappear. However, if the boreholes are offset as depicted as $EMY = 3$ and $EMY = 5$ in Figure 21, the anomalies of $\rho_s/\rho_t$ are positive, i.e., $\rho_s > \rho_t$.

**Error analysis**

There are three sources of error for calculating the potential by means of the surface charge integration described in this study. The first error arises in the integrations in equation (5). The second error is the accumulative error for solving the higher order linear equations (6). Even though these two kinds of errors are hard to estimate, their effect is minor with computation of adequate precision. The third error is the major one and arises in the discretization of the surface of the body. For solving the integral equation, the body surface is divided into a series of triangular subareas over which the surface charge density is assumed to be constant. However, the surface charge distribution is highly dependent upon the attitude of the body and the location of the body relative to the current source, especially when the current source is very close to the body. Of course, we can always assemble a polyhedron using more facets, but it will consume more computer time. Furthermore it may cause an increase in the second type of error because linear equations of higher order must be solved.

We checked the accuracy of the algorithm by two methods for each of three attitudes of a spheroid and two attitudes of an ellipsoid. As shown in Table 1, when the potential electrode is far away from the center of the body along the $z$-axis.

<table>
<thead>
<tr>
<th>Current Source Position</th>
<th>Potential Electrode Position</th>
<th>Apparent Resistivity</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Horizontal Spheroid</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$SBX = -3.0$</td>
<td>$EMX = 0$</td>
<td>1.00686</td>
</tr>
<tr>
<td>$SBY = 0$</td>
<td>$EMY = 0$</td>
<td></td>
</tr>
<tr>
<td>$SBZ = 0$</td>
<td>$EMZ = 50.0$</td>
<td></td>
</tr>
<tr>
<td><strong>Dipping Spheroid</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>($\beta = 45^\circ$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$SBX = -2.5$</td>
<td>$EMX = 0$</td>
<td>0.99765</td>
</tr>
<tr>
<td>$SBY = 0$</td>
<td>$EMY = 0$</td>
<td></td>
</tr>
<tr>
<td>$SBZ = 0$</td>
<td>$EMZ = 50.0$</td>
<td></td>
</tr>
<tr>
<td><strong>Vertical Spheroid</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>($\beta = 90^\circ$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$SBX = -1.0$</td>
<td>$EMX = 0$</td>
<td>1.00102</td>
</tr>
<tr>
<td>$SBY = 0$</td>
<td>$EMY = 0$</td>
<td></td>
</tr>
<tr>
<td>$SBZ = 0$</td>
<td>$EMZ = 50.0$</td>
<td></td>
</tr>
<tr>
<td><strong>Vertical Ellipsoid</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>($a = 6$, $b = 2$, $c = 0.2$, $\beta = 90^\circ$)</td>
<td>$SBX = -1.0$</td>
<td>1.00621</td>
</tr>
<tr>
<td>$SBY = 0$</td>
<td>$EMY = 0$</td>
<td></td>
</tr>
<tr>
<td>$SBZ = 0$</td>
<td>$EMZ = 50.0$</td>
<td></td>
</tr>
<tr>
<td><strong>Vertical Ellipsoid</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>($a = 2$, $b = 6$, $c = 0.2$, $\beta = 90^\circ$)</td>
<td>$SBX = -1.0$</td>
<td>1.00549</td>
</tr>
<tr>
<td>$SBY = 0$</td>
<td>$EMY = 0$</td>
<td></td>
</tr>
<tr>
<td>$SBZ = 0$</td>
<td>$EMZ = 50.0$</td>
<td></td>
</tr>
</tbody>
</table>
(EMZ = 50), the anomaly rapidly decreases and the apparent resistivity is very close to one. In addition, a comparison between the analytical solutions for a spherical body (Lytle, 1982; Siegel, 1952; and Dobey, 1980) and the numerical solution applied in this study for a horizontal spheroid indicates that the agreement is excellent on the condition that the current source is not on the surface of the body or approximately so (the errors are less than 1 or 2 percent).

In most cases, the main error appears as a shift of the baseline of the normalized apparent resistivity curves. The effects on the shape of the anomaly are less significant. So long as we appropriately adjust the discretization contour positions according to the variation of the surface charge distribution, a satisfactory result will be achieved.

We checked in principle the data of Figure 17, approximating the oblate spheroid by a summation of thin polygons, using the volume integral equation algorithm of Hohmann (1975). The results, while not shown here, were almost identical to those of Figure 17.

**ESTIMATION OF LOCATION OF CENTER, DIP, AND SIZE OF THE SEMIMAJOR AXES OF AN OBLATE BODY**

The cross-borehole and single-borehole modeling results illustrate that in some cases it is possible to determine the depth to the center or the depth of the upper edge of the body and to estimate its dip and the size of its semimajor axes. For a vertically dipping spheroid the depth to the center of the body and the size of semimajor axes can be determined very easily from the data of Figure 18. However, for a horizontal oblate spheroid, estimation of the size of the semimajor axes is not obvious: the only hope is to match curves by the modeling procedure (compare Figures 5 and 18 to perceive the problem).

For a dipping oblate body, which is the main target in this study, if two boreholes are located on opposite sides of the body and reasonably near it, an estimate of the depth to the center of the body, its size, and its dip can be obtained with a moving-bipole source and moving-bipole receiver. Model results for a dipping oblate body of β = 45 degrees with the moving-bipole source and moving-bipole receiver are presented in Figure 22. As mentioned by Daniels (1977), this is the “optimum” cross-borehole array configuration for eliminating singularities of the geometric factor. The negative peak accurately shows the depth to the center of the body, and meanwhile an estimate of size of the body may be obtained from the positions of the positive peaks. Once the size has been estimated, one may fix a current source at the depth to the center of the body in a borehole and make measurements of potential in another borehole with a moving electrode; then an estimate of dip may be achieved. Unfortunately, moving the source and receiver synchronously in two boreholes is difficult under typical field conditions.

A simple and practical method applied in this study is to interchange the roles of two boreholes; first use one as a potential measuring location, and then use it as a source location. As seen from Figure 23, we first fixed the current source at a depth below the center of the body in hole 1 and then measured the apparent resistivity with a moving electrode in hole 2. Next we used a current electrode located in hole 2, above the center of the body, and measured the potential in hole 1. The information about size, dip, and the center of the body are given by the peak positions, the asymmetry of the curves, and the crossover of the curves between the two peaks, respectively. The only assumption is that the body is below B2 and above B1. One could, of course, use continuums of locations of current and potential electrodes in both holes, if practical. No formal interpretation procedure is suggested for this technique because it would require a catalogue of curves which is beyond the scope of the present study. The dip vector of this body is parallel to the plane of the boreholes.

If the dip vector of the body is normal to the plane of the boreholes (Figure 24), the midpoint of the dashed line between the two peaks indicates the depth of the center of the body. Unfortunately, in this case there is ambiguity in estimating dip as -45 degrees or +45 degrees. No information on the size of the body is evident in Figure 24.

If the anomalous body is not located inside the two boreholes (Figure 25), an estimate of the direction of the anomalous body still can be made. As seen from Figure 25, for a conductive oblate body placed on the left-hand side of the two boreholes, the anomalies measured in both holes mainly appear as p1 < p2. This point can be taken as evidence that an anomalous body does not exist between the boreholes. Meanwhile, the larger anomaly measured from hole 1 indicates that it is nearer the body than is hole 2.

**CONCLUSIONS**

The algorithm for surface charge integration is a powerful tool to solve potential problems arising in resistivity. In fact, the method applied in this study can be used to compute the profile of apparent resistivity for arbitrarily shaped 3-D bodies if they can be represented by a polyhedron bounded by triangular facets. The model results indicate that the amplitude of apparent resistivity is highly dependent upon the locations of the current source and the borehole in which potential measurements are made, relative to the body, especially for a measuring borehole offset in the strike direction. If a resistivity contrast of 10 or more exists and if the boreholes are located at reasonable distances (e.g., EMX = EMY ≤ 6), the measured p1/p2 anomaly will be readily detectable above the noise, even though the ellipsoid is thin.

As seen from the typical cases studied, for a vertical spheroid and a vertical ellipsoid the depth to the center of the body and the size of the major axis can be determined with the simple configuration of a fixed current electrode and a moving potential electrode. For a dipping spheroid or ellipsoid, in which the dip vector of the body lies in the plane defined by the two boreholes, the depth to the center of the body can be determined accurately and estimates of the dip and the size of the major axis can be made. However, if the dip vector is normal to the plane defined by the two boreholes, it is more difficult to figure out the true dip of the body. It is also difficult to estimate the size of the body when the spheroid is horizontal.

It has been shown that in comparison with single-borehole measurements, the cross-borehole measuring technique is a more effective procedure in providing detectability of the anomalous amplitude and the dip of a spheroid or ellipsoid in the vicinity of a borehole. In most cases if the anomaly is detectable, then the depth to the center of the body can be located accurately; meanwhile the size of the major axis of the body and the dip of the body can be approximately outlined.

The strike of thin ellipsoids relative to the azimuth between
**Fig. 22.** Cross-borehole apparent resistivity profiles, for a spheroid dipping at \( \beta = 45 \) degrees, versus the \( L \)-spacing used in a configuration consisting of a moving-bipole source and a moving-bipole receiver. The midpoint of MN is the measuring point. All parameters are shown in the figure.

**Fig. 23.** Estimation of size, dip, and center of a spheroid obtained with an interchange measurement procedure using two boreholes. All parameters are shown in the figure.
Fig. 24. Estimation of the depth of the center of the body for a body whose dip vector is not parallel to the plane of the two boreholes used in cross-borehole measurements. All parameters are shown in the figure.

<table>
<thead>
<tr>
<th>Electrode Configuration</th>
<th>Body Size</th>
<th>Angles</th>
<th>Resistivity Contrast</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exchanging Measurement</td>
<td>a = 2</td>
<td>γ = 45°</td>
<td>$\frac{\rho_2}{\rho_1} = 0.10$</td>
</tr>
<tr>
<td>$SB_1 Y^0$ $SB_1 Z = 2.5$</td>
<td>b = 2</td>
<td>β = 0°</td>
<td></td>
</tr>
<tr>
<td>$SB_2 Y^0$ $SB_2 Z = -4.0$</td>
<td>c = 0.2</td>
<td>α = 0°</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 25. An illustration of the procedure to determine the direction to a thin spheroid from cross-borehole interchange measurements. All parameters are shown in the figure.

<table>
<thead>
<tr>
<th>Electrode Configuration</th>
<th>Body Size</th>
<th>Angles</th>
<th>Resistivity Contrast</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exchanging Measurement</td>
<td>a = 2</td>
<td>γ = 0°</td>
<td>$\frac{\rho_2}{\rho_1} = 0.10$</td>
</tr>
<tr>
<td>$SB_1 Y^0$ $SB_1 Z = 2.0$</td>
<td>b = 2</td>
<td>β = 45°</td>
<td></td>
</tr>
<tr>
<td>$SB_2 Y^0$ $SB_2 Z = -4.0$</td>
<td>c = 0.2</td>
<td>α = 0°</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 22. Cross-borehole apparent resistivity profiles, for a spheroid dipping at $\beta = 45$ degrees, versus the $L$-spacing used in a configuration consisting of a moving-bipole source, and a moving-bipole receiver. The midpoint of MN is the measuring point. All parameters are shown in the figure.

Fig. 23. Estimation of size, dip, and center of a spheroid obtained with an interchange measurement procedure using two boreholes. All parameters are shown in the figure.
two boreholes critically affects the size and shape of the resistivity anomaly for the cross-borehole technique.

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REFERENCES


