Step Responses for Two-Dimensional Electromagnetic Models

JOPIE I. ADHIDJAJA and GERALD W. HOHMANN
Department of Geology and Geophysics, University of Utah, Salt Lake City, UT 84112 (U.S.A.)
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ABSTRACT


We have computed the magnetic field step response for a 2-D body in a conductive host via numerical modeling. The numerical method for computing the step response is a modified version of our previous finite difference scheme that computes the impulse response. By comparing the responses of a suite of models, we observe that basically the step and the impulse responses are affected by the host in a similar manner; a homogeneous host delays and attenuates the peak responses, and a conductive overburden accentuates those effects even more. Both the step and impulse response exhibit similar shape of profiles and ability to detect a target. However, the step response exhibits profiles with a smaller dynamic range, and a detectability window that occurs earlier than the impulse response. As a result, measurements of step response require less time, which makes data collection more efficient, and which may minimize natural noise contamination. At early times, the step response of a good conductor stands out, while its impulse response is buried in noise. Therefore, the step response may have some practical advantages over the impulse response, especially for interpretation of a complex model. The late-time decay of 2-D response is closer to a power law than to an exponential. Although it has some limitations, the 2-D modeling is still useful for interpretation.

INTRODUCTION

In the time-domain electromagnetic (TEM) method the basic response can be measured as either the impulse or the step response of the ground. The response of a time-domain system with the source a constant current that is turned off abruptly (a step-function current), would be called an impulse response if it were measured with a coil, or a step response if it were measured with a magnetometer. They are related, one being the time derivative of the other. Theoretically, we could compute one from the other by differentiation or by integration. However, in some cases, it is difficult to compute accurately a step response by simply integrating an impulse response, because the impulse...
response data available are necessarily finite. A direct measurement of the step response, therefore, is preferable; in principle it can be achieved either by use of a magnetometer as receiver or by use of a triangular waveform for the source. Similarly, in numerical modeling, the step response corresponds to computing the magnetic field or the time derivative of magnetic field with an appropriate waveform as the source. Available time domain systems measure an approximation to the impulse response, except for the UTEM system, which measures an approximation to the step response.

Raiche (1983) observed that apparent resistivity for the step response is well behaved and single valued, and detects layering earlier than does the impulse response, for the sounding problem. This observation was confirmed later by Macnae and Lamontagne (1984). Spies and Eggers (1986) obtained similar results, and they argued the superiority of the step over the impulse response.

According to West et al. (1984), regardless of the signal-to-noise (S/N) ratio, the step response is more desirable for interpretational purposes, because it simplifies interpretation of data and improves the detection of good conductors in the presence of poorer ones. Sarma et al. (1976) made an analysis for the INPUT and NGRI systems (airborne) which use a repetitive half-sinusoidal waveform as a source. They considered the advantage or disadvantage of measuring either the magnetic field or its time derivative as the response, and also evaluated the S/N ratio for both measurements. General S/N analysis for frequency and time-domain systems, including the step and the impulse response measurements are given in McCracken et al. (1986a,b). More recently, Eaton and Hohmann (1987) compared signal/geologic noise ratios for step and impulse responses of a three-dimensional body in a half-space. The conclusion of an evaluation for the S/N ratio in Eaton and Hohmann (1987) slightly favors the impulse over the step response.

As a follow up of our previous work on two-dimensional modeling (Adhidjaja et al., 1985), we study step responses of conductors in a half-space with or without overburden, and in a complex geological model. We modified our previous 2-D finite difference (the Dufort-Frankel method) code to compute the step response directly. Here we describe the modification of the formulation and also present results of our study, emphasizing comparisons between step and impulse responses.

THEORETICAL FORMULATION

Following our previous formulation for a secondary field solution (see Adhidjaja et al., 1985), we start with Maxwell's equations,

\[ \nabla \times e(r,t) = -\mu_0 \frac{\delta h(r,t)}{\delta t} \]

(1)

\[ \nabla \times h(r,t) = \sigma e(r,t) + j^p \]

(2)

with an impressed electric current \( j^p \). For geophysical purposes \( \mu = \mu_0 \), and displacement current can be neglected. In the 2-D case under consideration, \( j^p \) is oriented in the y-direction, so that we can take advantage of the continuity of tangential electric component, and cast the solution in terms of \( e_y \).

The differential equation for the electric field is:

\[ \nabla^2 e_y - \sigma \mu_0 \frac{\delta e_y}{\delta t} = \mu_0 \frac{\delta j^p}{\delta t} \]

(3)

Next we write the fields as:

\[ e_y = e^p_y + e^s_y \]

and:

\[ h = h^p + h^s \]

where superscript \( p \) denotes primary and \( s \) denotes secondary field, and where the magnetic field has \( x \) and \( z \) components. Then the primary field, defined as the field in a half-space if it were homogeneous, satisfies:

\[ \nabla^2 e^p_y - \sigma^p \mu_0 \frac{\delta e^p_y}{\delta t} = \mu_0 \frac{\delta j^p}{\delta t} \]

(4)

where \( \sigma^p \) is the conductivity of the half-space. The secondary field satisfies:

\[ \nabla^2 e^s_y - \sigma \mu_0 \frac{\delta e^s_y}{\delta t} = \mu_0 (\sigma - \sigma^p) \frac{\delta e^p_y}{\delta t} \]

(5)

In the impulse response formulation we use a step current for the primary source \( (j^p) \); therefore \( \delta e^p_y/\delta t \) in eq. 5 is equivalent to electric field due to an impulse current. In the step response formulation we integrate eq. 5 with respect to time, so that the right-hand-side is the electric field due to a step current, and the resulting time derivative of the magnetic field computed from \( e^s_y \) is equivalent to the step response. By doing this, we preserve the original algorithm, and compute the step response via \( \nabla \times e_y = -\mu_0 \delta h^s/\delta t \). Thus our theoretical procedure is similar to that employed by the UTEM system.

The analytic expression for the electric field of a line source after a constant current, \( I \), is terminated at \( t=0 \) is:

\[
e_y(x,z,t) = \frac{I}{\pi \sigma} \left\{ \frac{\left( z^2 - x^2 \right)}{R^2} + 2\theta^2 z^2 \right\} \exp\left(-\theta^2 R^2\right) \frac{2\theta z \exp\left(-\theta^2 z^2\right)}{\sqrt{\pi R^2}} \bigg[ \theta R^2 \exp\left(\theta^2 + \frac{1}{R^2}\right) \bigg] + \frac{z^2 - x^2}{R^2} \operatorname{erfc}(\theta z) \right\} \]

(6)

where \( \theta = (\sigma^p \mu_0 / 4t)^{1/2} \), \( F(\theta x) \) is Dawson's integral, \( x \) is the horizontal dis-
distance from the line source, \( z \) is the depth in the earth, and \( R^2 = x^2 + z^2 \) (Oristaglio, 1982). This electric field is used as the source in eq. 5.

To compute the total magnetic field on the earth's surface, we add to the secondary field a primary field which is computed as the magnetic field due to the step current for a homogeneous earth. Expressions for horizontal and vertical components of the magnetic field for the line source over a half-space are:

\[
\begin{align*}
    h_x(x,t) &= \frac{I}{\pi \beta x} \left[ F(\beta x) \frac{1}{\beta^2 x^2} - \frac{1}{\beta x} \right] \\
    h_z(x,t) &= -\frac{I}{2\pi x} \left[ 1 + \exp \left( -\beta^2 x^2 \right) - 1 \right]
\end{align*}
\]

(7) (8)

We obtained these expressions by integrating with respect to time the corresponding expressions in Oristaglio and Hohmann (1984).

The Dufort-Frankel finite difference scheme is an unconditionally stable and explicit method which is suitable for our computational purposes. For a typical model, the time step used in the computation ranges from 0.1 \( \mu \)s to 40 \( \mu \)s, and the grid size is 80 \times 40 nodes, consisting of a fine and uniform grid within the area of interest, and a coarser and nonuniform grid toward the boundaries. The computation takes about 20 min of CPU time on a VAX 11/785 computer. More detailed discussions regarding the algorithm are given by Oristaglio and Hohmann (1984), and by Adhidjaja et al. (1985). Since we preserve most of the original algorithm, the only modification to the original computer program (TEM2D) is to use subroutine ELINE in place of subroutine EFIELD - the source in eq. 5.

NUMERICAL CHECK

As a check, we compare the step response of a model shown in Fig. 1 computed using our new algorithm with that computed using an algorithm given by Eaton and Hohmann (1987) for computing the step response from the impulse response. Their algorithm is similar to that of Levy (1984). They use the relation:

\[
p(t) = s(t) - s(t + t_r)
\]

(9)

where \( p(t) \) is the pulse response (the source is a constant current turned off with a pulse of length \( t_r \)) which is computed from an impulse response, and \( s(t) \) is the step response at time \( t \). Knowing \( p(t) \) at discrete times and using an initial guess for \( s(t_N) \), they extrapolate backward in time recursively by estimating \( s(t_{N-1}), s(t_{N-2}), \ldots \) and so forth. Their algorithm is already well tested against a known solution (P.A. Eaton, pers. commun., 1987), but it requires a large number of samples and a-priori knowledge of the late-time decay rate of the impulse response. As seen in Fig. 1, which is for a vertical conductor excited by two line sources, the agreement between the two is excellent. Other checks shown previously in Adhidjaja et al. (1985) validate the numerical solution.

MODEL STUDY

We carried out a model study similar to that of the impulse response in our previous paper (Adhidjaja et al., 1985). The particular models were selected for the purpose of comparing the step and impulse responses. All step responses are presented as profiles of magnetic field, while impulse responses are given as profiles of time derivative of magnetic field; the responses one would get from a system that uses a magnetometer or a coil for receiver and a step current for the source. The 2-D model with two line sources is used to simulate a field survey that employs a large fixed loop system where the target has a very long dimension in its strike direction, such as a dike or a fault plane, and measurements are taken along a line perpendicular to the center of the loop side.

Conductor in homogeneous host

Fig. 2 shows vertical magnetic field profiles for the model shown in the inset, plotted at seven delay times; the conductor is 20 m thick, and is located 300 m from the near line source. The solid lines denote positive values, dashed lines negative values. For this simple model the profiles show a typical cross-over
that at early times is located just beyond the body, but at later times migrates back toward the body. Except for the decay rates and their dynamic range the shapes of profiles are very similar to their impulse response counterparts (Fig. 3). Within the times plotted a subtle difference between the two is not discernible in the profiles, but becomes clear on vector plots shown later. The host delays the impulse response relative to the step response.

Fig. 4 shows profiles for the same model except the resistivity of the body is increased to $1\, \Omega \cdot \text{m}$. The profiles basically show the same features, but at about 9 ms the response of the body starts diminishing, indicated by the cross-over that migrates away from the body and then disappears. It is interesting to note that in the impulse response profiles (Fig. 5) the cross-over associated with the body starts diminishing at a later time (25 ms). Because the resistivity contrast is lower here, the host effect is more pronounced than in the previous model.

Fig. 6 shows profiles for the same model as that of Fig. 4, but the resistivity of the host has been reduced to $30\, \Omega \cdot \text{m}$. The profiles clearly fail to indicate the presence of this low-contrast body. For this model both the step and the impulse response (Fig. 7) fail to detect the body, because of a strong effect of the conductive host. To interpret the geometry of the conductor we need more information such as the horizontal component, or we need to strip off the host response.

From these results we see that basically the step and the impulse response are influenced by a conductive host in a similar manner. The host attenuates and delays the peak responses. Both the step and impulse response show similar profile shapes and ability to detect a target; the target is detected when the conductivity contrast is high, but not when the conductivity contrast is low. However, the step response exhibits a smaller dynamic range and simpler profile shapes.

Fig. 8 is a plot of decay curves for the secondary fields of bodies with different resistivities embedded in a conductive host. The plots clearly show that the decay rate of the step response is slower than its impulse response counterpart (Fig. 9), and the detectability window (a time interval in which the conductor response exceeds the half-space response) for the less conductive body occurs earlier in time but with a duration comparable to that of impulse response, on the log–log plot. The figures also show an important characteristic of the step response: the step response of a more conductive body always is larger than that of a less conductive body. In contrast, the impulse response (Fig. 9) of a more resistive body is larger at early times. For the step response, the early-time limit is identical to the in-phase frequency-domain inductive limit, and for a simple conductor in free space this is a function of geometry alone, while for the impulse response it is the same as the quadrature inductive limit which is scaled by the inverse of the time constant. Thus, at early times a poor conductor, which is characterized by a small time constant, has a larger impulse
Fig. 4. Vertical magnetic-field (step response) profiles for a moderate conductor (20 S).

Fig. 5. Vertical magnetic-field (impulse response) profiles, for the model in Fig. 4.

Fig. 6. Vertical magnetic-field (step response) profiles for a moderate conductor (20 S) in a very conductive half-space.

Fig. 7. Vertical magnetic-field (impulse response) profiles, for the model in Fig. 6.
response than a good conductor. Obviously, in an environment with complex geology at early times the responses of poor conductors obscure the response of a target (a good conductor), and make interpretation of impulse response more complicated (West et al., 1984; Dyck and West, 1984).

Fig. 10 shows horizontal-component profiles of the step response for a model with two bodies. The first body with a higher resistivity (poor conductor) is located at 200 m from the nearest line source, and the second one (good conductor) at 400 m. At 1 ms the response of the poor conductor is larger because it is closer to the transmitter, but by about 3 ms the anomaly of the good conductor is becoming clearly defined, while the anomaly of the poor one is diminishing. On the other hand, the impulse response profiles for the same model in Fig. 11 show a larger impulse response from the poor conductor at early times, while the response of the good conductor is not clear until at least 5 ms. Continuous normalization with the primary field (the half-space field), to remove the effect of the distance from the transmitter, does not alter much the relative sizes of the anomalies.

Comparison with 3-D model and late-time decay

Computing 2-D models is much cheaper and simpler than computing 3-D models. However, 2-D models are only approximations to more realistic 3-D models. Fig. 12 shows a comparison between profiles over 2-D and 3-D models. The 3-D solution was computed with the integral equation method of Newman and Hohmann (1988). The model is a vertical body, 25 m thick, 100 m in depth extent, with resistivity of 0.1 Ω m, embedded in a 1000 Ω m half-space. The corresponding 3-D model has the same cross-section but has a finite strike length of 800 m, and the source is a 500 m X 500 m loop.

As seen in Fig. 12 the amplitudes of the 2-D responses are much larger than those of the 3-D responses, because of the finite length of the 3-D body and the smaller source. In addition, the decay rates are markedly different; the 2-D responses decay much slower than the 3-D responses. Both the primary and secondary fields of the 2-D model decay slower than those of the 3-D case.

However, both the 2-D and 3-D profiles show the presence of the body indicated by a cross-over directly above it. The cross-over of the 2-D profiles persists within the whole time range (1–50 ms) plotted, while those of the 3-D profiles occur only within a time window (3–10 ms). For this high-contrast model the 2-D profiles show a dominant response of the body, while the 3-D profiles show more complex interaction between responses of the body and the host, because of a weaker response of the body.

In geophysical prospecting, the EM response for a 3-D model consists of vortex and galvanic (current channeling) effects which are associated with divergence-free and curl-free current sources (Lajoie and West, 1976). Equivalently, the sources can be viewed as due to magnetic and electric dipoles.
Fig. 10. Horizontal magnetic-field (step response) profiles for the model with a good conductor (66.7 S) and a poor conductor (20 S).

Fig. 11. Horizontal magnetic-field (impulse response) profiles for the model in Fig. 10.

Fig. 12. Comparison of step-response profiles for 2-D and 3-D models.

Fig. 13. Comparison of decay curves (secondary fields) for the 2-D and 3-D models showing the effects of varying the host resistivity ($\rho_h$), plotted for step responses at 170 m from the near line source. The resistivity of the body is 0.1 $\Omega$ m.
we would expect the normals at late times to converge to the center of the field vectors measured as profiles on the earth's surface. For the step response, magnetic field to locate the conductor by plotting the normals of the magnetic field vectors concentrated in a conductor at late times. Therefore, we may use the interpretation insight.

The physical properties of the conductor, but the response is still sensitive to conductivity changes, and the profiles, in some cases, resemble those of a corresponding 3-D model. Therefore, the 2-D models are useful for gaining interpretation insight.

Vector plots

For a 2-D model the currents in the earth are line currents, with the maximum concentrated in a conductor at late times. Therefore, we may use the magnetic field to locate the conductor by plotting the normals of the magnetic field vectors measured as profiles on the earth's surface. For the step response, we would expect the normals at late times to converge to the center of the conductor. However, as seen in Fig. 14, the normals at late times converge to the far edge of the conductor, due to the effect of the host. For the impulse response, where the normals are to the vectors of the magnetic field time derivatives, the vector plots (Fig. 15) are similar to those of the step response, converging to the far edge of the conductor. At early times, the normals are diffused because of the host effect; the normals of the step response start converging earlier than those of the impulse response. Nevertheless, at late times, the normals for both the step and impulse responses converge to the edge of the conductor.

Electric field in the earth due to a step or an impulse current

Snapshots of the electric field are very useful for studying the evolution of current distribution in the earth, which is very helpful for understanding the physics of the TEM process. Hence, they are useful in the interpretation of the TEM responses. Here, we study and compare the electric field due to a step or an impulse current in line sources over a homogeneous half-space.

Fig. 16 shows contour plots for an electric field, at four delay times, in a half-space with resistivity $300 \Omega \text{m}$, after a constant current in the transmitter was shut off. Similarly, Fig. 17 is a contour plot for an electric field in the half-space after an impulse current in the transmitter was shut off. In both figures the transmitter consists of two-line sources, and only half of the field is shown, because it is symmetric. Dashed lines denote negative values (electric field goes into the paper), and solid lines positive values.

If the magnetic field is measured as the response, these electric fields would be the primary electric fields for systems that measure the step and the impulse response, respectively. Figs. 16 and 17 explain why the detectability window of the step response comes earlier than that of the impulse response. The step-response electric field shows one maximum, which can be viewed as an equivalent current filament that flows into the page (negative sign, in the same direction as the current was in the transmitter), while the impulse-response electric field has positive and negative maxima, and the most intense maximum, in this case the negative one, lags that of the step response. Consequently it will be delayed in reaching and exciting a target. Notice that the impulse-response electric field is more intense, but also decays more rapidly than that of the step response. The contour intervals cover a dynamic range of $10^6$ for the impulse response compared to $10^4$ for the step response.

Overburden effect

In the following models we study the effect of overburden, which can be a serious problem for EM. The models are for overburden with a uniform resistivity, but the thickness may vary.
Fig. 14. Vector plot of magnetic-field normals for the step response. A circle with cross is one of the lines source in which the current flows into the page (negative sign).

Fig. 16. Contour plots of an electric field in the earth after a step current in two-line sources was shut off.

Fig. 17. Contour plots of an electric field in the earth after an impulse current in two-line sources was shut off.
Fig. 18. Vertical magnetic-field (step response) profiles for the overburden model. Thickness of the conductor is 20 m.

Fig. 19. Vertical magnetic-field (impulse response) profiles for the overburden model.

Fig. 20. Comparison of decay curves (total fields) of the responses, at 200 m from the nearest line source, for the overburden and the homogeneous host model.

Fig. 21. Horizontal magnetic-field (step response) profiles for the complex model.

Fig. 18 shows profiles for an overburden model with uniform thickness shown in the inset. These profiles resemble those of the homogeneous-host model (Fig. 2); they show a cross-over associated with the conductor. Here the effect
of the overburden is not very visible, it nearly disappears by 1 ms, but the response of the body seems to weaken by 35 ms. In contrast, the impulse response profiles (Fig. 19) show more complicated overburden and host effects. They show early-time cross-overs due to concentration of current in the overburden. The inducing field at early times tends to concentrate in the more conductive overburden before diffusing into the half-space and reaching the conductor. These effects are best observed in contour plots of an electric field in the earth (see Adhidjaja et al., 1985). However, the combined effects of conductive overburden and host on both the step and the impulse response are similar; they attenuate and delay the response of the body underneath.

Fig. 20 compares decay curves (total field) of the step responses for overburden and homogenous host models, where the resistivity of the overburden is 30 $\Omega$ m and that of the half-space is 300 $\Omega$ m. In this plot the overburden effect is more visible than in the profiles of Fig. 18; it attenuates and delays the peak responses with respect to that of the homogenous model.

Our last model is for multiple bodies beneath an overburden of variable thickness to simulate a search for a target in a typical environment, where there is differential weathering, and also geologic noise from poor conductors in bedrock. Profiles for the step response are shown in Fig. 21 (horizontal component) and Fig. 22 (vertical component); similar plots for the impulse
response are shown in Fig. 23 and Fig. 24. Both the step and the impulse response detect the good conductor at 400 m from the near line source. However, it is obvious that the step-response profiles are simpler and show the anomaly of the good conductor earlier than the impulse response. For interpretation purposes, the horizontal component is simpler and is less affected by primary fields for both the step and the impulse response, but peak responses associated with the target are usually rather broad, especially for a deep target. Similar to its impulse response, the horizontal component of the step-response primary field decays faster than the vertical component \( t^{-1.5} \) vs. \( t^{-1} \). Of course, a combination of both horizontal and vertical components will give the best result for interpretation.

CONCLUSIONS

From the model study we observe that the step and the impulse responses basically are similar, and both are influenced by the conductive host with or without an overburden in a similar manner. Both detect the target when the conductivity is high, but not when the conductivity is low. However, the step response has a smaller dynamic range, and shows an anomaly of a good conductor in an earlier time window.

The 2-D model is useful for gaining insight into interpretation, and in some cases it can be used, at least, for first-order approximation to a more realistic 3-D model. In addition, because it is cheaper to compute it can be used in routine modeling either for data interpretations or for survey design.

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