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Interpretation of MHD-sounding data from the Kola Peninsula by the electromagnetic migration method

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The electromagnetic migration method, based on principles analogous to those of seismic migration, is developed. The concept of migrated fields is based on the Stratton–Chu type integrals, written in reverse time for the observed electromagnetic fields. Four types of migrated fields, which form a system of migration transformations of the transient electromagnetic field, are introduced.

Study of the properties of the migrated fields by means of theoretical and model examples makes it possible to determine the optimal parameters of the migration procedure in which the anomalous field sources are localized by means of migration transformation.

Information about the experiment using the MHD-generator (experiment 'Chibini') carried out on the Kola Peninsula to study the geoelectrical structure of the Earth's crust and upper mantle is included. The electromagnetic migration method for the interpretation of MHD-sounding data recorded on the Kola Peninsula, along a profile crossing the mineral-rich region of the Imandra–Varzuga structure, permits us to determine the location of a conducting zone at the depth of 10 km in the Earth's crust.

1. Introduction

A new method of deep electromagnetic sounding of the Earth's crust and upper mantle utilizing a controlled source provided by MHD-generators has recently been developed in the U.S.S.R. The MHD-source is effective for geophysical exploration owing to its high power (up to 20 kA), and long current pulse duration (up to 6–8 s) (Gorbunov et al., 1982; Velikhov et al., 1982). One pulse of the MHD-generator propagates over an area of approximately 10^5 km^2 and reaches depths approaching 100 km in regions of high electrical resistivity (Gorbunov et al., 1982; Velikhov et al., 1982). The volume of the Earth's crust and mantle probed by the fields requires new techniques for data processing and interpretation.

Interpretation methods based on one-dimensional models of the Earth are no longer satisfactory and geophysicists must utilize two-and even three-dimensional modelling techniques. Electromagnetic migration is an example of the new methods (Zhdanov and Frenkel, 1983a,b; Frenkel, 1984; Zhdanov, 1984; Zhdanov and Frenkel, 1984; Velikhov et al., 1984). This method is similar to the seismic data interpretation method known as seismic migration (Berkhout, 1980; Gazdag and Squazzero, 1984). Seismic migration is one of the most effective seismic data transformation procedures that can be applied to determine the location of reflecting elements in the Earth. Migration is a mathematical procedure describing the process of reverse continuation from observation points to reflecting interfaces (Gazdag and Squazzero, 1984).

The concept of seismic migration may be re-

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stated as the restoration of a 'seismic image' at depth attained by oscillating the points of the Earth's surface in the reverse time domain. A wave field propagating in reverse time focuses on the reflectors that diffract the seismic waves. Drawing in space the amplitudes of these waves at the moment corresponding to their arrival at the given point of the medium we get the 'seismic image' of the section.

Various procedures for seismic migration have been developed. Among these, the method of wave front migration summation has been improved by representing the wave equation solution as Kirchhoff's integral * (Berkhout, 1980; Gazdag and Squazzero, 1984).

Note that the accuracy of migration strongly depends on precise determination of seismic velocities. Gazdag and Squazzero (1984) stressed that an ideal procedure is one that can determine the seismic velocities and migrate the data. Recursive step-by-step construction of cross-sectional parts by means of the migration procedure and velocity analysis is required for this procedure.

It is obvious that the basic concepts of seismic migration can also be used for working out methods of interpreting electromagnetic data. In fact the problem is the following in this case: to restore the 'geoelectric image' of the medium through the electromagnetic field (excited by a natural or artificial source) measured on the Earth's surface.

As in seismic migration, fictitious sources (extra currents and charges) are placed at the points of observation and are varied in reverse time according to the laws defined by the field measured at these points. Then the electromagnetic field initiated by these sources should illuminate the deep structure of the section.

The approach based on the Stratton–Chu type integrals, written for the observed electromagnetic fields in reverse time, was suggested for the practical realization of electromagnetic migration by Zhdanov and Frenkel (1983a,b). A migration procedure was shown to be the stable transformation of the electromagnetic field and allowed the approximate restoration of the distribution of anomalous field sources, in the electromagnetic sounding interpretation.

* It should be noted that the Stratton–Chu integral formulas, which underlie the electromagnetic migration method, are the analogue of Kirchhoff's integral in the theory of electromagnetic fields (Zhdanov and Frenkel, 1983a,b; Frenkel, 1984; Zhdanov, 1984; Zhdanov and Frenkel, 1984; Velikhov et al., 1984).

In this paper on the basis of Stratton–Chu type integrals, we introduce three new types of migration transformations of the transient electromagnetic fields forming together with the migration procedure considered in our previous works a system of migration transformations. The properties of the transformations are analysed in theoretical and model examples.

Note that there may be quite different approaches to electromagnetic field migration. For example, the finite difference technique, which is applied in seismic prospecting to the wave field migration in reverse time, may be used instead of Stratton–Chu type integrals. However, these problems are not considered in this study.

We now state the mathematical theory of the electromagnetic migration method and show how it was used for interpreting the MHD-sounding data from the Kola Peninsula.

2. The system of the transient electromagnetic field migration transformations

We first recall how the transient electromagnetic field migration transformation was introduced by Zhdanov and Frenkel (1983a,b, 1984).

The transient electromagnetic fields observed on the surface of the Earth Γ are designated by $\vec{E}(\vec{r}, t)$ and $\vec{H}(\vec{r}, t)$ *. We then change from the ordinary time t to the reverse time $\tau = T - t$ (T = the interval of field recording) and assign auxiliary fields $\vec{F}(\vec{r}, \tau)$ and $\vec{R}(\vec{r}, \tau)$ on Γ by the following formulas

$$\begin{aligned}
\vec{F}_{tg}(\vec{r}, \tau) &= \vec{E}_{tg}(\vec{r}, T-\tau) \\
F_n(\vec{r}, \tau) &= (1/c) E_n(\vec{r}, T-\tau) \\
\vec{R}_{tg}(\vec{r}, \tau) &= \vec{H}_{tg}(\vec{r}, T-\tau) \\
R_n(\vec{r}, \tau) &= -H_n(\vec{r}, T-\tau)
\end{aligned} \tag{1}$$

* A quasistationary model of the electromagnetic field is considered. Γ is assumed to be a piece-wise smooth closed surface passing through infinity.

where subscripts tg and n are the tangential and normal vectors' components and c is a positive migration constant.

The real fields $\vec{E}$ and $\vec{H}$ satisfy the two-dimensional consequences of Maxwell's equations everywhere on Γ

$$E_n = (-1/\sigma)\ \mathrm{div}^{\Gamma}\left[\vec{n} \times \vec{H}_{tg}\right];$$

$$\partial H_n/\partial t = (1/\mu_0)\ \mathrm{div}^{\Gamma}\left[\vec{n} \times \vec{E}_{tg}\right]$$

where σ = conductivity; μ_0 = permeability; div^{Γ} = surface divergence; $\vec{n}$ = external unit vector normal to Γ. Therefore the fields $\vec{F}$, $\vec{R}$ assigned by (1) satisfy the two-dimensional consequences of Maxwell's equations in reverse time on Γ (under the condition that conductivity in the lower half-space adjoining Γ is equal to $\sigma^m = c\sigma$)

$$F_n = (-1/\sigma^m)\ \mathrm{div}^{\Gamma}\left[\vec{n} \times \vec{R}_{tg}\right]$$
$$\partial R_n/\partial \tau = (1/\mu_0)\ \mathrm{div}^{\Gamma}\left[\vec{n} \times \vec{F}_{tg}\right] \qquad (2)$$

Consequently, these functions may be used as the densities of the Stratton–Chu type integrals, written for the reverse time τ

$$\vec{E}^m(\vec{r}', \tau) = \int_0^{\tau'} \iint_{\Gamma} \left\{ (\vec{n}\cdot\vec{F})\nabla G^m + [\vec{n}\times\vec{F}]\times\nabla G^m + [\vec{n}\times\vec{R}]\mu_0 \frac{\partial G^m}{\partial \tau} \right\} d\Gamma\, d\tau$$
$$\vec{H}^m(\vec{r}', \tau') = \int_0^{\tau'} \iint_{\Gamma} \left\{ (\vec{n}\cdot\vec{R})\nabla G^m + [\vec{n}\times\vec{R}]\times\nabla G^m + [\vec{n}\times\vec{F}]\sigma^m G^m \right\} d\Gamma\, d\tau \qquad (3)$$

where $G^m = G^m(\vec{r}', \tau' | \vec{r}, \tau)$, the Green's function of the diffusion equation (Morse and Feshbach, 1953).

According to the main properties of the Stratton–Chu type integrals the migrated fields $\vec{E}^m$ and $\vec{H}^m$ outside Γ satisfy Maxwell's equations

$$\mathrm{rot}\ \vec{H}^m = \sigma^m \vec{E}^m \qquad \mathrm{rot}\ \vec{E}^m = -\mu_0 \partial \vec{H}^m/\partial \tau'$$

Substituting (1) into (3) and replacing the variable τ in (3) by $T-t$ we obtain the final expressions for the migrated transformation of the observed electromagnetic fields (Zhdanov and Frenkel, 1983a,b)

$$\vec{E}^m(\vec{r}', T-t') = \int_{t'}^{T} \iint_{\Gamma} \left[\frac{(\vec{n}\cdot\vec{E})}{c} \nabla \tilde{G}^m + [\vec{n}\times\vec{E}]\times\nabla\tilde{G}^m + [\vec{n}\times\vec{H}]\mu_0 \frac{\partial \tilde{G}^m}{\partial t} \right] d\Gamma\, dt$$
$$\vec{H}^m(\vec{r}', T-t') = \int_{t'}^{T} \iint_{\Gamma} \left[-(\vec{n}\cdot\vec{H})\nabla\tilde{G}^m + [\vec{n}\times\vec{H}]\times\nabla\tilde{G}^m + [\vec{n}\times\vec{E}]\sigma^m \tilde{G}^m \right] d\Gamma\, dt \qquad (4)$$

where $\tilde{G}^m = \tilde{G}^m(\vec{r}', t' | \vec{r}, t)$, the conjugate to the fundamental Green's function of the diffusion equation.

The auxiliary fields $\vec{F}$ and $\vec{R}$ are not uniquely defined. However, if the introduced electromagnetic field ($\vec{E}$, $\vec{H}$) transformation is required to determine some new electromagnetic field, varying in the reverse time (τ) regime, there are only three other possible representations of the auxiliary fields $\vec{F}$, $\vec{R}$

$$\vec{F}_{tg}(\vec{r}, \tau) = -\vec{E}_{tg}(\vec{r}, T-\tau)$$
$$F_n(\vec{r}, \tau) = (1/c_2)E_n(\vec{r}, T-\tau)$$
$$\vec{R}_{tg}(\vec{r}, \tau) = \vec{H}_{tg}(\vec{r}, T-\tau) \qquad (5)$$
$$R_n(\vec{r}, \tau) = H_n(\vec{r}, T-\tau)$$

$$\vec{F}_{tg}(\vec{r}, \tau) = -\vec{E}_{tg}(\vec{r}, T-\tau)$$
$$F_n(\vec{r}, \tau) = E_n(\vec{r}, T-\tau)$$
$$\vec{R}_{tg}(\vec{r}, \tau) = c_3\vec{H}_{tg}(\vec{r}, T-\tau) \qquad (6)$$
$$R_n(\vec{r}, \tau) = H_n(\vec{r}, T-\tau)$$

$$\vec{F}_{tg}(\vec{r}, \tau) = \vec{E}_{tg}(\vec{r}, T-\tau)$$
$$F_n(\vec{r}, \tau) = E_n(\vec{r}, T-\tau)$$
$$\vec{R}_{tg}(\vec{r}, \tau) = c_4\vec{H}_{tg}(\vec{r}, T-\tau) \qquad (7)$$
$$R_n(\vec{r}, \tau) = -H_n(\vec{r}, T-\tau)$$

where c_2, c_3 and c_4 are some positive constants.

It is easy to ensure that each of the auxiliary field pairs $\vec{F}$ and $\vec{R}$ defined by (5)–(7) satisfies the conditions (2) if σ^m is replaced in the latter by $\sigma^{m2} = c_2\sigma$, $\sigma^{m3} = c_3\sigma$ and $\sigma^{m4} = c_4\sigma$. Thus three new types of transformations producing migrated fields $\vec{E}^{m2}\vec{H}^{m2}$, $\vec{E}^{m3}\vec{H}^{m3}$ and $\vec{E}^{m4}\vec{H}^{m4}$ may be introduced analogous to the fields $\vec{E}^m$ and $\vec{H}^m$ defined by the relations (4). These are

$$\vec{E}^{m2}(\vec{r}', T-t') = \int_{t'}^{T}\iint_{\Gamma}\left[\frac{(\vec{n}\cdot\vec{E})}{c_2}\nabla\tilde{G}^{m2} - [\vec{n}\times\vec{E}]\times\nabla\tilde{G}^{m2} + [\vec{n}\times\vec{H}]\mu_0\frac{\partial\tilde{G}^{m2}}{\partial t}\right]\mathrm{d}\Gamma\,\mathrm{d}t$$
$$\vec{H}^{m2}(\vec{r}', T-t') = \int_{t'}^{T}\iint_{\Gamma}\left[(\vec{n}\cdot\vec{H})\nabla\tilde{G}^{m2} + [\vec{n}\times\vec{H}]\times\nabla\tilde{G}^{m2} - [\vec{n}\times\vec{E}]\sigma^{m2}\tilde{G}^{m2}\right]\mathrm{d}\Gamma\,\mathrm{d}t \quad (8)$$

$$\vec{E}^{m3}(\vec{r}', T-t') = \int_{t'}^{T}\iint_{\Gamma}\left[(\vec{n}\cdot\vec{E})\nabla\tilde{G}^{m3} - [\vec{n}\times\vec{E}]\times\nabla\tilde{G}^{m3} + c_3[\vec{n}\times\vec{H}]\mu_0\frac{\partial\tilde{G}^{m3}}{\partial t}\right]\mathrm{d}\Gamma\,\mathrm{d}t$$
$$\vec{H}^{m3}(\vec{r}', T-t') = \int_{t'}^{T}\iint_{\Gamma}\left[(\vec{n}\cdot\vec{H})\nabla\tilde{G}^{m3} + c_3[\vec{n}\times\vec{H}]\times\nabla\tilde{G}^{m3} + [\vec{n}\times\vec{E}]\sigma^{m3}\tilde{G}^{m3}\right]\mathrm{d}\Gamma\,\mathrm{d}t \quad (9)$$

$$\vec{E}^{m4}(\vec{r}', T-t') = \int_{t'}^{T}\iint_{\Gamma}\left[(\vec{n}\cdot\vec{E})\nabla\tilde{G}^{m4} + [\vec{n}\times\vec{E}]\times\nabla\tilde{G}^{m4} + c_4[\vec{n}\times\vec{H}]\mu_0\frac{\partial\tilde{G}^{m4}}{\partial t}\right]\mathrm{d}\Gamma\,\mathrm{d}t$$
$$\vec{H}^{m4}(\vec{r}', T-t') = \int_{t'}^{T}\iint_{\Gamma}\left[-(\vec{n}\cdot\vec{H})\nabla\tilde{G}^{m4} + c_4[\vec{n}\times\vec{H}]\times\nabla\tilde{G}^{m4} + [\vec{n}\times\vec{E}]\sigma^{m4}\tilde{G}^{m4}\right]\mathrm{d}\Gamma\,\mathrm{d}t \quad (10)$$

where

$$\tilde{G}^{mi} = \left[-\left(\mu_0\sigma^{mi}\right)^{1/2}/8\pi^{3/2}(t-t')^{3/2}\right] \times \exp\left[-\mu_0\sigma^{mi}|\vec{r}-\vec{r}'|^2/4(t-t')\right]\kappa(t-t')$$

$$i = 2, 3, 4; \quad \mathcal{H}(t-t') = \begin{cases}1, & t > t'\\ 0, & t \leqslant t'\end{cases}$$

The following designations are assumed for uniformity:

$$\sigma^{m1} = \sigma^m; \quad c_1 = c; \quad \vec{E}^{m1} = \vec{E}^m;$$
$$\vec{H}^{m1} = \vec{H}^m; \quad \tilde{G}^{m1} = \tilde{G}^m.$$

The fields $\vec{E}^{mi}$ and $\vec{H}^{mi} (i = 1, 2, 3, 4)$ defined by formulas (4), (8)–(10) form the system of the transient electromagnetic field migration transformations.

Note that the electromagnetic migration is a stable procedure because the operators in the right-hand part of (4), (8)–(10) are limited (in space L_2).

It is necessary for practical utilization of migration transformations (i.e., for calculating the fields $\vec{E}^{mi}$ and $\vec{H}^{mi}$) to have information about conductivity of the Earth σ and the fields $\vec{E}(\vec{r}, t)$ and $\vec{H}(\vec{r}, t)$, which should be recorded on the Earth's surface by the synchronous observation technique.

3. Properties of the migrated fields

All the types of migrated fields defined above require the assignment of the migration constant $c(c_1, c_2, c_3, c_4)$. It may appear that the procedure is complicated by these additional parameters but Zhdanov and Frenkel (1983a,b, 1984) have shown that the migration constant provides more flexibility in locating conductive structures.

Let us enumerate some of the most important properties of the migrated fields for the two-dimensional case. We consider the theoretical model discussed by Zhdanov and Frenkel (1983a,b, 1984) in which the anomalous fields $\vec{E}$ and $\vec{H}$, excited by an infinitely long cable passing through the point $\vec{r}_0$ parallel to the axis Y, are recorded over the time interval $(0, T)$ on the horizontal profile Γ, coinciding with the coordinate axis X. The cable is immersed in an infinitely homogeneous medium with conductivity σ. The current in the cable is varied according to $J = j\delta(t)$, where $\delta(t)$ is the Dirac delta function and j is a constant. Let us study the properties of the fields $\vec{E}^{mi}$ and $\vec{H}^{mi}$, obtained as a result of migrating the fields $\vec{E}$ and $\vec{H}$ into the lower half-space.

Rewrite formulas (4), (8)–(10) in scalar form

$$\begin{pmatrix} H_x^{m1} \\ H_x^{m2} \\ H_x^{m3} \\ H_x^{m4} \end{pmatrix} = \int_{t'}^{T}\int_{-\infty}^{\infty}\left[H_x \begin{pmatrix} -\tilde{G},_z^{m1} \\ -\tilde{G},_z^{m2} \\ -c_3\tilde{G},_z^{m3} \\ -c_4\tilde{G},_z^{m4} \end{pmatrix} + H_z \begin{pmatrix} \tilde{G},_x^{m1} \\ -\tilde{G},_x^{m2} \\ -\tilde{G},_x^{m3} \\ \tilde{G},_x^{m4} \end{pmatrix} + E_y \begin{pmatrix} \sigma^{m1}\tilde{G}^{m1} \\ -\sigma^{m2}\tilde{G}^{m2} \\ \sigma^{m3}\tilde{G}^{m3} \\ \sigma^{m4}\tilde{G}^{m4} \end{pmatrix} \right] \mathrm{d}x\,\mathrm{d}t \qquad (11)$$

$$\begin{pmatrix} H_z^{m1} \\ H_z^{m2} \\ H_z^{m3} \\ H_z^{m4} \end{pmatrix} = \int_{t'}^{T}\int_{-\infty}^{\infty}\left[H_x \begin{pmatrix} \tilde{G},_x^{m1} \\ \tilde{G},_x^{m2} \\ c_3\tilde{G},_x^{m3} \\ c_4\tilde{G},_x^{m4} \end{pmatrix} + H_z \begin{pmatrix} \tilde{G},_z^{m1} \\ -\tilde{G},_z^{m2} \\ -\tilde{G},_z^{m3} \\ \tilde{G},_z^{m4} \end{pmatrix} \right] \mathrm{d}x\,\mathrm{d}t \qquad (12)$$

$$\begin{pmatrix} E_y^{m1} \\ E_y^{m2} \\ E_y^{m3} \\ E_y^{m4} \end{pmatrix} = \int_{t'}^{T}\int_{-\infty}^{\infty}\left[H_x\mu_0 \begin{pmatrix} -\tilde{G},_t^{m1} \\ -\tilde{G},_t^{m2} \\ -c_3\tilde{G},_t^{m3} \\ -c_4\tilde{G},_t^{m4} \end{pmatrix} + E_y \begin{pmatrix} -\tilde{G},_z^{m1} \\ \tilde{G},_z^{m2} \\ \tilde{G},_z^{m3} \\ -\tilde{G},_z^{m4} \end{pmatrix} \right] \mathrm{d}x\,\mathrm{d}t \qquad (13)$$

In formulas (11)–(13)

$$\tilde{G},_\alpha^{mi} = \frac{\partial \tilde{G}^{mi}}{\partial \alpha} \quad \alpha = x,\ z,\ t \quad i = 1, 2, 3, 4$$
$$H_x = -jG,_z \quad H_z = jG,_x \quad E_y = -j\mu_0 G,_t \qquad (14)$$

where

$$G = G(\vec{r},\ t\,|\,\vec{r}_0,\ 0) = \left(\frac{-1}{4\pi t}\right)\exp\left(\frac{-\mu_0\sigma\,|\vec{r}-\vec{r}_0|^2}{4t}\right)$$

Note that the functions G and $\tilde{G}$ are connected by the following relations (Morse and Feshbach, 1953)

$$\tilde{G}(\vec{r}',\ t'\,|\,\vec{r},\ t) = G(\vec{r}',\ -t'\,|\,\vec{r},\ -t) = G(\vec{r},\ t\,|\,\vec{r}',\ t') \qquad (15)$$

Zhdanov and Frenkel (1983a,b, 1984) showed that the migrated field components H_x^{m1} and E_y^{m1} possess a number of extremum properties that may be used for the determination of the anomalous field source location. In the lower half-space the extremum points are on the vertical half-axis passing through the cable, and in addition, the location of the cable coincides with the local extrema of the migrated field at the moment when the current in the source is switched on (i.e., when $t' = 0$). If migration is carried out into the medium with conductivity $\sigma^{m1} = 0.5\sigma$ (for the component H_x^{m1}) and $\sigma^{m1'} \equiv 0.55\sigma$ (for the component E_y^{m1}), the migrated field extremum is focussed on the anomalous field source.

We illustrate the focussing property of the migrated field by the isoline-maps of H_x^{m1} component, constructed for the case of field migration from an infinitely long cable, immersed in a homogeneous space with $\sigma = 0,1$ S m^{-1} ($\sigma^{m1} = 0.5\sigma = 0.05$ S m^{-1}). The isoline maps (Fig. 1) are constructed for a few moments of time (t')—proceeding from the end of field recording at time T to the instant the primary pulse is switched on at $t' = 0$. In terms of the reverse time τ, from the moment close to zero, i.e., the moment of switching on the migrated field sources on the observation surface, to $\tau' = T$, i.e., the moment when these sources are switched off. These maps visually demonstrate the forming process of the migrated 'image' of anomalous field local source, which in its turn is considered as an element of the 'geoelectric image' of inhomogeneous conducting medium. Actually, at $\tau' = 0$ the migrated field sources start working on the observation surface, and by the field extrema and isolines moving down one can judge about the process of migrated field propagation in the medium. The migrated field H_x^{m1} is focussed in the anomalous field source at the moment of switching on the current in it, i.e., when $t' = 0$ (Fig. 1).

Let us turn to consideration of the properties of the fields $\vec{\mathbf{E}}^{mi}$ and $\vec{\mathbf{H}}^{mi}$ ($i = 2, 3, 4$). In the lower half-space the extremum points of H_x^{mi} and E_y^{mi}

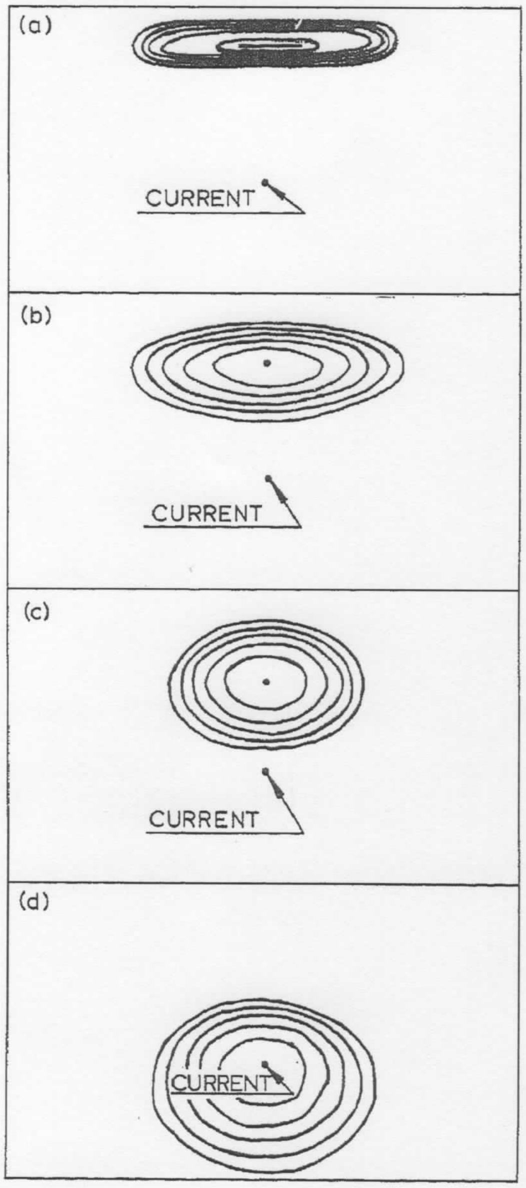

Fig. 1. Maps of H_x^{ml} isolines ($c_1 = 0.5$) constructed for an infinite cable embedded in a homogeneous space for different moments of time τ': (a) $\tau' = 0.5$ s ($t' = 9.5$ s); (b) $\tau' = 3$ s ($t' = 7$ s); $\tau' = T - t$; (c) $\tau' = 8$ s ($t' = 2$ s); (d) $\tau' = T = 10$ s ($t' = 0$). On each map the isoline step is equal to 0.1 of maximum value of the field at the corresponding extremum point.

components are on the vertical half-axis passing through the cable (just as for H_x^{ml} and E_y^{ml}). So consider the behaviour of the migrated field components H_x^{mi} and E_y^{mi} when $x' = x_0$. Without restricting generality we may take $x_0 = 0$. We calculate some integrals in expressions (11) and (13) when $t' = 0$, $T = \infty$, using (14) and the known tabulated integrals

$$I_H^{(1)} = \int_0^\infty \int_{-\infty}^\infty \left(-H_x \tilde{G},_z^{mi}\right) \mathrm{d}x\, \mathrm{d}t$$

$$= \left(j\mu_0^2 \sigma \sigma^{mi} z_0 z'/64\pi^2\right) \cdot \int_0^\infty t^{-4} \exp\left[\left(-\mu_0/4t\right)\left(\sigma z_0^2 + \sigma^{mi} z'^2\right)\right]$$

$$\times \int_{-\infty}^\infty \exp\left[\left(-\mu_0/4t\right) \cdot x^2\left(\sigma + \sigma^{mi}\right)\right] \mathrm{d}x\, \mathrm{d}t$$

$$= \left[j\mu_0^{3/2} \sigma \sigma^{mi}\left(\sigma + \sigma^{mi}\right)^{-1/2} z_0 z'/32\pi^{3/2}\right] \cdot \int_0^\infty t^{-7/2} \exp\left[\left(-\mu_0/4t\right)\left(\sigma z_0^2 + \sigma^{mi} z'^2\right)\right] \mathrm{d}t$$

$$= A z_0 z' B^{-5/2} \qquad (16)$$

where

$$A = 3j\sigma^{-1} c_i \left(1 + c_i\right)^{-1/2} / 4\pi\mu_0; \quad B = \left(z_0^2 + c_i z'^2\right)$$

Expressions for the other integrals in (11) and (13) may be calculated by analogy with the previous ones therefore only their final forms are given

$$I_H^{(2)} = \int_0^\infty \int_{-\infty}^\infty H_z \tilde{G},_x^{mi}\, \mathrm{d}x\, \mathrm{d}t = A\left(B/3\left(c_i + 1\right)\right) B^{-5/2} \qquad (17)$$

$$I_H^{(3)} = \int_0^\infty \int_{-\infty}^\infty E_y \tilde{G}^{mi} \sigma^{mi}\, \mathrm{d}x\, \mathrm{d}t = A\left(2B/3 - \left(B/3\left(c_i + 1\right)\right) - z_0^2\right) B^{-5/2} \qquad (18)$$

$$I_E^{(1)} = \int_0^\infty \int_{-\infty}^\infty \left(-H_x \tilde{G},_t^{mi} \mu_0\right) \mathrm{d}x\, \mathrm{d}t = A\left(c_i^2 + c_i\right)^{-1} \sigma^{-1} z_0 \left[z'^2 c_i \left(4c_i + 3\right) - z_0^2\left(c_i + 2\right)\right] B^{-7/2} \qquad (19)$$

$$I_E^{(2)} = \int_0^\infty \int_{-\infty}^\infty E_y \tilde{G},_z^{mi}\, \mathrm{d}x\, \mathrm{d}t = A\sigma^{-1}\left(c_i + 1\right)^{-1} z' \left[z'^2 c_i \left(2c_i + 1\right) - z_0^2\left(3c_i + 4\right)\right] B^{-7/2} \qquad (20)$$

The cancellations in (17)–(20) are assumed the same as in (16).

Note that

$$\begin{pmatrix} H_x^{m1} \\ H_x^{m2} \\ H_x^{m3} \\ H_x^{m4} \end{pmatrix} = I_H^{(1)} \begin{pmatrix} 1 \\ 1 \\ c_3 \\ c_4 \end{pmatrix} + I_H^{(2)} \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} + I_H^{(3)} \begin{pmatrix} 1 \\ -1 \\ 1 \\ 1 \end{pmatrix} \tag{21}$$

$$\begin{pmatrix} E_y^{m1} \\ E_y^{m2} \\ E_y^{m3} \\ E_y^{m4} \end{pmatrix} = I_E^{(1)} \begin{pmatrix} 1 \\ 1 \\ c_3 \\ c_4 \end{pmatrix} + I_E^{(2)} \begin{pmatrix} -1 \\ 1 \\ 1 \\ -1 \end{pmatrix} \tag{22}$$

From the necessary condition of local extremum for the migrated field component H_x^{m2}, i.e., from the condition $\partial H_x^{m2}/\partial z' = 0$ and differentiating the second row of (21), we obtain the following equation

$$AB^{-7/2}\left(2c_2^2 z'^3 - 4c_2 z'^2 z_0 - 3c_2 z' z_0^2 + z_0^3\right) = 0 \tag{23}$$

If the necessary condition of local extremum is demanded to be satisfied in the point that the cable passes through, $z' = z_0$ should be assumed in (23). Since $AB^{-7/2} \neq 0$, we obtain from (23) the second order algebraic equation

$$2c_2^2 - 7c_2 + 1 = 0 \tag{24}$$

Both solutions of the eq. 24 are positive, these are

$c_2^{(1)} \cong 0.15 \quad c_2^{(2)} \cong 3.35$

The found values of the migration constant c_2 are optimum, because with such values of c_2 the sufficient condition of local extremum is also satisfied in the point $z' = z_0$, i.e.

$$\partial^2 H_x^{m2}/\partial z'^2 \Big|_{\substack{t'=0,\ z'=z_0 \\ x'-x_0=0 \\ c_2=c_2^{(1)} \text{ or } c_2=c_2^{(2)}}} \neq 0$$

Consequently, if migration is carried out into the medium with conductivity $\sigma^{m2} \cong 0.15\sigma$ or $\sigma^{m2} \cong 3.35\sigma$, the component H_x^{m2}, restored in the lower half-space at the moment $t' = 0$, i.e., at the moment of switching on the current in the cable, has a local extremum coinciding with the location of the cable. Two optimum values of the constant c_2 suggest a more complicated dependence of H_x^{m2} on the migration constant than observed with the migrated field H_x^{m1} (Zhdanov and Frenkel, 1983a,b, 1984).

Optimum values of migration constants for the other migrated field components are calculated just as for the component H_x^{m2}.

The final results of these calculations are given in Table I.

We show by an example of the component H_x^{m1} how mistakes in the assignment of the embedding medium conductivity σ have an influence on the accuracy of determining the location of the anomalous field source.

From relations (16)–(18), (21) and condition

$$\partial H_x^{m1}/\partial z' \Big|_{\substack{x'-x_0=0 \\ t'=0}} = 0$$

comes the following algebraic equation

$$2c_1^2\alpha^3 + 4c_1\alpha^2 - 3c_1\alpha - 1 = 0 \tag{25}$$

where $\alpha = z'/z_0$. Obviously, if the optimum value of the migration constant $c_1 = 0.5$ is substituted in eq. 25, the unique positive root of (25) is $\alpha = 1$, i.e., H_x^{m1} has the local extremum in the point $z' = z_0$, which the cable passes through. Suppose now that the conductivity of the medium σ is

TABLE I

The optimum values of migration constants for different types of migrated transformations

Type of migrated transformation (i)	Optimum values of migration constants (c_i) for migrated field components	
	H_x^{mi}	E_y^{mi}
1	0.5	0.55 [a] 5.45 [a]
2	0.15 [a] 3.35 [a]	2
3	1	0.09 [a] 1.63 [a]
4	2/3	1/3

[a] These values of migration constants are approximate.

incorrectly assigned, for example, it is twice as much as the true value σ_{tr}: $\sigma = 2\sigma_{tr}$, then $\sigma^{m1} = \sigma_{tr}$ and we can assume in (25) $c_1 = 1$.

In this case (25) has the unique positive root $\alpha \cong 0.77$. Analogously, if $\sigma = 0.5\sigma_{tr}$, then $\sigma^{m1} = 0.25\sigma_{tr}$ and $c_1 = 0.25$. Then we determine $\alpha \cong 1.3$. Consequently, when the assigned conductivity of the medium is incorrect: $\sigma = 2\sigma_{tr}$ or $\sigma = 0.5\sigma_{tr}$, the shift of the local extremum of the migrated field component H_x^{m1}, restored at the moment $t' = 0$, is equal to $-0.23z_0$ (up-shift from the source) or to $+0.3z_0$ (down-shift from the source), respectively. Thus, substantial mistakes in the value of the embedding medium conductivity lead to relatively small mistakes in determining the location of the anomalous field sources.

We note that the migration method possesses high resolving power. Using it for separating closely located anomalous objects, we can determine the location of anomalous field sources disposed not only at the same but also at different depths. The conditions of such separation and other results of resolution study of the electromagnetic field migration method (by the example of the field $\vec{E}^{m1}$, $\vec{H}^{m1}$) were given in Frenkel (1984). In particular, it was shown that by means of the migrated transformation we could separate two sources of the anomalous field (cables) located at the same depth z_0 if the distance between them was more than $2z_0$ (the current in the cables varied according to the same law $J(t) = j\delta(t)$). This estimate agrees with the known condition for determining the possibility of separating neighbouring conducting objects excited by excessive currents when the field varies in time according to the harmonic law.

It should be emphasized that in studying the properties of the migrated transformations we have shown in principle the possibility of applying electromagnetic field transformations for localization of those regions occupied by the excessive currents. At the same time a detailed numerical investigation of the migrated field properties in models containing local geoelectric inhomogeneities should be carried out to determine the most effective migrated transformation.

4. The association of migrated field with the spatial distribution of the excessive currents *

The migrated field calculations for the models of Zhdanov and Frenkel (1983a,b, 1984) showed that conductive local inhomogeneities embedded in a homogeneous resistive media are mapped by the isolines of the migrated field. The method also locates geoelectric inhomogeneities embedded in a horizontally layered section. In analogy with seismic migration it is advisable in this case to take as σ the apparent conductivity of the Earth ($\sigma_a(\tau)$) obtained by electromagnetic sounding and time-averaged in the interval (o, τ')

$$\sigma = \frac{1}{\tau'} \int_0^{\tau'} \sigma_a(\tau)\, d\tau \qquad (26)$$

The migrated fields for the model with parameters presented in Fig. 2a (a case of E-polarization) were calculated by means of the MIGRA2 program (Frenkel, 1984). Migration was carried out into the homogeneous medium with conductivity σ, obtained by (26) at each moment of time. The moment of migrated field restoration (t') was determined as the time for which induced currents had maximum intensity, a technique suggested by Zhdanov and Frenkel (1983a, 1984).

The result of the migration (of the E_y^{m1} component) is presented in Fig. 2b. It is distinctly seen that the location and geometry of the rectangular insert are satisfactorily located by the migrated field isolines. This example demonstrates that the electromagnetic migration method is reliable for not only the homogeneous normal section but for the layered-homogeneous one as well.

It has been shown by means of theoretical examples that the migration procedure is most effective when the main part of the anomalous field ($\vec{E}^A$, $\vec{H}^A$) is transformed. According to Berdichevsky and Zhdanov (1984) the main part of the anomalous field is the field excited by the anomalous currents in an infinitely homogeneous space with the same conductivity as the lower half-space.

In the transient case, just as in the case of monochromatic fields (Berdichevsky and Zhdanov,

* Only the migrated fields $\vec{E}^{m1}$ and $\vec{H}^{m1}$ are used in the following discussion.

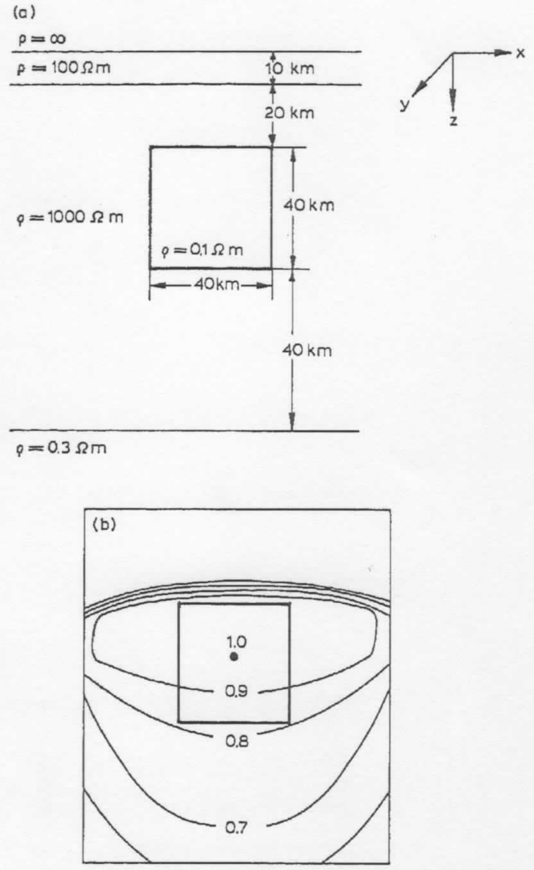

Fig. 2. (a) Two-dimensional model of a horizontally layered medium with a rectangular insert. (b) Map of E_y^{m1} isolines. Values of E_y^{m1} are normalized by the value of the field at the point of local extremum.

1984), the operators for determination of the main part of the anomalous field may be constructed by means of Stratton–Chu type integrals. The expression for the corresponding transformation is the following (Frenkel, 1984; Velikhov et al., 1984)

$$\begin{pmatrix}\vec{H}^A\\ \vec{E}^A\end{pmatrix} = 0.5\begin{pmatrix}\vec{H}\\ \vec{E}\end{pmatrix} + \int_0^{t'}\iint_\Gamma\left[\vec{n}\cdot\begin{pmatrix}\vec{H}\\ \vec{E}\end{pmatrix}\nabla G + \left[\vec{n}\times\begin{pmatrix}\vec{H}\\ \vec{E}\end{pmatrix}\right]\times\nabla G + \vec{n}\times\begin{pmatrix}\vec{E}\sigma G\\ \vec{H}\mu_0\partial G/\partial t\end{pmatrix}\right]\mathrm{d}\Gamma\,\mathrm{d}t \qquad (27)$$

Now consider the model of an infinitely thin ideally conducting screen S, embedded in a homogeneous space with conductivity σ. The field in the model is excited by an arbitrary source outside of S. The anomalous (secondary) electromagnetic field everywhere outside the screen is determined by the Stratton–Chu type integrals taken over the screen surface S (Zhdanov and Frenkel, 1983a,b; Velikhov et al., 1984)

$$\begin{pmatrix}\vec{H}^A\\ \vec{E}^A\end{pmatrix} = \int_0^{t'}\begin{pmatrix}-1\\ (1/\sigma)\ \mathrm{rot}\end{pmatrix}\mathrm{rot}\iint_S \vec{J}^E G\,\mathrm{d}S\,\mathrm{d}t$$

where $\vec{J}^E = [\vec{n}\times\vec{H}]$, the surface current induced on the screen by the field source. Note that in the case of a homogeneous medium the anomalous part of the field coincides with the main anomalous part, therefore we use index 'A' for its designation.

As a simplification consider that the current $\vec{J}^E$ is varying in time according to the delta-impulse law ($J^E(\vec{r}, t) = j\delta(t)$; $j =$ constant) in each point ($\vec{r}$) of the screen. Then, for example, in the two-dimensional case when the strike of the screen is parallel to the Y axis, it can be shown (Velikhov et al., 1984) that the migrated field has a local extremum in the intersection point of any vertical line with the screen S if the screen is of quasi-horizontal shape.

The structure of the migrated field becomes complicated in the case of screens of more complicated shape, but qualitatively the picture remains the same: the migrated field isolines outline the screen (Fig. 3). Thus, one can approximately

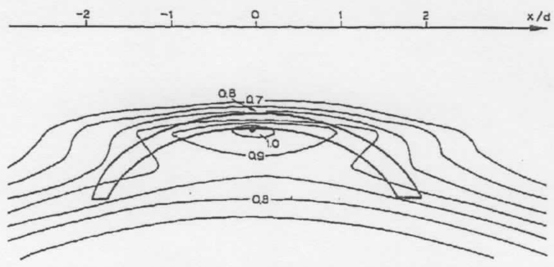

Fig. 3. The map of E_y^{m1} isolines for a two-dimensional model of a conducting thin screen ($\sigma_{screen}/\sigma = 10^4$). $d =$ distance from the observation surface to the top of the screen. The isolines are normalized as in Fig. 2.

describe highly conducting inhomogeneities of arbitrary shape by means of the ideally conducting screen model considered above, overlapping the screen with the inhomogeneity surface.

5. The interpretation of MHD-sounding data from the Kola Peninsula (experiment 'Chibini')

The migration technique of analysing the transient field excited by a powerful impulse source was applied to interpret the Kola Peninsula MHD-sounding data. The MHD-soundings began in 1976 to study the crystalline basement and to map the conducting graphitic structures associated with zones of mineralization (Gorbunov et al., 1982; Velikhov et al., 1982).

A unique feature of the Chibini experiment is the use of the Barents coastline around the Rybachyi and Srednyi Peninsulas as part of the current source by connecting the MHD-generator with the bays to the east and west of the isthmus between the Srednyi Peninsula and the land (Fig. 4). Gorbunov et al. (1982) emphasized that the main part of the current excited by the generator passes through the sea around the Rybachyi and Srednyi Peninsulas to form a current loop of 2000 km^2, which may be considered as a vertical magnetic dipole at great distances. The moment of the dipole can be as large as 10^{14} A-m. The creation of such a powerful impulse provides useful signals over most of the Kola Peninsula.

The data used in the migration technique were obtained on a profile that crossed the western portion of the Imandra–Varzuga structural zone and consisted of nine measurements of $\vec{E}$ and $\vec{H}$ profile A–A in Fig. 4). The experimental data along this profile were obtained by the Geological Institute of the Kola Branch of the U.S.S.R. Academy of Science.

The migration transformation is stable, therefore, noisy data can be processed without filtering (the transformation itself is a low pass filter of the fields $\vec{E}$ and $\vec{H}$). Only a limited number of measurements were available that had errors of 10–20%. Therefore the possibility of suppressing noise in the process of obtaining the 'geoelectric image' that results from the migration procedure is of great importance.

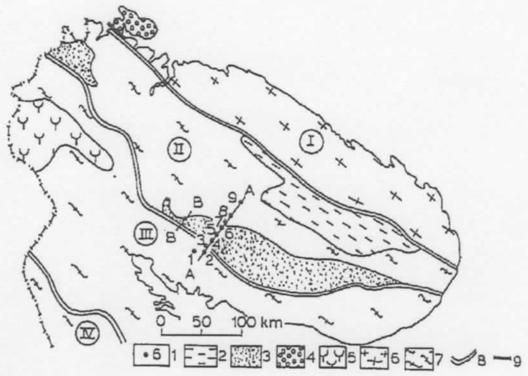

Fig. 4. Lay-out diagram of the stations for measuring the MHD-generator signals in the Kola Peninsula (A–A profile) (Gorbunov et al., 1982; Velikhov et al., 1982). The legend: 1–measurement stations and their ordinal number; 2–series Keiv; 3–Pechengo–Varzuga effusive-sedimentary complex; 4–subplatform terrigene complex of Srednyi and Ribachyi Peninsulas; 5–granulite complex; 6–Barents Sea granite-gneiss complex; 7–Kola–White Sea gneiss complex; 8–boundaries of megablocks (I–Murmanskyi, II–Central-Kola; III–White Sea; IV–North-Karelian); 9–location of MHD-generator.

Magnetovariational profiling (MVP) and magnetotelluric sounding (MTS) studies conducted earlier in the region (profile B–B in Fig. 4) (Zhamaletdinov et al., 1980) and the behaviour of the electromagnetic field excited by the MHD-generator in the same area (Gorbunov et al., 1982; Velikhov et al., 1982; Kirillov and Osipenko, 1984) showed that the Imandra–Varzuga structure could be considered two-dimensional. Therefore the MHD-sounding data along profile A–A were processed by the two-dimensional migration procedure (Frenkel, 1984).

The first stage of the interpretation consisted of determining the main part of the anomalous field according to (27). Graphs of the field $\vec{E}^{\wedge}$, $\vec{H}^{\wedge}$ are presented in Fig. 5.

The isoline map of the migrated horizontal magnetic component H_x^{m1} constructed for the time approximately corresponding to the maximum intensity of induction currents $t' = 1{,}2$ s (when σ^{m1}

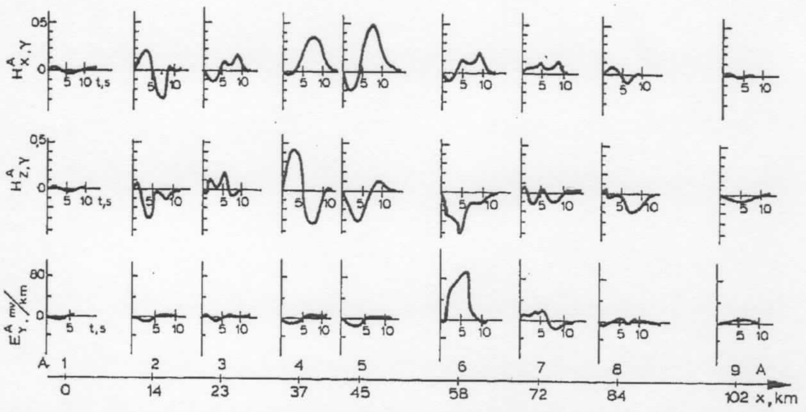

Fig. 5. Graphs of the main part of the anomalous field along profile A–A (numerals on x-line denote station locations shown in Fig. 4).

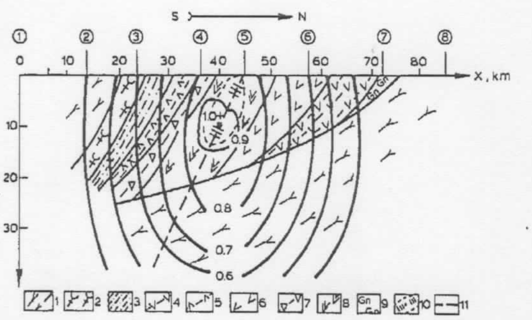

Fig. 6. Map of H_x^{m1} isolines constructed at the moment $t' = 1.2$ s and geological section in the Imandra–Varzuga structure region. Isolines are normalized as in Fig. 2. Legend: 1–plagiogranites, 2–granite–gneisses, 3–biotite gneisses, 4–vulcanites, 5–vulcanites and effusives, 6–effusives, 7–black shales, 8–basic vulcanites, 9–gabbro–norites, 10–conducting body, 11–isoline $H_z^{m1} = 0$.

$= 0.5\sigma = 0.001$ S m^{-1}) is shown in Fig. 6 *. The geological section in the region of the Imandra–Varzuga structure is also presented in Fig. 6 (Kirillov and Osipenko, 1984).

Figure 6 shows that the migrated field outlines the conducting zone at a depth of 10 km in the crust. This is in good agreement with the geological description as the conducting body is associated with a zone of sulphide-graphite shales and gneisses (Zhamaletdinov and Semenov, 1978).

It should be emphasized that the migration transformation only approximately defines the shape of geoelectric inhomogeneity. The boundaries of the conducting zone should be determined more precisely in the subsequent steps of the interpretation. This may be achieved by automatic trial and error methods, e.g., correction of the inhomogeneity shape using the method of tightening surfaces (Berdichevsky and Zhdanov, 1984). The model of geoelectric section obtained by the migrated transformation is a sufficiently accurate initial model that ensures the quick convergence of the tightening surfaces method.

* Using the horizontal electrical component of the migrated field E_y^{m1} for localization of the anomalous object gives practically the same result.

6. Conclusions

In this work we have suggested the system of migrated transformations based on the Stratton–Chu type integrals written in reverse time for the observed electromagnetic fields. It completes and generalizes the transformation types considered earlier (Zhdanov and Frenkel, 1983a,b).

We have found the optimum parameters of migrated transformations. Using these parameters in electromagnetic migration of the recorded fields we can determine the location of anomalous currents. A numerical experiment was carried out on a model consisting of a horizontally layered medium containing a highly conductive square insert. It was shown that the location and form of geoelectric inhomogeneity may be approximately determined by the extremum points and isolines of the migrated field just as in the case of a homogeneous embedding medium. Thus the migrated transformations may be used for constructing an initial model of geoelectric structure.

The results of practical utilization of the electromagnetic migration method for interpreting MHD-sounding data from the Kola Peninsula have been given. The migrated field outlined a conducting zone at a depth of 10 km in the Earth's crust. Thus we ascertained that migration of the electromagnetic field may be regarded as an effective transformation of the field distribution at the Earth's surface in MHD-sounding interpretation.

It should be noted that the migration method may be applied to interpret sounding data obtained from different sources of the transient electromagnetic fields that are not of such high power as MHD-generators. For example, the ordinary impulse generators usually used in electrical exploration are quite acceptable. However, the field recording should then be carried out using synchronous observation techniques. Both profile and area recording of the electromagnetic field may be applied. Correspondingly two- or three-dimensional migration procedures should be used for interpreting the data.

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