ELSEVIER

Journal of Applied Geophysics 37 (1997) 133–271

Journal of Applied Geophysics

Methods for modelling electromagnetic fields Results from COMMEMI—the international project on the comparison of modelling methods for electromagnetic induction

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Received 3 May 1995; accepted 25 August 1995

Abstract

This special issue is concerned with the present state of the art in methods of numerical modelling of geo-electromagnetic fields in inhomogeneous media. A theoretical overview is followed by specific applications of the various modelling methods and computer programs (developed throughout the world) to the geo-electric test models of the international project on the Comparison Of Modelling Methods for ElectroMagnetic Induction problems (COMMEMI). Numerous tables and diagrams provide a comparison of the results obtained by these different approaches. This material is intended for geophysicists dealing with the modelling and interpretation of geo-electromagnetic fields, for scientists involved in the testing of related software, for specialists in the field of computational geophysics, and for graduate and senior undergraduate students studying this branch of geophysics. © 1997 Elsevier Science B.V.

Keywords: numerical modelling; electromagnetic fields; geoelectromagnetic induction; geoelectrics; conductivity; finite differences; finite elements

1. Introduction

The development of methods for numerical modelling in geo-electrics has been carried out over several decades, but only during the last 10–15 years have numerical algorithms been developed for the solution of complex two-dimensional models. More recently, sufficiently reliable three-dimensional algorithms have been developed that the investigation of non-trivial, three-dimensional geoelectric structures is now perfectly feasible. There are dozens of algorithms and programs for the numerical modelling of electromagnetic fields in inhomogeneous media now available, so there is a need to undertake a comparative analysis of their accuracy, their computational efficiency and their

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0926-9851/97/$17.00
PII S0926-9851(97)00013-X

universality of application. Such an analysis is timely not only as a qualitative classification of existing modelling techniques but also for deciding what priorities exist in the future development of modelling methods.

Numerical modelling of electromagnetic fields has become an efficient tool in various theoretical investigations in geophysics, and two-dimensional modelling in particular is applied by hundreds of investigators in many organizations throughout the world. The consequences of choosing an inadequate or unsuitable method, or of applying a given program improperly, may therefore be quite substantial.

The original version of this special issue was published as a monograph in Russian, which was produced as part of an international project entitled "Comparison of Modelling Methods for Electromagnetic Induction" with the acronym COMMEMI. The project was initiated in 1983 by Working Group I-3 of the International Association of Geomagnetism and Aeronomy (IAGA) and was undertaken with wide international cooperation. Coordination of the project was provided by research workers at the Department of Deep Electromagnetic Studies at the Institute of the Earth Magnetism, Ionosphere and Radio Wave Propagation of the Academy of Sciences of the USSR (IZMIRAN) [1]. The monograph provided an opportunity to summarise the results of the project and to present in visual form the potential of modern methods for the numerical modelling of geophysical electromagnetic fields in horizontally inhomogeneous media.

Limitations of space do not permit the inclusion of a detailed theoretical survey of all existing approaches in the field. We believe that it is more important to confine ourselves to a brief description of the types of algorithms which were most often used in the comparative calculations, but to present in detail all the submitted results for a series of two- and three-dimensional test models, along with the conclusions of a statistical analysis of these results. This issue is therefore a practical guide on the application of numerical modelling methods in geomagnetic induction and related geophysical disciplines, and it contains numerical results which can serve as a standard against which new modelling methods can be tested.

2. Survey of methods for modelling inhomogeneous media in electromagnetism

2.1. The problem of numerical modelling in geoelectrics

In the method of numerical modelling a physical phenomenon under investigation is represented by a mathematical system which can be solved numerically. The results obtained from the mathematical investigation of such a model are then interpreted in terms of the original phenomenon and serve to develop an understanding of the physical processes involved. The most important task in geophysical modelling is the prediction of the geophysical field for a specified distribution of the physical parameters characterizing the region of the Earth in which the field exists. Geoelectric modelling involves calculating the total electromagnetic field in a model defined by a postulated distribution of electric and magnetic parameters in the medium under study, together with the exciting field. The calculated fields given by the particular technique under investigation can then be compared with the measured response.

A fundamental mathematical model in electrodynamics is represented by Maxwell's equations, which prescribe the analytical relationship in the form of a system of first order vector equations between the components of the magnetic and electric fields, and the parameters of the medium

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(electric conductivity, permittivity and magnetic permeability). A detailed analytic investigation of such models for the most general case is given in the monographs by Svetov (1984), Berdichevsky and Zhdanov (1984), Zhdanov (1986), Weaver (1994) and Zhdanov and Keller (1994). In the present work we shall be interested only in the following simplified form of Maxwell's equations:

$$\begin{aligned}
&\operatorname{curl}\mathbf{H} = \sigma\mathbf{E} + \mathbf{j}^s \\
&\operatorname{curl}\mathbf{E} = i\omega\mu\mathbf{H} \\
&\operatorname{div}\mathbf{H} = 0 \\
&\operatorname{div}\mathbf{E} = -(\mathbf{E}\cdot\operatorname{grad}\sigma + i\omega q^s)/\sigma
\end{aligned} \tag{1}$$

where $\mathbf{E}$ and $\mathbf{H}$ are complex vectors denoting the electric and magnetic fields, respectively, $\mathbf{j}^s$ is the density of extrinsic currents, and q^s is the density of extrinsic charges, and a time dependence $\exp(-i\omega t)$ is understood. Since it is these approximate equations that are generally used in problems of electromagnetic induction in the Earth, they are the equations on which COMMEMI was based.

Let us assume that the region of modelling consists of an upper half-space (atmosphere), and a lower conducting half-space (Earth) characterized by an inhomogeneous distribution of conductivity. The system of equations (Eq. (1)) in the conducting medium may be modified so that the first two equations involve only the electric and magnetic fields, respectively:

$$\begin{aligned}
&\operatorname{curl}\operatorname{curl}\mathbf{E} - i\omega\mu\sigma\mathbf{E} &&= i\omega\mathbf{j}^s \\
&\sigma\operatorname{curl}[(\operatorname{curl}\mathbf{H})/\sigma] - i\omega\mu\sigma\mathbf{H} &&= \sigma\operatorname{curl}(\mathbf{j}^s/\sigma).
\end{aligned} \tag{2}$$

Introducing the Laplacian operator in Eq. (2) with the aid of the identity:

$$\operatorname{curl}\operatorname{curl} \equiv \operatorname{grad}\operatorname{div} - \nabla^2$$

we obtain:

$$\begin{aligned}
&\nabla^2\mathbf{E} + \operatorname{grad}[(\mathbf{E}\cdot\operatorname{grad}\sigma + i\omega q^s)/\sigma] + i\omega\mu\sigma\mathbf{E} &&= -i\omega\mu\mathbf{j}^s \\
&\nabla^2\mathbf{H} + [(\operatorname{grad}\sigma)/\sigma]\times\operatorname{curl}\mathbf{H} + i\omega\mu\sigma\mathbf{H} &&= -\sigma\operatorname{curl}(\mathbf{j}^s/\sigma).
\end{aligned} \tag{3}$$

These two equations form the basis of the majority of algorithms for modelling magnetic fields in inhomogeneous media. Depending upon the particular problem under investigation, only one of Eq. (3) is solved and is supplemented by Eq. (1) as required.

In the nonconducting medium only the first equation in system (2) is of interest. Since no free charges exist in the medium, it reduces to:

$$\nabla^2\mathbf{E} = -i\omega\mu\mathbf{j}^s. \tag{4}$$

Finally we consider the equations for a stationary field which are obtained by passing to the limit $\omega \to 0$. The equation for the stationary magnetic field is obtained from the second Eq. (2):

$$\operatorname{curl}[(\operatorname{curl}(\mathbf{H})/\sigma] = \operatorname{curl}(\mathbf{j}^s/\sigma). \tag{5}$$

To simulate the electric field in regions of piecewise homogeneous media of uniform conductivity we may use the first equation from Eq. (2), which in this case reduces to Helmholtz's equation.

Let us now be more specific in our description of the analytic electrodynamic model expressed by system (1) by discussing the structure of the non-uniform conductivity distribution in the region being modelled, and the form of the primary electromagnetic field exciting it. The distribution of electrical conductivity is written as a superposition of normal and anomalous parts:

$$\sigma(\mathbf{r}) = \sigma^{\mathrm{n}}(\mathbf{r}) + \sigma^{\mathrm{a}}(\mathbf{r}). \tag{6}$$

The normal part is understood to be a (simpler) distribution of electric conductivity for which the solution of the modelling problem is known. Usually the spatial dimensionality of the normal conductivity is lower than that for the total conductivity. A classical example of a one-dimensional normal distribution is one that is divided into uniform horizontal layers. There may also be more complicated situations; for example, a two-dimensional distribution may play the role of the normal conductivity in a three-dimensional problem. Usually the normal distribution is chosen so that the anomalous part is described by a bounded function which differs from zero only within the limits of a finite region $\mathscr{V}^{\mathrm{a}}$.

In the majority of modelling methods dealing with electromagnetic fields in inhomogeneous media a limited set of rather simple primary source fields is employed (elementary electric and magnetic dipoles, finite distributions of dipoles, etc.). The simplest source is a uniform field or a 'plane wave'. With this idealization, which corresponds to the source being infinitely far from the modelling area, Eqs. (1)–(5) are assumed to be free from the terms involving external current and charge densities in the region under investigation. Associated with the decomposition (Eq. (6)) of the electrical conductivity, there is a separation of the electromagnetic field into normal and anomalous parts. Thus, when the model is excited by a plane wave, the normal field is the plane wave field in a medium with the normal distribution of electric conductivity. If the plane wave propagates perpendicularly into a normal medium comprising horizontally uniform layers, the normal field equations become one-dimensional and may be easily solved in explicit form (Weaver, 1994; Zhdanov and Keller, 1994). Equations for the anomalous fields $\mathbf{E}^{\mathrm{a}}$ and $\mathbf{H}^{\mathrm{a}}$ follow from Eq. (2) in the following form:

$$\begin{aligned} \nabla^2\mathbf{E}^{\mathrm{a}} + \mathrm{grad}[\mathbf{E}^{\mathrm{a}}\cdot(\mathrm{grad}\,\sigma)/\sigma] - i\omega\mu\sigma\mathbf{E}^{\mathrm{a}} &= -i\omega\mu\sigma^{\mathrm{a}}\mathbf{E}^{\mathrm{n}} - \mathrm{grad}[\mathbf{E}^{\mathrm{n}}\cdot(\mathrm{grad}\,\sigma^{\mathrm{a}})/\sigma] \\ \nabla^2\mathbf{H}^{\mathrm{a}} - [(\mathrm{grad}\,\sigma)/\sigma]\times\mathrm{curl}\,\mathbf{H}^{\mathrm{a}} - i\omega\mu\sigma\mathbf{H}^{\mathrm{a}} &= -i\omega\mu\sigma^{\mathrm{a}}\mathbf{H}^{\mathrm{n}} - \sigma\,\mathrm{grad}(\sigma^{\mathrm{a}}/\sigma)\times\mathbf{E}^{\mathrm{n}}. \end{aligned} \tag{7}$$

In the remainder of this section a brief theoretical analysis of various general methods for calculating the electromagnetic field in the model are discussed and some specific algorithms are described. Section 3 includes an analysis of the modelling programs used in the COMMEMI project, and in Sections 4 and 5, where the results of the comparative study are given, the advantages and limitations of these various programs are discussed. The majority of the various approaches to solving problems of numerical modelling fall into two basic classes—the integral equation method and methods based on differential equations governing the behaviour of the field, as described, for example, in the surveys by Varentsov (1983), Zhdanov and Spichak (1984), Hohmann (1983, 1987), Kaikkonen (1986), Chave and Booker (1987), Červ and Pek (1990), Wannamaker (1991) and Xiong (1992).

2.2. Integral equation method

Since the mid-1960's the Fredholm integral equation method has been successfully applied to the problem of modelling of electromagnetic fields in inhomogeneous geoelectric media. The method of volume integral equations, which is widely applicable, has become one of the most effective means for solving two- and three-dimensional problems involving media with local heterogeneities of rather complex structure (Dmitriev, 1969; Weidelt, 1975; Hohmann, 1975, 1983; Dmitriev et al., 1977; Hvozdara, 1981; Ting and Hohmann, 1981; Wannamaker et al., 1984a,b; Dmitriev and Zaharov, 1987; Hvozdara et al., 1987; Wannamaker, 1991; Zhdanov and Spichak, 1992; Xiong, 1992; Xiong and Tripp, 1993a). The method of surface integral equations, which has performed well for the class of homogeneous inclusions of simple form was developed in parallel (Kaufman, 1974; Taborovsky, 1975). The first reliable results from the numerical modelling of electromagnetic fields in geoelectric problems in both two-dimensional (Dmitriev, 1969; Hohmann, 1971) and three-dimensional media (Weidelt, 1975; Hohmann, 1975) were obtained by means of the method of integral equations, and there are now many different computer programs available based on this method. The general theoretical principles of the approach are outlined below.

2.2.1. Volume integral equations

Let us confine ourselves to the analysis of a geoelectric model in which the anomalous conductivity in Eq. (6) is concentrated in a local (and in the general case multiply connected) region $\mathscr{V}^{a}$, while the distribution of normal conductivity is a one-dimensional horizontally layered structure.

The main advantage of the method of volume integral equations is that the most tedious part of the modelling problem is restricted to just the anomalous region $\mathscr{V}^{a}$. This simplification is achieved simply by converting the differential Eq. (2) into an integral equation by volume integration, an application of Green's theorems, and the introduction of elementary or fundamental solutions (Green's operators) (Berdichevsky and Zhdanov, 1981; Berdichevsky and Zhdanov, 1984; Svetov, 1984; Dmitriev and Zaharov, 1987; Zhdanov and Keller, 1994; Weaver, 1994).

The electromagnetic field in the model can be represented by the operator (Weidelt, 1975):

$$\mathbf{U}(\mathbf{r}) = \mathbf{U}^{n}(\mathbf{r}) + \int_{\mathscr{V}^{a}} \mathbf{G}^{u}(\mathbf{r}|\mathbf{r}') \cdot \mathbf{j}(\mathbf{r}')\,\mathrm{d}\mathscr{V}' \tag{8}$$

where $\mathbf{U} = \mathbf{E}$ or $\mathbf{H}$ is the total vector field, $\mathbf{U}^{n}$ is the corresponding normal vector field, $\mathbf{j} = \sigma^{a}\mathbf{E}$ is the density of excess conduction currents, and $\mathbf{G}^{u}$ is the Green tensor (electric $\mathbf{G}^{e}$ or magnetic $\mathbf{G}^{h}$) for the normal structure. The Fredholm vector equation of second order for the electric field in $\mathscr{V}^{a}$ is therefore given by:

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}^{n}(\mathbf{r}) + \int_{\mathscr{V}^{a}} \mathbf{G}^{e}(\mathbf{r}|\mathbf{r}') \cdot \mathbf{j}(\mathbf{r}')\,\mathrm{d}\mathscr{V}'. \tag{9}$$

The primary or source field is completely taken care of by the normal field.

The vector components $\mathbf{G}_{i}^{e}$ and $\mathbf{G}_{i}^{h}$ of the tensor satisfy the first of Eq. (2) and the second of Eq. (1) for the electromagnetic field:

$$\begin{aligned} \nabla^{2}\mathbf{G}_{i}^{e}(\mathbf{r}|\mathbf{r}') + i\omega\mu\sigma^{n}(\mathbf{r})\mathbf{G}_{i}^{e}(\mathbf{r}|\mathbf{r}') \quad &= -i\omega\mu\delta(\mathbf{r}-\mathbf{r}')\hat{\mathbf{x}}_{i} \\ \mathbf{G}_{i}^{h}(\mathbf{r}|\mathbf{r}') \quad &= \operatorname{curl}\mathbf{G}_{i}^{e}(\mathbf{r}|\mathbf{r}')/i\omega\mu \end{aligned}$$

and give the electromagnetic field at the point that originates with a current dipole of unit moment placed at the point $\mathbf{r}'$ and oriented along axis $\hat{\mathbf{x}}_{i}$.

The traditional approach to the numerical solution of integral Eq. (9) involves subdividing region $\mathscr{V}^{a}$ into prismatic cells $\mathscr{V}_{k}$ $(k = 1, 2, \ldots, K)$ and assuming that the functions $\mathbf{E}(\mathbf{r})$ and $\sigma^{a}(\mathbf{r})$ are constant in each cell. In this case the problem defined by Eq. (9) is reduced to a linear system with the following structure:

$$\sum_{k=1}^{K} \left(\sigma^{ak}\mathbf{G}^{e}_{ik} - \delta_{ik}\mathbf{I}\right)\cdot\mathbf{E}_k = -\mathbf{E}^{n}_{k}, \quad i=1,2,\ldots,K \tag{10}$$

where

$$\mathbf{G}^{u}_{ik} = \int_{\mathscr{V}_k} \mathbf{G}^{u}(\mathbf{r}_i|\mathbf{r}')\,\mathrm{d}\mathscr{V}', \quad i=1,2,\ldots,K;\ k=i=1,2,\ldots,K \tag{11}$$

and $\mathbf{E}_k = \mathbf{E}(\mathbf{r}_k)$, $\mathbf{E}^{n}_{k} = \mathbf{E}^{n}(\mathbf{r}_k)$, $\sigma^{ak} = \sigma^{a}(\mathbf{r}_k)$. Here $\mathbf{r}_k$ is the centre of the cell $\mathscr{V}_k$, $\mathbf{I}$ is the unit tensor, and δ_{ik} is the Kronecker delta. The discrete representation of Eq. (8) takes the form:

$$\mathbf{U}_i = \mathbf{U}^{n}_{i} + \sum_{k=1}^{K} \mathbf{G}^{u}_{ik}\cdot\left(\sigma^{ak}\mathbf{E}_k\right) \tag{12}$$

where i is the index of the observation point.

The discrete problem Eqs. (10)–(12) has virtually the same form in most of the algorithms based on the volume integral equation method. What differences there are affect principally the accuracy of solution and the computational effectiveness. They usually arise in the way the Green's tensor is represented and in the different methods of integrating Eq. (11) (Weidelt, 1975; Ting and Hohmann, 1981; Hvozdara, 1981; Das and Verma, 1981a,b, 1982; Wannamaker et al., 1984b; Hvozdara et al., 1987; Wannamaker, 1991; Xiong, 1992; Xiong and Tripp, 1993a). Most of the computational resources are required for solving the linear system Eq. (10) even for models of average complexity. Altogether there are $3K$ scalar equations, a number which can typically go into the many hundreds, although the size of the system can sometimes be reduced by taking into consideration the symmetries, if any, in the problem (Hvozdara, 1981; Hohmann, 1983; Tripp and Hohmann, 1984; Xiong and Tripp, 1993b). Direct and iterative methods are equally applicable for solving such problems. For systems of just a few hundred equations the method of direct elimination is preferable (Hohmann, 1983; Hvozdara et al., 1987). When solving larger systems on computers of average capacity it is necessary to switch to iterative techniques such as the block method of over relaxation (Hvozdara and Varentsov, 1988) or the Gauss–Seidel method (Hvozdara, 1981) and others (Xiong, 1992).

When transferring to a two-dimensional model in which the field and model parameters are independent of the spatial variable y the method of integral equations is separated into two independent sub-problems, corresponding to the cases of E- and H-polarization of the field. In E-polarization, the scalar equation for the electric field component E_y follows from the general Eq. (9) (Dmitriev et al., 1977; Dmitriev and Mershikova, 1979; Berdichevsky and Zhdanov, 1981, 1984; Zhdanov and Keller, 1994); in H-polarization a system of two equations for components E_x and E_z must be solved. Direct methods are usually used to solve numerical problems in two-dimensions.

2.2.2. Surface integral equations

This approach offers the possibility of reducing still further the region over which the basic modelling problem is solved by reducing it, in the case of a single homogeneous inclusion, to the surface $\partial\mathscr{V}^{a}$ of the anomaly or, when the anomalous region $\mathscr{V}^{a}$ is divided into homogeneous elements, to the boundary surfaces where the elements adjoin each other. A system of Fredholm equations is defined on this surface (Taborovsky, 1975; Dmitriev and Zaharov, 1987). We shall not go into further details of this method here since no algorithms of this class were used in the project. It should be emphasized, however, that the usefulness of this approach, even for models with inhomogeneities of only average complexity, is very limited because the number of elements of discretization on the boundary surfaces becomes great compared with that required in the volume integral method. Moreover, problems of numerical stability arise because the system of Fredholm equations includes equations of first order (Dmitriev and Zaharov, 1987). The situation for stationary fields is somewhat different; for to solve the three-dimensional problem of an anomalous inclusion of uniform conductivity in an otherwise homogeneous medium, it is sufficient to employ just one scalar Fredholm equation of second order on the inclusion surface. Solutions of stationary problems obtained by the method of surface integral equations were used in the three-dimensional part of the project for comparison with the low frequency results obtained by the more general methods. Therefore let us briefly discuss one such solution, due to Hvozdara (1982, 1983, 1985).

The electric field $\mathbf{E}$ in each inhomogeneous region is expressed in terms of the potential U, defined by:

$$\mathbf{E}(\mathbf{r}) = \operatorname{grad} U(\mathbf{r}).$$

Outside the inhomogeneity the potential takes the form:

$$U(\mathbf{r}) = U^{\mathrm{n}}(\mathbf{r}) + \frac{1}{4\pi}\int_{\partial\mathscr{V}^{\mathrm{a}}} f(\mathbf{r}')\frac{\partial G(\mathbf{r}|\mathbf{r}')}{\partial n'}\,\mathrm{d}\mathscr{S}' \tag{13}$$

where f is the density of the dipole (double) layer sources distributed on the inhomogeneity surface $\mathscr{S}$ and oriented in the direction of the outward normal, and where the normal derivative of the Green's function G specifies the potential of an elementary dipole placed at the point $\mathbf{r}'$. The prime on the element of area $\mathrm{d}\mathscr{S}'$ indicates surface integration over the primed coordinates.

It follows that on the surface of the inhomogeneity (Hvozdara, 1982):

$$f(\mathbf{r}) = 2\beta\left[U^{\mathrm{n}}(\mathbf{r}) - q\right] + \frac{\beta}{2\pi}\int_{\partial\mathscr{V}^{\mathrm{a}}} f(\mathbf{r}')\frac{\partial G(\mathbf{r}|\mathbf{r}')}{\partial n'}\,\mathrm{d}\mathscr{S}', \tag{14}$$

$$q = \frac{1}{\mathscr{S}}\int_{\partial\mathscr{V}^{\mathrm{a}}} U^{\mathrm{n}}(\mathbf{r})\,\mathrm{d}\mathscr{S}, \qquad \beta = -\sigma^{\mathrm{a}}/(2\sigma^{\mathrm{n}} + \sigma^{\mathrm{a}}).$$

When the inhomogeneity is embedded in a homogeneous half-space, the Green's function takes the simple analytic form:

$$G(\mathbf{r}|\mathbf{r}') = \frac{1}{|\mathbf{r}-\mathbf{r}'|} + \frac{1}{|\mathbf{r}-\mathbf{r}^{+}|}, \quad \mathbf{r}' = (x', y', z'), \quad \mathbf{r}^{+} = (x', y', -z').$$

With a plane wave source the normal potential is chosen to be proportional to the horizontal coordinate that corresponds to the polarization of the field, thereby ensuring the uniformity of the normal electric field.

Eq. (13) is discretized in the same manner as Eq. (9). The resulting linear system may be solved either by direct or by iterative methods. It is significant that the number of real equations per element of discretization is here six times less than with Eq. (9).

The value of the potential U on the Earth's surface is obtained by substituting the f given by the discrete solution of Eq. (13) back into Eq. (12). The corresponding electric fields are then determined either by a direct difference calculation of the potential gradient, or according to Eq. (13) with the gradient taken under the integral sign.

The anomalous magnetic field, corresponding to the anomalous current density $\mathbf{j}^{a} = (\sigma^{n} + \sigma^{a})$ in the inhomogeneity, satisfies the equations:

$$\operatorname{curl} \mathbf{H}^{a} = -\sigma \operatorname{grad} U^{a}, \qquad \operatorname{div} \mathbf{H}^{a} = 0$$

and are also represented in terms of surface integral operators (Hvozdara, 1983).

2.3. Differential equation methods

Methods of modelling electromagnetic fields based on the boundary-value problems that follow from Maxwell's differential equations, have been intensively developed over the last 15 years. Most of the two-dimensional problems of geoelectrics, as well as a number of important ones in higher dimensions (quasi three-dimensional and three-dimensional problems) have been solved by such methods.

Finite differences and finite elements are most often used for the numerical solution of differential equations. It is difficult to distinguish between these two approaches, especially with rectangular meshes (elements) when the resulting discrete systems of equations are essentially the same. A number of combined approaches have also been developed, for example the variation difference scheme (Vanyan et al., 1984). Therefore, we first discuss the general problem of setting up the boundary-value problems which are relevant to both finite differences and finite elements. In Section 2.3.2 we examine in more detail the finite difference approach and in Section 2.3.3 we briefly consider the essential steps in the finite element method.

2.3.1. Representation of modelling problems as boundary-value problems

In the methods of finite differences or finite elements it is first necessary to write down the complete and consistent system of electromagnetic field equations in the limited region of modelling $\mathscr{V}$ which has an anomalous electrical conductivity σ^{a}. Within the region of modelling it is usual to specify the second order partial differential equation for either the (total or anomalous) electric or magnetic field $\mathbf{U}$ as:

$$L[\mathbf{U}(\mathbf{R})] = \mathbf{R}(\mathbf{r}), \quad \mathbf{r} \in \mathscr{V}. \tag{15}$$

Eq. (15) is supplemented with a boundary-value condition, i.e. with an additional equation for the field U on the boundary $\partial\mathscr{V}$ of the region $\mathscr{V}$:

$$L_{b}[\mathbf{U}(\mathbf{r})] = \mathbf{R}_{b}(\mathbf{r}), \quad \mathbf{r} \in \partial\mathscr{V}. \tag{16}$$

The system of equations defined by Eqs. (15) and (16) forms a boundary-value problem.

In three dimensions the equations for the vector boundary-value problem satisfied by the electric field are usually treated with the first equation in Eq. (3) or Eq. (7) playing the role of Eq. (15). The magnetic field is then found from the second Eq. (1).

In the two-dimensional case, when the distribution of the electric conductivity in the model does not vary along axis Oy, the modelling problem is separated into two independent scalar boundary-value problems corresponding to the two polarizations of the field—electric (EP) and magnetic (HP). In this case Eq. (15) takes the following general form:

$$L[U(\mathbf{r})] = \operatorname{div}(p \operatorname{grad} U) + qU = R(\mathbf{r}), \quad \mathbf{r} \in \mathscr{V} \tag{17}$$

and in accordance with Eq. (1):

$$V_{x} = -\partial U/\partial z, \qquad V_{z} = \partial U/\partial x. \tag{18}$$

In the case of E-polarization:

$$U = E_{y}, \quad V_{x} = i\omega\mu H_{x}, \quad V_{z} = i\omega\mu H_{z}, \quad p = 1, \quad q = i\omega\mu\sigma,$$

while for H-polarization:

$$U = H_y, \quad V_x = \sigma E_x, \quad V_z = \sigma E_z, \quad p = 1/\sigma, \quad q = i\omega\mu.$$

In the problem for the total field ($U = U^{\mathrm{t}}$) the right-hand side of Eq. (17) is identically equal to zero.

The quantity R for the anomalous field problem ($U = U^{\mathrm{a}} = U^{\mathrm{t}} - U^{\mathrm{n}}$) takes the form:

$$R = \begin{cases} \sigma^{\mathrm{a}} E_y^{\mathrm{n}} & \text{EP} \\ \operatorname{div}\left[(\sigma^{\mathrm{a}}/\sigma)\operatorname{grad} H_y^{\mathrm{n}}\right] & \text{HP}. \end{cases}$$

The classical statements of the boundary-value problems of electrodynamics are based on an application of Dirichlet boundary-value conditions of the first, second and the third order, prescribed generally by means of linear combinations of the field itself and its derivative normal to the boundary (Tikhonov and Samarsky, 1972).

The simplest Dirichlet condition of the first order requires specification of field boundary values. Usually the boundary of the modelling region is set so far from the conductivity anomaly that it is possible to neglect the anomalous field there. It is then possible to choose the normal field values as the boundary values—usually one-dimensional, but sometimes two-dimensional if the normal distribution of electric conductivity is two-dimensional. Such is the approach taken in the majority of algorithms, including those of participants in the COMMEMI project (Sections 3.3.2 and 3.3.3). In the general case, where the modelling region of finite dimensions, it is very approximate—the error in the boundary conditions are of order $O(1/|\mathbf{r}|)$. Note that in some cases, however, such conditions are quite precise; for example, those on the surface of an ideal conductor in the two-dimensional case of E-polarization, or on the surface of an ideal insulator (in particular on the Earth's surface) in the case of H-polarization.

Another possibility is to apply the second order condition which requires the normal derivative of the solution to vanish on the boundary. This condition is natural and is therefore widely used in algorithms based on the method of finite elements. In a number of approaches, the two types of conditions are combined on different sections of the boundary (Vardanyanz, 1978, 1979, 1983). It should be noted, however, that application of the aforementioned boundary conditions requires the dimensions of the modelling region to exceed the dimensions of the inhomogeneous region $\mathscr{V}^{\mathrm{a}}$ many times over, which results in an unreasonably large number of discrete problems to be solved. Any attempt to decrease the number by choosing a coarser discretization of the medium near the boundaries introduces considerable difficulties when approximating boundary-value problems.

It is very helpful to reduce the extent of the modelling region by taking into consideration the boundary-value operator L_{b} defining the spatial structure of the anomalous magnetic field. The most obvious method that comes to mind is an application of various integral representations (Weidelt, 1975). For example it is possible to use operator (10), connecting the electric field on the boundary with the field inside the modelling region (Lee et al., 1981; Petrick et al., 1981). However in this case the structure of the resulting discrete system of equations is highly complicated, and is treated in the special class of hybrid schemes discussed in Section 2.4. Another approach is to apply the boundary conditions at a relatively small distance from the inhomogeneities in the medium by analyzing the asymptotic behaviour of the electromagnetic field far away from the geoelectrical anomalies (Berdichevsky and Zhdanov, 1981, 1984).

A first step in this direction was taken by Weaver and Brewitt-Taylor (1978) for the two-dimensional case of E-polarization. Asymptotic boundary value conditions were obtained in the nonconducting half-space in the form of a differential equation of first order. In a subsequent paper

(Varentsov and Golubev, 1980a,b) this result was generalized to the three-dimensional case, and written down in the following form:

$$L_{\mathrm{b}}^{\mathrm{a}0}[\mathbf{U}-\mathbf{U}^{\mathrm{n}}]=0, \qquad L_{\mathrm{b}}^{\mathrm{a}0}\equiv(1+\mathbf{r}\cdot\mathrm{grad}) \quad (|\mathbf{r}|\gg 1). \tag{19}$$

The error in condition (19) is $O(1/|\mathbf{r}|^2)$. In the work of Zhdanov et al. (1982a,b) and Berdichevsky and Zhdanov (1984), two-dimensional and three-dimensional asymptotic boundary-value conditions of higher order were constructed for a non-conducting half-space.

The analogue of conditions (19) for a conducting half-space was obtained for the two-dimensional case by Varentsov and Golubev (1985) in the form:

$$L_{\mathrm{b}}^{\mathrm{a}\sigma}[U-U^{\mathrm{n}}]=0,\ L_{\mathrm{b}}^{\mathrm{a}\sigma}\equiv\left(k|\mathbf{r}|+\frac{1}{2}+\mathbf{r}\cdot\mathrm{grad}\right) \quad (|k\mathbf{r}|\gg 1) \tag{20}$$

where $k^2=\mathrm{i}\,\omega\mu\sigma$ (Re $k>0$), and the corresponding three-dimensional form by Spichak (1985), as follows

$$L_{\mathrm{b}}^{\mathrm{a}\sigma}[\mathbf{U}-\mathbf{U}^{\mathrm{n}}]=0,\ L_{\mathrm{b}}^{\mathrm{a}\sigma}(1-ikr+\mathbf{r}\cdot\mathrm{grad}). \tag{21}$$

Finally, Varentsov and Golubev (1985) have specified conditions of the form (20) and (21) on the surface of a stratified conducting base of the model.

The ensemble of aforementioned asymptotic boundary value conditions in the upper and lower half-spaces, in combination with the one-dimensional equations for the normal field on the vertical side boundaries, provide possibilities for decreasing the modelling region. To a certain extent this approach was taken in all of the algorithms used in the COMMEMI project (see Section 3.3.3).

The advantage of first order asymptotic boundary conditions lies in their similarity to the classical Dirichlet boundary conditions of the third order (the difference is that an oblique rather than a normal boundary derivative is used) so that they do not complicate the discrete approximatization of the problem to be solved.

2.3.2. *Finite-difference approximation of boundary value problems*

In the majority of algorithms the finite-difference approximation of a three-dimensional boundary value problem is based on a rectangular, uneven mesh:

$$\Sigma=\{(x_i,y_j,z_k),\ i=1,2,\dots,N_x;\ j=1,2,\dots,N_y;\ k=1,2,\dots,N_z\}.$$

It is necessary to find the linear relation between the values U_{ij} of the function U at the nodes of the mesh Σ, according to the boundary-value problem defined by Eqs. (15) and (16). Various approaches are used for this purpose. The simplest one is based on a direct replacement of the differential operators in the boundary-value problem by analogous difference expressions. This approach was adopted widely in the first finite difference algorithms for geoelectrics (Jones and Pascoe, 1971, 1972; Pascoe and Jones, 1972; Praus, 1976; Yudin and Kazanceva, 1977). It should be noted that these early finite difference schemes were not always of high quality; the nature of the errors of approximation arising with inappropriate averaging of the discrete values of electric conductivity was clarified in the middle of the 1970's (Williamson et al., 1974; Jones and Thomson, 1974; Brewitt-Taylor and Weaver, 1976; Červ and Praus, 1978).

The most convenient means of obtaining a finite difference approximation turns out to be the balance method (Samarsky, 1984). In this method the local approximations of the equations at the nodes of the mesh are used to construct a quadrature solution of integral identities that hold in this problem. For example it is possible to use the integral identities obtained by integrating Eqs. (15) and (16) over the elementary cell S_{ijk}^{+} of the auxiliary mesh Σ^{+} which is composed of the central points in the cells of the mesh Σ (Berdichevsky and Zhdanov, 1981; Zhdanov et al., 1982a,b). In the two-dimensional case the identity takes the form (Varentsov and Golubev, 1980a):

$$\int_{\partial \mathscr{S}_{ik}^{+}} p(\partial U/\partial n)\,\mathrm{d}\ell + \int_{\mathscr{S}_{ik}^{+}} qU\,\mathrm{d}\mathscr{S} = \int_{\mathscr{S}_{ik}^{+}} R\,\mathrm{d}\mathscr{S}. \tag{22}$$

Substituting discrete values of U and R at the nodes of mesh Σ, and the nodal values of p and q on the mesh Σ^{+}, we arrive by quadratures at the system of five-point difference equations that approximate the two-dimensional problem to second order of accuracy (Varentsov and Golubev, 1980a, 1985; Zhdanov et al., 1982a,b). In the three-dimensional case the simplest application of the balance method produces seven-point difference schemes (Zhdanov et al., 1982a,b; Spichak, 1983; Zhdanov and Spichak, 1992). The key point in the balance method is that the field components and conductivity values are defined at nodes on their own separate grids. This approach is also called the method of 'staggered grids' (Mackie et al., 1993, 1994; Weaver, 1994). Application of variation integral identities (considered in Section 2.3.6) yields the so-called variation difference schemes (Vanyan et al., 1984).

Note that the three approaches we have considered here will all yield the same coefficients, or ones that differ only by second order quantities, in the difference equations for two-dimensional structures. For example, the coefficients in the five-point difference equations for two-dimensional E-polarization problems are identical in each method (Brewitt-Taylor and Weaver, 1976; Červ and Praus, 1978; Varentsov and Golubev, 1980a; Yudin, 1981a,b, 1983; Zhdanov and Keller, 1994).

2.3.3. Numerical methods for the solution of difference systems

The system of equations obtained by discretization has the matrix representation:

$$\mathbf{AU} = \mathbf{R} \tag{23}$$

where the matrix $\mathbf{A}$ is complex and band diagonal. In the two-dimensional case the matrix has dominant symmetrical and hermitian parts and is characterized by diagonal dominance. In the three-dimensional case, however, the last property may not be true (Spichak, 1983; Zhdanov and Spichak, 1992).

The dimension of Eq. (23) varies in two-dimensional problems from several hundreds in the simplest applications to many thousands in intricate models of complicated regional structures. In three-dimensional problems, the system represents many thousands of equations in even the simplest problems, and the band width of the matrix is also bigger than in the two-dimensional case.

The methods of solving these types of linear systems arising in problems of geoelectrics have been discussed in papers by Varentsov and Golubev (1985) and Červ and Segeth (1982). In the two-dimensional case they were studied in detail. The direct methods seem to be more accurate and effective here. Examples of direct methods are the Gaussian symmetrical decomposition (Červ and Segeth, 1982), the block elimination algorithm (Samarsky and Nikolaev, 1977; Varentsov and Golubev, 1982), and the specialized Gauss block algorithm of asymmetrical factorization without choice of leading element (Varentsov and Golubev, 1982). The last algorithm provides the possibility of solving Eq. (23) in N complex operations of addition and multiplication where:

$$N = N_{\max} N_{\min}^{3}, \quad N_{\max} \equiv \max(N_x, N_z), \quad N_{\min} \equiv \min(N_x, N_z).$$

The iterative method most widely applied to difference problems of this class is over-relaxation, but for the majority of two-dimensional applications it is inferior to direct methods (Varentsov and

Golubev, 1982; Červ and Segeth, 1982). The great difficulties encountered in the iterative solutions of two-dimensional problems were clearly demonstrated in a paper by Müller and Losecke (1975). The basic problem is bound up with the complexity of specifying effective criteria for terminating the iterations given their slow convergence under complex geoelectric conditions (Berdichevsky and Zhdanov, 1984).

To improve the convergence of the over-relaxation method, a certain decomposition of the modelled region using the alternating Schwartz method has been proposed by Yudin (1981b, 1983) as a way of decreasing the size of the difference system. A more general approach to optimization of iteration methods—solving the boundary-value problem on a converging sequence of grids—was demonstrated within the framework of the method of alternating directions by Vardanyanz (1978).

Another line of attack is to combine the advantages of both the direct and iteration methods. An example of such an approach in two-dimensions (Meijerink and Van der Vorst, 1981) is a quick, approximate (incomplete) five-diagonal factorization of the system matrix in Eq. (23):

$$\mathbf{A} \approx \mathbf{L}^* \mathbf{U}^*$$

together with the subsequent solution of the equivalent system with the matrix $(\mathbf{L}^*)^{-1}\mathbf{A}(\mathbf{U}^*)^{-1}$ by means of three-layer iteration methods—conjugate directions, Chebyshev three-layer, etc. (Varentsov, 1985; Smith and Booker, 1988).

In three-dimensional problems, direct methods become too cumbersome for most present-day computers, and it is evidently timely to implement them on powerful supercomputers. The disadvantages associated with traditional iteration methods are even more aggravated in three dimensions (Yudin, 1983; Spichak, 1983; Zhdanov and Spichak, 1992). A very promising innovation which offers great hope in these problems is the introduction of multiple-grid methods.

2.3.4. Transformation of difference solution

As already noted above, it is sufficient to solve a boundary-value problem for one scalar component of the electromagnetic field in each of the two-dimensional polarizations or one vector component in the three-dimensional case. Other components may be calculated by means of simple differentiation according to Eq. (18). Two problems arise here: the accuracy, and the stability, of the discrete representations of Eq. (18). A simple difference approximation of the derivatives on rough grids gives rise to large errors. In the two-dimensional case, judicious design of the numerical grid can reduce such errors considerably. Alternative methods have also been proposed for improving the accuracy of numerical differentiation (Weaver et al., 1985, 1986; Weaver, 1994). Therefore, the problem in two-dimensions is not critical.

In the three-dimensional case, however, it is still necessary to solve the problem on rough grids and much attention has to be paid to devising reliable methods of numerical differentiation. A radical solution here is to apply the integral operator (9) to recalculate excess currents by a difference solution in the anomalous region $\mathscr{V}^{a}$ into the required solution components, but that constitutes a departure from the pure difference problem and falls under the class of hybrid schemes which are discussed in Section 2.4. The integral estimation of the vertical magnetic component is:

$$(H_z)_{ijk} = \frac{1}{\mathscr{V}} \int_{\mathscr{S}^+_{ijk}} H_z \,\mathrm{d}\mathscr{V} = \frac{1}{i\omega\mu\mathscr{V}} \int_{\partial\mathscr{S}^+_{ijk}} (\mathrm{d}\mathscr{S} \times \mathbf{E})_z \tag{24}$$

where $\mathscr{V}$ is the volume of the cell $\mathscr{V}_{ijk}$, is of interest with the horizontal magnetic components found by Hilbert transformation (Zhdanov, 1984; Zhdanov and Spichak, 1992).

Finally, in both the two-dimensional and three-dimensional cases, it may be appropriate to construct local analytic approximations of the difference solution, thereby providing the possibility of calculating the required derivatives explicitly. In two dimensions, the solution approximated by a plane wave field has been used successfully (Jones and Pascoe, 1971; Varentsov and Golubev, 1985). Such an approach with the possible use of inclined plane waves is also deserving of attention in the three-dimensional case.

2.3.5. Control of modelling accuracy

The most important problem is control of numerical modelling accuracy. Errors are generated while setting up the boundary-value problem, forming its discrete approximations, and finally at the stage of conversion to a finite difference solution.

When applying asymptotic boundary conditions and a direct method for solving the linear system, the dominating errors are those involved with the discrete approximation of the field equations. The natural way of controlling them is through a grid convergence analysis.

In the two-dimensional case of E-polarization there are two useful integral tests of accuracy—the Hilbert transform connecting magnetic field components on the Earth's surface, and the integral Eq. (10). The misfit obtained with these integral tests on the finite difference solution characterizes the modelling error (Zhdanov et al., 1982a,b).

Finally, an important way of investigating and controlling accuracy is to compare the performance of various methods and modelling problems on a series of test models. Such is the essence of the international project on the comparison of modelling methods for electromagnetic induction problems (COMMEMI), described in this article.

2.3.6. Method of finite elements

The method of finite elements is one of the basic methods for numerical solution of boundary-value problems. Its origin can be traced back to Courant (1943), the present name having been introduced in the publication of Turner et al. (1956). The method became widely known through the work of Zienkiewicz and Cheung (1967) and Zenkevich (1975). Pilot studies on applying the method to problems of geoelectrics appeared quite later (Coggon, 1971; Silvester and Haslam, 1972; Reddy and Rankin, 1973). At present a number of different programs exist for modelling electromagnetic anomalies with finite elements in both two and three dimensions (Kisak and Silvester, 1975; Rodi, 1976; Kaikkonen, 1977; Reddy et al., 1977; Pridmore et al., 1981; Wannamaker et al., 1987). The most complete theory of the method applied to electrodynamics is stated in the monograph by Silvester and Ferrari (1990).

The basic principle in the method of finite elements is a division of the region in which the boundary value problem is solved, into a number of comparatively small sub-regions (finite elements), and the approximation of the unknown function in each element as a combination of some basic functions.

Let us consider the simplest application of the method to a two-dimensional boundary-value problem in geoelectrics in which the modelling area is approximated by triangular elements of the first order and excitation is by an incident E-polarized plane wave. The local inhomogeneity $\mathscr{V}^{a}$ is taken to be near the centre of the model. The electric field satisfies the two-dimensional scalar Helmholtz Eq. (17). At the upper and lower bounds of the modelling region $\mathscr{V}$ (located quite far from the local inhomogeneity) Dirichlet boundary conditions of the first order are specified. Neumann boundary conditions $\partial E_y / \partial n = 0$ are specified on vertical boundaries.

Let us divide the solution region into triangular elements and consider one of them with top coordinates $(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)$, and let us suppose that inside the triangle the field is approximated by the equation:

$$E_y = a + bx + cz. \tag{25}$$

Thus the true solution is replaced by a piecewise planar approximation which is continuous across the sides of the triangular elements.

Eq. (25) may be rewritten for an elementary triangle in the form:

$$E_y = \sum_{i=1}^{3} E_y^i \alpha_i(x,z) \tag{26}$$

where the E_y^i represent the values of the field at the vertices of the triangle, and where α_i are linear functions dependent only upon the positions of the triangle's vertices. In this approximation the solution of the boundary-value problem reduces to the determination of the field values E_y^i at the top points of triangular elements which cover the region. This transformed problem is usually solved by variational methods or by methods of weighted residuals.

Consider first the variational methods, in which the boundary-value problem for the differential equation is regarded as corresponding to a stationary functional of the unknown function. That function for which the functional is minimized, and which satisfies the boundary conditions, is the solution of the boundary-value problem. The functional for the two-dimensional problem above takes the form:

$$F(E_y) = \frac{1}{2}\int_{\mathscr{V}} \operatorname{div}(E_y \operatorname{grad} E_y) d\mathscr{V} = \frac{1}{2}\int_{\mathscr{V}}\left[(\operatorname{grad} E_y)\cdot(\operatorname{grad} E_y) - k^2E_y^2\right] \mathrm{d}\mathscr{V}. \tag{27}$$

By solving the discrete problem for the functional minimum Eq. (27) in which E_y is approximated according to Eq. (26), we determine the E_y values at the top points, i.e. we obtain the approximate solution of the boundary-value problem. The functional Eq. (19) may be represented as:

$$F(E_y) = \sum_{\ell} F^{\ell}(E_y) = \frac{1}{2}\sum_{\ell}\int_{\mathscr{V}_\ell}\left[(\operatorname{grad} E_y)\cdot(\operatorname{grad} E_y) - k^2E_y^2\right] \mathrm{d}\mathscr{V}. \tag{28}$$

The component of the functional for an elementary triangle $\mathscr{V}_\ell$ takes the form:

$$F^{\ell}(E_y) = \frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3} E_y^i E_y^j \int_{\mathscr{V}_\ell}\left[(\operatorname{grad} \alpha_i)\cdot(\operatorname{grad} \alpha_j) - k^2\alpha_i\alpha_j\right] \mathrm{d}\mathscr{V}. \tag{29}$$

Introducing matrix elements defined by:

$$\mathscr{S}_{ij}^{\ell} = \int_{\mathscr{V}_\ell}\left[(\operatorname{grad} \alpha_i)\cdot(\operatorname{grad} \alpha_j) - k^2\alpha_i\alpha_j\right] \mathrm{d}\mathscr{V}, \tag{30}$$

we may express Eq. (29) in the matrix form:

$$F^{\ell}(E_y) = \frac{1}{2}\mathbf{E}^{\mathrm{T}}\mathbf{S}^{\ell}\mathbf{E} \tag{31}$$

where $\mathbf{S}^{\ell}$ is the 3×3 matrix defined in Eq. (30), and $\mathbf{E}^{\mathrm{T}}$ means the transpose of the matrix. Elements of the matrix $\mathbf{S}^{\ell}$ are easily calculated and Eq. (31) provides the possibility of obtaining the approximate value of the stationary functional for each element. Specific forms of the matrices $\mathbf{S}^{\ell}$ for the cases of contracting and noncontracting elements is given in the monograph by Silvester and Ferrari (1990). In the final result, the functional is expressed by the values of all the top points. The problem of minimizing this quadratic functional is equivalent to solving the linear system:

$$\partial F / \partial E_y^i = 0, \quad i = 1,2,\ldots,N \tag{32}$$

where N is the total number of top points in the region. The functional Eq. (19) is a positively defined one and Eq. (32) would have the trivial solution $E_y^i = 0$ if the Dirichlet boundary conditions were not taken into account at the upper and lower boundaries. In accordance with these conditions the values of E_y^i at the top points belonging to these boundaries are considered to be known and the corresponding equations are excluded from Eq. (32). Thus the dimension of the system of linear equations obtained from Eq. (32) is equal to the total number N of the top points of region $\mathscr{V}$ minus the number of top points belonging to the horizontal boundaries. It should be noted that homogeneous Neumann conditions on the vertical side boundaries are natural for this problem and are satisfied automatically.

The matrix for the system of Eq. (32) possesses band type structure and is solved by known direct or iterative methods. The solution of Eq. (32) gives the values E_y^i at all top points and hence the piecewise-planar approximation E_y in the whole region.

The piecewise-planar approximation is often a rather rough one for obtaining a good approximation to the modelled function and it is therefore necessary to keep the element dimensions small. In addition, it introduces discontinuities in the spatial derivatives of the solution which do not correspond to the nature of the electromagnetic field. The obvious method of improving the approximation is to use triangular elements of higher order. This means that additional points are added to the triangular elements and the function E_y is approximated on the element by a combination of basic isoclines subject to the condition that the solution and its derivatives are continuous at the element boundaries. The order of polynomials is determined by the number of additional points. Such an approximation provides a more exact solution and reduces the number of elements needed to cover the region. At present there exist programs for modelling electromagnetic anomalies with sixth order triangular elements.

In three-dimensional models, the region of solution is usually divided into tetrahedric elements and rectangular prisms. The unknown components of the vector field are approximated to by means of three-dimensional basis functions. For the variational definition of a problem, the functional which minimizes the field energy is usually employed (Coggon, 1971), i.e.:

$$F = \frac{1}{2} \int \left[k^2 \mathbf{E} \cdot \mathbf{E} - (\operatorname{curl} \mathbf{E}) \cdot (\operatorname{curl} \mathbf{E}) \right] \mathrm{d}\mathscr{V} \tag{33}$$

where $\mathbf{E}$ is the unknown vector field. The problem (as in the two-dimensional case) reduces to the solution of a system of linear equations, with the difference that the system is now so much larger that direct methods of solution, even with the most modern computers, is practically impossible.

It is appropriate to mention a second method of boundary-value solution in the method of finite elements—the method of weighted residuals. It is usually used when construction of the stationary functional for the problem is not helpful or when it is difficult.

Consider, for example, the equation for H_y in the two-dimensional case of H-polarization:

$$L[H_y] = R. \tag{34}$$

Let us determine the residual $T = L[H_y] - R$, where H_y is the approximated solution constructed on finite elements. The boundary-value problem Eq. (34) is replaced by the equation:

$$\int_{\mathscr{V}} WT \, \mathrm{d}\mathscr{V} = 0 \tag{35}$$

where W is a weight function. The boundary conditions are analogous to those in the case of E-polarization considered above. The well-known Galerkin method is a modification of the method of weighted residuals in which the basis functions are also taken as the weights. As shown by many authors the resulting system of linear equations in this case is analogous to the system obtained as a result of the variational approach. Thus both methods give similar approximations to the unknown function.

In conclusion we attempt to list the advantages and disadvantages of the method of finite elements compared with other methods, primarily the method of finite differences. Its principal advantage over finite-differences is the possibility it gives of designing a model which more naturally approximates to the intricate boundaries of a real inhomogeneity. A second advantage is that an approximation to the unknown function is constructed on the whole region of the solution, not just at the discrete nodes of the mesh as in the finite difference method. This means that once the solution has been obtained, it is not necessary to resort to further approximations of the field before it can be further treated in some mathematical manner.

A disadvantage of the finite element method is the complicated procedure required to subdivide the region properly into triangular elements. Formalization of this procedure does not always ensure the required quality of triangulation, while manual triangulation has the disadvantage of requiring much time spent on tedious work. A poor choice of mesh for the finite elements often leads to considerable field distortions; for example asymmetry can appear in symmetric models, and so on.

Elements which are greatly stretched in one direction can seriously affect the accuracy of the solution, just as in the finite-difference method. In three-dimensional problems, where fields have strong gradients, the tetrahedral elements often do not meet the requirements of the approximation to the extent that researchers have been forced to use a division into rectangular prisms, which nullifies, of course, the principal advantage of the finite element method. In a number of two-dimensional applications, it is necessary to resort to rectangular elements as well. The large dimensions of the matrices arising in three-dimensional problems render direct methods of solution practically impossible, and demand the use of iterative methods with a resulting loss of accuracy.

The use of natural Neumann boundary conditions by the majority of researchers is especially a source of uncertainty. This practice can be explained in all probability by the fact that the finite-element method originated in mechanics, where the objects of investigation are considerably limited in size, and has only recently entered geoelectrics. In typical problems of geoelectric modelling, Neumann boundary conditions may be applied only at very great distances from the inhomogeneity. This causes the finite elements to be excessively prolate with a resulting deterioration of the approximation. Conversely, if the boundary conditions themselves are seriously error-prone, then the accuracy of solution also deteriorates.

From all that has been said above, it follows that subject to the application of more exact boundary conditions and the creation of effective algorithms for subdividing the region into elements, the method of finite elements is one of the basic tools for the solution of numerical modelling problems in geoelectrics.

2.4. Hybrid schemes

Let us consider modelling algorithms which combine elements of both integral equation and differential equation methods. As we have remarked already, the natural way of constructing hybrid schemes is to select integral boundary conditions, created by the operator (9), while setting up the modelling boundary-value problem. This idea was developed by a number of authors (Petrick et al., 1981; Lee et al., 1981; Best et al., 1985). The first numerical results were presented by Lee et al. (1981).

With this approach, systems of discrete linear equations of a specific kind are formed. For the electric field they are:

$$\begin{pmatrix} \mathbf{A}_{ii} & \mathbf{A}_{ib} \\ \mathbf{A}_{bi} & \mathbf{A}_{bb} \end{pmatrix} \begin{pmatrix} \mathbf{E}_i \\ \mathbf{E}_b \end{pmatrix} = \begin{pmatrix} \mathbf{R}_i \\ \mathbf{R}_b \end{pmatrix} \tag{36}$$

where indices i and b refer to field values at the interior and boundary points, respectively. The square matrix $\mathbf{A}_{ii}$ has banded structure while $\mathbf{A}_{bb}$ is the unit matrix. The rectangular matrices $\mathbf{A}_{ib}$ and $\mathbf{A}_{ib}$ are, respectively, dense and sparse.

Solving system (36) with respect to the interior field (Lee et al., 1981) we obtain:

$$(\mathbf{A}_{ii} - \mathbf{A}_{ib}\mathbf{A}_{bi})\mathbf{E}_i = \mathbf{R}_i - \mathbf{A}_{ib}\mathbf{R}_b.$$

In the above mentioned work the authors proposed a special method for the solution of this problem, based on the inversions of the banded matrix $\mathbf{A}_{ii}$ and subsequent iteration. Compared with the standard three-dimensional approach, the number of internal equations is considerably reduced and is determined by the discretization of the inhomogeneous region. We note, however, that there are difficulties with the convergence of iterations in this method despite the application of special procedures for convergence acceleration (Lee et al., 1981).

Another approach is based on the solution of system (36) for the boundary values of the field (Best et al., 1985):

$$(\mathbf{I} - \mathbf{A}_{bi}\mathbf{A}_{ib}^{-1})\mathbf{E}_b = \mathbf{R}_b - \mathbf{A}_{bi}\mathbf{A}_{ib}^{-1}\mathbf{R}_i.$$

The dimension of the dense system obtained is determined by the number of boundary equations, and we have here some analogy with the method of surface integral equations. Therefore it is necessary to investigate the problem of stability of the solution for this system. To solve it, we may apply the direct methods when the number of equations is approximately equal to 1000. Solving the problem by means of the first of Eq. (36) we may recover the solution in terms of interior values.

One more example of a hybrid scheme is the algorithm of Yudin (1983) in which in the background of the three-dimensional iterative solution a redefinition of the Dirichlet boundary conditions of the first order is formulated by analytic continuation. In this manner it is possible to contract the boundary of the model to a position considerably closer to the inhomogeneities. There remains only the problem of ensuring convergence of such a procedure, but numerical experiments with analogous two-dimensional schemes represented in the COMMEMI project look quite hopeful.

2.5. Analytical and quasi-analytical methods

The first mathematical models of non-uniform geoelectric structures were solved analytically at the beginning of the 1960s, when direct methods of numerical solution on the computer were not feasible. The analytical models provided the initial impetus for studying the properties of electromagnetic fields in non-uniform media. They are still relevant today as benchmarks against which the more complicated and general methods of modelling can be tested. It turned out, that such solutions could be obtained only for a very limited class of the simplest two-dimensional models [see the catalogue of results by Porstendorfer (1976)], namely a two-dimensional model, uniform along the horizontal axis Oy, and consisting of flat layer, lying on a perfectly conducting or a perfectly insulating half-space. The layer can be divided by several vertical boundaries into segments with different uniform electrical conductivities. The model is excited by a vertically incident H-polarized plane wave.

The solution for the simplest case of one vertical boundary was derived in a paper by D'Erceville and Kunetz (1962) and then generalized (Rankin, 1962) to the case of a dike in an otherwise homogeneous layer. More general formulae for a model in which the layer consisted of three segments of different electrical conductivities were obtained by Wait and Spies (1974). All these results have since been used by many researchers and, in particular, solutions for model 2D-0 have been calculated analytically by such methods and then proposed as the first test for the COMMEMI project (Weaver et al., 1985, 1986). One can also find an overview of different analytical solutions for some simple models in the recent books by Weaver (1994) and Zhdanov and Keller (1994).

Let us consider briefly the basic steps in the construction of the analytical solution for the three-segment model (Weaver, 1994). Suppose that a horizontal layer lies on a perfectly conducting lower half-space. This layer consists of three different constant conductivities σ_1, σ_2 and σ_3 from left to right. The segments are divided by planes $x=-a$ and $x=a$, and the thickness of the layer is d. For the H-polarization case under consideration, Helmholtz equations for the magnetic field H_y are valid in each segment, i.e.:

$$\nabla^2 H_y^j + k_j^2 H_y^j = 0, \quad j = 1,2,3 \tag{37}$$

where $k_j^2 = i\omega\mu_0\sigma_j$ is the squared wave number, and H_y^j is the magnetic field in the jth segment. The magnetic field satisfies the following boundary conditions:

1. $H_y^j = H_0$ (const) when $z=0$;
2. $\partial H_y^j/\partial z = 0$ when $z=d$;
3. $H_y^1 = H_y^2$ when $x=-a$, and $H_y^2 = H_y^3$ when $x=a$;
4. $\sigma_2\ (\partial H_y^1/\partial x) = \sigma_1\ (\partial H_y^2/\partial x)$ when $x=-a$, and $\sigma_3\ (\partial H_y^2/\partial x) = \sigma_2\ (\partial H_y^3/\partial x)$ when $x=a$.

The first condition is explained by the absence of a vertical component of current at the Earth's surface, the others by continuity of the tangential components of electric and magnetic fields across conductivity boundaries.

Let us seek a solution of the form:

$$H_y^j(x,z) = H_y^{(\mathrm{n})}(z) + f_j(x,z)$$

subject to normalization $H_y^{(\mathrm{n})j}(0) = 1$ where $H_y^{(\mathrm{n})j}$ is the one-dimensional solution (the normal field) for the jth segment given by:

$$H_y^{(\mathrm{n})j} = \cosh\left[(d-z)k_j\right]/\cosh(dk_j).$$

The function f_j is a solution of Eq. (37) and satisfies the conditions $f_j = 0$ when $z=0$, and $\mathrm{d}f_j/\mathrm{d}z = 0$ when $z=d$. The function may be found by the method of separation of the variables. The z-dependent part of the function is expressed as a Fourier series and the x-dependence may be expressed in terms of exponential functions. In general, the solution in the three segments takes the form:

$$
\begin{aligned}
f_1(x,z) &= \sum_{m=0}^{\infty} P_m \exp(n_1^m x)\sin(l_m z) \\
f_2(x,z) &= \sum_{m=0}^{\infty} [Q_m(n_2^m x) + R_m(-n_2^m x)](l_m z) \\
f_3(x,z) &= \sum_{m=0}^{\infty} S_m \exp(-n_3^m x)\sin(l_m z)
\end{aligned}
\tag{38}
$$

where $l_m = (2m+1)\pi/2d$ and $n_j^m = (l_m^2 + k_j^2)^{1/2}$. The coefficients P_m, Q_m, R_m and S_m are determined by matching the Fourier series at the boundaries $x = \pm a$ with the aid of boundary conditions (3) and (4). See the paper by Weaver et al. (1985) and Weaver (1994) for details.

To obtain the solution for a model in which the basement is perfectly insulating, it is necessary to change boundary condition 2 to $H_y^j = 0$ when $z = d$. The derivation of the solution remains the same with the final formulae differing by the fact that d is replaced by $d/2$ and the summation of the Fourier series is over only the even numbered terms.

Thus the solution in both cases has the following form:

$$
H_y^j(x,z) = H_y^{(n)j} + \sum_{m=0}^{\infty} B_m^j(x)\sin(l_m z).
\tag{39}
$$

Equations for the other components of the field, E_x and E_z, are obtained from Eq. (39) by differentiating H_y^j. Also the infinite series on the right-hand side of Eq. (39) is easily evaluated on the computer to any desired accuracy with minimal demand on cpu time. This makes the analytical model particularly attractive as a test of numerical solutions of the forward problem in geoelectrics.

Attempts to obtain an analytical solution for the same model in the E-polarization mode have not been successful because in this case there is no simple boundary condition on the Earth's surface. By means of a number of transformations of the original problem, however, it is possible to reduce the problem to the solution of a simple one-dimensional integral equation which involves minimal numerical calculations. Because of this, such solutions have been named 'quasi-analytical'. The first results for the layer divided by one vertical boundary were obtained by Weidelt (1966), with further developments by Mann (1970), Klügel (1977) and Rodemann (1978).

In the work of Weaver et al. (1986) and Weaver (1994) the numerical solution of the integral equation was simplified considerably with the aid of some complicated mathematical transformations and a general solution for the three-segment layer was obtained. An outline is now presented of the solution by successive approximations of the E-polarization problem for the model considered in this section (Weidelt, 1966; Weaver, 1994). In each segment of the model, the two-dimensional Helmholtz equation is satisfied by the component E_y of the electric field, subject to the following boundary conditions:

1. $E_y^j = 0$ when $z = d$;
2. $E_y^1 = E_y^2$ when $x = -a$, and $E_y^2 = E_y^3$ when $x = a$;
3. $\partial E_y^1/\partial x = \partial E_y^2/\partial x$ when $x = -a$, and $\partial E_y^2/\partial x = \partial E_y^3/\partial x$ when $x = a$.

The first boundary condition represents the vanishing of the electric field on the surface of a perfect conductor, and the others the continuity of the tangential components of the electric and magnetic fields at the segment boundaries. The additional condition is $\partial E_y/\partial x \to 0$ when $|x| \to \infty$, which corresponds to the vanishing of the vertical component of the magnetic field at infinity. As it was already noted, in the case of E-polarization no component of the surface field is constant and we are forced to use an integral boundary condition based on the Hilbert transform, namely:

$$H_x(x,0) = H_x^0 + \mathscr{K}H_z(x,0), \tag{40}$$

where

$$\mathscr{K}H_z(x,0) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{H_z(\xi,0)}{x-\xi}\,\mathrm{d}\xi.$$

and H_x^0 is the surface magnetic field for a homogeneous slab. Let us introduce the Green's function $G_{ij}(\mathbf{r}|\mathbf{r}')$ where the position vector belongs to the ith segment, and $\mathbf{r}'$ to the jth segment. The Green's function is the solution of the equation:

$$(\nabla^2 + k_i^2)G_{ij}(\mathbf{r}|\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}') \tag{41}$$

in the ith segment, and satisfies the following boundary conditions:

$$G_{ij}(x,d|\mathbf{r}') = 0,\ \partial G_{ij}(x,0|\mathbf{r}')/\partial z = 0.$$

If we multiply Eq. (41) by E_y^i, the Helmholtz equation by G_{ij} and then integrate over the ith segment, we obtain:

$$\int\int\left[E_y^i(\mathbf{r})\nabla^2 G_{ij}(\mathbf{r}|\mathbf{r}') - G_{ij}(\mathbf{r}|\mathbf{r}')\nabla^2 E_y^i(\mathbf{r})\right]\mathrm{d}x\,\mathrm{d}z = \begin{cases} E_y^i(\mathbf{r}') & (i=j) \\ 0 & (i \neq j) \end{cases} \tag{42}$$

The left side of Eq. (42) can be transformed to a closed linear integral around the segment boundary by means of Green's formula as follows:

$$\sum_i \int_{\mathscr{C}_i}\left[E_y^i(\mathbf{r})\frac{\partial G_{ij}(\mathbf{r}|\mathbf{r}')}{\partial n} - G_{ij}(\mathbf{r}|\mathbf{r}')\frac{\partial E_y^i(\mathbf{r})}{\partial n}\right]\mathrm{d}\ell_i = E_y^j(\mathbf{r}') \tag{43}$$

Here the contours $\mathscr{C}_1$ and $\mathscr{C}_3$ extend to infinity and $\partial/\partial n$ means the normal derivative on the contour.

Eq. (43) is considerably simplified if we take into consideration all the boundary conditions for E_y and G_{ij}, i.e.:

$$\int_{-\infty}^{-a} G_{1j}(x,0|\mathbf{r}')\frac{\partial E_y^1(x,0)}{\partial z}\,\mathrm{d}x + \int_{-a}^{a} G_{2j}(x,0|\mathbf{r}')\frac{\partial E_y^2(x,0)}{\partial z}\,\mathrm{d}x$$
$$+\int_{a}^{\infty} G_{3j}(x,0|\mathbf{r}')\frac{\partial E_y^3(x,0)}{\partial z}\,\mathrm{d}x = E_y^j(\mathbf{r}') \tag{44}$$

which, with an application of Maxwell's equations, can be written in general as:

$$E_y(\mathbf{r}) = -i\omega\mu\int_{-\infty}^{\infty} G(x,z,u)H_x(u,0)\,\mathrm{d}u \tag{45}$$

where $G = G_{ij}$. Differentiating Eq. (45) in accordance with Maxwell's equations, we obtain for the other components:

$$H_x(\mathbf{r}) = \int_{-\infty}^{\infty}[\partial G(\mathbf{r},u)/\partial z]H_x(u,0)\,\mathrm{d}u, \tag{46}$$

$$H_z(\mathbf{r}) = -\int_{-\infty}^{\infty}[\partial G(\mathbf{r},u)/\partial x]H_x(u,0)\,\mathrm{d}u. \tag{47}$$

Substituting Eq. (47) in Eq. (40) we obtain:

$$H_x(x,0) = H^0 - \mathscr{K}\int_{-\infty}^{\infty} [\partial G(\mathbf{r},u)/\partial x] H_x(u,0)\,\mathrm{d}u, \tag{48}$$

which leads to Weidelt's scheme of successive approximations:

$$H_x^{n+1}(x,0) = H^0 - \mathscr{K}\int_{-\infty}^{\infty} [\partial G(\mathbf{r},u)/\partial x] H_x^n(u,0)\,\mathrm{d}u. \tag{49}$$

Choosing the first approximation to be $H_x^0(x,0) = H^0$, we determine $H_x(x,0)$ as:

$$H_x(x,0) = \lim_{N\to\infty} H_x^n(x,0). \tag{50}$$

This scheme generally converges. The full electromagnetic field in the conducting layer is then determined from Eqs. (45)–(47). Other researchers have since devised a number of alternative schemes of successive approximations based on Eqs. (45)–(47).

The solution is completed by determining the Green's functions. They are found from Eq. (41) by separation of the variables, also taking into account the boundary conditions for the Green's functions. The form of the Green's function is (Weaver, 1994):

$$G_{ij}(x,z,u) = \frac{1}{d}\sum_{m=0}^{\infty} G_{ij}^{m*}(x,u)\cos(l_m z) \tag{51}$$

where

$$G_{ij}^{m*}(x,u) = A_m^{ij}\exp(\eta_i^m x) + B_m^{ij}\exp(-\eta_i^m x) - \delta_{ij}\exp\left(-\eta_j^m|x-u|\right)/\eta_j^m$$

and δ_{ij} is the Kronecker delta. Thus, all the functions in Eqs. (45)–(50) are expressed analytically so that the successive approximations can be calculated on the computer without difficulty.

The matter is much more complicated when the basement of the model is a perfect insulator. In this case the procedure is simplified if the layer thickness exceeds several skin depths in the layer, which means that actually the underlying uniform half-space can be ignored, i.e. it is a particular case of the previous model corresponding to $d \to \infty$.

We have examined methods of obtaining analytical and quasi-analytical solutions for the simplest models. Evidently the ease with which these solutions can be generated on the computer and the high accuracy that is attainable, makes them ideal models for testing two-dimensional modelling programs which are based on the various methods of numerical analysis. Therefore the first test in the two-dimensional part of the COMMEMI project becomes model 2D-0.

3. Comparison of modelling programs

3.1. Program comparison

3.1.1. Sources of error

The various schemes for modelling electromagnetic fields numerically in inhomogeneous media reduce in essence to the following steps:

- Construct a discrete model of the geoelectric medium;
- Approximate the electromagnetic field equations in the model by a system of discrete equations;
- Obtain the solution of the discrete system of equations;
- Perform various operations on the solution obtained.

The errors associated with the approximations above accumulate at every stage of the scheme and impose an ultimate limit on the accuracy of the numerical modelling. The principal sources of error will now be considered for each step of the calculation in turn.

3.1.1.1. Constructing a discrete model. The initial geoelectric model is designed as an assemblage of conducting bodies in each of which the electrical conductivity is uniform. Regular or irregular meshes composed of discrete elements of simple shape—triangles and rectangles in two dimensions, tetrahedra and hexahedra in three dimensions—are most often used. Elements of simple shape are preferred because the more complicated the design of the model, the greater the difficulties arising in the subsequent calculation of the field.

Certain shapes of the elements are more appropriate for some models than for others. For example, two-dimensional structures in which the conductivity boundaries are curvilinear can be more effectively modelled with triangular elements than with rectangular ones. Note, however, that whatever the discretization scheme used, it will always be impossible to model certain geoelectric objects of intricate shape without introducing some errors. By increasing both the size of the mesh and the complexity of its elements, it is possible, in theory, to reduce such errors; but it should be remembered that such measures may introduce further problems at later stages of the calculation because of the enlargement and structural alteration of the system of equations and a consequent deterioration of their conditioning and numerical stability which can severely affect the accuracy of the final results.

3.1.1.2. Approximating by a system of discrete equations. Various approximations of the original electromagnetic field equations may be chosen. The resulting discrete system of equations depends on several factors, ranging from the basic choice of method, i.e. differential or integral equation, to the way the problem is represented numerically, i.e. grid design and size, and to the assumptions made when determining the coefficients in the system of equations. The final result is a compromise between having the best possible approximation to the geometry of the real geoelectric structure and being able to solve a large system of linear equations whose coefficient matrix may be dense and complex. A majority of researchers therefore limit their attention to models with piecewise constant distributions of electrical conductivity which, at least in the differential equation method, lead to systems of equations with banded matrices.

3.1.1.3. Solving the system of equations. At this stage of the procedure, the number of possible directions that can be taken expands because, even within the confines of one overall approach, there are several numerical methods that can be used. It is again necessary to choose between general but less efficient methods, and specialized techniques which are more limited in application but more effective under particular circumstances. For example, over-relaxation is an iterative technique in widespread use because the class of problems to which it can be applied in practice is not restricted. However, its effectiveness in solving systems with complex rather than real coefficients still leaves much to be desired. More efficient methods under development are dependent on a number of conditions being satisfied which impose limits on the range of choices available in the earlier stages of the modelling procedure.

Much the same can be said about techniques for solution by direct elimination. Thus, the methods designed to handle dense matrices can always be used to solve systems of up to a few hundred equations, but more efficient methods which take advantage of the sparsity of matrices are restricted to certain selected modelling procedures. The nature of the non-vanishing elements in the coefficient matrix can be more readily used to advantage in iterative schemes than in the direct methods of solution but the uncertainty in how to formalize the criterion for completion of the iterative process can greatly reduce the effectiveness of such schemes, to the extent that it is quite possible to obtain different and incorrect results depending on how proficient the different users of the program are in numerical methods.

As a rule, modellers have developed their own numerical procedures to meet their own particular needs. This is partly because for many specific applications to real geoelectric structures, the effectiveness of universal software packages is lower than that of specialized algorithms. There is always the fear, however, that the performance levels of such locally developed, specialized programs will not match those of the standard packages when subjected to general comparison tests.

3.1.1.4. Performing computational operations on the solution. The system of equations yields a basic solution in one or several field components. The other field components then have to be obtained from a further calculation in which the possibility of additional errors being introduced arises. A typical example is in the finite difference method where numerical differentiation of the basic solution must be undertaken.

It is clear from what has been said that errors are accumulated at each stage of the modelling procedure. A theoretical estimation of the probable error is very complicated, however. It is therefore usual in practice to evaluate the accuracy of a given program experimentally. For each model, what appears to be most difficult is making a balanced choice which is suitable for all approximations of the model, the discretization of the equations, and the method of their solution, directed towards obtaining an effective algorithm.

Programs for modelling electromagnetic fields are quite intricate and their testing requires a great deal of additional work which is not always feasible for the programmer. As a rule, the complex components are thoroughly studied, but final testing requires a knowledge of the actual field for a whole range of models, which complicates the checking of a program taken in isolation. Another difficulty in error estimation is the fact that results may vary considerably from one computing system to the next if they use different word lengths and different round-off procedures when executing arithmetical operations.

Because of the difficulty in making a priori estimates of modelling errors, it is recognized that it is most effective to incorporate internal diagnostics and error analysis in the program itself. However, for the majority of algorithms it is simply not possible to adopt this all-embracing and reliable approach to error estimation. The most general method is to study a convergent sequence of discretizations. An important method of control is to compare results with some standard calculations. For a number of simple models we may obtain analytic solutions, but good agreement with such analytic results does not guarantee that the program is equally accurate for more complicated models.

Thus, it is rather difficult to ensure complete control of accuracy, and the quality of numerical modelling algorithms may depend ultimately on the qualifications of their authors. Given this conclusion, the most acceptable way of ensuring quality control in modelling is to compare directly the results of different programs. Comparison of a considerable number of modelling results for a set of models provides the following opportunities:

- to reveal those programs that give results in closest agreement;
- to set numerical standards, based on simple and complicated models, and to select those solutions that are most probably the ones of greatest accuracy;
- to compile the statistics of the set of all solutions obtained and to determine the reasons for deviations in some results;

- to make recommendations on the application of modelling methods and the various techniques used in their implementation.

The success of this exercise will depend to a considerable extent on the set of test models chosen and the method of analysing the results obtained.

3.1.2. A review of the program comparisons

One of the most important stages of software development is program testing. Since the development of a comprehensive set of real test models is not a simple task, previous checks used on software have been conducted without the availability of standard solutions. In such cases, problems which were most interesting from a practical point of view were solved by all available methods.

Originally comparisons were made with the sole purpose of ensuring the identity of the programs operating under complicated situations. Eventually, the community of participants was enlarged and the number of models investigated was increased so that the comparisons took on a new character—they formed a set of numerical standards against which new approaches could be tested. With the increase in the number of participants, there appeared yet one more development, namely the standardization of various facets of the comparison project, which included modelling methods, formats for the presentation of results, and methods of ranking them, etc.

One of the first of such projects was a comparison of a number of algorithms of two-dimensional modelling on a rather simple geoelectric model—a homogenous, vertically elongated rectangular conducting anomaly embedded in a uniform medium. The electromagnetic field was excited by an infinitely long cable placed on the Earth's surface. Ten participants presented a description of their algorithms and their calculated results. All these papers were published in a special issue of Geophysics with a preceding reviewing article by Ward (1981). This exercise provided all those who were interested in the problem an opportunity not only to have their own algorithms publicized, but also to select those that showed the most promising trends for further development.

Later, a similar project was initiated by the Commissions for Planetary Geophysics of the various Academies of Sciences in Eastern European countries for two-dimensional models excited by a vertically incident plane wave. Well-known scientists working in geoelectrics suggested models which were of interest to them, and then all the participants in the comparison test calculated the responses for these models. A study of the results obtained by the various programs exposed the basic difficulties that had arisen during the modelling. Foremost among such difficulties were the problems of approximating intricate geoelectric structures, the utilization of very non-uniform grids, devising criteria for ensuring the accuracy of iterative processes, and so on. A summary of this project was discussed at the conference of the Commissions on Geophysics of the Academies of Science in Dresden, Germany in 1978. Even with only qualitative, visual comparisons of the results, precise information was exchanged, calculation techniques were improved, and modelling algorithms were perfected.

At the beginning of the 1980's one more comparison of programs was carried out in the former USSR under the supervision of Professor M.N. Berdichevsky. The basic objective of this project was not only to confirm the equivalence of different algorithms, but also to study the possibility of their application to extreme theoretical models, as well as to the modelling and interpretation of real regional sections. Theoretical models were devised with complicated structures possessing large conductivity gradients in the horizontal direction and requiring study for a very wide range of frequencies. Such models required different discretizations of the geoelectric medium for the various frequencies in the range studied. As a result of this comparison those conductive bodies which are the most difficult to model were discovered, and those programs which most often gave deviations from the results of many of the others were exposed. Unfortunately this particular project was not completed and no recommendations of a constructive kind were ever published.

Experiments on the comparison of numerical modelling programs were also carried out for three-dimensional models, one of the first examples being the comparison of methods of modelling in three-dimensional media with controlled sources discussed at the international working conference on electromagnetic modelling in Berkeley, USA, in 1978.

Previous experience gained in these early comparisons of modelling programs was invaluable in stimulating expertise in the design of models, the presentation of results, and in setting new standards for future projects of this kind.

3.2. The international project COMMEMI

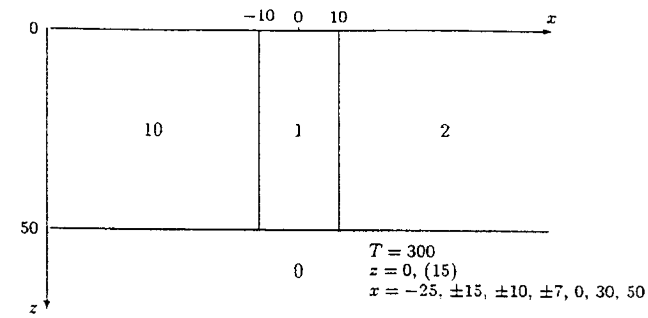
3.2.1. Project aims

The international project on the Comparison Of Modelling Methods for ElectroMagnetic Induction (COMMEMI) was proposed and adopted at the VIth International Workshop in Electromagnetic Induction (Victoria, B.C., Canada, 1982) under the aegis of Working Group I-3 (since renamed I-2) of the International Association on Geomagnetism and Astronomy (IAGA). The basic aim of this project was to compare the accuracy and effectiveness of various algorithms and programs for the numerical modelling of electromagnetic fields in inhomogeneous media. In all the models devised for COMMEMI it was assumed that the source field is a vertically incident plane wave with a time-dependence exp $(-i\omega t)$ in the quasi-stationary approximation.

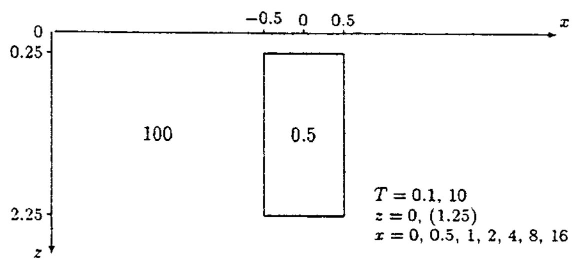
A number of essential, practical problems were investigated in the project—evaluating the practicability of specific programs, comparing results of numerical modelling with analytical solutions, comparing the performance of different algorithms on a set of simple models, and finally comparing their modelling capability for real geoelectrical structures. The accumulation of a large number of results for a whole range of models enables one to identify those algorithms which are reliable and readily useable in a variety of situations. There arises, also, the possibility of examining how the approximation of the geoelectric fields affects the computed model fields, and which programs are most efficient at solving the systems of linear equations, and calculating the field components and transfer functions. The availability of so many results, obtained by participants using, in the majority of cases, quite different programs, also raises the possibility of preparing some standard solutions for certain complex models and, in the case of those solutions that do not conform to the standard solution, of ascertaining which properties of the algorithm or its method of application explain the deviation from the standard solution. Thus, another purpose of the project was to assemble results for complex standard models which cannot be solved analytically. Such standards are very useful for developing and testing new methods and algorithms for the modelling of electromagnetic fields.

It should be noted that general consent on what field components and transfer functions are needed in practice has gradually evolved over the years, and that the comparison projects have contributed to the standardization of the output data. Some programs have appeared, however, with their own individual peculiarities in the way they present results, thereby hindering their comparison with others.

An important objective of the project was to identify general programs which perform precisely and reliably when modelling a variety of geoelectric structures. Quantitative measures of program effectiveness were not estimated since this is a task which would require a special investigation. Calculations were carried out on various computers; to compare their speed is a difficult problem in



(a)



(b)

Fig. 1. COMMEMI test models (a) 2D-0 (upper diagram), and (b) 2D-1 (lower diagram). Coordinates are given in kilometres, periods in seconds, and resistivities in ohm-metres. The periods and the coordinates of the observation points for which the fields are to be calculated are stated in brackets.

itself. Many participants did not present information about the speed of computation, since it depends very much on grid size and it would always be necessary to select the grid of least dimension for which accuracy is not sacrificed, to obtain the most rapid results. Some participants were opposed in principle to any comparison of computing speeds, as they considered such information to be primarily of commercial interest and incompatible with the purely research aims of the project.

3.2.2. Structure of the test models

In 1983 a set of two-dimensional and three-dimensional models together with brief instructions describing the parameters of the models and standard formats for the presentation of results, were distributed to potential participants. Some suggestions for modifying the project were received along with the first results. As a result of this feedback from the early participants a new set of models was proposed in 1985, including six two-dimensional and two three-dimensional structures. Some were rather simple theoretical models but the last two approached real geoelectrical structures in complexity.

Model 2D-0 in Fig. 1a is a layer, consisting of three segments of different conductivities, lying on a perfectly conducting basement. The analytical solutions (Weaver et al., 1985, 1986) for this model enable one to estimate how well the original geoelectric problem is described by the discrete equations used by the participants. This model is useful for checking the basic correctness of a program.

Model 2D-1 in Fig. 1b is a symmetrical, rectangular, highly conducting anomaly embedded in a uniform conducting half-space. The anomalous field is quite large, the contrast in electric conductivity

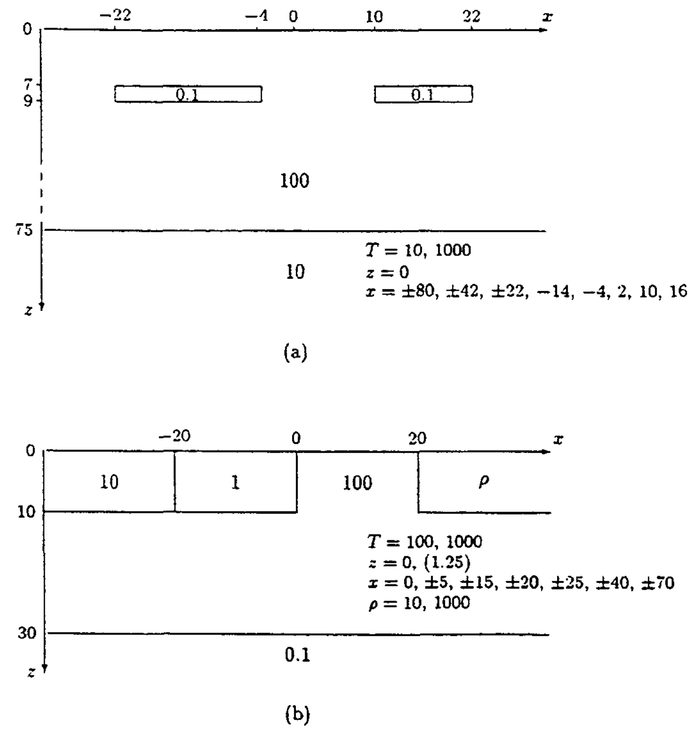


Fig. 2. COMMEMI test models (a) 2D-2 (upper diagram),and (b) 2D-3 (lower diagram).

is considerable, and the anomaly is located near the surface. In spite of its apparent simplicity this model imposes high requirements on the ability of algorithms to approximate the model, and on the method of solving linear systems with highly varying coefficients.

Model 2D-2 in Fig. 2a is characterized by an even higher contrast of electrical conductivity, and anomalies of small thickness but considerable length. The resulting system of linear equations also has a high condition number.

In Model 2D-3 shown in Fig. 2b all the anomalies are surficial. There are four distinct regions. Those at the edges of the model have equal conductivities in the first version, but in the second the conductivity of the right-hand region is decreased considerably. There are large variations in the H-polarization field in this model. It may also be possible to apply thin sheet modelling algorithms to this model. The greatest difficulties arise in the differentiation of the basic field at points where the vertical boundaries separating regions of different conductivity reach the surface.

In contrast to the previous models, Model 2D-4 (Fig. 3) has a rather more complicated structure, including inclined boundaries and a thin surficial layer. In this model the result is affected not only by the choice of discrete equations and the method of solution, but also by the capability of the program to approximate different structures which include, perhaps, factors which are more subjective in nature.

Model 2D-5 (Fig. 4) is the most complicated model of all. It is an idealized representation of the Carpathian geomagnetic anomaly, in which the Ukrainian shield, characterized by highly resistive rocks down to large depths, is located in the region $x > 0$, and the Pennon depression, which has a quite thick conductive jacket of residual rocks, occupies $x < 0$. The model is complicated in structure, requires a detailed representation of the section, and necessitates the use of a grid with a large number

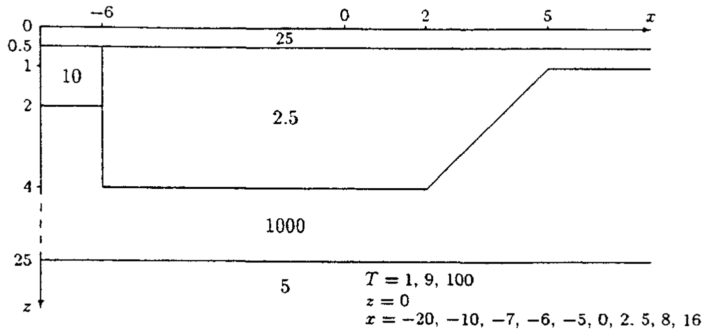


Fig. 3. COMMEMI test model 2D-4.

of nodes because of its large dimensions. Thus, it is suitable for determining how well programs perform when modelling real geoelectric structures.

Three-dimensional models are represented by two configurations. Model 3D-1 in Fig. 5a is a rectangular insert in a homogenous half-space. The section of this model in the plane xOz is the same as model 2D-1, thereby providing a link between the two- and three-dimensional parts of COMMEMI. Although the model is geometrically simple, it has a rather high contrast of electric conductivity and the inhomogeneity is placed near the surface, resulting in a relatively large anomalous field. Two versions of the model are proposed: first an insert whose horizontal dimensions are in the ratio 1:2, and secondly, a more elongated insert for which the corresponding ratio is 1:10. The field values are calculated only on the surface along the coordinate axes.

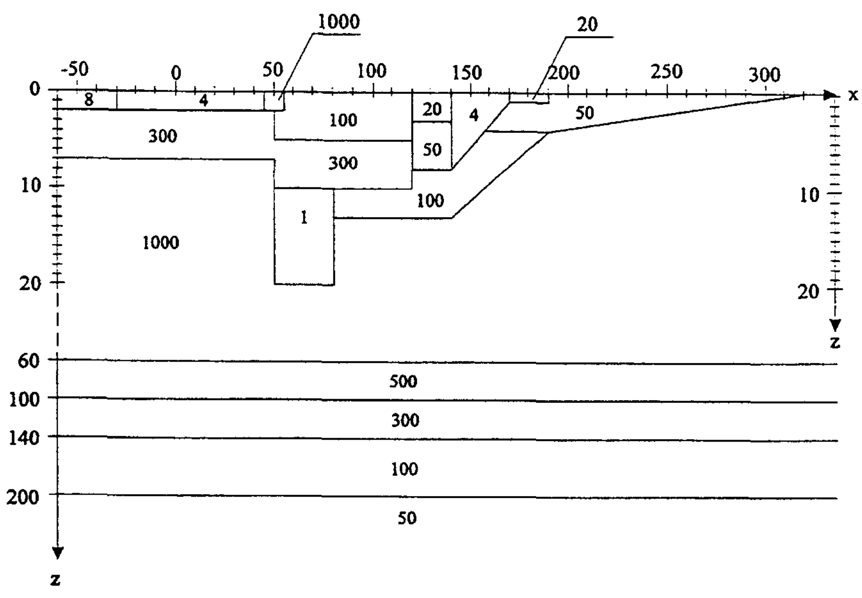


Fig. 4. COMMEMI test model 2D-5.

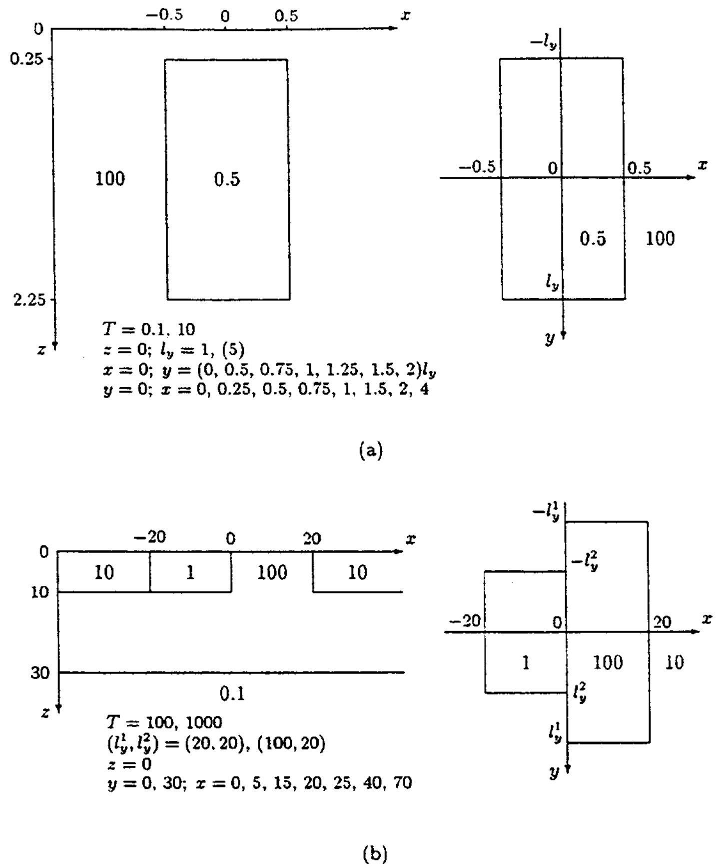


Fig. 5. COMMEMI test models (a) 3D-1 (upper diagram), and (b) 3D-2 (lower diagram).

The second model 3D-2 given in Fig. 5b is much more complicated. It consists of two perpendicular, rectangular inserts in the upper layer of a three-layer section. The section along the x-axis reproduces exactly the first version of model 2D-3. Two versions of the model corresponding to different elongation of the inserts are again considered.

3.2.3. Presentation of results

The two-dimensional calculations were made for both polarizations of the field and for both a standard set, and an additional set of periods and observation points. The following field components were compared:

$$\tilde{E}_y = E_y/E_y^{\mathrm{n}},\ \tilde{H}_x = H_x/H_x^{\mathrm{n}},\ \tilde{H}_z = H_z/H_z^{\mathrm{n}},\quad \text{(EP)}$$

$$\tilde{H}_y = H_y/H_y^{\mathrm{n}},\ \tilde{E}_x = E_x/E_x^{\mathrm{n}},\quad \text{(HP)}$$

where E_x^{n}, E_y^{n}, H_x^{n}, H_y^{n}, are components of the normal field at $x = -\infty$, $z = 0$ on the surface of the left-hand, one-dimensional section of the model. At points where the electric field in HP was discontinuous, the two limiting values of field were presented as far as possible. When anomalous fields were calculated, the results were converted back into total fields.

The electromagnetic fields were tabulated as complex values in the form (Re, Im). Apparent resistivities were calculated according to the formulae:

$$\rho_a = |Z|^2/(\omega\mu)$$

where $Z = -E_y/H_x$ for E-polarization, and $Z = E_x/H_y$ for H-polarization. In the case of three-dimensional calculations, two polarizations of the normal field were defined as follows:

$$\mathbf{E}^n = (E^n,0,0), \mathbf{H}^n = (0,H^n,0), \quad \text{(EXN)}$$

$$\mathbf{E}^n = (0,E^n,0), \mathbf{H}^n = (H^n,0,0), \quad \text{(EYN)}$$

and designated EXN and EYN, respectively, as indicated. The components of the total electric and magnetic fields were normalized by the complex values E^n and H^n, and again presented in the complex form (Re, Im). At points on the surface where the electric field is discontinuous, the limiting values on each side of the discontinuity were recorded.

In addition to the field components themselves apparent resistivity values were calculated from the impedance tensor. Since the designated points in Model 3D-1 lie on the coordinate axes which are in planes of vertical symmetry, only the off-diagonal elements of the impedance tensor are non-vanishing in the given coordinate system, from which we obtain two principal apparent resistivities ρ_a^{xy} and ρ_a^{yx}.

All results were stored in a data base ready for further processing. Along with the results, the participants' numbers given in Tables 1 and 2 were stored, as well as the period, components, identification, depth and the horizontal coordinates of the points where the fields were determined. In this way it was possible to process the results automatically and to apply various comparison algorithms.

The main purpose of the data bank was to provide ready access to all the existing results for any specified period, level, components and coordinates. Secondly it facilitated the application of algorithms which calculated the statistical characteristics of the results, and finally it facilitated procedures for printing tables and plotting diagrams. It therefore provided the authors with the opportunity to test and choose from several algorithms for calculating the statistics and different ways of plotting the diagrams, and also simplified the inclusion of new results and the re-organization of the tables.

For the present, we have restricted our statistical calculations at each point to the mean and standard deviation δ_0 of all the submitted results for the separate real and imaginary parts of each component. The standard deviation characterizes to some extent the quality of the model calculations, but the mean values may be considerably distorted by outliers, especially when the number of available results for a particular model is small. In order to improve the quality of the analysis by reducing the influence of such outliers, new statistics were calculated by first removing all results displaced more than $2\delta_0$ from the mean value, and then calculating a new mean and standard deviation δ_1 for the reduced data set.

It should be noted that this algorithm does not necessarily guarantee accurate mean values. If several results still differ markedly from the mean, then the standard deviation δ_0 may be great without these values being rejected. Therefore an examination of the tables of components above is not always convenient for determining numerical benchmarks. For this reason, graphical diagrams of results are presented at the end of this article.

The minimum and the maximum values are shown on the diagrams by dashed lines with the mean value and error bars defining the interval $2\delta_1$ depicted by three continuous lines. In the lower part of

Table 1
List of COMMEMI participants (two-dimensional calculations)

No.	Country (Organization)	Method of calculation	Authors of Programs	Authors of Calculations	Calculated models
1	2	3	4		5
1	Canada (Univ. Victoria), Switzerland (Neuchâtel)	A	Weaver, Fischer, LeQuang		2D-0
2	Canada, Switzerland	FD(D)	Weaver, Brewitt–Taylor	Weaver, Fischer, LeQuang	2D-1, 2D-3
3	Canada (Univ. Victoria)	FD(D)	Weaver, Brewitt–Taylor	Weaver	2D-0–2D-5
4	Finland (Univ. Oulu)	FD(D)	Weaver, Brewitt–Taylor	Kaikkonen et al.	2D-2, 2D-5
5	Finland	FE(D)	Kaikkonen		2D-4 (EP)
6	Poland (Inst. Geophys. Acad. Sci.)	FD(D)	Tarlowskii	Jankowskii et al.	2D-1–2D-5
7	Hungary (Inst. Geod. Geophys. Sci.)	FD(I)	Tatralya	Ádám et al.	2D-2
8	Russia (IZMIRAN)	FD(D)	Varentsov, Golubev		2D-0–2D-5
9	Russia (IZMIRAN)	IE(D)	Varentsov, Golubev, Chernyak		2D-1, 2D-2
10	Uzbekistan (Tashkent State Univ.)	FD(I)	Varentsov, Golubev	Belyavskiy	2D-1–2D-4
11	Russia (St. Petersburg State Univ.)	FD(I,SG)	Varadanyanz		2D-1–2D-5
12	Czechoslovakia (Inst. Geophys. Acad. Sci.)	FD(I)	Červ, Pek, Praus		2D-1–2D-5
13	Germany (Freiberg, Acad. Mines)	FD(I)	Veller, Porstendorfer, Rosler		2D-1–2D-5
14	Russia (MGRI)	FD(I)	Yudin, Ananevich, Veselovskiy		2D-1–2D-4
15	Russia (MGRI)	FD(I)	Yudin, Ananevich, Veselovskiy		2D-1
16	Russia (MGRI)	FD(I)	Yudin, Kazantsev	Kuznetsov et al.	2D-1–2D-5
17	USA (Univ. Utah)	FE(D)	Wannamaker et al.		2D-0, 2D-1, 2D-3, 2D-5
18	Germany (Göttingen)	FD(I)	Schmucker		2D-1–2D-5
19	Germany (Göttingen)	IE(D)	Flüche		2D-1–2D-3
20	Russia (Moscow State Univ.)	FD(I)	Dmitriev, Barashkov		2D-0–2D-4 (HP)
21	Canada (Univ. Victoria)	IE(D) (TS)	Weaver et al.		2D-1–2D-3 (HP)
22	Canada (Univ. Alberta)	IE(D) (TS)	Weaver et al.	McKirdy	2D-0–2D-4
23	Canada (Univ. Victoria)	FE(D)	Kizak, Silvester	Weaver	2D-0
24	Russia (Moscow State Univ.)	IE(D)	Dmitriev, Mershikova		2D-0
25	Russia (Krasnoyarsk Computation Centre)	IE(I)	Baburina, Bersenev		2D-0–2D-5
26	Russia (Krasnoyarsk Computation Centre)	FE(I,D,SG)	Bogdanov		2D-0–2D-4
27	Russia (IZMIRAN)	FE(I,SG)	Varentsov, Golubev		2D-1, 2D-4

For a list of abbreviations, see Table 2.

the diagrams the results are presented in larger scale for selected coordinates. Each point has its own scale and the various results obtained after the rejection of outliers are denoted by different symbols. New average values, and the interval of root mean square errors δ_1 are given. It is easy to determine

Table 2
List of COMMEMI participants (three-dimensional calculations)

No. 1	Country (Organization) 2	Method of calculation 3	Authors of programs and calculations 4	Calculated models 5
1	Czechoslovakia (Inst. Geophys., Acad. Sci.), Russia (IZMIRAN)	IE(D)	Varentsov, Hvodzara	3D-1
2	Czechoslovakia (Inst. Geophys., Acad. Sci.), Russia (IZMIRAN)	IE(D,S)	Varentsov, Hvodzara	3D-1
3	Russia (IZMIRAN)	FD(D)	Zhdanov, Spichak	3D-1, 3D-2
4	Russia (MGRI)	FD(I)	Yudin, Ananevich, Veselovskiy	3D-1
5	Russia (MGRI)	FE(I)	Yudin, Veselovskiy, Ananevich	3D-1
6	Russia (Moscow State Univ.)	IE(D)	Dmitriev, Pozdnyakova	3D-1
7	USA (Univ. Utah)	IE(D)	Wannamaker, Hohmann et al.	3D-1
8	Canada (Univ. Victoria)	IE(I,TS)	Weaver et al.	3D-2
9	Germany (Göttingen)	IE(D,S)	Xiong, Schmucker	3D-1
10	Russia (UTE)	FD(I)	Druskin et al.	3D-1
11	Russia (Moscow State Univ.)	IE(D,S)	Yakovlev, Modin	3D-1
12	Russia (IO.AS)	IE(I,TS)	Weidelt, Palshin	3D-2
13	Canada (Univ. Alberta)	IE(I,TS)	McKirdy et al.	3D-2
14	Russia (IZMIRAN)	IE(I,TS)	Singer, Fainberg	3D-2
15	Hungary (Univ. Budapest), Russia (Moscow State Univ.)	IE(I)	Farzan, Dmitriev	3D-1

Abbreviations: A = analytic solution; FE = finite element method; TS = thin sheet approximation; I = iterative solution; FD = finite difference method; IE = integral equation method; S = stationary field approximation; D = direct solution; SG = solution on a sequence of grids.

from these diagrams the behaviour of the various solutions along the profile and to discern the group of solutions which are nearly coincident.

3.3. Packages of modelling programs

The methods for solving problems of modelling electromagnetic fields in inhomogeneous media described in Section 2 were only theoretical in nature. Concrete results are obtained with the aid of computer programs, and various practical realizations of the same theoretical method, in the form of a computer code, depend on several factors including the algorithm chosen, the numerical method and programming technique employed, and above all on the skill of the programmer. Therefore, when comparing algorithms in future we shall always refer to a particular program written by a specified author rather than a theoretical method.

The first programs for modelling in two dimensions based on the three fundamental methods—the method of integral equations (Dmitriev, 1969; Hohmann, 1971; Kaufman, 1974), the method of finite elements (Coggon, 1971; Silvester and Haslam, 1972) and the method of finite differences (Jones and Price, 1970; Jones and Pascoe, 1971; Madden and Swift, 1972; Pascoe and Jones, 1972; Červ and Praus, 1972)—were already in use at the very beginning of the 1970's. Since then, all three methods have been extensively developed, but not to the same degree for two-dimensional modelling, since programs based on the method of integral equations are now only rarely used in two dimensions. This is because of difficulties encountered in this method with complex geoelectric models, especially those that extend laterally beyond the limits of the host normal section. It should be noted, however, that our fundamental understanding of electromagnetic field distortions in inhomogeneous media were formed over the last twenty five years with the aid of programs of this type which are very effective in yielding results of high accuracy for simple bounded bodies of a local nature.

A wider selection of programs has been based on the method of finite elements. Such programs permit flexibility in model design with the use of triangular finite elements generated automatically in some cases. It becomes necessary in the finite element method, however, to solve linear systems with irregular structures. The increased complexity of the method of finite elements has meant that the development of general programs has been carried out by mathematicians who, having only a general knowledge of potential applications, do not always appreciate fully the effectiveness and convenience of such methods for solving particular problems in geoelectrics. Programs intended specifically for geoelectrics, which have usually been developed by geophysicists themselves, have tended to be coded only for rectangular finite elements (which are the simplest to handle) and have therefore failed to take full advantage of the flexibility in design offered by this method.

In widest use are finite-difference programs written especially for the solution of geoelectric problems. Because of their simplicity, accessibility and reliability, and the ease with which they can be modified to accommodate new features, the majority of COMMEMI participants favoured them.

For modelling in three dimensions, however, the distribution of methods used among the various participants was somewhat different with the greatest number of programs being based on the integral equation method. As already noted in Section 2.2, this method requires only that the problem be solved within the boundaries of an anomalous body, which enables one to use quite fine and accurate discretizations. Finite difference and finite element programs were used only rarely by participants in the three-dimensional part of the project.

The different modelling programs employed by the various participants in COMMEMI will now be discussed in detail.

3.3.1. Programs using integral equation methods

3.3.1.1. Two-dimensional programs. The first programs based on this method were intended for application to a narrow class of geoelectrical anomalies of simple shape (Dmitriev, 1969; Kaufman, 1974; Taborovsky, 1975). Without alteration these programs permitted only a limited set of model parameters to be varied, e.g. the diameter and depth of a circular cylinder, its electrical conductivity, and the conductivity of the host half-space or, in another example, the dimensions of a rectangular insert in a surficial layer, its electrical conductivity, and the electrical conductivities and thicknesses of three layers of the normal section. In the last case, if one wanted to include a four-layer normal section, it was necessary to rewrite the program. More flexible and universal programs did not appear until later.

First of all, attention is drawn to program $N24$ (Dmitriev and Mershikova, 1979, 1980). The applicability of this program has been considerably enhanced in recent years and it now affords the possibility of treating homogeneous inclusions embedded in an arbitrary stratified section.

A similar program is No. 9, which offers the capability of modelling several inhomogeneous inclusions in an arbitrary stratified medium. Finally, it should be mentioned that the only solution of an H-polarization problem by the integral equation method, for which a vector problem has to be solved instead of scalar one, was submitted by participant 19 (Flüche, Germany).

The complexity associated with the application of the integral equation method to various models with different normal sections has meant that the method has had limited appeal in two-dimensional studies. A more promising approach can be taken when the upper inhomogeneous part of the model can be approximated by a infinitely thin sheet or, more accurately, by a thin layer of finite thickness. This approach is represented by results 21 and 22 (Green and Weaver, 1978; McKirdy and Weaver, 1984). Thin sheet programs can effectively model fields over a quite complicated surficial structure of variable electric conductance, which may extend to infinity on the right and left sides of the model, not only at low frequencies but also in a wider band.

In all two-dimensional programs the discrete systems were solved by means of direct methods.

3.3.1.2. Three-dimensional programs. For three-dimensional calculations the majority of participants used the method of integral equations. With present computing facilities, however, it is still not possible to model real distributions of electric conductivity within the limits of this method and the programs have been used primarily for studying simple three-dimensional synthetic models which provide information on various three-dimensional responses and are also useful for testing methods of interpretation.

In such investigations it is possible to limit models to those involving local bodies of simple shape, for which the effectiveness of the integral equation method is not in question, especially if model symmetries are taken into account with a consequent and considerable simplification of the calculations.

Participants 1 and 2 presented two programs of three-dimensional modelling; one for the general case of an alternating field (Hvozdara, 1981; Hvozdara et al., 1987; Hvozdara and Varentsov, 1988) and another for the particular limiting case of a stationary field (Hvozdara, 1982, 1983, 1985).

In program 1, the analytical potential of the integral equation method was exploited to the full for those models comprising an inhomogeneous insert in a homogeneous half-space. In particular, the algorithm took into account the properties of the Green's operators and their volume integrals for a one-dimensional conducting half-space, thereby reducing the number of dimensions in which the numerical procedures were carried out. Thus it was possible with this program to undertake a detailed study of model 3D-1 with only moderate demands on computing resources. Algorithm 5 (Dmitriev and Pozdnyakova, 1989) resembles the aforementioned one closely, but is more universal since it can include an arbitrary layered structure for the normal section. The results of the earlier algorithm 15 (Dmitriev and Farzan, 1980) turned out to be very rough and were not considered. Number 7, a U.S. program (Wannamaker et al., 1984a,b) seemed to be more general, since it allowed for a non-uniform discretization of the inhomogeneity. Computations of the coefficients for the discrete system were based on a three-dimensional interpolation of previously tabulated values of the Green's operator. An arbitrary stratified normal section as well as the non-uniform structure of the insert can be accommodated in this program. The results obtained by participants 9 (Xiong et al., 1986) were obtained on a supercomputer and involved a maximum discretization of model 3D-1. In all of the aforementioned programs, symmetries of the geoelectric model were taken into consideration.

A second solution valid for stationary fields was submitted by participants 11 (Yakovlev and Modin, 1988).

Algorithms based on the method of integral equations applied to the class of thin sheet models, were employed by participants 8 (Dawson and Weaver, 1979; Agarwal and Weaver, 1987), 12 (Weidelt, 1977), 13 (McKirdy et al., 1985) and 14 (Zinger and Fainberg, 1985; Weaver, 1994). Results obtained with these algorithms were for model 3D-2, but their analysis lies outside the scope of this article.

Except for programs 9 and 14 the algorithms based on the integral equations used direct methods to solve the discrete linear systems. Program 1 provides an option of selecting both direct and iterative methods of solution.

3.3.2. *Programs using finite difference methods*

3.3.2.1. Two-dimensional programs. A considerable number of results obtained with two-dimensional finite-difference programs was submitted to COMMEMI. Some of them were produced by different versions of programs prepared by the same authors. Several algorithms, developed at different times, have similar structures and differ only in isolated but not very essential details. Results 2, 3, 4, 6, 8, 12 were obtained by means of such algorithms. They are characterized by the fact that the systems of difference equations are all solved by elimination methods which yield stable high quality calculations. The second main group of programs (7, 10, 11, 13–16, 20, 27) employed iterative methods of solution. The results for this group were very variable in quality.

Let us characterize the main programs briefly. Results 2–4 were obtained with a Canadian program (Brewitt-Taylor and Weaver, 1976; Weaver and Brewitt-Taylor, 1978; Weaver, 1986) which has been widely distributed around the world. The specific features of this program are—the systems of linear equations are solved by elimination, asymptotic boundary conditions of the first order are applied in the atmosphere and Dirichlet conditions of the first order on other boundaries, the method of differentiating of the finite difference solution for calculating the secondary components of the field takes into account the boundaries of separation between regions of different conductivity.

Result 6 was obtained with a Polish program (Tarlowsky, 1977) which also employed a direct method for solving the system of finite difference equations, and boundary conditions of the first order. The calculations were performed on very fine grids with the number of nodes exceeding 5000.

A rather old Hungarian program (Tatrallyay, 1978) was used to obtain result 7. It also employed the usual Dirichlet boundary conditions of the first order, while the difference system was solved by means of an iterative method of over-relaxation.

The same iterative method was also applied in program 10 (Varentsov and Golubev, 1980a,b, 1982; Zhdanov et al., 1982a,b). Asymptotic boundary conditions in the nonconducting atmosphere were used, as in program 2. Perfection of this program led to a new development labelled 8—a package of modelling programs called FDM2D (Varentsov and Golubev, 1982, 1985). This package features an effective treatment of the finite difference system by direct solution, as well as successive applications of asymptotic boundary conditions, including those in a homogeneous conducting or nonconducting basement (as well as in a non-homogeneous basement by taking into account the apparent electric conductivity at boundary points). The FDM2D package also offers the opportunity to plot the solution for both the total and the anomalous fields, to take into account the model symmetries, to minimize computation requirements by optimally enumerating the difference equations and to calculate accurately the derivatives of the discrete solution by taking into special consideration those points where possible breakdowns occur. This package was made available to all geophysical organizations belonging to the former Soviet Union.

Algorithm 27 (Varentsov, 1985) differs from algorithm 10 in only one essential detail: the direct method of solution is replaced by a hybrid method which combines the advantages of both direct and iterative methods. Here we use incomplete factorization of the matrix of the difference system, and then a conjugate gradient iterative scheme which provides for quick and reliable convergence of the process. This program also includes the option of performing the calculations on a converging sequence of grids.

Algorithm 11 (Vardanyanz, 1976, 1979, 1983) is very interesting and the program has been used by a number of researchers. Its main characteristics are the use of boundary conditions of second order (vanishing derivative of the normal field) on the side boundaries and impedance boundary conditions (of third order) on the lower boundary, as well as the method of solving the finite difference equations—on a converging sequence of grids using an iterative method of variable directions.

Algorithm 12 is based on the exclusion methods of Červ and Praus (1972, 1978) and Červ et al. (1984); it uses Dirichlet boundary conditions of first order. In program 13, developed in Germany (the former GDR), over-relaxation and Dirichlet boundary conditions of the first order are used. Algorithm 14 (Yudin (1981b, 1983) is widely known in the countries of the former USSR. It employs an iterative solution based on the Gauss–Seidel method and over-relaxation.

Dirichlet boundary conditions of the first order are dynamically corrected during iterations by the Schwartz method. The results are normalized by the values of the normal field. Calculation 14.1 was performed by running this same algorithm on another computer (IMB PC).

Calculation 16 is based on an old adaptation (Yudin and Kazanceva, 1977) of the well-known Jones program (Jones and Pascoe, 1971). The adapted program inherited many of the disadvantages of its prototype and in spite of its widespread use in the former USSR during the 1970's and early 1980's it is seldom used today.

Algorithm 18 (Schmucker, Germany) is characterized by the application of integral boundary conditions which reduce the regions of finite difference modelling, and by direct solution of the system of difference equations on a uniform grid with equal step sizes.

Program 20 (Dmitriev and Barashkov, 1969) has been devised specifically for the case of H-polarization. Here the difference equations with Dirichlet boundary conditions of the first order are solved by the method of variable directions.

3.3.2.2. Three-dimensional programs. Algorithm 3 (Zhdanov and Spichak, 1980, 1992; Zhdanov et al., 1982a,b; Spichak, 1983) is characterized primarily by asymptotic boundary conditions in the non-conducting atmosphere, a seven-point vector finite difference scheme and an iterative solution by the method of over-relaxation. It is similar in structure to the corresponding two-dimensional program and forms part of a powerful program package.

The results labelled 4 were obtained with the program of Yudin (1981b, 1983) and Vanyan et al. (1984). In this approach, as in the two-dimensional algorithm numbered 14, the simplest iterative solution is formed, and in the process of iteration Dirichlet boundary conditions of the first order are refined by the Schwartz method.

Finally, we consider the results labelled 10 (Druskin and Knizhnerman, 1987). Their method is unusual in that the calculations for a harmonic field are reached via a program for modelling non-stationary fields in a temporary area. A stabilized non-stationary solution for harmonic excitation of the model is considered. The spatial approximation of the operator for non-stationary modelling is performed by the method of finite differences. The vector differential equation in time domain is approximated in the specified time range by the Galerkin method and is solved by means of the Lanczos scheme (Druskin and Knizhnerman, 1994).

3.3.3. Programs using the finite element method

3.3.3.1. Two-dimensional programs. These programs were submitted to the COMMEMI project by six participants. A leading program in the category is number 23 from Canada (Kisak and Silvester, 1975;

Kisak et al., 1977). It represented a particular version of a general purpose finite element package (Silvester and Ferrari, 1990) with standard subroutines—description of the model, triangulation of the area of modelling, formulation and solution of the system of discrete equations. In this application triangular elements of high order were used, which provided accurate approximations with natural boundary conditions.

On the contrary program 17 from the United States (Wannamaker et al., 1986, 1987) is a specialized program specifically designed for the solution of geo-electric problems and it does not use standard software for application of the finite element method. Triangular elements are used; the approximation is carried out by the method of weighted errors with linear functions. Accurate calculation of the discontinuous spatial derivatives in the solution as well as the inclusion and analysis of topographic effects at the boundary between Earth and atmosphere are incorporated in the program.

A scheme with rectangular elements is also found in algorithm 15 (Yudin, 1981a,b, 1983), but the approximation is carried out on the basis of a variational principle with linear basis functions horizontally and exponential functions vertically. The solution of the discrete system is the same as for the finite difference program numbered 14. The other version of these calculations, 15.1, was obtained by adapting the same algorithm for running on another computer (IBM PC).

Triangular elements with linear basis functions are employed in program 5 from Finland (Kaikkonen, 1977, 1983). The solution of the discrete system is carried out by direct elimination and Dirichlet boundary conditions of the first order are used.

Algorithm 25 (Alekseev et al., 1986; Bersenev, 1988) is also characterized by choosing triangular elements with linear basis functions. Approximation is carried out by the Galerkin method and the linear system is solved by the method of over-relaxation.

The results of participant 26 (Bogdanov, 1988) were obtained by a program which differs from the previous one only in the way the system of equations is solved. Here the classic scheme of solution on converging grids was used with direct solutions obtained on coarse grids. The method of over-relaxation was employed for the required iterations.

The finite-element algorithms 15 and 17 are structurally very close to the finite-difference algorithms 6, 12 and 14, respectively.

3.3.3.2. Three-dimensional programs. In the three-dimensional case only one algorithm, number 5 (Yudin, 1981b; Vanyan et al., 1984), is based on the method of finite elements. Rectangular prisms were used with linear basis functions in the horizontal direction and exponential functions vertically. The approximation of the model was developed in the same manner as in its two-dimensional analogue 15. Dirichlet boundary conditions of the first order were corrected during the process of solution by over-relaxation using the Schwartz method as in the three-dimensional finite difference method 4.

4. Comparison of 2D results

4.1. Model calculations

4.1.1. Participants

The set of two-dimensional data comprises 27 results submitted by participants from 10 countries. While most of these calculations were performed by the authors of the algorithms themselves, in several cases the calculations presented were performed by other researchers some of whom were, in fact, authors in their own right of different algorithms. Only a few participants submitted calculations for the entire set of two-dimensional models, the majority limiting themselves to individual calculations.

To make the assimilation of the material more convenient, essential information on the participants is given in Table 1, in the following format:
1. participant number, which serves as an identification of the results;
2. country and institution where the calculations were performed;
3. method of calculation and the symbol used to identify the result on the diagrams in Appendix C; the method of solving the system of linear equations is shown in brackets;
4. author of the program used and, if they are not the same, the name of the contributor who performed the calculations;
5. list of models for which results were submitted.

The characteristics of the various algorithms have been described in Section 3.3.

Participants from the former USSR and Canada are featured most widely. The spectrum of methods used is very wide. Five algorithms are based on the method of integral equations, three of them suitable for arbitrary models, and two for use in the thin sheet approximation. Six submissions used the finite-element approach. The most popular method turned out to be the finite-difference method for which there were fifteen contributions.

The results are collected together by components in various tables which are enumerated successively and presented in Appendix B. These tables also include information on the coordinates along the profile. Appendix C contains diagrams and plots of the most interesting field components.

4.1.2. Analytical model

4.1.2.1. Model 2D-0. This model is a single slab consisting of three segments of different electrical conductivities lying on the surface of a perfect conductor as shown in Fig. 1a. The field in this model is calculated for one period ($T = 300$ s) on the Earth's surface and also at the depth of 15 km (Tables B.1–B.5). The complications that arise in the calculations for this model are determining the values of field components over the vertical boundaries separating the three homogeneous segments and in handling the perfect conductor at the base of the model.

The results, some of which were published earlier, were contributed by ten participants. There is one analytical (quasianalytic in case of EP) solution (1), one algorithm (22) using the method of integral equations (approximation with thick layer), four finite-element calculations (17, 23, 25, 26) and four finite-difference calculations (3, 6, 18, 20). Three participants (20, 25, 26) used iterative methods for the solution of the system of linear equations; the others all used direct methods.

In the case of E-polarization, eight results were presented, all of them with solutions for both the Earth's surface ($z = 0$) and for a depth of 15 km. The comparatively small number of contributors for this model is explained by two factors: first, the model was included in COMMEMI considerably later, when many of the most punctual participants had already submitted their results; second, the model has different geoelectric sections on the left and right which poses difficulties for the method of integral equations.

Let us consider the calculations as presented. Component E_y has been calculated by all algorithms without noticeable deviations from the standard solution (1), as is well seen on the plot in the upper part of Fig. 6. The dashed lines on this plot connect the maximum and the minimum values for each

Table 3
Correspondence between participant numbers and the letters used in the diagrams in Appendices C and E

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

point of comparison. Although the maximum and the minimum values may belong to different solutions at the various points, on average they predict a rough estimate of the deviations in solutions. The three continuous lines display the mean value E_y for all sections and the error interval δ_1 (see Section 3.2.3) on either side of it. They give a qualitative picture of the root-mean-square range of the solutions. Specific information about the distribution of various solutions is found on the diagram in the lower part of Fig. 6. The results for a number of comparison points are denoted by letters which correspond to the different participants according to the key given in Table 3. The mean value and error interval δ_1 are given there also. Reference to the lower diagram suggests that some of the data (1, 3, 8—symbols A, C, H) are almost completely confined to points near the mean value, and that data (23) (symbol W) are consistently lower, but by a small value. The deviation of calculation (22) (symbol V) from the mean value cannot be predicted as some points lie above the mean value and some below it. It should be noted that values which do not lie in the range $2\delta_0$ are not shown on the diagram.

Complete information about the behaviour of E_y may be obtained from Table B.1 where all the separate real and imaginary parts of the submitted results are tabulated. Following the field values at each point the mean value and standard deviation δ_0 are recorded. While calculating the statistics of the data, all results differing from the mean value by more than $2\delta_0$ were discarded for a subsequent calculation of the statistics on the remaining limited sample. Thus, a marked difference between δ_1 and δ_0 indicates the presence of considerable scatter in the results.

An analysis of the tabulated results suggests a possible overestimation of the imaginary part of E_y in algorithm 18 (Table B.1) at the points of horizontal discontinuity in the electric conductivity, but it is almost invisible on the diagram itself.

Errors in the absolute value of the component E_y at a depth of 15 km are nearly the same as those on the surface with all the algorithms yielding similar results. This testifies to the absence of fundamental errors in the approximation of the boundary conditions on the lower boundary.

The magnetic field components in this polarization are characterized by a great range of values. The component H_x has standard deviations δ_0 which are of an order higher than those of E_y. Even after the rejection of highly outlying values, the error remains large. Sometimes three results deviate at the same time from the standard solution and δ_0 is so big that outlying data are not rejected for the repeated statistical analysis. Most of the deviations from the mean are observed in algorithms 25, 26 and more rarely 23.

The range of differences between the values obtained for the vertical magnetic component H_z has almost the same magnitude as for H_x it being especially great at the points on the vertical conductivity boundaries (Fig. 7). At these points H_z reaches its extrema, and their amplitudes very much depend upon the method of numerical differentiation used and the grid spacing around the boundary under consideration. The majority of results fall below the analytic solution (how far depends on the quality of numerical differentiation) and only the integral equation method of algorithm 22 gives values exceeding the analytic ones. Apparent resistivities are close over the whole profile (see Fig. 8), excluding the very left side.

In the case of H-polarization the range of the different values of E_x among ten results is very small (around 0.005). A considerable deviation at the left end of the profile is seen in the results generated by algorithm 23 (Fig. 9). Large deviations from the standard solution arise only in algorithm 25. The values of the apparent resistivities simply confirm their expected physical behaviour.

In summary, the following conclusions may be drawn about model 2D-0. Several algorithms (3, 8, 17, 20) give results which are very close to the standard solution 1. Algorithms 18 and 22 have moderate deviations at some points. In H-polarization algorithm 23 gives significant deviations associated with errors in the calculation of H_y in the Earth near the left boundary of the modelling region (evidently due to the boundary conditions applied there). Algorithms 25 and 26 have multiple deviations from the standard solution, which most probably arise with the differentiation of the discrete solution.

4.1.3. Models with simple geoelectric structures

4.1.3.1. Model 2D-1. This model has the simplest geometrical shape. It comprises a symmetrical rectangular insert embedded in a homogeneous half-space as shown in Fig. 1b. Calculations for this model were carried out for two periods, $T = 0.1$ s and 10 s on the Earth's surface and along two lines intersecting the insert at different depths inside the Earth. Results were received from twenty participants, seven calculations being presented for the two levels inside the Earth. Three algorithms (9, 19, 22) are based on the method of integral equations, five use finite elements (15, 17, 23, 25, 26), and twelve use finite differences (2, 3, 6, 8, 10–14, 18, 20, 27). In seven cases out of twenty, the algorithms used iterative methods for the solution of the linear systems; the others used methods of direct elimination. It is of interest to note that the values of δ_1 calculated after rejection of the outlying results, show negligible decrease (Table B.6) which is explained by the small number of participants.

For the E-polarized field at period $T = 0.1$ s the value of δ_0 for the component E_y is always less than 0.022; considerable deviations from the mean result were obtained by participants 10, 18 and 24. Algorithm 10 uses an iterative solution and the decreased accuracy is explained by an insufficient number of iterations and a rough grid. A similar effect in algorithm 24 is probably caused by the rough approximation of the anomalous body (4×6 elements). In this case the skin-depth turns out to be less than the cell dimensions in the insert. For the longer period, the results calculated with this algorithm more nearly coincide with those found by the other participants. In the results presented by participant 18, the anomalous E_y is too high and this is evidently associated with the strong influence exerted by the side boundary where integral boundary conditions are used. Errors in H_x are roughly double on an average. Since algorithms 14 and 15 used a nonstandard normalization of the field amplitude with respect to the normal field, it is only possible to compare them by apparent resistivities. Results 25 and 26 appear to have considerable errors as in the case of model 2D-0. Discrepancies in H_z are lower and δ_1 does not exceed 0.03. Deviations from the mean value in the H-polarization results are generally prevalent but only sharp deviations are rejected (Table B.7). We refer here to the results 6, 18, and 26 over the centre of the insert. The E-polarization apparent resistivities show a large spread. The minimum values of apparent resistivity over the insert are given by algorithms 14 and 18, the maximum ones by 10 and 24.

The picture changes significantly for the period $T = 10$ s. The field penetrates into the insert and becomes horizontally more smooth, so that the mechanism producing the errors becomes different. In the case of E-polarization, the maximum deviations in E_y are observed in algorithms 10, 13, 15, 25, and 26 which solve the system iteratively (Table B.9). However, iteration is not in itself a cause of errors, because the interactive solution (14), which continued for a long time, or those which employ special methods (11 and 27), yield results that cannot be distinguished from those obtained by direct methods. Errors in the magnetic components of the field are mainly inherited from the grid-generated solution. The greatest deviations here are also associated with those algorithms which employ an iterative solution (see Table B.9). Errors noticeable in the field components are somewhat smoothed out in the apparent resistivities although a number of them, specifically 14, 15, 25, 26, do deviate from the mean values quite considerably (Table B.11).

On the whole, the H-polarized fields in model 2D-1 caused less trouble for the participants. Differences in the results at depths are much bigger than those obtained on the surface, and their number varies from eight to three. Thus, it is only marginally possible to rely on the statistical estimates. These results are presented in Tables B.12–B.15.

4.1.3.2. Model 2D-2. In this model, depicted in Fig. 2a, calculations of 16 participants are presented. Among them are four algorithms using the method of integral equations (19, 21, 22, 24), three using the method of finite elements (15, 25, 26), and nine using finite differences (3, 6, 8, 10, 12, 13, 14, 18, 20). In eight of these algorithms iterative methods of solution were employed.

Modelling was carried out for the two periods 10 s and 1000 s. In E-polarization and with $T = 10$ s, the values of δ_0 and δ_1 for the component E_y are higher than those for the previous models, i.e. about 0.02–0.03 (Table B.16). It is difficult to draw firm conclusions from the mean solution because the individual results do not fall into concentrated groups, although only a few particular algorithms give solutions outside the confidence range of $2\delta_0$. Since the magnetic components do not have large amplitudes in this range, the small absolute values of the errors are not in themselves proper indicators of the scatter in the results. The number of apparent resistivities included in Table B.17 is more complete with the addition of algorithms 5 and 7 and contributions from participants 14, 15, 16, all of which had been omitted in the comparison of the magnetic components because of nonstandard normalizations. The results are distributed quite uniformly within the limits of the confidence interval. The H-polarization anomalous fields are small and errors in them reach 25% of the maximum anomaly. The apparent resistivities on the other hand, coincide quite well but only eight results are included (Table B.17).

For $T = 1000$ s some researchers also used algorithms based on the thin sheet approximation applied to the upper inhomogeneous part of the model. The majority of such algorithms give very similar values of E_y with only algorithm 21 differing noticeably because of an extremely crude and inappropriate approximation of the anomalous region by a surface rather than a buried thin sheet, which essentially results in the electric field being overestimated. Another group of participants (16, 25, 26) have underestimated the value of electric field (Table B.18).

The magnetic field components are characterized by large errors, reaching 0.14. The thin sheet algorithm 21 is also very noticeable here, and a number of other algorithms overestimate the values of H_x and H_z (13, 25, 26). When calculating apparent resistivities the errors are smoothed out and are not big (Table B.20). It may be noted from Fig. 14 that the results of several algorithms are grouped together at the majority of points in the profile (code names C, F, J, K, L, N—participants 3, 6, 8, 9, 11, 12). The other algorithms give differing results, many of them considerably smaller at all points.

For H-polarization the spread of values is very great because of algorithm 21 (Fig. 15). The curves describing the maximum and the minimum values differ a great deal from the mean and exceed the error considerably. The same picture is apparent in the results for apparent resistivity. The maximum deviations in the results for this model arise in some algorithms because of the decision to use an iterative solution. The influence of the approximation of thin layers of high conductivity contrast cannot be neglected either.

4.1.3.3. Model 2D-3. This model, shown in Fig. 2b, comes in two versions (A) and (B). In (A) the electric conductivities of the surficial layer on the left and right sides are equal, while in (B) they differ. Calculations were carried out for periods $T = 100$ and 1000 s. Seventeen results are presented, three of which (19, 21, 22) were obtained by the method of integral equations, four (15, 17, 25, 26) by the method of finite elements and ten (2, 3, 6, 8, 10, 11, 12, 13, 16, 18) by the method of finite differences. This model is one of the most effective ones for making comparisons. Let us consider the two versions separately. In version (A), for *E*-polarization and period $T = 100$ s, the errors in component E_y do not exceed 0.01 or approximately 2% of the anomalous field (Table B.21). The magnetic component H_x has errors of the same order and only at a few points, −25, −20, −15 km, is the error larger. Component H_z has large errors, most notably at the points −20 and 0 km, where vertical boundaries between regions of different electrical conductivities are present. Over each such boundary, H_z attains an extremum whose magnitude becomes greater with increasing conductivity contrast.

It is evident that the smaller the grid spacings near the boundary the greater the accuracy in the calculated value of H_z. To evaluate qualitatively the influence of grid spacing on the value of H_z the results of algorithm 8 are entered in Table B.24 for two different spacings (Fig. 16)—version 8.4 with gridpoint separations of 2 km near the contact and version 8.5 with spacings of 0.15 km. It is clear that the finer spacings bring the values of H_z closer to the results presented by participants 2 and 3 whose method of numerical differentiation accounts more accurately for the different resistivities on either side of the contact. In this example the statistical characteristics only indicate where the results have maximum divergence but they are unable to reveal the correct values because of the asymmetrical distribution of data. The values of apparent resistivities are obtained with such relatively small errors (Table B.24) that it is impossible to separate the participants (Fig. 17).

The model is no less interesting in the *H*-polarization mode. When discussing model 2D-0 we touched on the problem of calculating the electric field directly over the vertical contacts where E_x has a discontinuity. All participants used one-sided derivatives with that theoretical model. For model 2D-3, however, there are other variations: some participants presented both one-sided derivatives, while others presented only one of them either on one particular side or the maximum one of the two (Fig. 18). The remainder of the participants calculated the derivative over the contact in the same way as at normal points, thereby obtaining an average result. In Table B.22 the values of E_x are tabulated for most of the points of comparison. The greatest errors are observed over the vertical contacts (Fig. 19). All the values of E_x (both one-sided and average values) over the contacts have been brought together in a special Table B.23 for comparison. The apparent resistivities for this polarization coincide well also (Table B.24), with a greater number of participants having calculated them.

The number of results at longer periods increases with the additional appearance of two thin sheet solutions. Even so, the thin sheet conditions are barely satisfied at this period for the application of algorithm 21, but the model is well-suited for the more accurate program used by participant 22 in which the thin sheet is allowed to have some vertical structure. The submitted results reflect this difference. The apparent resistivities in *E*-polarization agree well across the whole profile, with discrepancies existing only for participants 21 and 26 (Table B.26). For *H*-polarization, the results of participants 13, 16, 21 and 26 differ quite considerably.

In the second version—model 2D-3b—the resistivity of the block on the right of the surface layer is changed to 1000 Ω m. Only nine participants submitted results, less than for model 2D-3a. However, the results are of high quality, with only some deviations evident in those given by algorithm 21 and at some points by algorithms 25 and 26. For the period of 100 s, components E_y and H_x (Table B.27) exhibit very good agreement, in which the differences between the five algorithms 3, 8, 17, 18 and 22 do not exceed 0.02, and only those results associated with algorithms 25 and 26 have consistent deviations. The discrepancies are more pronounced in H_z over the vertical contact, just as in version 3a. The apparent resistivities are also very similar (Fig. 20). Results from the H-polarization calculations are very good as well (Fig. 21). At the period of 1000 s the same general trends hold except that the results produced by algorithm 27 now contribute to an increase in error. The results are given in Tables B.31 and B.32.

The model under consideration here is particularly informative about the accuracy of the methods for differentiating the solution obtained on the grid. It becomes very evident that the magnetic components derived from E_y in an E-polarized field are dependent on the grid spacing near the Earth's surface. Basic errors in the component H_x arise as a result of the nonlinear behaviour of the electric field in the conductor due to exponential attenuation. This effect is mostly noticeable at short periods and is further confirmed by a decrease in the error of H_x relative to E_y at longer periods for which the exponential attenuation is reduced. The main method of overcoming such errors is by decreasing the vertical grid spacings near the surface and using differentiation procedures which take into account the exponential behaviour of the field. The horizontal spacing of the grid near vertical conductivity boundaries is of greatest importance in calculating the H_z component. The departures of the values at these points are very large, and the methods of combating the errors are much the same —designing fine grids and using specialized formulae which take into account the discontinuity of electrical conductivity at the boundary.

In the case of H-polarization the vertical conductivity boundaries exert an even greater influence. Straightforward central differences at the boundary points are the least accurate derivative formulae to use because they give an averaged result, and this average depends upon the scheme for computing the difference coefficients (and consequently upon how the electrical conductivities are averaged) and on how the grid is designed. When taking one-sided derivatives it is useful to calculate both derivatives or the maximum one of the two, since in cases of high conductivity contrasts the smaller derivative may have considerable errors.

The difficulties encountered in estimating the statistical parameters for the results of this model calculation are very apparent from a visual inspection. First, we see that the distribution of results for component H_z are asymmetrical, because all the errors have the same sign. The values δ are considerably distorted and can therefore only provide a qualitative estimate of the spread since the true solution is obviously displaced from the mean one. These distortions are aggravated by the presence of some algorithms with sharply differing results, namely 21, 25 and 26. In fact, δ_0 is so great, that not all of the poor results are rejected, and the final value δ_1 does not reflect the real accuracy of modelling. Under these circumstances it is possible to isolate a number of algorithms whose results agree with each other to a very high degree of accuracy and the majority of which also performed well when compared with the analytic solution for model 2D-0. Generally these are the algorithms (3, 6, 8, 11, 12, 17, 22) which solve the linear system by a direct rather than a iterative method. The similarity of their outputs allows one to select the most probable solution even for models for which there are no reliable statistics or in which even up to one-half of the solutions are substantially different.

4.1.4. Complex geoelectric structures

Prior to a detailed comparison of the results we comment briefly on the set of calculations received for complex geoelectric structures. In model 2D-4, sixteen submissions were received (Table 1). Twelve of them were obtained by finite-difference programs, three by the method of finite elements, and one by the method of integral equations for which the model was represented by a thick inhomogeneous layer. In seven cases the systems of linear equations were solved by direct methods, the rest by iteration. For model 2D-5 only nine sets of calculations were submitted, seven of which were by the method of finite differences and two by finite elements. In seven of these submissions, direct methods were used for the solution of the system of linear equations.

Thus, fewer submissions were received for models with complex geoelectric structures (especially model 2D-5) than for the earlier simpler models (2D-0–2D-3), a fact that is readily explained by the complexity of model design required. It should also be noted that not all contributors submitted complete sets of calculations; some of them limited themselves only to one of the polarizations, while others sent in calculations for selected periods and points along the profile. A number of submissions were excluded from the analysis because, in one way or another, they did not meet the defined project format (e.g. magnetic fields were presented by participants 14 and 16 in nonstandard form). The smallness of the sample of calculations hinders their statistical analysis somewhat, but since a large proportion of them are finite-difference calculations coupled with a direct solution of the resulting system of linear equations which performed well on the simple models, it is still possible to carry out an analysis with some confidence. The results of calculations for these models are presented in Tables B.33–B.41 and in Figs. 22–25.

The calculations for these final two 2D models reveal some specific problems which cause additional errors that are not present in the simple models. The basic problem is the complexity of the discrete representation of the geoelectric structure. Even in model 2D-4 (see Fig. 3) which is not an unduly complicated one, the errors associated with the geometry of the structure (especially the inclined boundaries) are superimposed on the usual errors due to other sources. To describe complex models the researcher has to use grids of large dimension, which often exceed the capabilities of the computational facilities available. In this event the model must be simplified or the region modelled must be reduced, either choice leading to the introduction of new modelling errors. The rate of convergence is sharply decreased and the time taken for one iteration is longer, when iterative solutions of the large systems of linear equations arising from complex structures are used. It is therefore extremely difficult for inexperienced users of such programs to obtain a precise result by iteration.

4.1.4.1. Model 2D-4. Let us consider the differences apparent in the calculations for model 2D-4. Modelling was carried out for both polarizations at the periods $T = 1$, 9 and 100 s. The best agreement is reached with the primary field component E_y in E-polarization at the period $T = 1$ s where the error is only 1–2%, but as the period increases up to 100 s, this error in E_y grows up to 5–6%. In the case of the secondary fields this value reaches 5–10%, and for the secondary components H_z and E_x the error exceeds this value at a number of points.

Errors in the apparent resistivities were within the limit of 10%, and were greater for the H-polarization calculations than for those in E-polarization. It should be noted that the errors are most apparent in the region of the vertical contact at $x = 6$ km (especially for H-polarization at period $T = 9$ s) and also in the region of the inclined boundary at $x = 2$–5 km. The former errors are connected with the sharp discontinuity in the electric field across the boundary separating the two regions of strongly differing conductivities (the influence of this boundary is most noticeable at the Earth's surface for the period of $T = 9$ s); the latter are associated with the different representations of the inclined boundary by various researchers. Considerable errors often appear on boundaries in the model which are principally attributed to non-precise boundary conditions.

Let us now consider in detail the results obtained by the various programs. Good agreement was given by programs 3, 4, 6, 8, 11, 12, 14 (FD), 22 in practically all sets of calculations, and these results were taken as the standard for comparison purposes. Results 13, 19, 20 gave large errors only in some examples, while the rest differed from the main group in many of the results.

The largest errors present in the calculations are from program 26. The character of the distribution of these program errors (considerable systematic deviation along the whole profile, including the boundaries of the modelling region, and approximation to the main results at the turning points) testify to the presence of two main causes of errors. The first, which is typical for finite-difference algorithms, originates with the use of non-precise boundary conditions, and gives errors on the boundaries of the modelled region; the second is the use of an insufficient number of iterations for solving the system of linear equations. Evidently the method of converging nested grids, with only a few iterations on the original rough grid, is not wholly effective. Typical shortcomings of programs 25 and 26 are evident in the errors of calculations for the derived fields at some neighbouring points on the profile where equal values are obtained which most probably indicates a very crude approximation over large elements.

Significant errors caused by an insufficient number of iterations are typical for results 10, 14.1 (FE), 16, 25 and to a lesser degree 13 and 20. Non-precise boundary conditions account for inaccuracies near the model boundaries in results 14.1 and 25. Failure to model the geoelectric structure adequately is especially noticeable in the finite-difference results of participant 18. This is possible because the errors are mainly observed in the region of the vertical and inclined contacts against a background of very insignificant errors from other sources. It is also observed that in the application of program 18 the right edge of the modelling region was not always placed sufficiently far away so that the boundary conditions turned out to be inaccurate. It should be noted once more that while calculating the field H_z (values in Table B.35 are corrected) the signs were changed occasionally in program 11. At the same time it is the only one of the above mentioned iterative programs in which mistakes due to a small number of iterations are not present.

In summary, analysis of the calculations for model 2D-4 made it possible to identify and classify the errors of widest scatter and thence to plot a satisfactorily narrow band of 'best coincidence' for all sets of calculations.

4.1.4.2. Model 2D-5. In this model (see Fig. 4), the complexity of the geoelectric section influences the accuracy of results to an even greater degree. As a rule an error is increased by 1–3% compared with model 2D-4 and at some typical points, for example $x = -50$ km, it may reach much more. The results of calculations by programs 3, 6, 8, 11, 17 are taken as a standard since they agree well.

Program 12 gives small errors in some cases, but the results obtained by programs 4, 18 and 25 look worse. As we mentioned, the differences in model design are especially apparent at such points as $x = -50$ km in the H-polarization model. Around this point a surface anomaly of very small dimensions and large contrast of electric conductivity is inserted. Thus it is not surprising that there is a disparity between the values of E_x around this point as calculated by the various researchers. The greatest departures at this point belong to results 4 and 18. Evidently the basic errors in programs 4, 12 and 18 are explained simply in terms of shortcomings in the model design which are, in turn, a consequence of insufficient grid dimensions. The errors in 25 are due to non-precise boundary conditions and an insufficient number of iterations as in model 2D-4. On the whole the results of calculations for model 2D-5 may be considered satisfactory and suitable for producing standard curves against which other modelling programs can be tested.

Summing up the results of our analysis of calculations for complex models, we note that the quality of results turned out to be much better than might have been expected. Two reasons are offered in explanation: the most effective programs employed direct methods of solution for the system of linear equations in general, and the participants paid close attention to model design and exploited the potential of their computing facilities to the full.

4.2. Analysis of results

4.2.1. Typical misfits of results and their sources

With the exception of occasional mishaps such as faulty input–output information, failure during the solution of a program, small programming errors, etc., all errors may be subdivided into three main groups:

1. those introduced when setting up the modelling program;
2. those arising in the discretization of the program;
3. those associated with the solution of the discrete program.

Random errors are not subject to systematic study and therefore will not be discussed here.

Errors in the first group can arise when certain assumptions and simplifications are introduced. One such example is provided by algorithm 21 in which the conducting medium is approximated by a thin sheet. Since the thin sheet conditions are not properly satisfied by models 2D-2 and 2D-3, errors of type (1) are bound to occur. Another assumption in differential equation methods is how the boundary conditions are determined. Usually the 1D solutions on the side boundaries of the model, or equality of the normal derivatives to zero there, are used as boundary conditions. The validity of such conditions is evidently only exact when the side boundaries are at infinity. Inaccurate prescriptions of the boundary conditions may cause considerable deviations of the calculated results from the true solution.

Errors in the second group arise when coarse subdivisions of the region into discrete elements are used. Several factors are involved here:

- Approximation of the inhomogeneities. Too small a number of elements decreases the field anomaly. Thus in simple models, too rough a subdivision turned out to be an essential source of error, even if applied only to the row of elements (cells) on the boundary of the anomalous region.
- Presence of considerably elongated elements. Such elements usually appear towards the boundaries of the grid. To satisfy the boundary conditions more exactly, the edge of the grid should be well removed from any inhomogeneities, a condition which is usually fulfilled by means of a progressive increase in the spacing of the grid points out towards the edge. Thus, the rectangular cells at the level of inhomogeneities become very elongated. Since coefficients in the system of linear equations are proportional to the square of cells' sides, the conditioning of the matrix of the linear system deteriorates due to the elongation of the cells. In iterative methods of solution, the field anomaly behaves as though it is not spread across the short sides of the cell.
- Derived fields. A rough subdivision of the grid near the Earth's surface introduces errors during the calculation of derivatives. Even if the accuracy of the horizontal derivatives is not affected by the grid spacing, the vertical derivatives may still be disturbed since they are also sensitive to the electric conductivity of the surface layer. For highly conducting surface material, more accurate subdivisions are required.
- Approximation of inclined boundaries. In complex models inclined boundaries cannot be represented properly by rectangular subdivisions of the geoelectric structure.

It should be noted that in practically all these cases, the same problems also arise if the region is divided into triangular elements.

The third group of errors arise mainly in the application of iterative methods of solution. The majority of iterative methods used in geoelectrics are not equipped with sufficiently reliable stoppage criteria so that the number of iterations performed is usually limited by some technological or economic factor. Thus, many algorithms (10, 13, 15, 25, 26) do not take the iterative process far enough to solve complex problems, and produce a solution which has not converged to a steady state. The most successful methods seem to be applications of more detailed iterative schemes (27) and the organization of iterations on a nested sequence of grids (11, 25, 26). In general the most common cause of modelling errors is the early cessation of an iterative solution, while second in frequency of occurrence is a poorly chosen representation of the modelled structure by discrete elements. Other types of error have considerably less impact compared with those occurring in these two classes.

Several algorithms (3, 6, 8, 11, 12, 17, 22) which are based on different approaches and methods of solution yield results which are in close agreement for all models. It seems evident, therefore, that they define the actual solution to within the error bounds specified by the standard deviation of this set of results. The results from this group of algorithms testify to the fact that all approaches are feasible if they are properly realized and are implemented by well-qualified specialists. A next step in the comparison of the various programs is to check their effectiveness and workability in the hands of researchers who do not have a great deal of experience in modelling. At this stage of the investigation it would be appropriate not only to compare the 'user-friendliness' of the programs themselves but also the documentation provided for the general user.

In conclusion let us note that algorithms 25 and 26 gave results which are considerably worse than those expected on the basis of errors in program realization alone, and that algorithm 21 also did not perform well because, despite the guidelines circulated to the participants in COMMEMI, it turned out that the use of thin sheets was completely inappropriate for those two models where it was suggested, with a resulting exaggeration of the field anomalies. However, algorithm 22 which models a surficial inhomogeneous layer of *finite* thickness, yielded results which compare well with the best calculations provided by the more rigorous and universal methods.

4.2.2. Main problems in 2D modelling

There are two main concerns which should be considered separately from all the other problems arising in 2D modelling. They are the absence of precise criteria for evaluating modelling accuracy, and the great number of diverse modelling programs which have been developed by various researchers in an uncoordinated manner. It is known, for example, that for historical and economic reasons, several tens of programs for geoelectrical modelling have been independently produced in many different scientific and industrial research centres throughout the world. More than ten such programs exist in the former Soviet Union alone. The absence of analytical solutions and other criteria for testing the accuracy of the majority of these programs meant that individual researchers considered their solutions to be the correct ones, even though, in some cases, they differed from others. A result of the completion of COMMEMI is that a set of various standard test results (including rather complex models) has been obtained, which allows the adjustment of existing and new electromagnetic modelling programs and the determination of their effectiveness. Several shortcomings in existing programs have already been exposed in the course of this study. But on the other hand, there were authors of specific algorithms and programs who submitted excellent material for analysis.

Let us now discuss the less significant difficulties encountered in two-dimensional modelling. One of them was caused by the lack of adequate computational facilities for the task at hand. Reasonably fast computers with sufficient memory were not available in all the research centres with the result that it was not possible to simulate properly some of the complex structures; the consequent necessary simplifications of the models introduced undesirable errors. One solution here has been the creation of processing centres such as ILONEM in Oulu, Finland where powerful computers are available, although recent advances in desk-top computing power will eventually eliminate this particular problem. Another problem is the unreliability of solutions generated by programs which use iterative schemes that stop iterating before proper convergence is achieved. For now it is evidently simpler to abandon all use of iterative schemes in favour of direct methods of solution rather than to persevere with the search for reliable criteria for stopping the iterative procedure.

The problem of determining precise boundary conditions should also be noted, as we have already seen that non-exact boundary conditions can be the cause of considerable errors. Since the simplest boundary conditions are almost always inapplicable in geoelectric modelling, authors of programs are forced to use more complicated conditions (asymptotic, integral, etc.). Great attention should be paid to this problem when modelling programs are developed.

The fact that different users often obtain different results even when applying the same program is usually explained by the level of experience of the user, the care put into the design of the discrete model, and the choice of regimes and parameters for numerical calculation.

This section is concluded with the recommendation that the leading modellers in the field of electromagnetic induction be urged to cooperate closely on the most effective way of eliminating the problems alluded to here.

5. Comparison of 3D results

5.1. Model calculations

5.1.1. Participants

The set of results for three-dimensional models is much more limited than those for two-dimensional models, and the total number of participants listed in Table 2 is only half the number who took part in the 2D calculations. For the comparison project only a very limited program of testing (Section 3.2) was offered since three-dimensional modelling is still extremely demanding on computing resources. Nevertheless virtually no one was able to complete all the tests, and many participants presented only single results.

In the end it was only possible to compare the various methods and algorithms in a fairly presentable manner and to draw reasonable statistical conclusions for the first and simplest model 3D-1. The largest number of results included here were submitted by scientists from the former USSR, and Czechoslovakia. Analysis of test calculations for the more complicated model 3D-2 was abandoned because of insufficient results and the discrepancy between them.

In the aggregate of methods in Table 2 one class of algorithm dominates—that based on the method of integral equations (see Section 2.2). Algorithms of this class were used by six participants to solve model 3D-1, four of whom used the method of volume (vector) integral equations for alternating magnetic fields. The four algorithms in question are 1, 6, 7 and 9 (by analogy with the practice in Section 4 we shall continue to refer to the results obtained by the various participants and programs according to the code number of the participant indicated in Table 2), which are very similar in their structure. The first two algorithms, which employ a homogeneous discretization of the geoelectric inhomogeneity in each direction, are closely related. Algorithm 7 differs from them mainly because an inhomogeneous scheme of discretization is used (see Section 2.2.1). The first three algorithms (1, 6 and 7) apply direct solvers to the system of linear equations, while the fourth one (9) utilizes an iterative method of solution. Two other algorithms (2 and 4) are based on the method of surface integral equations for stationary electromagnetic fields (see Section 2.2.2). We note in passing that the application of the method of integral equations to model 3D-2 was even more varied with participants 8 and 12–14 using different variations of the thin sheet version of this method. In the majority of calculations made with the integral equation method, the symmetry in model 3D-1 was exploited.

Calculations performed with algorithms based on differential equation methods, which dominated the 2D part of COMMEMI, were presented in the three-dimensional comparison by only four of the participants. Nevertheless they demonstrated a very wide range of approach. For example, the finite difference schemes submitted by participants 3–5 are markedly different. Even more specific, and difficult to compare with the previous ones, is method 10, which is also based on finite differences.

A summary of those results for model 3D-1 received before June 1, 1989 is presented in Appendix D (Tables D.1–D.24) and Appendix E (Figs. 26–35). The format of the tables and figures is similar to that used in Section 4. Three-dimensional calculations are also presented for two field polarizations as in the two-dimensional case, but there are now a greater number of field components to be compared along the two coordinate axes (see Section 3.2). The reader is reminded that the participant number in the tables coincides with the number given in the list of participants. Where a participant submitted several versions of results obtained with the same algorithm, the number of the calculation is specified by an additional digit appearing after the participant number and separated from it by a decimal point (for example, 1.2 indicates participant 1, version 2). The symbols on the figures in Appendix E are related to the participant number according to Table 3.

5.1.2. Model 3D-1 (variant A)

This model is a prismatic conducting insert in a homogenous half-space (see Fig. 5a) as described in Section 3.2. Its section in the plane $y=0$ coincides with the two-dimensional model 2D-1, investigated in Section 4.1.3. Calculations for this model were carried out along the two coordinate axes on the Earth's surface for two periods ($T=0.1$ and 10 s) and for two different lengths of the insert parallel to the axis Oy (Fig. 2). Even though the structure of this model is very simple, it is characterized by a large contrast in electric conductivity ($\sigma_1/\sigma^n=200$) and the nearness of inhomogeneity to the surface of observation, both of which render the numerical calculations quite difficult to perform.

Let us consider the first variant of model 3D-1A with an isometric insert ($l_y=1$) of dimensions $1\times2\times2$ km, with the reminder that in accordance with the estimation made in Section 4.1.1, the skin-depth for the short period $T=0.1$ s is many times less than the dimensions of the insert, so that the essential skin-effect attenuation occurs within the volume of the insert itself, thus creating the anomalous electromagnetic field. For the long period, $T=10$ s, the skin-effect is much reduced, the field varies smoothly, with the galvanic mode dominating, the anomalous magnetic field becomes negligibly small, and the electric field loses its frequency dependence and behaves much like the limiting solution for a stationary field ($\omega=0$). Because of this, the results of the test calculations for the period $T=0.1$ s provide information on the quality of the various schemes of approximation for dealing with a strongly damped electromagnetic field, while those for the period $T=10$ s inform us, above all, on the ability of the various algorithms to take into account the stationary component of the solution, and for those algorithms based on differential equations, on their ability to choose correctly the dimensions of the modelling area and the type of boundary operators on its boundary.

For the period $T = 0.1$ s, the number of different calculations varies from 5 to 10 (Tables D.1–D.4, D.9, D.10 and Figs. 26–30). Each table contains the mean values and standard errors δ_0 and δ_1 (defined in Section 3.1.2), while graphs of mean values, the intervals of width $2\delta_1$ of best fit, and the bounds of maximum spread, are shown in the figures.

The most complete comparisons of results are those for electric fields and apparent resistivities. Best agreement is found for the field E_y in E_y-polarization, i.e. when the source electric field is in the y-direction (see Table D.2; Fig. 28). Here the results 8 of participants 1, 3–7, 9, and 10 are confined to the range with error δ_1 which does not exceed the values 0.03–0.04. Good agreement is found for all the components calculated by algorithms based on the method of integral equations (1, 6, 7, 9). Results 1.1 and 6 obtained by closely related algorithms with similar discretizations of the insert (with homogenous spacing of 0.25 km in all directions and dimensions of $4 \times 8 \times 8 = 256$) are almost identical. Result 1 differs from result 1.1 in that the discretization is twice as fine—the vertical grid spacing of 0.125 km becoming comparable with the skin-depth. This allows the values of the components of the electromagnetic field in the region above the insert to be specified within a range of about 0.02–0.05.

The dimensions of the discretization of the inhomogeneity in solution 9 are equal to $8 \times 16 \times 10 = 1280$, which is the finest discretization among all the solutions. Constant spacings in each direction are, respectively, equal to 0.125, 0.125 and 0.1 km here. Solution 7 is obtained with a reasonably detailed discretization of the inhomogeneity with 216 elements and a minimum spacing of 0.125 km in each direction. An earlier, somewhat rougher calculation by this participant is present in the alternative solution 7.1. Calculations 7 and 7.1 usually agree to an accuracy of 1%; qualitatively the later results look better. The field components computed in 1, 7 and 9 differ by values within the limits of 0.01–0.03 in the case of E_y-polarization, and within the limits of 0.01–0.05 in the case of E_x-polarization. Here it should be borne in mind that the lower part of the inhomogeneity in solution 7 is approximated less precisely. The apparent resistivities (Tables D.9 and D.10) calculated by the method of integral equations lie in a band of width of 1–2 Ω m (E_y-polarization) and 2–5 Ω m (E_x-polarization). It is important to emphasize that the results 1.6, 7 and 9 practically coincide outside the boundary of the insert.

Results computed by the method of integral equations show a greater scatter. Solution 10 was presented only for E_y-polarization and it agrees qualitatively with the trend exhibited by solutions 1, 6, 7 and 9, but it differs from them systematically by values of up to 0.05–0.07 for the electric fields, and 0.03–0.05 for the magnetic fields. Solutions 3–5 show satisfactory agreement with the previous ones only for the electric fields. For the magnetic fields such agreement is found only for the components Re H_x and Re H_z (Table D.2; Figs. 29 and 30).

Analysis of the deviations of results 3–5 from the mean values of the electric field calculations in 1, 6, 7 and 9, shows that the maximum divergences occur at the points of discontinuity in the variation of electric conductivity—i.e. at the vertical boundaries of the insert ($x = 0.5$ km, $y = 1.0$ km). In the region of the insert boundaries we observe an underestimation of the fields. These effects are primarily associated with the rough discretization of the insert (for example, only $2 \times 2 \times 4 = 16$ elements in solution 3) and, possibly, an insufficiently accurate approximation of the electric field in the region of sharp changes in the electric conductivity. At the same time, in the surrounding region of the model, the divergences of the results disappear with increasing distance from the insert. For such high quality calculations the problem of choice of boundary conditions is not essential; the solution in the host region is greatly influenced by the geometry of the numerical grid.

For the second period, $T = 10$ s, the overall picture is not very different. As usual the results of algorithms using the method of integral equations (1, 6, 7, 9 in Tables D.5, D.6, D.11, D.12) agree well, with the respective values of the anomalous magnetic fields differing by extremely small absolute values which do not exceed 0.02–0.04. These results for the magnetic field indicate a low level of computational errors in the method of integral equations. Slightly less impressive are the results for the electric field and apparent resistivities in the dc approximation (2 and 11). Solution 2 virtually coincides with solution 1.2, both obtained by means of the same algorithm 1 for the extremely long period $T = 10^5$ s for which the field is also essentially dc. In this connection the dc solution 11 in E_y-polarization seems to be overestimated. Results 7, and especially 7.1, seem unreliable at points over the insert which in a number of cases gave electric field and apparent resistivity values considerably lower than those obtained in solutions 1, 6, 9 and is very close to the dc solutions 1.2 and 2. Solution 10 is in better agreement with solutions 1, 6, 7 and 9 for all components in E_y-polarization than for the previous period; for the electric field and apparent resistivity, it is practically impossible to distinguish the results. The finite difference algorithms 3–5 were not able to produce satisfactory results for this period.

5.1.3. Model 3D-1 (variant B)

The second variant of model 3D-1 (Fig. 10a) contains an insert which is prolate along axis Oy ($l_y = 5$) and has dimensions of $1 \times 10 \times 2$ km. The other parameters in model 3D-1 remain unchanged.

This model is more complex than the previous one to a certain extent, since it requires a large number of discrete elements to reach comparable accuracy in the electromagnetic field approximation. At the same time the field is of quasi two-dimensional character within the largest part of the insert. The degree of two-dimensionality of the field over the centre of the insert is controlled by the ratio of the half-length of the insert and the wavelength in the medium (Ting and Hohmann, 1981; Vanyan et al., 1984). For periods $T = 0.1$ and 10 s, this ratio is equal to 0.5 and 5, respectively. Thus, for the first period in E_y-polarization, the field in the central part of the inhomogeneity is of pronounced two-dimensional character, and for the second it is essentially three-dimensional.

Because of this, the tables of results for this model (Tables D.13–D.24) contain both three-dimensional and two-dimensional solutions, the latter indicated by the notation 2D and calculated only for an insert infinitely elongated along the axis Oy. This solution is the result of participant 8 in model 2D-1. For the period $T = 0.1$ s only the three-dimensional calculations 1, 6 and 7 can be compared with the two-dimensional one; all of them are obtained by the method of integral equations. The insert was subdivided uniformly into $4 \times 10 \times 8 = 320$ and $4 \times 8 \times 8 = 256$ elements, as well as non-uniformly into 240 elements. Model symmetry was taken into account in obtaining the solutions. Within the limits of algorithm 1, direct and iterative methods of solution of the linear system were used, with both sets of results not differing significantly. The three-dimensional results 1, 6 and 7 agreed satisfactorily as in model 3D-1A with the electric and magnetic fields not usually differing by more than 0.02–0.03, and the apparent resistivities by not more than 2–3 Ω m. Only for the electric field and apparent resistivity in the E_x-polarization does solution 7 give results considerably lower than the other ones. The structure of the solution on axis Ox is close to being two-dimensional outside the boundaries of insert, but over the insert the discrepancies between two-dimensional and three-dimensional fields may reach a value of 0.1 (Tables D.13 and D.14). The corresponding differences for the apparent resistivities do not exceed 10 Ω m.

For the second period $T = 10$ s there are four three-dimensional solutions, since the dc solution was added to the three mentioned above. For both polarizations, solutions 1, 6 and 7 gave the components of the electromagnetic field in the range 0.2–0.5, and several Ω m for apparent resistivities. Let us note, however, that in solution 7, the electric field and apparent resistivity values calculated along the axis Oy over the inhomogeneity, lie below those given by the dc solutions 2 and 1.2. Solution 2 corresponds qualitatively to solutions 1, 6 and 7, but has more pronounced extrema. The latter fact may be partially explained by the more detailed discretization, $8 \times 20 \times 16 = 2560$, used in the dc calculations.

Solutions obtained by differential equation methods were not submitted for this model.

5.2. Conclusions

The first conclusion to be drawn from the comparison of the three-dimensional calculations is that with model 3D-1A, which is simple in form but non-trivial in geo-electric structure (because of the sharp contrast in electrical conductivities and the nearness of the insert to the Earth's surface), we found a qualitative agreement between the results of several algorithms of various kinds. The second conclusion is that the majority of these algorithms belong to the class of integral equation methods. Let us emphasize that the discrepancies among the latter algorithms are within 1% and are determined primarily by the fineness of discretization of the insert. A good correspondence between the results of three algorithms based on the method of integral equations (1, 6 and 7) was also demonstrated for model 3D-1B. These algorithms performed successfully for the whole program of calculations with model 3D-1.

It should be noted, however, that even for this simple model, calculations involving the method of integral equations resulted in linear systems comprising 200–300 complex equations which is quite demanding on computer resources. In order to apply this method to more complicated configurations, therefore, such as models with inhomogeneous inserts or a whole group of anomalous domains, it is necessary to improve its efficiency considerably.

As we have mentioned, the principal demands on computing resources in this approach are associated with the solution of the system of linear equations. For a detailed analysis of more complex models, therefore, one will need access to increased computing power as well as special processors. Some advantage may be gained if the direct methods for solving the linear systems are replaced by iterative ones, but there arises the danger here of a loss of accuracy and reliability. An alternative approach is to decrease the dimension of the linear system by using discrete elements of higher order when approximating the integral equation, but at present this possibility has not been properly investigated and it is difficult to estimate what advantages it offers.

The third conclusion refers to the differential equation method itself. The comparison revealed certain shortcomings in their theoretical and, above all, their practical development. None of the traditional algorithms of this class were applied to the complete set of comparisons in any of the test models. Thus, it seems impossible to present an appropriately detailed, well-informed analysis of the potential suitability of specific programs, or of differential equation methods in general, because of the paucity of material available. As disadvantages of the algorithms we draw attention, first of all, to the large errors generated in the calculation of the magnetic field components from the electric field by difference formulae. The difference formulae in the region of conductivity discontinuities should be also improved. Finally, it is evidently necessary to perfect the iterative schemes for the solution of the of system difference equations—the Gauss–Seidel method and over-relaxation— since their slow and unreliable convergence places limits on the size of the grids that can be used. It seems appropriate to apply more efficient iterative methods, as well as to use algorithms devised for the solution of systems of difference equations on a sequence of grids.

The results of algorithm 10, in which the steady-state, time-harmonic solution was sought as part of a more general treatment of the problem of modelling arbitrary nonstationary magnetic fields, are also very interesting. The difference approximation is also used in this algorithm as the operator on the spatial part of the field. In spite of the rather overly complicated manner in which the problem is stated in this method (for the purpose of modelling *harmonic* fields), satisfactory results were obtained quite uniformly for all the calculated components.

The final conclusion we make is that it is necessary to continue the comparative calculations for three-dimensional models—in particular, to extend the set of solutions for model 3D-1 and also to assemble calculations for model 3D-2 where there exists the possibility of including results provided by a large number of thin sheet algorithms as well.

Acknowledgements

The authors are grateful to all participants in the COMMEMI project whose materials have been used in the preparation of this article. They are especially thankful to Drs. M. Hvozdara, A. Ádám, S.E. Hjelt and B.A. Hobbs, who actively assisted in the organization and implementation of the International Project.

Appendix A

This article has five appendices containing the material which constitutes the project: Appendix B contains the tables of data which compare the calculated components of the electromagnetic fields and the apparent resistivities for two-dimensional models; Appendix C includes the plots and diagrams of the electromagnetic field components for the two-dimensional models; Appendices D and E provide the analogous material for the three-dimensional models. Appendix F contains a late submission of calculations for Model 3D-2 by Z. Xiong (University of Utah). They are tabulated separately and have not been analysed or even referred to in the main text of this article.

The first line of each table for the two-dimensional models (Appendix B) contains the name of the model followed by (in brackets): the field polarization index, the period T, and the level (z = const.) for which the field is calculated. For example, the statement: Model 2D-0 (EP, $T = 300$, $z = 15$) in a first line indicates that the values given are for the components and apparent resistivity associated with the E-polarized field in Model 2D-0, calculated at the depth of 15 km and for a period of 300 s. The identification HP is used to designate the corresponding H-polarized field. The only difference in the first line of the tables for the three-dimensional models (Appendix D) is in the polarization indices, the two designations of polarization used here being EXN and EYN which indicate that the model is excited by an electric field oriented in the direction of the axes Ox and Oy, respectively. In the second line of all tables, the positions x at which the data are presented are given in kilometres. For example, the line reads: "participant/$x = 0$, 0.25, 0.5, ..." meaning that the data are given for the positions $x = 0$ km, $x = 0.25$ km, $x = 0.5$ km and so on. The data values corresponding to these positions are tabulated in the underlying columns in the main part of the table.

In the three-dimensional tables the field is presented along the axis Oy as well. In this case it is the y-coordinates of the points where data are tabulated that are indicated in the second line of the table. The format of the tables is the same as for the other two- and three-dimensional results with each section corresponding to one of the field components at the stated period and depth. The first line in the table of each section indicates the name of field component; for example, Re E_y means the real part of the component E_y.

In Tables 1 and 2, corresponding to two-dimensional and three-dimensional models, respectively, the first columns contain the names of participants, and the name of their countries is given in the second column. The results of the calculations then follow. Each table is concluded with four lines containing some statistics relating to the comparison of the results. They include the mean values and standard deviations from the mean results, compared at each point both prior to and after the rejection of large deviations (see Section 3). If there is no rejection of deviations only two lines of statistical information are given. In cases where the table is continued to another page, the second line with the positions of the data values is repeated at the beginning of the new page. A dashed line in the table instead of a numerical value means that this participant has not provided results for this particular point. The tables were computer-compiled, which, it is hoped, has avoided the possibility of misprints. The tables were re-formatted for this English version, however, and the numerical results re-entered in them by hand; but they have been independently checked and it is again hoped that no typographical errors have occurred.

The plots and diagrams of the absolute values of the electromagnetic field components for two- and three-dimensional models are given in Appendices C and E, respectively. The format of the figures in both appendices is identical. Graphs depicting the collective behaviour along the profile Ox of all the results for each normalized field component (in the case of three-dimensional models possibly along Oy as well) are placed in the upper part of the figures. The central continuous line goes through the mean values of the field at the points of calculation. Two continuous lines which lie above and below the central line, indicate the limits of the band of width 2δ, δ being the standard deviation, obtained after rejecting outliers at each point of comparison. The broken lines are drawn through the maximum and minimum values at each point. The horizontal scale gives the coordinates of the points of comparison in kilometres.

A detailed comparison of the results of the various participants at selected points along the profile is displayed in the lower part of each figure. In Fig. 6, for example, the lower diagram is divided into seven columns in each of which the results at one of seven different points on the profile are compared. The coordinate (in kilometres) of the point on the profile is indicated at the foot of each column, and the magnitude of the field component is measured on the left-hand vertical scale of the column. The horizontal dashed line marks the mean value of the field at the given point, and the two continuous lines indicate the band of width 2δ obtained after rejecting outliers. The results of the various participants at the given point is indicated by Roman letters defined according to Table 2.3.

The model name, the type of polarization, the field component name, the depth z, and the period T for which the results are presented, are indicated in the caption for each figure.

Appendix B. Results for two-dimensional models

Table B.1. Model 2D-0 (EP, $T = 300$, $z = 0$)

Participant/x =	−25.0	−15.0	−10.0	−7.0	0.0	7.0	10.0	15.0	30.0	50.0
Re E_y										
1 Canada	0.727	0.606	0.529	0.483	0.413	0.397	0.403	0.418	0.436	0.441
3 Canada	0.726	0.605	0.528	0.482	0.412	0.396	0.402	0.417	0.435	0.441
8 Russia	0.728	0.606	0.528	0.482	0.411	0.396	0.402	0.417	0.439	0.440
18 Germany	0.725	0.604	0.528	—	0.411	—	0.402	0.416	0.43	0.441
22 Canada	0.731	0.608	0.528	0.479	0.408	0.394	0.402	0.419	0.437	0.441
23 Canada	0.714	0.593	0.522	0.475	0.406	0.392	0.397	0.413	0.438	0.441
25 Russia	0.722	0.602	0.528	0.483	0.415	0.395	0.399	0.412	—	0.434
26 Russia	0.734	0.608	0.532	0.486	0.413	0.397	0.402	0.417	—	0.435
Average (0)	0.726	0.604	0.528	0.481	0.411	0.395	0.401	0.416	0.437	0.439
St. dev. (0)	0.006	0.005	0.003	0.004	0.003	0.002	0.002	0.002	0.001	0.003
Average (1)	0.726	0.606	0.529	0.481	0.411	0.395	0.402	0.416	0.437	0.439
St. dev. (1)	0.006	0.002	0.001	0.004	0.003	0.002	0.001	0.002	0.001	0.003
Im E_y										
1 Canada	−0.106	−0.056	0.006	0.043	0.065	0.050	0.034	0.012	0.000	0.000
3 Canada	−0.103	−0.053	0.009	0.046	0.068	0.053	0.037	0.015	0.003	0.002
8 Russia	−0.104	−0.055	0.008	0.044	0.066	0.051	0.035	0.013	0.001	0.002
18 Germany	−0.096	−0.044	0.024	—	0.087	—	0.054	0.028	0.014	0.013
22 Canada	−0.107	−0.059	0.006	0.045	0.063	0.048	0.032	0.010	−0.001	0.000
23 Canada	−0.097	−0.048	0.010	0.046	0.068	0.055	0.040	0.019	0.007	0.011
25 Russia	−0.089	−0.045	0.009	0.046	0.065	0.052	0.041	0.016	—	−0.001
26 Russia	−0.095	−0.042	0.008	0.047	0.071	0.054	0.038	0.015	—	0.003
Average (0)	−0.100	−0.050	0.010	0.045	0.069	0.052	0.039	0.016	0.004	0.004
St. dev. (0)	0.006	0.006	0.006	0.001	0.008	0.002	0.007	0.006	0.006	0.005
Average (1)	−0.100	−0.050	0.008	0.045	0.067	0.052	0.037	0.014	0.004	0.004
St. dev. (1)	0.006	0.006	0.002	0.001	0.003	0.002	0.003	0.003	0.006	0.005
Re H_x										
1 Canada	0.863	0.929	1.140	1.295	1.313	1.242	1.159	1.058	1.031	1.032
3 Canada	0.863	0.926	1.132	1.298	1.314	1.244	1.156	1.058	1.031	1.032
8 Russia	0.861	0.922	1.124	1.307	1.321	1.248	1.157	1.056	1.031	1.032
18 Germany	0.859	0.918	1.144	—	1.324	—	1.163	1.055	1.031	1.033
22 Canada	0.860	0.914	1.135	1.321	1.316	1.247	1.151	1.052	1.031	1.032
23 Canada	0.860	0.926	1.070	1.300	1.310	1.250	1.170	1.070	1.040	1.030
25 Russia	0.890	0.956	1.061	1.274	1.269	1.218	1.115	1.073	—	1.021
26 Russia	0.895	0.977	0.977	1.216	1.284	1.247	1.114	1.114	—	1.026
Average (0)	0.869	0.933	1.098	1.287	1.306	1.242	1.148	1.067	1.032	1.030
St. dev. (0)	0.015	0.022	0.058	0.034	0.019	0.011	0.021	0.020	0.004	0.004
Average (1)	0.869	0.927	1.115	1.299	1.306	1.246	1.148	1.060	1.031	1.031
St. dev. (1)	0.015	0.014	0.035	0.015	0.019	0.003	0.021	0.008	0.000	0.002
Im H_x										
1 Canada	0.016	0.073	0.031	0.008	0.092	0.095	0.099	0.093	0.050	0.027
3 Canada	0.018	0.075	0.034	0.004	0.091	0.093	0.100	0.093	0.051	0.026
8 Russia	0.017	0.076	0.046	0.004	0.094	0.096	0.103	0.094	0.040	0.028
18 Germany	0.021	0.085	0.020	—	0.077	—	0.091	0.094	0.049	0.024
22 Canada	0.013	0.072	0.029	0.003	0.099	0.095	0.099	0.089	0.038	0.026
23 Canada	0.017	0.079	0.088	0.014	0.100	0.100	0.101	0.103	0.053	0.045
25 Russia	0.028	0.078	0.059	0.033	0.076	0.096	0.092	0.093	—	0.019
26 Russia	0.009	0.057	0.057	0.030	0.083	0.095	0.088	0.088	—	0.019
Average (0)	0.017	0.074	0.046	0.014	0.089	0.096	0.097	0.093	0.047	0.027
St. dev. (0)	0.006	0.008	0.022	0.013	0.009	0.002	0.005	0.005	0.006	0.008
Average (1)	0.017	0.077	0.046	0.014	0.089	0.095	0.097	0.092	0.047	0.024
St. dev. (1)	0.006	0.004	0.022	0.013	0.009	0.001	0.005	0.002	0.006	0.004

Table B.1 (continued)

Participant/x =	−25.0	−15.0	−10.0	−7.0	0.0	7.0	10.0	15.0	30.0	50.0
Re H_z										
1 Canada	−0.175	−0.335	−0.492	−0.321	−0.079	0.073	0.150	0.078	0.008	−0.002
3 Canada	−0.180	−0.340	−0.483	−0.325	−0.081	0.075	0.145	0.080	0.009	0.000
8 Russia	−0.170	−0.335	−0.476	−0.322	−0.079	0.074	0.142	0.078	0.006	−0.002
18 Germany	−0.167	−0.332	−0.459	—	−0.069	—	0.150	0.091	0.026	0.009
22 Canada	−0.178	−0.343	−0.519	−0.328	−0.074	0.078	0.159	0.078	0.014	−0.001
23 Canada	−0.137	−0.284	−0.381	−0.273	−0.060	0.070	0.116	0.069	0.006	−0.001
25 Russia	−0.187	−0.316	−0.396	−0.285	−0.085	0.043	0.132	0.086	—	−0.010
26 Russia	−0.140	−0.364	−0.364	−0.299	−0.049	0.060	0.109	0.109	—	−0.012
Average (0)	−0.167	−0.331	−0.446	−0.308	−0.072	0.068	0.138	0.084	0.012	−0.002
St. dev. (0)	0.018	0.023	0.058	0.022	0.012	0.012	0.018	0.012	0.008	0.006
Average (1)	−0.167	−0.338	−0.446	−0.308	−0.072	0.072	0.138	0.080	0.012	−0.002
St. dev. (1)	0.018	0.014	0.058	0.022	0.012	0.006	0.018	0.007	0.008	0.006
Im H_z										
1 Canada	−0.117	−0.088	−0.015	−0.091	−0.092	−0.041	−0.056	0.000	0.008	−0.002
3 Canada	−0.114	−0.083	−0.010	−0.087	−0.092	−0.043	−0.056	0.002	0.008	−0.004
8 Russia	−0.121	−0.090	−0.004	−0.094	−0.093	−0.040	−0.049	0.002	0.008	−0.003
18 Germany	−0.102	−0.066	0.008	—	−0.080	—	−0.039	0.009	0.021	0.011
22 Canada	−0.117	−0.090	0.012	−0.100	−0.092	−0.034	−0.052	0.003	0.008	−0.002
23 Canada	−0.116	−0.101	−0.067	−0.104	−0.085	−0.026	−0.020	0.011	0.011	0.000
25 Russia	−0.120	−0.092	−0.053	−0.101	−0.104	−0.046	−0.044	−0.013	—	0.002
26 Russia	−0.121	−0.081	−0.082	−0.096	−0.088	−0.051	−0.021	−0.021	—	0.003
Average (0)	−0.116	−0.086	−0.026	−0.096	−0.091	−0.040	−0.042	−0.001	0.011	0.001
St. dev. (0)	0.006	0.010	0.036	0.006	0.007	0.008	0.015	0.011	0.005	0.005
Average (1)	−0.118	−0.086	−0.026	−0.096	−0.091	−0.040	−0.042	−0.001	0.011	−0.001
St. dev. (1)	0.003	0.010	0.036	0.006	0.007	0.008	0.015	0.011	0.005	0.003

Table B.2. Model 2D-0 (HP, T = 300, z = 0)

Participant/x =	−25.0	−15.0	−10.0	−7.0	0.0	7.0	10.0	15.0	30.0	50.0
Re E_x										
1 Canada	1.006	1.076	1.230	0.235	0.301	0.288	0.247	0.443	0.424	0.426
3 Canada	1.003	1.071	1.218	0.239	0.301	0.289	0.246	0.442	0.426	0.426
8 Russia	1.007	1.076	1.222	0.238	0.302	0.290	0.251	0.443	0.426	0.427
17 USA	1.005	1.076	1.230	0.235	0.300	0.287	0.247	0.443	0.424	0.426
18 Germany	1.007	1.070	—	—	0.303	—	—	0.439	0.426	0.426
20 Russia	1.010	1.080	1.230	0.237	0.301	0.288	0.249	0.442	0.427	0.426
22 Canada	1.004	1.066	1.220	0.254	0.306	0.294	0.245	0.438	0.426	0.426
23 Canada	0.940	1.021	1.105	0.236	0.299	0.291	0.254	0.443	0.426	0.426
25 Russia	1.001	1.052	1.166	0.216	0.281	0.269	—	0.423	—	0.408
26 Russia	1.008	1.085	1.164	0.244	0.303	0.291	—	0.442	—	0.426
Average (0)	0.999	1.067	1.198	0.237	0.300	0.287	0.248	0.440	0.426	0.424
St. dev. (0)	0.021	0.019	0.044	0.010	0.007	0.007	0.003	0.006	0.001	0.006
Average (1)	1.006	1.072	1.210	0.240	0.302	0.290	0.248	0.442	0.426	0.426
St. dev. (1)	0.003	0.009	0.028	0.006	0.002	0.002	0.003	0.002	0.001	0.000
Im E_x										
1 Canada	0.033	0.047	−0.009	−0.048	−0.033	−0.025	−0.007	0.002	−0.009	−0.011
3 Canada	0.029	0.046	−0.007	−0.052	−0.036	−0.028	−0.008	0.001	−0.011	−0.014
8 Russia	0.031	0.047	−0.005	−0.049	−0.034	−0.026	−0.011	0.001	−0.010	−0.012
17 USA	0.031	0.047	−0.009	−0.048	−0.029	−0.020	−0.005	0.006	−0.007	−0.007
18 Germany	0.032	0.050	—	—	−0.056	—	—	−0.013	−0.022	−0.024
20 Russia	0.028	0.046	−0.007	−0.048	−0.033	−0.025	−0.007	0.001	−0.008	−0.011
22 Canada	0.030	0.050	−0.004	−0.052	−0.029	−0.024	−0.006	0.002	−0.008	−0.011
23 Canada	0.005	0.004	0.004	−0.053	−0.034	−0.027	−0.013	0.002	−0.007	−0.011
25 Russia	0.070	0.109	0.079	0.007	0.020	0.033	—	0.059	—	0.034
26 Russia	0.031	0.040	0.032	−0.054	−0.036	−0.028	—	0.000	—	−0.011
Average (0)	0.031	0.049	0.008	−0.044	−0.030	−0.019	−0.008	0.006	−0.010	−0.008
St. dev. (0)	0.018	0.025	0.030	0.019	0.019	0.020	0.003	0.019	0.005	0.015
Average (1)	0.031	0.042	−0.001	−0.051	−0.036	−0.025	−0.008	0.000	−0.009	−0.012
St. dev. (1)	0.002	0.015	0.014	0.003	0.008	0.003	0.003	0.005	0.002	0.005

Table B.3. Model 2D-0 (T = 300, z = 0)

Participant/x =	−25.0	−15.0	−7.0	7.0	10.0	15.0	30.0
ρ_a (EP)							
1 Canada	8.00	4.70	1.50	1.10	1.30	1.70	2.00
3 Canada	7.90	4.70	1.50	1.10	1.30	1.70	1.90
8 Russia	8.00	4.70	1.50	1.10	1.30	1.70	2.00
18 Germany	8.00	4.70	1.60	1.10	1.30	1.70	1.90
22 Canada	8.10	4.88	1.45	1.11	1.34	1.73	1.97
23 Canada	7.60	4.50	1.50	1.10	1.30	1.60	1.90
25 Russia	7.33	4.35	1.59	1.17	1.41	1.61	—
26 Russia	7.50	4.26	1.77	1.13	1.43	1.53	—
Average (0)	7.80	4.60	1.55	1.11	1.34	1.66	1.94
St. dev. (0)	0.29	0.21	0.10	0.03	0.05	0.07	0.05
Average (1)	7.80	4.60	1.52	1.11	1.34	1.66	1.94
St. dev. (1)	0.29	0.21	0.05	0.01	0.05	0.07	0.05
ρ_a (HP)							
20 Russia	11.50	12.75	17.00	1.01	0.68	2.15	2.03
22 Canada	11.08	12.52	16.35	1.04	0.66	2.11	1.99
25 Russia	11.05	12.30	15.00	0.87	—	2.01	—
26 Russia	11.20	13.00	—	1.02	—	2.15	—
Average (0)	11.21	12.64	16.12	0.98	0.67	2.11	2.01
St. dev. (0)	0.21	0.30	1.02	0.08	0.01	0.07	0.03
Average (1)	11.21	12.64	16.12	0.98	0.67	2.11	2.01
St. dev. (1)	0.21	0.30	1.02	0.08	0.01	0.07	0.03

Table B.4. Model 2D-0 (EP, T = 300, z = 15.0)

Participant/x =	−25.0	−15.0	−10.0	−7.0	0.0	7.0	10.0	15.0	30.0	50.0
Re E_y										
1 Canada	0.337	0.204	0.106	0.049	−0.016	−0.017	−0.004	0.019	0.041	0.046
3 Canada	0.335	0.203	0.105	0.048	−0.017	−0.018	−0.005	0.018	0.040	0.045
8 Russia	0.337	0.204	0.106	0.049	−0.017	−0.018	−0.005	0.018	0.044	0.045
18 Germany	0.333	0.200	0.103	—	−0.017	—	−0.005	0.016	0.039	0.043
22 Canada	0.337	0.203	0.107	0.052	−0.011	−0.014	−0.002	0.019	0.041	0.046
23 Canada	0.338	0.205	0.106	0.049	−0.018	−0.019	−0.006	0.018	0.043	0.044
25 Russia	0.329	0.203	0.104	0.052	−0.019	−0.015	−0.004	0.019	—	0.048
26 Russia	0.338	0.202	0.107	0.050	−0.017	−0.017	−0.004	0.020	—	0.047
Average (0)	0.335	0.203	0.105	0.050	−0.017	−0.017	−0.004	0.018	0.041	0.045
St. dev. (0)	0.003	0.002	0.001	0.002	0.002	0.002	0.001	0.001	0.002	0.002
Average (1)	0.336	0.203	0.105	0.050	−0.017	−0.017	−0.004	0.018	0.041	0.045
St. dev. (1)	0.002	0.002	0.001	0.002	0.001	0.002	0.001	0.001	0.002	0.002
Im E_y										
1 Canada	0.130	0.131	0.143	0.144	0.119	0.102	0.101	0.103	0.112	0.122
3 Canada	0.131	0.132	0.144	0.144	0.118	0.102	0.101	0.103	0.112	0.121
8 Russia	0.131	0.132	0.144	0.144	0.119	0.102	0.101	0.103	0.116	0.122
18 Germany	0.133	0.133	0.144	—	0.116	—	0.099	0.102	0.113	0.121
22 Canada	0.132	0.134	0.144	0.143	0.117	0.102	0.102	0.105	0.115	0.122
23 Canada	0.132	0.134	0.145	0.146	0.120	0.104	0.103	0.106	0.120	0.127
25 Russia	0.135	0.134	0.147	0.145	0.122	0.104	0.102	0.104	—	0.121
26 Russia	0.137	0.134	0.149	0.149	0.122	0.105	0.104	0.104	—	0.121
Average (0)	0.133	0.133	0.145	0.145	0.119	0.103	0.102	0.104	0.115	0.122
St. dev. (0)	0.002	0.001	0.002	0.002	0.002	0.001	0.002	0.001	0.003	0.002
Average (1)	0.133	0.133	0.145	0.145	0.119	0.103	0.102	0.104	0.115	0.121
St. dev. (1)	0.002	0.001	0.002	0.002	0.002	0.001	0.002	0.001	0.003	0.001

Table B.5. Model 2D-0 (HP, $T = 300$, $z = 15.0$)

Participant/x =	−25.0	−15.0	−10.0	−7.0	0.0	7.0	10.0	15.0	30.0	50.0
Re E_x										
1 Canada	0.514	0.500	0.475	0.029	0.007	0.007	0.013	0.038	0.045	0.046
3 Canada	0. 513	0. 500	0.476	0.030	0.009	0.008	0.015	0.039	0.047	0.048
8 Russia	0.509	0.488	0.459	0.050	0.013	0.010	0.015	0.033	0.044	0.045
17 USA	0.515	0.501	0.478	0.030	0.007	0.008	0.014	0.039	0.047	0.048
18 Germany	0.515	0.503	—	—	0.012	—	—	0.044	0.049	0.051
22 Canada	0.515	0.496	0.482	0.042	0.011	0.010	0.014	0.036	0.044	0.046
25 Russia	0.470	0.557	0.472	0.022	0.002	0.007	—	0.045	—	0.053
26 Russia	0.530	0.520	0.493	0.047	0.016	0.015	—	0.044	—	0.055
Average (0)	0.510	0.508	0.476	0.036	0.010	0.009	0.014	0.040	0.046	0.049
St. dev. (0)	0.017	0.022	0.010	0.011	0.004	0.003	0.001	0.004	0.002	0.004
Average (1)	0.516	0.501	0.476	0.036	0.010	0.008	0.014	0.040	0.046	0.049
St. dev. (1)	0.007	0.010	0.010	0.011	0.004	0.001	0.001	0.004	0.002	0.004
Im E_x										
1 Canada	0.306	0.331	0.354	0.048	0.059	0.060	0.060	0.119	0.117	0.117
3 Canada	307	0.331	0.353	0.048	0.058	0.060	0.060	0.118	0.116	0.117
8 Russia	0.301	0.327	0.352	0.051	0.066	0.067	0.065	0.124	0.122	0.122
17 USA	0.306	0.330	0.355	0.048	0.058	0.061	0.060	0.118	0. 117	0.117
18 Germany	0. 306	0. 330	—	—	0.057	—	—	0.114	0.113	0.113
22 Canada	0.306	0.331	0.349	0.048	0.060	0.061	0.059	0.117	0.117	0.117
25 Russia	0.319	0.324	0.355	0.055	0.056	0.063	—	0.124	—	0.119
26 Russia	0.304	0.322	0.353	0.054	0.070	0.070	—	0.123	—	0.121
Average (0)	0.307	0.328	0.353	0.050	0.060	0.063	0.061	0.120	0.117	0.118
St. dev. (0)	0.005	0.004	0.002	0.003	0.005	0.004	0.002	0.004	0.003	0.003
Average (1)	0.305	0.328	0.353	0.050	0.060	0.063	0.061	0.120	0.117	0.118
St. dev. (1)	0.002	0.004	0.002	0.003	0.005	0.004	0.002	0.004	0.003	0.003

Table B.6. Model 2D-1 (EP, $T = 0.1$, $z = 0$)

Participant/x =	0.0	0.5	1.0	2.0	4.0	8.0	16.0
Re E_y							
2 Canada	0.399	0.482	0.660	0.870	0.986	0.997	1.000
3 Canada	0.401	0.485	0.664	0.872	0.988	1.000	1.000
6 Poland	0.400	0.484	0.679	0.914	1.000	1.000	1.000
8 Russia	0.384	0.471	0.654	0.868	0.985	0.997	0.998
8.1 Russia	0.385	0.472	0.656	0.870	0.987	1.000	1.000
9 Russia	0.401	0.493	0.672	0.880	0.995	1.000	1.000
10 Uzbekistan	0.440	0.520	0.680	0.880	0.990	0.996	0.997
11 Russia	0.386	0.473	0.655	0.869	0.987	0.999	1.000
12 Czech	0.393	0.477	0.659	0.871	0.987	0.998	0.998
13 Germany	0.397	0.480	0.659	0.870	0.985	0.997	0.998
14 Russia	0.354	0.446	0.638	0.852	0.975	0.998	1.000
15 Russia	0.390	0.477	0.654	0.866	0.995	1.000	1.000
14.1 Russia	0.399	0.479	0.657	0.869	0.987	1.000	1.000
15.1 Russia	0.401	0.487	0.667	0.877	0.990	1.000	1.000
17 USA	0.405	0.488	0.665	0.875	0.990	1.000	1.000
18 Germany	0.345	0.440	0.637	0.856	—	—	—
19 Germany	0.422	0.502	0.675	0.881	0.993	—	—
24 Russia	0.449	0.524	0.690	0.893	1.000	1.004	1.001
25 Russia	0.389	0.468	0.636	0.849	0.979	0.994	1.000
26 Russia	0.374	0.460	0.587	0.820	0.999	0.998	1.001
27 Russia	0.399	0.479	0.657	0.869	0.987	1.000	1.000
27.1 Russia	0.399	0.479	0.657	0.869	0.987	1.000	1.000
27.2 Russia	0.399	0.479	0.657	0.869	0.987	1.000	1.000
Average (0)	0.396	0.480	0.657	0.870	0.989	0.999	1.000
St. dev. (0)	0.022	0.019	0.020	0.017	0.006	0.002	0.001
Average (1)	0.396	0.478	0.660	0.870	0.990	0.999	1.000
St. dev. (1)	0.017	0.012	0.014	0.010	0.005	0.001	0.001

Table B.6 (continued)

Participant/x =	0.0	0.5	1.0	2.0	4.0	8.0	16.0
Im E_y							
2 Canada	−0.115	−0.129	−0.169	−0.148	−0.060	−0.011	0.000
3 Canada	−0.116	−0.131	−0.172	−0.151	−0.064	−0.013	−0.004
6 Poland	−0.112	−0.127	−0.169	−0.131	−0.028	−0.006	−0.001
8 Russia	−0.098	−0.115	−0.163	−0.147	−0.060	−0.010	−0.001
8.1 Russia	−0.099	−0.116	−0.165	−0.149	−0.063	−0.013	−0.003
9 Russia	−0.116	−0.150	−0.183	−0.154	−0.062	−0.010	−0.004
10 Uzbekistan	−0.120	−0.120	−0.150	−0.130	−0.040	0.000	0.009
11 Russia	−0.098	−0.116	−0.163	−0.149	−0.063	−0.013	−0.003
12 Czech	−0.103	−0.119	−0.164	−0.147	−0.060	−0.010	−0.001
13 Germany	−0.101	−0.113	−0.154	−0.137	−0.054	−0.008	−0.001
14 Russia	−0.095	−0.104	−0.155	−0.145	−0.065	−0.012	0.010
15 Russia	−0.115	−0.130	−0.168	−0.144	−0.051	0.000	0.000
14.1 Russia	−0.112	−0.123	−0.166	−0.149	−0.063	−0.012	−0.002
15.1 Russia	−0.116	−0.129	−0.169	−0.149	−0.061	−0.012	−0.003
17 USA	−0.117	−0.131	−0.170	−0.151	−0.062	−0.012	−0.004
18 Germany	−0.096	−0.106	−0.157	−0.140	—	—	—
19 Germany	−0.152	−0.160	−0.185	−0.156	−0.063	—	—
24 Russia	−0.173	−0.177	−0.193	−0.157	−0.062	−0.010	−0.001
25 Russia	−0.095	−0.110	−0.145	−0.141	−0.059	−0.000	0.000
26 Russia	−0.088	−0.103	−0.120	−0.143	−0.066	−0.006	0.001
27 Russia	−0.112	−0.124	−0.166	−0.149	−0.063	−0.013	−0.003
27.1 Russia	−0.113	−0.124	−0.166	−0.150	−0.063	−0.013	−0.003
27.2 Russia	−0.113	−0.124	−0.166	−0.149	−0.063	−0.013	−0.003
Average (0)	−0.112	−0.125	−0.164	−0.146	−0.059	−0.010	−0.001
St. dev. (0)	0.019	0. 017	0.014	0.007	0.009	0.004	0.004
Average (1)	−0.107	−0.123	−0.166	−0.148	−0.061	−0.011	−0.002
St. dev. (1)	0.010	0.014	0.011	0.005	0.004	0.003	0.002
Re H_x							
2 Canada	1.419	1.308	0.957	0.904	0.971	0.997	1.000
3 Canada	1.422	1.310	0.950	0.904	0.970	0.999	1.000
6 Poland	1.422	1.311	0.944	0.911	0.985	0.992	0.992
8 Uzbekistan	1.470	1.370	0.928	0.894	0.968	0.997	0.999
8.1 Russia	1.470	1.370	0.930	0.895	0.970	0.999	1.000
8.2 Russia	1.510	1.370	0.956	0.907	0.972	0.999	1.000
9 Russia	1.483	1.256	0.952	907	0.978	1.000	1.000
10 Russia	1.380	1.280	0.960	0.900	0.960	0.980	0.980
11 Russia	1.440	1.330	0.951	0.902	0.970	0.998	1.000
12 Czech	1.430	1.320	0.953	0.899	0.965	0.991	0.992
13 Germany	1.424	1.322	0.961	0.905	0.971	0.999	1.000
17 USA	1.430	1.310	0.957	0.903	0.972	0.999	1.000
18 Germany	1.443	1.356	0.944	0.897	—	—	—
19 Germany	1.373	1.277	0.966	0.909	0.974	—	—
24 Russia	1.347	1.266	0.977	0.919	0.980	1.003	1.001
25 Russia	1.406	1.149	0.986	0.927	0.967	1.000	1.000
26 Russia	1.330	1.077	1.077	0.983	0.960	1.000	1.000
27 Russia	1.440	1.350	0.963	0.905	0.971	0.999	1.000
27.1 Russia	1.440	1.350	0.963	0.905	0.971	0.999	1.000
27.2 Russia	1.440	1.350	0.963	0.905	0.971	0.999	1.000
Average (0)	1.426	1.302	0.962	0.909	0.971	0.997	0.998
St. dev. (0)	0.044	0.074	0.030	0.019	0.006	0.005	0.005
Average (1)	1.431	1.323	0.956	0.905	0.970	0.998	0.999
St. dev. (1)	0.038	0.036	0.014	0.008	0.005	0.003	0.003

Table B.6 (continued)

Participant/x =	0.0	0.5	1.0	2.0	4.0	8.0	16.0
Im H_x							
2 Canada	0.371	0.262	0.107	−0.021	−0.045	−0.014	−0.009
3 Canada	0.373	0.261	0.104	−0.025	−0.044	−0.014	−0.005
6 Poland	0.379	0.270	0.105	−0.028	−0.019	0.002	0.006
8 Uzbekistan	0.402	0.267	0.108	−0.023	−0.043	−0.011	−0.001
8.1 Russia	0.400	0.265	0.106	−0.025	−0.045	−0.013	−0.003
8.2 Russia	0.376	0.258	0.101	−0.023	−0.043	−0.013	−0.003
9 Russia	0.341	0.289	0.106	−0.023	−0.045	−0.010	0.000
10 Russia	0.420	0.300	0.140	0.008	−0.017	0.010	0.020
11 Russia	0.375	0.255	0.108	−0.020	−0.044	−0.013	−0.003
12 Czech	0.376	0.267	0.117	−0.011	−0.033	−0.003	0.006
13 Germany	0.400	0.282	0.125	−0.007	−0.034	−0.008	−0.001
17 USA	0.382	0.272	0.111	−0.022	−0.045	−0.012	−0.004
18 Germany	0.409	0.254	0.104	0.016	—	—	—
19 Germany	0.378	0.277	0.104	−0.023	−0.045	—	—
24 Russia	0.380	0.282	0.104	−0.023	−0.043	−0.011	−0.001
25 Russia	0.354	0.180	0.093	0.000	−0.032	−0.008	−0.001
26 Russia	0.312	0.115	0.115	0.049	−0.032	−0.007	−0.001
27 Russia	0.379	0.259	0.107	−0.021	−0.044	−0.013	−0.003
27.1 Russia	0.379	0.259	0.107	−0.021	−0.044	−0.013	−0.003
27.2 Russia	0.379	0.259	0.107	−0.021	−0.044	−0.013	−0.003
Average (0)	0.378	0.257	0.109	−0.013	−0.039	−0.009	0.000
St. dev. (0)	0.024	0.041	0.010	0.019	0.009	0.006	0.006
Average (1)	0.382	0.264	0.107	−0.016	−0.041	−0.010	−0.002
St. dev. (1)	0.018	0.024	0.007	0.012	0.005	0.004	0.004
Re H_z							
2 Canada	0.000	0.343	0.278	0.069	−0.010	−0.003	−0.001
3 Canada	0.000	0.346	0.283	0.071	−0.010	−0.003	−0.001
6 Poland	0.000	0.347	0.264	0.039	−0.009	−0.001	0.000
8 Uzbekistan	0.000	0.375	0.291	0.061	−0.012	−0.003	0.000
8.1 Russia	0.000	0.377	0.293	0.062	−0.012	−0.003	0.000
9 Russia	0.000	0.360	0.264	0.060	0.014	—	—
10 Russia	0.000	0.290	0.240	0.060	−0.007	−0.003	0.000
11 Russia	0.000	0.355	0.292	0.073	−0.009	0.003	0.000
12 Czech	0.000	0.354	0.287	0.073	−0.009	−0.003	0.000
13 Germany	0.000	0.336	0.282	0.070	−0.009	−0.002	0.000
17 USA	0.000	0.339	0.279	0.076	−0.008	−0.004	−0.001
18 Germany	0.000	0.424	0.302	0.067	0.000	—	—
19 Germany	0.000	0.306	0.259	0.059	−0.012	—	—
24 Russia	0.000	0.278	0.243	0.054	−0.015	−0.003	0.000
25 Russia	0.000	0.326	0.262	0.067	−0.005	−0.002	0.000
26 Russia	0.000	0.229	0.229	0.192	−0.013	−0.001	0.000
27 Russia	0.000	0.346	0.283	0.067	−0.013	−0.003	0.000
27.1 Russia	0.000	0.346	0.283	0.067	−0.013	−0.003	0.000
27.2 Russia	0.000	0.346	0.283	0.067	−0.013	−0.003	0.000
Average (0)	0.000	0.338	0.274	0.071	−0.009	−0.002	0.000
St. dev. (0)	0.000	0.041	0.020	0.030	0.006	0.002	0.000
Average (1)	0.000	0.339	0.276	0.065	−0.010	−0.003	0.000
St. dev. (1)	0.000	0.027	0.017	0.009	0.004	0.001	0.000

Table B.6 (continued)

Participant/x =	0.0	0.5	1.0	2.0	4.0	8.0	16.0
Im H_z							
2 Canada	0.000	0.196	0.220	0.142	0.040	0.003	0.001
3 Canada	0.000	0.196	0.218	0.142	0.040	0.003	0.001
6 Poland	0.000	0.198	0.220	0.117	0.015	0.001	0.000
8 Uzbekistan	0.000	0.193	0.220	0.141	0.036	0.002	0.000
8.1 Russia	0.000	0.193	0.220	0.141	0.036	0.002	0.000
9 Russia	0.000	0.201	0.233	0.142	0.036	—	—
10 Russia	0.000	0.190	0.210	0.130	0.030	0.003	0.000
11 Russia	0.000	0.193	0.218	0.142	0.040	0.003	0.000
12 Czech	0.000	0.192	0.219	0.143	0.040	0.003	0.000
13 Germany	0.000	0.198	0.219	0.140	0.038	0.002	0.000
17 USA	0.000	0.196	0.219	0.143	0.044	0.003	−0.001
18 Germany	0.000	0.227	0.225	0.140	0.000	—	—
19 Germany	0.000	0.213	0.232	0.143	0.037	—	—
24 Russia	0.000	0.217	0.235	0.144	0.037	0.002	0.000
25 Russia	0.000	0.213	0.199	0.134	0.047	0.002	0.001
26 Russia	0.000	0.173	0.173	0.163	0.037	0.002	−0.001
27 Russia	0.000	0.201	0.221	0.141	0.036	0.002	0.000
27.1 Russia	0.000	0.201	0.221	0.141	0.036	0.002	0.000
27.2 Russia	0.000	0.201	0.221	0.141	0.036	0.002	0.000
Average (0)	0.000	0.200	0.218	0.141	0.035	0.002	0.000
St. dev. (0)	0.000	0.012	0.013	0.008	0.011	0.001	0.001
Average (1)	0.000	0.200	0.221	0.141	0.037	0.002	0.000
St. dev. (1)	0.000	0.008	0.008	0.003	0.007	0.001	0.001

Table B.7. Model 2D-1 (HP, $T = 0.1$, $z = 0$)

Participant/x =	0.0	0.5	1.0	2.0	4.0	8.0	16.0
Re E_x							
2 Canada	0.282	0.681	0.975	0.995	1.002	1.003	1.003
3 Canada	0.282	0.679	0.974	0.992	0.998	1.000	1.000
6 Poland	0.266	0.659	0.960	0.978	0.984	0.984	0.984
8 Uzbekistan	0.290	0.689	0.971	0.993	1.000	1.000	1.000
8.1 Russia	0.289	0.689	0.971	0.993	1.000	1.000	1.000
10 Russia	0.320	0.690	0.930	0.960	0.970	0.970	0.970
11 Russia	0.289	0.687	0.971	0.992	0.999	1.000	1.000
12 Czech	0.289	0.680	0.970	0.992	1.000	1.000	1.000
17 USA	0.278	0.670	0.975	0.992	0.999	1.000	1.000
18 Germany	0.315	0.734	0.966	0.991	—	—	—
19 Germany	0.249	0.656	1.003	0.997	0.999	—	—
20 Russia	0.290	0.725	0.980	0.993	0.997	0.999	0.999
25 Russia	0.289	0.761	0.911	0.994	0.989	0.997	0.995
26 Russia	0.313	0.841	0.909	0.987	0.999	1.001	1.000
Average (0)	0.289	0.703	0.962	0.989	0.995	0.996	0.996
St. dev. (0)	0.019	0.049	0.027	0.010	0.009	0.010	0.009
Average (1)	0.292	0.692	0.962	0.991	0.997	0.999	0.998
St. dev. (1)	0.015	0.030	0.027	0.005	0.005	0.005	0.005

Table B.7 (continued)

Participant/x =	0.0	0.5	1.0	2.0	4.0	8.0	16.0
Im E_x							
2 Canada	−0.140	−0.059	0.003	0.000	−0.004	−0.003	0.000
3 Canada	−0.139	−0.057	0.006	0.003	−0.001	0.000	0.000
6 Poland	−0.126	−0.045	0.021	0.016	0.015	0.015	0.015
8 Uzbekistan	−0.155	−0.063	0.004	0.002	−0.001	0.000	0.000
8.1 Russia	−0.154	−0.063	0.004	0.002	−0.001	0.000	0.000
10 Russia	−0.130	−0.040	0.020	0.030	0.024	0.030	0.030
11 Russia	−0.156	−0.064	0.004	0.003	−0.001	0.000	0.000
12 Czech	−0.148	−0.058	0.011	0.006	−0.001	0.000	0.000
17 USA	−0.139	−0.059	0.007	0.003	−0.001	0.000	0.000
18 Germany	−0.190	−0.068	0.000	0.002	—	—	—
19 Germany	−0.124	−0.059	0.015	0.008	0.000	—	—
20 Russia	−0.140	−0.061	0.006	0.002	−0.001	−0.001	−0.001
25 Russia	−0.121	−0.005	0.016	0.027	0.029	0.027	0.012
26 Russia	−0.156	−0.036	−0.016	0.001	−0.002	−0.001	−0.002
Average (0)	−0.144	−0.053	0.007	0.008	0.004	0.006	0.004
St. dev. (0)	0.018	0.017	0.009	0.010	0.011	0.012	0.010
Average (1)	−0.141	−0.056	0.009	0.006	0.002	0.003	0.002
St. dev. (1)	0.013	0.010	0.007	0.008	0.008	0.009	0.006

Table B.8. Model 2D-1 (T = 0.1, z = 0)

Participant/x =	0.0	0.5	1.0	2.0	4.0	8.0	16.0
ρ_a (EP)							
2 Canada	8.03	14.01	49.99	95.18	103.20	99.60	100.30
3 Canada	8.07	14.10	51.50	95.70	104.00	100.00	100.00
6 Poland	8.00	14.00	54.20	102.70	103.10	101.70	101.60
8 Uzbekistan	6.80	12.13	52.05	96.78	103.60	99.97	99.78
8.1 Russia	6.80	12.14	52.15	97.03	103.85	100.21	100.01
8.2 Russia	6.55	12.09	49.42	94.67	103.27	100.11	99.98
9 Russia	7.53	15.99	52.87	96.96	103.69	100.00	100.00
10 Russia	10.00	16.48	51.52	97.68	106.49	103.28	103.46
11 Russia	7.14	12.80	49.70	95.60	104.00	100.00	100.00
12 Czech	7.50	13.37	50.00	96.66	105.00	101.40	101.10
13 Germany	7.68	13.31	48.76	94.67	103.10	99.72	99.62
14 Russia	5.24	9.15	53.20	96.90	104.00	101.00	100.00
15 Russia	7.46	13.60	54.70	97.40	105.00	101.00	100.00
14.1 Russia	7.83	13.07	49.77	95.87	103.86	100.00	100.00
15.1 Russia	7.87	13.93	52.69	97.35	104.05	100.00	100.00
17 USA	8.06	14.11	50.53	96.81	104.40	100.10	99.69
18 Germany	5.70	10.74	47.82	93.44	—	—	—
19 Germany	9.92	16.26	51.89	96.82	104.14	—	—
24 Russia	11.82	18.15	53.23	97.22	104.21	100.20	100.01
25 Russia	7.60	17.10	43.40	86.30	102.80	98.70	100.10
26 Russia	7.92	18.90	30.50	71.50	109.00	99.60	100.00
27 Russia	7.71	12.95	48.94	94.97	103.50	100.18	100.00
27.1 Russia	7.71	12.95	48.94	94.98	103.50	100.19	100.00
27.2 Russia	7.71	12.95	48.94	94.98	103.50	100.19	100.00
Average (0)	7.78	13.93	49.86	94.92	104.14	100.33	100.26
St. dev. (0)	1.34	2.26	4.79	5.67	1.32	0.92	0.84
Average (1)	7.60	13.92	50.70	95.94	103.92	100.18	100.10
St. dev. (1)	1.04	1.82	2.48	2.75	0.80	0.65	0.44

Table B.8 (continued)

Participant/x =	0.0	0.5	1.0	2.0	4.0	8.0	16.0
ρ_a(HP)							
2 Canada	9.93	46.80	95.10	99.10	100.00	101.00	101.00
3 Canada	9.86	46.40	94.80	98.30	99.70	100.00	100.00
6 Poland	8.65	43.59	92.28	95.67	96.87	96.87	96.87
8 Uzbekistan	10.79	47.85	94.32	98.60	99.96	100.24	100.23
8.1 Russia	10.73	47.83	94.32	98.60	99.96	100.24	100.23
11 Russia	10.80	47.70	94.40	98.50	99.80	100.00	100.00
12 Czech	10.55	46.59	94.16	98.47	101.00	100.40	100.40
14 Russia	10.90	49.50	94.30	98.50	99.70	100.00	100.00
15 Russia	8.23	47.60	94.20	97.80	99.50	99.80	100.00
14.1 Russia	10.22	48.57	94.75	98.54	99.66	99.92	99.91
15.1 Russia	9.50	45.28	93.65	98.16	99.79	100.00	100.00
17 USA	9.64	45.19	95.13	98.48	99.78	100.00	100.00
18 Germany	13.54	54.37	93.22	98.28	100.00	100.00	100.00
20 Russia	10.10	45.90	95.10	98.40	99.10	99.50	99.90
25 Russia	9.80	57.90	83.00	98.80	97.90	99.40	98.90
26 Russia	12.20	70.90	82.40	97.40	99.80	100.00	99.90
Average (0)	10.34	49.50	92.82	98.22	99.53	99.84	99.83
St. dev. (0)	1.26	6.71	4.02	0.78	0.94	0.87	0.89
Average (1)	10.13	48.07	94.27	98.40	99.71	100.03	100.03
St. dev. (1)	0.96	3.65	0.79	0.40	0.64	0.37	0.42

Table B.9. Model 2D-1 (EP, $T = 10.0$, $z = 0$)

Participant/x =	0.0	0.5	1.0	2.0	4.0	8.0	16.0
Re E_y							
3 Canada	0.423	0.440	0.475	0.543	0.640	0.751	0.865
6 Poland	0.424	0.441	0.479	0.562	0.689	0.803	0.908
8 Uzbekistan	0.419	0.435	0.470	0.538	0.634	0.747	0.862
8.1 Russia	0.423	0.439	0.475	0.543	0.640	0.754	0.869
9 Russia	0.420	0.436	0.472	0.555	0.639	0.753	0.869
10 Russia	0.440	0.460	0.498	0.570	0.680	0.790	0.896
11 Russia	0.426	0.443	0.478	0.546	0.643	0.756	0.871
12 Czech	0.429	0.445	0.478	0.544	0.640	0.752	0.867
13 Germany	0.524	0.546	0.592	0.678	0.791	0.896	0.964
14 Russia	0.445	0.461	0.495	0.561	0.655	0.768	0.878
15 Russia	0.475	0.492	0.531	0.598	0.690	0.794	0.893
14.1 Russia	0.410	0.427	0.463	0.532	0.631	0.746	0.866
15.1 Russia	0.422	0.439	0.474	0.543	0.642	0.757	0.874
17 USA	0.432	0.449	0.484	0.553	0.651	0.765	0.869
18 Germany	0.410	0.427	0.462	0.530	0.628	0.753	0.861
19 Germany	0.422	0.439	0.474	0.543	0.640	—	—
22 Canada	0.419	0.437	0.474	0.543	0.641	0.754	0.868
24 Russia	0.423	0.440	0.476	0.544	0.642	0.757	0.873
25 Russia	0.468	0.486	0.530	0.611	0.730	0.864	0.989
26 Russia	0.476	0.494	0.533	0.612	0.733	0.863	0.988
27 Russia	0.428	0.444	0.479	0.547	0.643	0.756	0.870
27.1 Russia	0.429	0.445	0.480	0.548	0.644	0.757	0.871
27.2 Russia	0.429	0.446	0.481	0.548	0.645	0.757	0.871
Average (0)	0.435	0.453	0.489	0.561	0.661	0.777	0.888
St. dev. (0)	0.027	0.028	0.030	0.034	0.041	0.043	0.039
Average (1)	0.431	0.448	0.485	0.555	0.655	0.762	0.878
St. dev. (1)	0.019	0.019	0.021	0.023	0.030	0.016	0.023

Table B.9 (continued)

Participant/x =	0.0	0.5	1.0	2.0	4.0	8.0	16.0
Im E_y							
3 Canada	0.228	0.213	0.184	0.132	0.066	0.003	−0.035
6 Poland	0.222	0.207	0.175	0.111	0.028	−0.027	−0.047
8 Uzbekistan	0.225	0.211	0.181	0.130	0.064	0.001	−0.038
8.1 Russia	0.228	0.213	0.184	0.132	0.065	0.001	−0.039
9 Russia	0.220	0.212	0.182	0.130	0.062	−0.001	−0.041
10 Russia	0.306	0.290	0.260	0.220	0.150	0.080	0.030
11 Russia	0.228	0.213	0.184	0.132	0.065	0.001	−0.039
12 Czech	0.229	0.215	0.187	0.136	0.070	0.005	−0.036
13 Germany	0.368	0.350	0.314	0.249	0.164	0.081	0.023
14 Russia	0.225	0.210	0.180	0.128	0.064	0.000	−0.034
15 Russia	0.169	0.146	0.115	0.075	0.028	−0.017	−0.038
14.1 Russia	0.241	0.227	0.200	0.152	0.091	0.033	−0.003
15.1 Russia	0.230	0.216	0.187	0.136	0.071	0.009	−0.027
17 USA	0.234	0.219	0.189	0.137	0.071	0.065	0.028
18 Germany	0.230	0.215	0.186	0.136	0.070	0.003	−0.029
19 Germany	0.226	0.212	0.182	0.131	0.064	—	—
22 Canada	0.228	0.213	0.182	0.129	0.063	0.000	−0.039
24 Russia	0.224	0.210	0.180	0.129	0.062	−0.002	−0.041
25 Russia	0.308	0.294	0.262	0.207	0.134	0.061	0.005
26 Russia	0.295	0.281	0.254	0.199	0.122	0.053	0.004
27 Russia	0.228	0.213	0.184	0.132	0.065	0.000	−0.040
27.1 Russia	0.228	0.214	0.184	0.132	0.065	0.000	−0.040
27.2 Russia	0.229	0.214	0.184	0.132	0.065	0.000	−0.039
Average (0)	0.241	0.226	0.197	0.145	0.077	0.016	−0.023
St. dev. (0)	0.041	0.041	0.040	0.038	0.034	0.031	0.025
Average (1)	0.235	0.221	0.195	0.140	0.069	0.009	−0.029
St. dev. (1)	0.031	0.031	0.027	0.031	0.024	0.024	0.019
Re H_x							
3 Canada	3.112	2.645	1.933	1.348	1.042	0.943	0.932
6 Poland	3.100	2.649	1.904	1.257	0.977	0.931	0.948
8 Uzbekistan	3.210	2.690	1.930	1.340	1.040	0.938	0.931
8.1 Russia	3.240	2.710	1.940	1.340	1.040	0.938	0.931
8.2 Russia	3.220	2.710	1.950	1.350	1.040	0.939	0.931
9 Russia	3.080	2.640	1.950	1.360	1.050	0.944	0.884
10 Russia	3.360	2.880	2.130	1.470	1.110	0.990	0.970
11 Russia	3.120	2.660	1.950	1.360	1.040	0.942	0.933
12 Czech	3.230	2.720	1.950	1.360	1.060	0.947	0.938
13 Germany	4.016	3.386	2.424	1.606	1.160	1.005	0.981
17 USA	3.180	2.710	1.980	1.360	1.040	0.943	0.938
18 Germany	3.060	2.608	1.923	1.351	0.981	0.938	0.933
19 Germany	3.090	2.640	1.951	1.362	1.046	—	—
22 Canada	3.157	2.656	1.912	1.338	1.038	0.940	0.931
24 Russia	3.090	2.644	1.956	1.366	1.051	0.946	0.936
25 Russia	3.435	2.527	2.092	1.576	1.230	1.046	1.001
26 Russia	3.173	2.167	2.167	1.754	1.189	1.047	1.002
27 Russia	3.180	2.700	1.950	1.350	1.040	0.937	0.929
27.1 Russia	3.190	2.700	1.960	1.350	1.040	0.938	0.931
27.2 Russia	3.190	2.700	1.960	1.350	1.040	0.938	0.931
Average (0)	3.222	2.687	1.996	1.397	1.063	0.957	0.943
St. dev. (0)	0.209	0.211	0.123	0.117	0.063	0.036	0.028
Average (1)	3.180	2.677	1.973	1.379	1.046	0.947	0.939
St. dev. (1)	0.095	0.069	0.073	0.083	0.040	0.020	0.015

Table B.9 (continued)

Participant/x =	0.0	0.5	1.0	2.0	4.0	8.0	16.0
Im H_x							
3 Canada	0.304	0.282	0.246	0.162	0.092	0.046	0.009
6 Poland	0.288	0.271	0.235	0.141	0.068	0.028	−0.004
8 Uzbekistan	0.315	0.289	0.251	0.163	0.092	0.047	0.010
8.1 Russia	0.317	0.290	0.252	0.163	0.092	0.047	0.009
8.2 Russia	0.311	0.287	0.249	0.161	0.091	0.046	0.009
9 Russia	0.307	0.287	0.248	0.164	0.087	0.048	0.007
10 Russia	0.680	0.590	0.450	0.260	0.140	0.070	0.030
11 Russia	0.294	0.274	0.242	0.160	0.091	0.045	0.008
12 Czech	0.303	0.281	0.247	0.162	0.094	0.047	0.011
13 Germany	0.625	0.529	0.379	0.192	0.075	0.030	0.013
17 USA	0.319	0.295	0.254	0.164	0.090	0.045	0.018
18 Germany	0.332	0.302	0.262	0.173	0.086	0.049	0.014
19 Germany	0.317	0.292	0.250	0.164	0.092	—	—
22 Canada	0.319	0.286	0.244	0.158	0.090	0.045	0.008
24 Russia	0.312	0.286	0.245	0.161	0.090	0.044	0.007
25 Russia	0.503	0.387	0.316	0.208	0.112	0.040	0.003
26 Russia	0.450	0.298	0.298	0.232	0.109	0.047	0.003
27 Russia	0.300	0.277	0.243	0.160	0.091	0.047	0.009
27.1 Russia	0.299	0.277	0.243	0.160	0.092	0.047	0.010
27.2 Russia	0.299	0.277	0.242	0.160	0.091	0.047	0.010
Average (0)	0.360	0.318	0.270	0.173	0.093	0.046	0.010
St. dev. (0)	0.114	0.087	0.054	0.029	0.014	0.008	0.007
Average (1)	0.327	0.291	0.254	0.165	0.091	0.045	0.009
St. dev. (1)	0.056	0.025	0.020	0.014	0.009	0.004	0.004
Re H_z							
3 Canada	0.000	0.897	1.048	0.840	0.514	0.251	0.093
6 Poland	0.000	0.900	1.041	0.757	0.374	0.166	0.051
8 Uzbekistan	0.000	0.902	1.050	0.829	0.497	0.245	0.089
8.1 Russia	0.000	0.911	1.060	0.837	0.502	0.247	0.089
9 Russia	0.000	0.908	1.060	0.840	0.505	0.237	0.068
10 Russia	0.000	0.910	1.065	0.840	0.510	0.250	0.090
11 Russia	0.000	0.892	1.050	0.837	0.509	0.250	0.094
12 Czech	0.000	0.835	0.998	0.827	0.511	0.253	0.096
13 Germany	0.000	1.146	1.341	1.041	0.585	0.229	0.059
17 USA	0.000	0.910	1.060	0.847	0.519	0.261	0.133
18 Germany	0.000	0.907	1.040	0.826	0.494	0.226	0.087
19 Germany	0.000	0.909	1.060	0.841	0.509	—	—
22 Canada	0.000	0.979	1.082	0.845	0.518	0.259	0.098
24 Russia	0.000	0.909	1.060	0.841	0.509	0.249	0.092
25 Russia	0.000	1.209	1.185	0.940	0.625	0.283	0.126
26 Russia	0.000	1.057	1.057	1.065	0.534	0.258	0.124
27 Russia	0.000	0.920	1.060	0.835	0.498	0.243	0.090
27.1 Russia	0.000	0.921	1.070	0.837	0.499	0.243	0.090
Average (0)	0.000	0.944	1.077	0.861	0.511	0.244	0.092
St. dev. (0)	0.000	0.093	0.073	0.075	0.047	0.023	0.021
Average (1)	0.000	0.917	1.062	0.839	0.512	0.249	0.092
St. dev. (1)	0.000	0.045	0.035	0.033	0.021	0.013	0.021

Table B.9 (continued)

Participant/x =	0.0	0.5	1.0	2.0	4.0	8.0	16.0
Im H_z							
3 Canada	0.000	0.060	0.107	0.131	0.113	0.088	0.064
6 Poland	0.000	0.055	0.105	0.123	0.097	0.075	0.049
8 Uzbekistan	0.000	0.061	0.111	0.135	0.113	0.088	0.064
8.1 Russia	0.000	0.062	0.111	0.135	0.113	0.088	0.064
9 Russia	0.000	0.065	0.115	0.136	0.116	0.091	0.065
10 Russia	0.000	0.130	0.201	0.190	0.130	0.070	0.031
11 Russia	0.000	0.056	0.104	0.128	0.110	0.087	0.064
12 Czech	0.000	0.054	0.099	0.123	0.108	0.086	0.063
13 Germany	0.000	0.115	0.175	0.154	0.076	0.022	0.006
17 USA	0.000	0.066	0.113	0.134	0.114	0.088	0.067
18 Germany	0.000	0.073	0.122	0.142	0.129	0.091	0.069
19 Germany	0.000	0.070	0.118	0.138	0.116	—	—
22 Canada	0.000	0.071	0.116	0.133	0.113	0.089	0.065
24 Russia	0.000	0.070	0.117	0.136	0.115	0.089	0.065
25 Russia	0.000	0.188	0.205	0.204	0.161	0.093	0.054
26 Russia	0.000	0.184	0.184	0.192	0.143	0.092	0.062
27 Russia	0.000	0.058	0.106	0.127	0.108	0.086	0.064
27.1 Russia	0.000	0.057	0.106	0.127	0.108	0.086	0.064
Average (0)	0.000	0.082	0.127	0.143	0.115	0.083	0.058
St. dev. (0)	0.000	0.042	0.035	0.025	0.017	0.016	0.016
Average (1)	0.000	0.069	0.118	0.136	0.115	0.087	0.061
St. dev. (1)	0.000	0.021	0.024	0.016	0.011	0.006	0.009

Table B.10. Model 2D-1 (HP, T = 10.0, z = 0)

Participant/x =	0.0	0.5	1.0	2.0	4.0	8.0	16.0
Re E_x							
3 Canada	0.126	0.687	1.069	1.079	1.039	1.014	1.006
6 Poland	0.118	0.659	1.088	1.082	1.035	1.019	1.016
8 Uzbekistan	0.124	0.672	1.073	1.084	1.043	1.018	1.009
8.1 Russia	0.124	0.672	1.073	1.084	1.043	1.018	1.009
10 Russia	0.176	0.680	1.040	1.070	1.040	1.030	1.030
11 Russia	0.122	0.666	1.070	1.080	1.030	1.010	1.000
12 Czech	0.138	0.690	1.080	1.090	1.050	1.020	1.010
17 USA	0.113	0.644	1.070	1.080	1.030	1.010	1.000
18 Germany	0.143	0.711	1.048	1.073	1.034	1.008	1.001
19 Germany	0.095	0.613	1.080	1.080	1.040	—	—
20 Russia	0.120	0.690	1.100	1.080	1.040	1.020	1.010
22 Canada	0.101	0.692	1.078	1.075	1.034	1.009	1.001
26 Russia	0.151	0.872	0.966	1.062	1.031	1.007	1.000
Average (0)	0.127	0.688	1.064	1.078	1.038	1.015	1.008
St. dev. (0)	0.021	0.060	0.033	0.007	0.006	0.007	0.009
Average (1)	0.123	0.673	1.072	1.080	1.037	1.014	1.006
St. dev. (1)	0.016	0.026	0.016	0.005	0.005	0.005	0.006

Table B.10 (continued)

Participant / x =	0.0	0.5	1.0	2.0	4.0	8.0	16.0
Im E_x							
3 Canada	−0.032	−0.015	0.000	0.006	0.004	0.000	−0.002
6 Poland	−0.029	−0.006	0.014	0.020	0.015	0.012	0.010
8 Uzbekistan	−0.031	−0.012	0.005	0.011	0.009	0.005	0.003
8.1 Russia	−0.031	−0.012	0.005	0.011	0.009	0.005	0.003
10 Russia	−0.030	−0.010	0.003	0.009	0.008	0.007	0.010
11 Russia	−0.031	−0.012	0.004	0.011	0.008	0.004	0.002
12 Czech	−0.030	−0.012	0.003	0.010	0.008	0.005	0.002
17 USA	−0.031	−0.013	0.003	0.010	0.008	0.004	0.001
18 Germany	−0.032	−0.012	0.002	0.009	0.007	0.004	0.002
19 Germany	−0.028	−0.014	0.002	0.010	0.008	—	—
20 Russia	−0.029	−0.017	−0.007	−0.001	−0.002	−0.006	−0.006
22 Canada	−0.033	−0.012	0.005	0.010	0.008	0.004	0.001
26 Russia	−0.032	−0.009	−0.005	0.002	0.003	0.001	0.000
Average (0)	−0.031	−0.012	0.003	0.009	0.007	0.004	0.002
St. dev. (0)	0.001	0.003	0.005	0.005	0.004	0.004	0.004
Average (1)	−0.031	−0.012	0.002	0.009	0.007	0.005	0.002
St. dev. (1)	0.001	0.002	0.004	0.003	0.002	0.003	0.004

Table B.11. Model 2D-1 (T = 10.0, z = 0)

Participant / x =	0.0	0.5	1.0	2.0	4.0	8.0	16.0
ρ_a (EP)							
3 Canada	2.36	3.38	6.83	16.90	37.80	63.30	86.20
6 Poland	2.40	3.30	7.10	20.50	49.50	74.30	92.40
8 Uzbekistan	2.17	3.19	6.74	16.86	37.60	64.30	85.76
8.1 Russia	2.18	3.21	6.80	17.06	38.19	64.47	87.40
8.2 Russia	2.21	3.21	6.74	16.99	38.10	64.34	87.28
9 Russia	2.35	3.33	6.62	17.32	37.13	63.46	96.84
10 Russia	2.44	3.42	6.66	16.75	38.74	64.01	85.34
11 Russia	2.37	3.37	6.77	16.70	37.80	64.00	86.90
12 Czech	2.24	3.26	6.81	16.86	36.89	62.89	85.49
13 Germany	2.48	3.58	7.46	19.96	48.29	80.09	96.70
14 Russia	1.98	3.05	7.02	18.40	40.30	66.90	87.70
15 Russia	1.70	3.76	10.20	24.20	45.30	70.60	89.30
14.1 Russia	2.23	3.20	6.51	16.09	35.67	59.81	81.69
15.1 Russia	2.29	3.28	6.68	16.66	37.42	62.99	85.58
17 USA	2.35	3.33	6.68	16.80	37.78	63.28	83.45
18 Germany	2.33	3.31	6.59	16.13	41.26	64.25	85.36
19 Germany	2.38	3.37	6.66	16.58	37.52	—	—
22 Canada	2.26	3.31	6.94	17.19	38.15	64.18	87.07
24 Russia	2.38	3.36	6.66	16.51	37.42	63.84	87.06
25 Russia	2.60	4.90	7.80	16.50	36.10	68.50	97.50
26 Russia	3.05	6.75	7.28	13.20	38.80	68.10	97.30
27 Russia	2.30	3.31	6.80	17.10	38.59	64.92	87.89
27.1 Russia	2.30	3.31	6.80	17.12	38.58	64.90	87.82
27.2 Russia	2.30	3.32	6.82	17.17	38.67	65.00	87.83
Average (0)	2.32	3.53	7.00	17.31	39.23	65.76	88.52
St. dev. (0)	0.23	0.77	0.74	1.97	3.52	4.30	4.49
Average (1)	2.31	3.39	6.86	17.19	38.35	65.11	88.52
St. dev. (1)	0.12	0.36	0.30	1.09	1.96	3.03	4.49

Table B.11 (continued)

Participant/x =	0.0	0.5	1.0	2.0	4.0	8.0	16.0
ρ_a (HP)							
3 Canada	1.68	47.20	114.00	116.00	108.00	103.00	101.00
6 Poland	1.47	43.48	118.39	117.19	107.16	103.95	103.14
8 Uzbekistan	1.63	45.23	115.17	117.44	108.84	103.56	101.90
8.1 Russia	1.63	45.23	115.17	117.44	108.84	103.56	101.90
11 Russia	1.58	44.60	114.00	116.00	108.00	102.00	101.00
12 Czech	2.00	47.66	115.90	118.50	109.70	104.50	102.80
14 Russia	1.59	43.90	107.00	110.00	103.00	99.70	100.00
15 Russia	1.37	54.00	108.00	116.00	106.00	101.00	99.80
14.1 Russia	1.72	47.66	119.40	121.90	113.10	107.60	105.70
15.1 Russia	1.31	42.72	114.70	115.80	107.10	101.90	100.30
17 USA	1.37	41.47	114.40	115.60	107.00	101.80	100.00
18 Germany	2.14	50.52	109.74	115.04	106.97	101.65	100.28
20 Russia	1.75	52.20	144.00	145.00	135.00	128.00	125.00
22 Canada	1.12	47.96	116.20	115.60	106.92	101.87	100.26
26 Russia	2.37	76.10	93.30	113.00	106.00	101.00	100.00
Average (0)	1.65	48.66	114.62	118.03	109.44	104.34	102.87
St. dev. (0)	0.33	8.36	10.30	7.89	7.40	6.81	6.33
Average (1)	1.60	46.70	114.01	116.11	107.62	102.65	101.29
St. dev. (1)	0.27	3.64	3.69	2.67	2.25	1.94	1.67

Table B.12. Model 2D-1 (EP, T = 10.0, z = 1.25)

Participant/x =	0.0	0.5	1.0	2.0	4.0	8.0	16.0
Re E_y							
3 Canada	0.205	0.255	0.338	0.433	0.558	0.676	0.792
8 Uzbekistan	0.202	0.251	0.333	0.438	0.553	0.672	0.788
11 Russia	0.206	0.256	0.339	0.445	0.561	0.680	0.797
13 Germany	0.226	0.296	0.412	0.553	0.692	0.808	0.882
18 Germany	0.193	0.243	0.325	0.429	0.546	0.678	0.788
22 Canada	0.205	0.255	0.339	0.444	0.559	0.678	0.795
25 Russia	—	0.273	0.372	0.497	0.638	0.782	0.910
26 Russia	0.218	0.280	0.370	0.501	0.641	0.781	0.910
Average (0)	0.208	0.264	0.354	0.467	0.594	0.719	0.833
St. dev. (0)	0.011	0.018	0.029	0.045	0.055	0.059	0.057
Average (1)	0.208	0.264	0.345	0.467	0.594	0.719	0.833
St. dev. (1)	0.011	0.018	0.018	0.045	0.055	0.059	0.057
Im E_y							
3 Canada	0.368	0.336	0.279	0.207	0.131	0.068	0.031
8 Uzbekistan	0.364	0.333	0.277	0.205	0.130	0.065	0.029
11 Russia	0.369	0.337	0.280	0.207	0.131	0.065	0.028
13 Germany	0.531	0.494	0.426	0.336	0.239	0.152	0.095
18 Germany	0.363	0.334	0.280	0.209	0.135	0.067	0.038
22 Canada	0.366	0.335	0.278	0.205	0.129	0.064	0.028
25 Russia	—	0.422	0.364	0.289	0.206	0.130	0.076
26 Russia	0.451	0.417	0.361	0.281	0.197	0.124	0.076
Average (0)	0.402	0.376	0.318	0.242	0.162	0.092	0.050
St. dev. (0)	0.065	0.061	0.058	0.052	0.044	0.037	0.027
Average (1)	0.402	0.376	0.318	0.242	0.162	0.092	0.050
St. dev. (1)	0.065	0.061	0.058	0.052	0.044	0.037	0.027
Re H_x							
3 Canada	0.722	0.753	0.794	0.817	0.827	0.836	0.854
8 Uzbekistan	1.337	0.534	0.644	0.755	0.807	0.830	0.853
11 Russia	0.724	0.755	0.796	0.818	0.827	0.836	0.855
13 Germany	1.358	1.158	0.950	0.985	0.992	0.981	0.952
18 Germany	0.716	0.746	0.786	0.811	0.821	0.832	0.852
22 Canada	0.829	0.780	0.754	0.810	0.825	0.835	0.854
25 Russia	1.754	0.758	0.830	0.715	0.757	0.862	0.932
26 Russia	0.087	0.598	0.760	0.880	0.894	0.904	0.920
Average (0)	0.941	0.760	0.789	0.824	0.844	0.864	0.884
St. dev. (0)	0.518	0.184	0.085	0.081	0.071	0.053	0.043
Average (1)	0.941	0.703	0.789	0.824	0.823	0.848	0.884
St. dev. (1)	0.518	0.096	0.085	0.081	0.040	0.027	0.043

Table B.12 (continued)

Participant/x =	0.0	0.5	1.0	2.0	4.0	8.0	16.0
Im H_x							
3 Canada	0.366	0.298	0.201	0.140	0.113	0.094	0.071
8 Uzbekistan	0.543	0.234	0.171	0.131	0.111	0.094	0.072
11 Russia	0.372	0.303	0.203	0.141	0.113	0.094	0.071
13 Germany	0.695	0.475	0.294	0.219	0.178	0.144	0.108
18 Germany	0.368	0.302	0.207	0.147	0.121	0.099	0.066
22 Canada	0.378	0.282	0.189	0.137	0.112	0.093	0.071
25 Russia	0.717	0.300	0.211	0.134	0.113	0.108	0.067
26 Russia	0.160	0.228	0.194	0.145	0.108	0.093	0.075
Average (0)	0.450	0.303	0.209	0.149	0.121	0.102	0.075
St. dev. (0)	0.189	0.076	0.037	0.029	0.023	0.018	0.014
Average (1)	0.450	0.278	0.197	0.139	0.113	0.096	0.070
St. dev. (1)	0.189	0.033	0.014	0.006	0.004	0.006	0.003

Table B.13. Model 2D-1 (HP, $T = 10.0$, $z = 1.25$)

Participant/x =	0.0	0.5	0.5	1.0	2.0	4.0	8.0	16.0
Re E_x								
3 Canada	0.006	—	—	1.056	0.997	0.956	0.935	0.928
8 Uzbekistan	0.014	—	1.090	1.064	1.001	0.959	0.938	0.931
11 Russia	0.006	0.719	—	1.050	0.995	0.952	0.931	0.923
18 Germany	0.005	0.545	—	1.056	0.994	0.951	0.929	0.922
22 Canada	—	—	1.075	1.052	0.993	0.952	0.930	0.923
25 Russia	0.011	—	1.078	1.039	0.986	0.930	0.907	0.933
26 Russia	0.014	—	1.094	1.054	0.997	0.950	0.926	0.921
Average (0)	0.009	0.632	1.084	1.053	0.995	0.950	0.928	0.926
St. dev. (0)	0.004	0.123	0.009	0.008	0.005	0.009	0.010	0.005
Average (1)	0.009	0.632	1.084	1.053	0.995	0.953	0.931	0.926
St. dev. (1)	0.004	0.123	0.009	0.008	0.005	0.003	0.004	0.005
Im E_x								
3 Canada	0.001	—	—	0.087	0.081	0.076	0.073	0.070
8 Uzbekistan	−0.003	—	0.101	0.095	0.086	0.081	0.078	0.075
11 Russia	0.001	0.064	—	0.092	0.085	0.080	0.077	0.074
18 Germany	0.001	0.047	—	0.090	0.083	0.079	0.076	0.074
22 Canada	—	—	0.095	0.092	0.085	0.080	0.076	0.074
25 Russia	−0.002	—	0.093	0.088	0.092	0.094	0.090	0.063
26 Russia	−0.003	—	0.098	0.089	0.079	0.076	0.075	0.073
Average (0)	−0.001	0.056	0.097	0.090	0.084	0.081	0.078	0.072
St. dev. (0)	0.002	0.012	0.004	0.003	0.004	0.006	0.006	0.004
Average (1)	−0.001	0.056	0.097	0.090	0.084	0.079	0.076	0.073
St. dev. (1)	0.002	0.012	0.004	0.003	0.004	0.002	0.002	0.002

Table B.14. Model 2D-1 (EP, $T = 10.0$, $z = 6.0$)

Participant/x =	0.0	0.5	1.0	2.0	4.0	8.0	16.0
Re E_y							
8 Uzbekistan	0.314	0.315	0.318	0.329	0.362	0.433	0.529
11 Russia	0.320	0.321	0.324	0.335	0.368	0.440	0.537
13 Germany	0.444	0.442	0.441	0.445	0.466	0.520	0.585
Im E_y							
8 Uzbekistan	0.266	0.265	0.264	0.258	0.241	0.213	0.195
11 Russia	0.266	0.266	0.264	0.258	0.242	0.213	0.194
13 Germany	0.387	0.387	0.385	0.378	0.355	0.312	0.269

Table B.15. Model 2D-1 (HP, $T = 10.0$, $z = 6.0$)

Participant/x =	0.0	0.5	1.0	2.0	4.0	8.0	16.0	16.0
Re E_x								
8 Uzbekistan	0.619	0.620	0.622	0.627	0.638	0.646	0.646	0.646
11 Russia	0.609	0.610	0.612	0.620	0.633	0.641	0.640	0.640
Im E_x								
8 Uzbekistan	0.252	0.252	0.252	0.253	0.254	0.255	0.254	0.254
11 Russia	0.251	0.251	0.251	0.252	0.254	0.254	0.254	0.254

Table B.16. Model 2D-2 (EP, $T = 10.0$, $z = 0$)

Participant/x =	−42.0	−22.0	−14.0	−4.0	2.0	10.0	16.0	22.0	42.0	80.0
Re E_y										
3 Canada	0.974	0.727	0.649	0.669	0.704	0.682	0.676	0.735	0.976	1.009
6 Poland	0.971	0.723	0.646	0.666	0.699	0.678	0.673	0.730	0.972	1.005
8 Uzbekistan	0.972	0.722	0.646	0.664	0.698	0.676	0.672	0.730	0.973	1.002
10 Russia	0.910	0.650	0.610	0.630	0.680	0.670	0.650	0.690	0.920	0.970
12 Czech	0.970	0.715	0.640	0.656	0.689	0.668	0.664	0.721	0.971	1.004
13 Germany	0.966	0.707	0.531	0.550	0.684	0.661	0.656	0.714	0.968	1.003
14 Russia	0.958	0.676	0.603	0.615	0.649	0.629	0.633	0.687	0.961	1.000
15.1 Russia	0.969	0.713	0.640	0.657	0.691	0.669	0.665	0.721	0.971	1.010
15.2 Russia	0.977	0.742	0.664	0.688	0.723	0.701	0.692	0.750	0.978	1.010
16 Russia	—	0.702	0.631	0.649	0.681	0.659	0.657	0.714	—	—
18 Germany	—	0.684	0.607	0.624	0.657	0.636	0.632	0.691	—	—
19 Germany	0.980	0.742	0.661	0.685	0.721	0.699	0.691	0.748	—	—
24 Russia	0.982	0.735	0.650	0.675	0.710	0.684	0.679	0.751	0.983	1.009
25 Russia	0.966	0.719	0.643	0.657	0.687	0.669	0.667	0.723	0.968	1.006
26 Russia	0.965	0.698	0.641	0.656	0.682	0.668	0.669	0.721	0.967	1.006
Average (0)	0.966	0.710	0.631	0.649	0.690	0.670	0.665	0.722	0.967	1.003
St. dev. (0)	0.018	0.025	0.033	0.034	0.020	0.019	0.018	0.021	0.016	0.011
Average (1)	0.971	0.715	0.638	0.656	0.693	0.673	0.665	0.722	0.972	1.006
St. dev. (1)	0.007	0.020	0.019	0.021	0.018	0.016	0.018	0.021	0.006	0.003
Im E_y										
3 Canada	−0.128	−0.257	−0.285	−0.303	−0.310	−0.297	−0.280	−0.256	−0.126	−0.033
6 Poland	−0.129	−0.254	−0.284	−0.302	−0.309	−0.296	−0.279	−0.255	−0.128	−0.023
8 Uzbekistan	−0.128	−0.258	−0.286	−0.303	−0.310	−0.297	−0.280	−0.257	−0.127	−0.026
10 Russia	−0.120	−0.260	−0.280	−0.290	−0.290	−0.280	−0.270	−0.240	−0.100	−0.025
12 Czech	−0.128	−0.256	−0.282	−0.299	−0.308	−0.293	−0.277	−0.255	−0.127	−0.021
13 Germany	−0.121	−0.248	−0.275	−0.291	−0.299	−0.285	−0.270	−0.247	−0.121	−0.019
14 Russia	−0.126	−0.268	−0.297	−0.311	−0.318	−0.302	−0.285	−0.259	−0.120	−0.008
15.1 Russia	−0.112	−0.235	−0.263	−0.278	−0.281	−0.272	−0.257	−0.234	−0.107	0.000
15.2 Russia	−0.099	−0.228	−0.258	−0.274	−0.279	−0.267	−0.251	−0.226	−0.097	0.000
16 Russia	—	−0.249	−0.276	−0.291	−0.299	−0.285	−0.270	−0.248	—	—
18 Germany	—	−0.247	−0.272	−0.289	−0.297	−0.283	−0.268	−0.247	—	—
19 Germany	−0.127	−0.265	−0.295	−0.315	−0.321	−0.308	−0.290	−0.264	—	—
24 Russia	−0.127	−0.258	−0.286	−0.302	−0.305	−0.277	−0.236	−0.264	−0.125	−0.021
25 Russia	−0.124	−0.252	−0.281	−0.295	−0.303	−0.289	−0.274	−0.250	−0.122	−0.013
26 Russia	−0.133	−0.258	−0.287	−0.306	−0.315	−0.301	−0.281	−0.255	−0.133	−0.011
Average (0)	−0.123	−0.253	−0.280	−0.297	−0.303	−0.289	−0.271	−0.250	−0.119	−0.017
St. dev. (0)	0.009	0.011	0.011	0.011	0.012	0.012	0.014	0.011	0.012	0.010
Average (1)	−0.125	−0.255	−0.282	−0.297	−0.303	−0.289	−0.274	−0.252	−0.119	−0.017
St. dev. (1)	0.005	0.008	0.009	0.011	0.012	0.012	0.010	0.009	0.012	0.010

Table B.16 (continued)

Participant/x =	−42.0	−22.0	−14.0	−4.0	2.0	10.0	16.0	22.0	42.0	80.0
Re H_x										
3 Canada	0.958	1.043	1.077	1.022	0.944	1.029	1.081	1.049	0.959	1.005
6 Poland	0.949	1.041	1.075	1.020	0.944	1.027	1.078	1.048	0.950	0.995
8 Uzbekistan	0.958	1.047	1.079	1.024	0.944	1.031	1.084	1.052	0.959	1.003
10 Russia	0.930	1.050	1.050	1.020	0.930	0.990	1.050	1.040	0.950	0.990
12 Czech	0.943	1.040	1.070	1.020	0.943	1.030	1.080	1.050	0.944	0.988
13 Germany	0.952	1.048	1.074	1.025	0.944	1.032	1.080	1.053	0.953	1.001
18 Germany	—	1.058	1.087	1.036	0.953	1.043	1.093	1.063	—	—
19 Germany	0.959	1.033	1.075	1.014	0.940	1.020	1.077	1.036	—	—
24 Russia	0.961	1.033	1.081	1.010	0.937	1.027	1.123	1.040	0.963	1.005
25 Russia	0.956	1.046	1.062	1.003	0.969	1.039	1.064	1.045	0.963	1.001
26 Russia	0.961	1.051	1.058	1.029	0.998	1.033	1.056	1.039	0.967	1.003
Average (0)	0.953	1.045	1.072	1.020	0.950	1.027	1.079	1.047	0.956	0.999
St. dev. (0)	0.010	0.008	0.011	0.009	0.019	0.014	0.019	0.008	0.008	0.006
Average (1)	0.955	1.045	1.072	1.020	0.945	1.031	1.074	1.045	0.956	0.999
St. dev. (1)	0.006	0.008	0.011	0.009	0.010	0.006	0.013	0.006	0.008	0.006
Im H_x										
3 Canada	−0.059	0.137	0.178	0.121	0.071	0.131	0.179	0.140	−0.058	−0.034
6 Poland	−0.048	0.148	0.189	0.132	0.084	0.142	0.189	0.151	−0.048	−0.015
8 Uzbekistan	−0.058	0.142	0.182	0.124	0.073	0.134	0.184	0.145	−0.058	−0.022
10 Russia	−0.028	0.190	0.180	0.140	0.090	0.130	0.190	0.170	−0.031	−0.013
12 Czech	−0.004	0.159	0.192	0.140	0.094	0.150	0.194	0.162	−0.039	−0.007
13 Germany	−0.047	0.160	0.196	0.140	0.091	0.150	0.198	0.162	−0.047	−0.018
18 Germany	—	0.146	0.183	0.127	0.079	0.138	0.185	0.149	—	—
19 Germany	−0.058	0.134	0.182	0.121	0.069	0.131	0.182	0.135	—	—
24 Russia	−0.057	0.136	0.190	0.126	0.074	0.141	0.157	0.138	−0.056	−0.022
25 Russia	−0.041	0.148	0.171	0.115	0.097	0.144	0.170	0.142	−0.038	−0.014
26 Russia	−0.028	0.134	0.148	0.125	0.109	0.127	0.135	0.099	−0.026	−0.013
Average (0)	−0.043	0.149	0.181	0.128	0.085	0.138	0.178	0.145	−0.045	−0.018
St. dev. (0)	0.018	0.017	0.013	0.009	0.013	0.008	0.018	0.019	0.012	0.008
Average (1)	−0.047	0.144	0.184	0.128	0.085	0.138	0.183	0.149	−0.045	−0.015
St. dev. (1)	0.012	0.010	0.007	0.009	0.013	0.008	0.012	0.012	0.012	0.005

Table B.17. Model 2D-2 (T = 10.0, z = 0)

Participant/x =	−42.0	−14.0	−4.0	2.0	10.0	16.0	22.0
ρ_a (EP)							
3 Canada	105.00	42.10	51.00	66.10	51.50	44.50	54.10
5 Finland	87.80	41.50	50.40	62.00	50.20	43.90	53.50
6 Poland	106.30	41.90	50.50	65.00	50.90	44.20	53.60
7 Hungary	—	40.80	48.60	63.80	48.90	42.80	—
8 Uzbekistan	104.47	41.66	50.07	65.03	50.45	43.85	53.12
10 Russia	97.32	39.70	45.38	62.60	52.89	43.51	48.06
11 Russia	105.00	41.80	49.90	64.50	50.60	44.10	53.20
12 Czech	107.50	41.17	48.65	63.47	49.03	43.22	51.96
13 Germany	104.40	38.76	47.52	81.92	47.68	41.78	50.38
14 Russia	105.00	38.90	43.90	60.40	44.00	40.30	45.50
15.1 Russia	104.80	39.40	46.50	61.70	46.90	41.30	49.30
15.2 Russia	105.40	41.40	51.70	69.10	52.00	43.50	54.10
16 Russia	—	39.33	46.92	62.83	46.20	41.05	49.52
18 Germany	—	39.87	47.46	62.14	47.86	41.96	51.10
19 Germany	105.79	44.08	54.51	70.12	55.17	47.07	57.64
Average (0)	103.23	40.82	48.87	65.38	49.62	43.14	51.79
St. dev. (0)	5.45	1.47	2.70	5.29	2.84	1.69	3.07
Average (1)	104.63	40.59	48.46	64.20	49.62	42.85	52.28
St. dev. (1)	2.59	1.21	2.29	2.76	2.84	1.34	2.57

Table B.17 (continued)

Participant/x =	−42.0	−14.0	−4.0	2.0	10.0	16.0	22.0
ρ_a (HP)							
5 Finland	108.50	44.60	72.10	97.00	75.10	54.90	76.80
6 Poland	95.20	40.65	63.83	85.81	66.30	49.45	68.02
8 Uzbekistan	98.79	43.35	66.86	88.97	69.39	52.15	71.13
11 Russia	98.00	43.40	66.90	88.60	69.10	52.20	70.80
12 Czech	98.62	44.45	68.82	89.67	71.53	54.14	73.66
16 Russia	—	43.19	60.53	84.58	59.10	47.75	65.10
18 Germany	—	46.56	72.89	95.64	76.08	56.87	78.17
20 Russia	99.60	41.80	64.70	87.90	67.00	50.05	68.60
Average (0)	99.78	43.50	67.08	89.77	69.20	52.19	71.54
St. dev. (0)	4.53	1.80	4.16	4.39	5.38	3.04	4.46
Average (1)	99.78	43.50	67.08	89.77	69.20	52.19	71.54
St. dev. (1)	4.53	1.80	4.16	4.39	5.38	3.04	4.46

Table B.18. Model 2D-2 (EP, T = 1000.0, z = 0)

Participant/x =	−42.0	−22.0	−14.0	−4.0	2.0	10.0	16.0	22.0	42.0	80.0
Re E_y										
3 Canada	0.581	0.418	0.360	0.360	0.377	0.385	0.400	0.442	0.594	0.743
6 Poland	0.593	0.427	0.369	0.368	0.385	0.394	0.409	0.451	0.606	0.759
8 Uzbekistan	0.582	0.419	0.361	0.361	0.377	0.386	0.401	0.442	0.595	0.743
9 Russia	—	0.419	0.361	0.360	0.377	0.386	0.401	0.442	—	—
11 Russia	0.583	0.417	0.359	0.359	0.375	0.384	0.400	0.441	0.596	0.747
12 Czech	0.589	0.423	0.365	0.364	0.382	0.390	0.405	0.447	0.602	0.755
13 Germany	0.617	0.432	0.367	0.366	0.384	0.394	0.410	0.457	0.630	0.796
15.1 Russia	0.582	0.415	0.357	0.356	0.372	0.383	0.399	0.440	0.595	0.749
15.2 USR	0.585	0.417	0.360	0.358	0.373	0.384	0.401	0.441	0.597	0.752
16 Russia	—	0.477	0.407	0.405	0.425	0.437	0.457	0.507	—	—
19 Germany	0.579	0.411	0.352	0.351	0.368	0.377	0.393	0.426	—	—
21 Canada	0.562	0.315	0.176	0.241	0.310	0.270	0.234	0.343	0.576	0.739
22.1 Canada	0.588	0.426	0.368	0.367	0.383	0.391	0.405	0.447	0.600	0.750
24 Russia	0.585	0.420	0.362	0.361	0.378	0.386	0.401	0.443	0.598	0.749
25 Russia	0.691	0.488	0.417	0.414	0.433	0.444	0.464	0.513	0.704	0.915
26 Russia	0.689	0.483	0.419	0.411	0.429	0.447	0.472	0.522	0.704	0.916
Average (0)	0.600	0.425	0.360	0.363	0.383	0.390	0.403	0.450	0.615	0.778
St. dev. (0)	0.040	0.039	0.054	0.038	0.029	0.039	0.052	0.041	0.041	0.063
Average (1)	0.586	0.433	0.372	0.371	0.388	0.398	0.415	0.457	0.599	0.753
St. dev. (1)	0.012	0.026	0.022	0.021	0.022	0.024	0.026	0.030	0.013	0.015
Im E_y										
3 Canada	0.109	0.199	0.228	0.230	0.225	0.228	0.224	0.202	0.112	0.037
6 Poland	0.107	0.198	0.228	0.230	0.225	0.227	0.229	0.202	0.110	0.033
8 Uzbekistan	0.104	0.195	0.224	0.226	0.221	0.224	0.220	0.198	0.107	0.033
9 Russia	—	0.193	0.223	0.225	0.220	0.223	0.218	0.195	—	—
11 Russia	0.103	0.194	0.223	0.226	0.221	0.223	0.219	0.197	0.106	0.030
12 Czech	0.113	0.205	0.234	0.236	0.231	0.234	0.230	0.208	0.115	0.038
13 Germany	0.150	0.239	0.267	0.270	0.266	0.269	0.265	0.244	0.152	0.071
15.1 Russia	0.122	0.211	0.239	0.243	0.238	0.241	0.236	0.216	0.125	0.050
15.2 Russia	0.116	0.203	0.230	0.233	0.229	0.232	0.227	0.207	0.119	0.045
16 Russia	—	0.251	0.283	0.288	0.283	0.286	0.281	0.256	—	—
19 Germany	0.102	0.194	0.223	0.225	0.220	0.223	0.219	0.196	—	—
21 Canada	0.119	0.277	0.338	0.309	0.272	0.314	0.345	0.285	0.121	0.035
22.1 Canada	0.098	0.185	0.213	0.216	0.211	0.213	0.210	0.189	0.101	0.028
24 Russia	0.103	0.193	0.222	0.224	0.219	0.222	0.218	0.196	0.105	0.031
25 Russia	0.135	0.239	0.273	0.276	0.271	0.274	0.268	0.244	0.138	0.036
26 Russia	0.145	0.255	0.287	0.293	0.290	0.289	0.282	0.256	0.150	0.039
Average (0)	0.116	0.214	0.246	0.247	0.240	0.245	0.243	0.218	0.120	0.039
St. dev. (0)	0.017	0.028	0.034	0.030	0.026	0.031	0.036	0.029	0.017	0.011
Average (1)	0.114	0.210	0.240	0.243	0.240	0.241	0.236	0.214	0.120	0.036
St. dev. (1)	0.014	0.023	0.025	0.026	0.026	0.025	0.025	0.024	0.017	0.006

Table B.18 (continued)

Participant / x =	−42.0	−22.0	−14.0	−4.0	2.0	10.0	16.0	22.0	42.0	80.0
Re H_x										
3 Canada	1.038	1.846	2.312	1.986	1.697	1.936	2.124	1.805	1.030	0.917
6 Poland	1.048	1.892	2.357	2.020	1.731	1.972	2.164	1.849	1.040	0.926
8 Uzbekistan	1.028	1.858	2.325	1.984	1.690	1.938	2.138	1.818	1.022	0.920
9 Russia	—	1.851	2.299	1.974	1.695	1.932	2.118	1.808	—	—
11 Russia	1.030	1.850	2.310	1.980	1.700	1.930	2.130	1.820	1.020	0.914
12 Czech	1.050	1.880	2.340	2.020	1.720	1.960	2.150	1.830	1.040	0.925
13 Germany	1.105	1.998	2.480	2.131	1.831	2.086	2.289	1.957	1.096	0.966
19 Germany	1.036	1.865	2.320	1.988	1.705	1.943	2.132	1.838	—	—
21 Canada	0.868	1.705	3.302	1.792	0.864	1.976	3.522	1.751	0.869	0.882
22 Canada	1.027	1.855	2.355	1.984	1.631	1.945	2.199	1.830	1.021	0.917
22.1 Canada	1.033	1.826	2.274	1.972	1.686	1.930	2.110	1.798	1.027	0.918
24 Russia	1.036	1.853	2.302	1.975	1.695	1.929	2.115	1.812	1.029	0.918
25 Russia	1.204	2.208	2.571	2.179	2.040	2.246	2.308	2.084	1.194	1.023
26 Russia	1.242	2.100	2.372	2.254	2.090	2.125	2.095	1.827	1.230	1.021
Average (0)	1.057	1.899	2.423	2.017	1.698	1.989	2.257	1.845	1.051	0.937
St. dev. (0)	0.090	0.125	0.265	0.110	0.276	0.095	0.370	0.082	0.091	0.044
Average (1)	1.058	1.875	2.355	2.016	1.762	1.969	2.159	1.826	1.051	0.937
St. dev. (1)	0.053	0.092	0.082	0.068	0.142	0.063	0.067	0.046	0.091	0.044
Im H_x										
3 Canada	0.058	−0.008	0.023	0.035	0.025	−0.040	−0.090	−0.072	0.487	0.413
6 Poland	0.061	−0.004	0.025	0.038	0.027	−0.039	−0.089	−0.070	0.052	0.045
8 Uzbekistan	0.062	−0.005	0.025	0.038	0.028	−0.038	−0.089	−0.070	0.054	0.045
9 Russia	—	−0.001	0.025	0.037	0.022	−0.050	−0.084	−0.051	—	—
11 Russia	0.057	−0.003	0.029	0.040	0.026	−0.037	−0.084	−0.068	0.048	0.040
12 Czech	0.061	−0.016	0.030	0.037	0.026	−0.041	−0.088	−0.077	0.053	0.046
13 Germany	0.081	0.082	0.154	0.140	0.104	0.053	0.015	0.006	0.070	0.050
19 Germany	0.057	0.002	0.031	0.044	0.030	−0.033	−0.082	−0.040	—	—
21 Canada	0.073	−0.212	0.150	−0.051	0.100	−0.139	−0.156	−0.258	0.072	0.043
22 Canada	0.057	−0.020	0.029	0.033	0.028	−0.043	−0.095	−0.085	0.049	0.040
22.1 Canada	0.055	−0.002	0.039	0.044	0.031	−0.028	−0.066	−0.061	0.047	0.039
24 Russia	0.056	−0.004	0.023	0.037	0.026	−0.037	−0.086	−0.067	0.048	0.040
25 Russia	0.036	−0.004	0.034	0.032	0.023	−0.054	−0.092	−0.080	0.024	0.012
26 Russia	0.030	−0.010	0.005	0.014	−0.006	−0.054	−0.081	−0.061	0.014	0.011
Average (0)	0.057	−0.015	0.044	0.037	0.035	−0.041	−0.083	−0.075	0.085	0.069
St. dev. (0)	0.013	0.062	0.046	0.038	0.030	0.038	0.035	0.057	0.128	0.109
Average (1)	0.059	0.001	0.026	0.036	0.024	−0.041	−0.085	−0.061	0.048	0.037
St. dev. (1)	0.011	0.025	0.008	0.008	0.010	0.008	0.007	0.023	0.017	0.013
Re H_z										
3 Canada	−0.626	−0.948	−0.405	0.261	0.183	0.064	0.454	0.869	0.601	0.271
6 Poland	−0.627	−0.969	−0.409	0.273	0.187	0.058	0.458	0.889	0.603	0.272
8 Uzbekistan	−0.608	−0.957	−0.404	0.269	0.186	0.057	0.451	0.879	0.585	0.235
9 Russia	—	−0.968	−0.408	0.272	0.185	0.055	0.460	0.890	—	—
11 Russia	−0.630	−0.965	−0.409	0.265	0.184	0.062	0.458	0.885	0.605	0.274
12 Czech	−0.637	−0.936	−0.407	0.259	0.186	0.081	0.460	0.863	0.611	0.279
13 Germany	−0.662	−1.040	−0.440	0.279	0.194	0.068	0.494	0.955	0.635	0.282
19 Germany	−0.631	−0.983	−0.413	0.278	0.188	0.054	0.461	0.929	—	—
21 Canada	−0.655	−3.048	−0.463	2.152	0.318	−1.876	0.460	3.042	0.630	0.262
22 Canada	−0.621	−1.047	−0.413	0.348	0.196	−0.025	0.454	0.977	0.597	0.273
22.1 Canada	−0.615	−0.960	−0.408	0.260	0.177	0.052	0.454	0.887	0.594	0.270
24 Russia	−0.625	−0.971	−0.407	0.278	0.187	0.053	0.455	0.890	0.597	0.267
25 Russia	−0.769	−1.096	−0.349	0.295	0.263	0.173	0.599	0.998	0.765	0.433
26 Russia	−0.817	−0.815	−0.536	0.068	0.242	0.301	0.587	1.044	0.853	0.432
Average (0)	−0.656	−1.122	−0.419	0.397	0.205	−0.059	0.479	1.071	0.640	0.296
St. dev. (0)	0.063	0.558	0.041	0.509	0.041	0.528	0.049	0.570	0.083	0.065
Average (1)	−0.642	−0.973	−0.410	0.262	0.197	0.081	0.460	0.920	0.620	0.269
St. dev. (1)	0.043	0.066	0.025	0.063	0.026	0.078	0.011	0.057	0.050	0.013

Table B.18 (continued)

Participant/x =	−42.0	−22.0	−14.0	−4.0	2.0	10.0	16.0	22.0	42.0	80.0
Im H_z										
3 Canada	0.019	0.036	−0.003	0.034	0.056	0.070	0.019	−0.061	−0.035	0.015
6 Poland	0.018	0.036	−0.003	0.034	0.057	0.071	0.019	−0.062	−0.034	0.015
8 Uzbekistan	0.018	0.038	−0.004	0.033	0.055	0.070	0.019	−0.064	−0.033	0.021
9 Russia	—	0.034	−0.003	0.037	0.060	0.064	−0.001	−0.065	—	—
11 Russia	0.016	0.030	−0.005	0.037	0.058	0.070	0.020	−0.058	−0.032	0.016
12 Czech	0.022	0.030	−0.005	0.037	0.058	0.069	0.020	−0.057	−0.038	0.014
13 Germany	−0.006	−0.029	−0.030	0.061	0.074	0.073	0.045		−0.016	0.011
19 Germany	0.012	0.030	−0.007	0.035	0.059	0.073	0.022	−0.045	—	—
21 Canada	0.012	0.258	−0.156	0.032	0.073	0.286	0.144	−0.415	−0.030	0.022
22 Canada	0.018	0.046	−0.011	0.033	0.056	0.081	0.023	−0.080	−0.034	0.015
22.1 Canada	0.011	0.026	−0.010	0.034	0.054	0.066	0.022	−0.052	−0.026	0.018
24 Russia	0.015	0.035	−0.005	0.032	0.056	0.072	0.020	−0.061	−0.031	0.017
25 Russia	0.000	0.004	−0.007	0.051	0.062	0.079	0.030	−0.036	−0.024	0.019
26 Russia	0.018	0.031	−0.001	0.037	0.077	0.043	0.029	−0.058	−0.055	0.006
Average (0)	0.013	0.043	−0.018	0.038	0.061	0.085	0.031	−0.080	−0.032	0.016
St. dev. (0)	0.008	0.065	0.040	0.008	0.008	0.059	0.034	0.098	0.009	0.004
Average (1)	0.015	0.027	−0.007	0.036	0.060	0.069	0.022	−0.055	−0.030	0.017
St. dev. (1)	0.006	0.019	0.007	0.005	0.006	0.009	0.010	0.016	0.006	0.003

Table B.19. Model 2D-2 (HP, T = 1000.0, z = 0)

Participant/x =	−42.0	−22.0	−14.0	−4.0	2.0	10.0	16.0	22.0	42.0	80.0
Re E_x										
3 Canada	1.096	0.905	0.392	0.932	1.201	0.942	0.613	0.903	1.078	1.023
6 Poland	1.095	0.859	0.373	0.897	1.200	0.914	0.592	0.862	1.076	1.017
8 Uzbekistan	1.107	0.870	0.378	0.910	1.214	0.927	0.600	0.874	1.089	1.031
12 Czech	1.090	0.921	0.436	0.913	1.160	0.948	0.677	0.913	1.070	1.020
19 Germany	1.086	0.817	0.356	0.815	1.130	0.798	0.533	0.796	1.051	1.011
20 Russia	1.090	0.840	0.358	0.860	1.195	0.900	0.570	0.840	1.080	1.008
21 Canada	1.193	0.033	0.013	0.030	1.765	0.025	0.011	0.031	1.173	1.040
22 Canada	1.095	0.902	0.300	0.918	1.248	0.923	0.483	0.894	1.076	1.019
22.1 Canada	1.094	0.831	0.396	0.823	1.181	0.849	0.618	0.911	1.071	1.018
25 Russia	1.081	0.869	0.385	0.936	1.177	0.949	0.615	0.883	1.061	1.003
26 Russia	1.083	0.933	0.414	0.923	1.201	0.932	0.642	0.931	1.063	1.010
Average (0)	1.101	0.798	0.346	0.814	1.243	0.828	0.541	0.803	1.081	1.018
St. dev. (0)	0.031	0.257	0.116	0.263	0.176	0.270	0.183	0.259	0.032	0.011
Average (1)	1.092	0.875	0.379	0.893	1.191	0.908	0.594	0.881	1.071	1.016
St. dev. (1)	0.008	0.039	0.037	0.044	0.032	0.049	0.055	0.040	0.011	0.008
Im E_x										
3 Canada	0.009	0.002	−0.023	0.004	0.018	0.005	−0.012	0.001	0.007	0.001
6 Poland	0.021	0.010	−0.019	0.013	0.031	0.013	−0.006	0.009	0.019	0.012
8 Uzbekistan	0.023	0.010	−0.019	0.013	0.032	0.014	−0.006	0.010	0.021	0.014
12 Czech	0.012	0.004	−0.020	0.004	0.017	0.005	−0.008	0.003	0.010	0.004
19 Germany	0.012	0.000	−0.024	0.000	0.015	−0.005	−0.017	−0.005	0.006	0.003
20 Russia	0.013	0.002	−0.023	0.006	0.023	0.006	−0.012	0.002	0.011	0.002
21 Canada	0.030	0.001	0.000	0.001	0.056	0.001	0.000	0.001	0.028	0.013
22 Canada	0.016	0.005	−0.027	0.005	0.028	0.005	−0.017	0.007	0.014	0.007
22.1 Canada	0.017	0.000	−0.023	0.000	0.023	−0.001	−0.011	0.007	0.013	0.006
25 Russia	0.018	0.010	−0.015	0.014	0.025	0.014	−0.004	0.009	0.015	0.010
26 Russia	0.007	0.001	−0.025	0.001	0.014	0.000	−0.015	0.001	0.006	0.000
Average (0)	0.016	0.004	−0.020	0.006	0.026	0.005	−0.010	0.004	0.014	0.007
St. dev. (0)	0.007	0.004	0.007	0.005	0.012	0.006	0.006	0.005	0.007	0.005
Average (1)	0.015	0.004	−0.022	0.006	0.023	0.005	−0.010	0.004	0.012	0.007
St. dev. (1)	0.005	0.004	0.004	0.005	0.006	0.006	0.006	0.005	0.005	0.005

Table B.20. Model 2D-2 (T = 1000.0, z = 0)

Participant/x =	−42.0	−14.0	−4.0	2.0	10.0	16.0	22.0
ρ_a (EP)							
3 Canada	20.30	2.14	2.90	4.20	3.36	2.93	4.54
5 Finland	16.10	2.00	2.90	3.80	3.00	2.90	4.80
6 Poland	20.70	2.10	2.90	4.20	3.30	2.90	4.50
8 Uzbekistan	20.69	2.10	2.89	4.20	3.33	2.87	4.46
9 Russia	—	2.14	2.90	4.16	3.34	2.91	4.48
11 Russia	20.80	2.12	2.89	4.17	3.34	2.90	4.47
12 Czech	20.47	2.14	2.91	4.20	3.37	2.94	4.53
13 Germany	20.35	2.06	2.80	4.02	3.23	2.82	4.34
15.1 Russia	20.90	1.82	2.68	4.21	3.05	2.51	3.97
15.2 Russia	21.30	1.80	2.74	4.44	3.09	2.50	3.99
16 Russia	—	1.96	2.75	4.11	3.18	2.72	4.42
19 Germany	20.16	2.03	2.76	3.97	3.19	2.79	4.09
21 Canada	22.60	0.70	2.50	11.70	2.30	0.70	3.30
22 Canada	20.87	2.01	2.88	4.53	3.29	2.68	4.39
22.1 Canada	20.89	2.19	2.92	4.22	3.35	2.94	4.57
Average (0)	20.47	1.95	2.82	4.68	3.18	2.67	4.32
St. dev. (0)	1.45	0.37	0.12	1.95	0.27	0.56	0.36
Average (1)	20.84	2.04	2.84	4.17	3.24	2.81	4.40
St. dev. (1)	0.64	0.12	0.08	0.18	0.12	0.15	0.23
ρ_a (HP)							
5 Finland	79.00	10.40	54.00	92.00	56.00	25.70	50.50
6 Poland	75.37	8.74	50.52	90.49	52.52	22.11	46.66
8 Uzbekistan	77.00	9.02	51.98	95.55	54.01	22.61	47.91
11 Russia	77.30	9.18	52.60	92.80	54.00	22.90	48.10
12 Czech	74.06	11.96	52.37	84.90	56.38	28.77	52.32
16 Russia	—	4.02	34.52	75.73	31.33	11.60	33.77
20 Russia	79.00	8.50	51.50	95.00	53.70	21.60	47.20
22 Canada	75.26	5.71	52.87	97.80	53.43	14.69	50.19
22.1 Canada	75.10	9.87	42.52	87.56	45.31	23.98	52.12
Average (0)	76.51	8.60	49.21	90.20	50.74	21.55	47.64
St. dev. (0)	1.86	2.40	6.45	6.74	7.95	5.30	5.60
Average (1)	76.51	8.60	51.05	92.01	53.17	21.55	49.37
St. dev. (1)	1.86	2.40	3.59	4.28	3.43	5.30	2.20

Table B.21. Model 2D-3A (EP, T = 100.0, z = 0)

Participant/x =	−40.0	−25.0	−20.0	−15.0	−5.0	0.0	5.0	15.0	20.0	25.0
Re E_y										
2 Canada	0.848	0.622	0.489	0.375	0.401	0.535	0.671	0.847	0.904	0.943
3 Canada	0.848	0.623	0.489	0.375	0.401	0.535	0.672	0.848	0.904	0.943
6 Poland	0.851	0.625	0.491	0.376	0.402	0.536	0.673	0.849	0.905	0.944
8 Uzbekistan	0.853	0.624	0.488	0.373	0.399	0.532	0.670	0.847	0.903	0.943
10 Russia	0.870	0.640	0.500	0.380	0.400	0.540	0.680	0.860	0.920	0.950
11 Russia	0.851	0.624	0.490	0.376	0.403	0.536	0.672	0.849	0.905	0.944
12 Czech	0.851	0.623	0.488	0.373	0.399	0.534	0.671	0.848	0.905	0.944
15 Russia	0.864	0.635	0.499	0.383	0.410	0.547	0.687	0.866	0.920	0.956
16 Russia	0.872	0.635	—	0.381	0.404	0.538	0.675	0.852	0.907	0.946
17 USA	0.851	0.625	0.491	0.376	0.402	0.535	0.672	0.848	0.904	0.942
18 Germany	0.845	0.619	0.487	0.373	0.400	0.535	0.670	0.845	0.901	0.940
19 Germany	0.859	0.637	0.502	0.389	0.415	0.548	0.686	0.861	0.916	0.953
22 Canada	0.851	0.627	0.490	0.372	0.397	0.533	0.672	0.849	0.906	0.945
25 Russia	0.827	0.607	0.493	0.386	0.413	0.531	0.656	0.830	0.894	0.936
26 Russia	0.842	0.590	0.488	0.388	0.408	0.527	0.664	0.847	0.907	0.939
Average (0)	0.852	0.624	0.492	0.378	0.404	0.536	0.673	0.850	0.907	0.945
St. dev. (0)	0.011	0.012	0.005	0.006	0.005	0.006	0.008	0.008	0.007	0.005
Average (1)	0.854	0.626	0.491	0.378	0.403	0.535	0.674	0.851	0.907	0.944
St. dev. (1)	0.009	0.008	0.004	0.006	0.005	0.005	0.006	0.006	0.007	0.004

Table B.21 (continued)

Participant/x =	−40.0	−25.0	−20.0	−15.0	−5.0	0.0	5.0	15.0	20.0	25.0
Im E_y										
2 Canada	−0.104	−0.065	0.021	0.078	0.065	−0.035	−0.180	−0.259	−0.226	−0.176
3 Canada	−0.101	−0.064	0.023	0.079	0.067	−0.034	−0.180	−0.260	−0.226	−0.176
6 Poland	−0.103	−0.065	0.020	0.075	0.062	−0.037	−0.182	−0.262	−0.228	−0.177
8 Uzbekistan	−0.102	−0.063	0.024	0.078	0.065	−0.032	−0.178	−0.259	−0.225	−0.175
10 Russia	−0.090	−0.060	0.030	0.080	0.070	−0.030	−0.180	−0.260	−0.220	−0.170
11 Russia	−0.102	−0.063	0.023	0.079	0.066	−0.035	−0.179	−0.260	−0.227	−0.177
12 Czech	−0.101	−0.062	0.024	0.079	0.066	−0.032	−0.178	−0.259	−0.225	−0.175
15 Russia	−0.079	−0.040	0.044	0.096	0.083	−0.011	−0.155	−0.235	−0.202	−0.152
16 Russia	−0.090	−0.047	—	0.095	0.083	−0.015	−0.159	−0.237	−0.204	−0.155
17 USA	−0.103	−0.065	0.020	0.074	0.061	−0.038	−0.181	−0.260	−0.226	−0.176
18 Germany	−0.095	−0.056	0.032	0.086	0.073	−0.024	−0.174	−0.258	−0.225	−0.175
19 Germany	−0.104	−0.071	0.011	0.064	0.051	−0.046	−0.187	−0.262	−0.227	−0.176
22 Canada	−0.103	−0.068	0.020	0.074	0.062	−0.036	−0.184	−0.263	−0.227	−0.176
25 Russia	−0.081	−0.029	0.036	0.097	0.079	−0.001	−0.131	−0.213	−0.198	−0.155
26 Russia	−0.091	−0.019	0.029	0.086	0.073	−0.013	−0.147	−0.231	−0.205	−0.169
Average (0)	−0.097	−0.056	0.026	0.081	0.068	−0.028	−0.172	−0.252	−0.219	−0.171
St. dev. (0)	0.008	0.015	0.008	0.009	0.009	0.012	0.016	0.015	0.011	0.009
Average (1)	−0.098	−0.058	0.024	0.081	0.070	−0.030	−0.175	−0.255	−0.219	−0.172
St. dev. (1)	0.007	0.012	0.006	0.009	0.008	0.010	0.012	0.011	0.011	0.008
Re H_x										
2 Canada	0.911	0.941	1.234	1.485	1.509	1.189	0.820	0.746	0.812	0.897
3 Canada	0.913	0.941	1.234	1.484	1.508	1.189	0.821	0.747	0.812	0.897
6 Poland	0.905	0.942	1.228	1.437	1.459	1.187	0.827	0.745	0.809	0.890
8 Uzbekistan	0.912	0.938	1.241	1.498	1.524	1.188	0.817	0.740	0.806	0.900
10 Russia	0.900	0.910	1.200	1.390	1.410	1.160	0.800	0.730	0.800	0.880
11 Russia	0.914	0.945	1.230	1.480	1.510	1.180	0.825	0.747	0.814	0.900
12 Czech	0.898	0.937	1.200	1.390	1.410	1.160	0.827	0.746	0.806	0.883
17 USA	0.912	0.946	1.268	1.494	1.517	1.223	0.823	0.742	0.811	0.898
18 Germany	0.908	0.943	1.266	1.483	1.509	1.225	0.824	0.747	0.814	0.899
19 Germany	0.915	0.449	1.249	1.465	1.489	1.210	0.827	0.749	0.818	0.902
22 Canada	0.914	0.939	1.244	1.482	1.508	1.223	0.822	0.746	0.817	0.902
25 Russia	0.932	1.175	1.175	1.301	1.484	1.155	0.812	0.767	0.866	0.910
26 Russia	0.991	1.126	1.126	1.439	1.320	1.054	0.876	0.857	0.885	0.885
Average (0)	0.917	0.933	1.223	1.448	1.474	1.180	0.825	0.755	0.821	0.896
St. dev. (0)	0.024	0.166	0.039	0.057	0.060	0.045	0.017	0.032	0.025	0.009
Average (1)	0.911	0.974	1.231	1.461	1.486	1.191	0.820	0.746	0.815	0.896
St. dev. (1)	0.009	0.084	0.027	0.038	0.040	0.025	0.008	0.008	0.017	0.009
Im H_x										
2 Canada	−0.021	0.094	0.134	0.248	0.188	0.089	0.145	0.091	0.016	−0.049
3 Canada	−0.018	0.096	0.135	0.249	0.189	0.089	0.146	0.092	0.018	−0.049
6 Poland	−0.010	0.104	0.152	0.277	0.220	0.106	0.149	0.095	0.024	−0.040
8 Uzbekistan	−0.021	0.097	0.145	0.263	0.200	0.105	0.149	0.096	0.024	−0.052
10 Russia	0.000	0.120	0.170	0.310	0.260	0.130	0.160	0.100	0.030	−0.030
11 Russia	−0.018	0.097	0.138	0.250	0.189	0.092	0.147	0.094	0.018	−0.050
12 Czech	0.000	0.111	0.171	0.304	0.251	0.126	0.149	0.097	0.031	−0.028
17 USA	−0.018	0.099	0.132	0.262	0.200	0.087	0.150	0.095	0.019	−0.050
18 Germany	−0.023	0.101	0.117	0.244	0.182	0.060	0.149	0.095	0.019	−0.049
19 Germany	−0.019	0.093	0.131	0.250	0.191	0.083	0.140	0.087	0.014	−0.052
22 Canada	−0.019	0.092	0.130	0.256	0.197	0.068	0.145	0.093	0.012	−0.053
25 Russia	0.017	0.150	0.150	0.191	0.274	0.103	0.162	0.101	0.043	−0.022
26 Russia	0.076	0.138	0.138	0.267	0.164	0.134	0.135	0.061	0.043	0.043
Average (0)	−0.006	0.107	0.142	0.259	0.208	0.098	0.148	0.092	0.024	−0.037
St. dev. (0)	0.027	0.018	0.016	0.029	0.033	0.023	0.007	0.010	0.010	0.026
Average (1)	−0.013	0.104	0.142	0.265	0.208	0.098	0.148	0.095	0.024	−0.044
St. dev. (1)	0.012	0.014	0.016	0.022	0.033	0.023	0.007	0.004	0.010	0.011

Table B.21 (continued)

Participant/x =	−40.0	−25.0	−20.0	−15.0	−5.0	0.0	5.0	15.0	20.0	25.0
Re H_z										
2 Canada	−0.072	−0.321	−0.567	−0.145	0.240	0.731	0.428	0.093	−0.044	−0.041
3 Canada	−0.070	−0.322	−0.571	−0.145	0.242	0.735	0.430	0.092	−0.046	−0.043
6 Poland	−0.068	−0.313	−0.528	−0.140	0.236	0.688	0.420	0.089	−0.040	−0.043
8 Uzbekistan	−0.061	−0.312	−0.558	−0.121	0.212	0.717	0.414	0.091	−0.043	−0.041
8.4 Russia	—	—	−0.517	—	—	0.675	—	—	−0.039	—
8.5 Russia	—	—	−0.561	—	—	0.720	—	—	−0.043	—
10 Russia	−0.060	−0.290	−0.540	−0.130	0.220	0.700	0.400	0.090	−0.050	−0.050
12 Czech	−0.070	−0.319	−0.503	−0.148	0.245	0.661	0.427	0.092	−0.031	−0.042
17 USA	−0.070	−0.314	−0.531	−0.147	0.244	0.691	0.420	0.089	−0.042	−0.042
18 Germany	−0.057	−0.313	−0.483	−0.135	0.233	0.645	0.426	0.093	−0.033	−0.041
19 Germany	−0.062	−0.305	−0.483	−0.139	0.234	0.640	0.413	0.083	−0.042	−0.047
22 Canada	−0.074	−0.327	−0.525	−0.146	0.241	0.691	0.426	0.088	−0.041	−0.044
25 Russia	−0.092	−0.340	−0.340	−0.319	0.123	0.503	0.304	0.076	−0.022	−0.024
26 Russia	−0.147	−0.280	−0.280	−0.233	0.185	0.468	0.372	0.073	−0.025	−0.025
Average (0)	−0.075	−0.313	−0.499	−0.162	0.221	0.662	0.407	0.087	−0.039	−0.040
St. dev. (0)	0.024	0.016	0.086	0.057	0.035	0.080	0.036	0.007	0.008	0.008
Average (1)	−0.069	−0.316	−0.516	−0.148	0.230	0.677	0.416	0.089	−0.040	−0.042
St. dev. (1)	0.010	0.013	0.060	0.029	0.018	0.060	0.017	0.005	0.007	0.006
Im H_z										
2 Canada	−0.109	−0.162	−0.071	−0.138	0.261	−0.042	0.087	0.161	0.222	0.144
3 Canada	−0.111	−0.163	−0.072	−0.139	0.156	0.043	0.087	0.161	0.222	0.144
6 Poland	−0.110	−0.169	−0.110	−0.141	0.158	0.004	0.092	0.158	0.204	0.140
8 Uzbekistan	−0.110	−0.173	−0.096	−0.154	0.177	−0.017	0.096	0.157	0.212	0.140
8.4 Russia	—	—	−0.131	—	—	−0.030	—	—	0.205	—
8.5 Russia	—	—	−0.091	—	—	−0.023	—	—	0.213	—
10 Russia	−0.100	−0.170	−0.100	−0.130	0.150	−0.008	0.090	0.150	0.200	0.130
12 Czech	−0.111	−0.166	−0.121	−0.137	0.152	0.023	0.089	0.159	0.197	0.141
17 USA	−0.111	−0.168	−0.109	−0.137	0.151	0.001	0.092	0.158	0.207	0.140
18 Germany	−0.103	−0.162	−0.129	−0.139	0.165	0.041	0.088	0.157	0.199	0.139
19 Germany	−0.110	−0.171	−0.138	−0.140	0.157	0.044	0.097	0.159	0.199	0.138
22 Canada	−0.111	−0.167	−0.123	−0.142	0.156	0.016	0.092	0.160	0.208	0.145
25 Russia	−0.107	−0.139	−0.139	−0.132	0.092	0.051	0.107	0.167	0.167	0.123
26 Russia	−0.143	−0.143	−0.143	−0.125	0.109	0.101	0.107	0.167	0.135	0.135
Average (0)	−0.111	−0.163	−0.112	−0.138	0.157	0.015	0.094	0.160	0.199	0.138
St. dev. (0)	0.011	0.011	0.024	0.007	0.040	0.039	0.007	0.005	0.023	0.006
Average (1)	−0.108	−0.165	−0.112	−0.136	0.148	0.008	0.094	0.160	0.204	0.140
St. dev. (1)	0.004	0.008	0.024	0.005	0.025	0.031	0.007	0.004	0.014	0.004

Table B.22. Model 2D-3A (HP, T = 100.0, z = 0)

Participant/x =	−40.0	−25.0	−20.0	−15.0	−5.0	0.0	5.0	15.0	20.0	25.0
Re E_x										
2 Canada	0.997	1.002	1.134	0.265	0.250	0.043	4.029	3.991	4.376	0.705
3 Canada	0.997	1.001	1.134	0.265	0.250	0.043	4.017	3.986	4.370	0.706
6 Poland	0.987	0.995	1.132	0.254	0.239	—	3.990	3.950	4.260	0.669
8 Uzbekistan	0.999	1.010	1.150	0.266	0.239	0.052	4.020	4.110	4.300	0.715
10 Russia	0.960	0.970	—	0.260	0.240	—	4.020	3.990	—	0.700
12 Czech	0.999	1.010	—	0.266	0.251	—	4.030	3.980	—	0.708
17 USA	0.997	1.005	—	0.267	0.253	—	3.999	3.955	—	0.712
18 Germany	0.987	0.997	—	0.262	0.247	—	3.996	3.937	—	0.704
22 Canada	0.998	1.001	1.138	0.267	0.254	0.043	3.996	3.950	4.321	0.720
25 Russia	0.976	0.990	1.070	0.246	0.235	0.063	4.266	4.226	—	0.679
26 Russia	0.997	1.017	1.046	0.264	0.250	0.074	4.312	4.194	—	0.629
Average (0)	0.990	1.000	1.115	0.262	0.246	0.053	4.061	4.024	4.325	0.695
St. dev. (0)	0.012	0.012	0.040	0.007	0.007	0.013	0.114	0.103	0.049	0.027
Average (1)	0.993	1.003	1.115	0.264	0.246	0.053	4.036	4.024	4.325	0.702
St. dev. (1)	0.008	0.008	0.040	0.004	0.007	0.013	0.082	0.103	0.049	0.016

Table B.22 (continued)

Participant/x =	−40.0	−25.0	−20.0	−15.0	−5.0	0.0	5.0	15.0	20.0	25.0
Im E_x										
2 Canada	−0.080	−0.113	−0.174	−0.073	−0.085	−0.005	−0.381	−0.437	−0.580	−0.185
3 Canada	−0.079	−0.112	−0.174	−0.074	−0.086	−0.005	−0.378	−0.434	−0.577	−0.185
6 Poland	−0.062	−0.094	−0.154	−0.055	−0.068	—	−0.353	−0.401	−0.534	−0.171
8 Uzbekistan	−0.076	−0.109	−0.179	−0.071	−0.094	−0.016	−0.364	−0.513	−0.548	−0.187
10 Russia	−0.040	−0.070	—	−0.050	−0.060	—	−0.150	−0.220	—	−0.160
12 Czech	−0.075	−0.109	—	−0.062	−0.075	—	−0.377	−0.426	—	−0.183
17 USA	−0.076	−0.108	—	−0.069	−0.081	—	−0.371	−0.418	—	−0.187
18 Germany	−0.076	−0.111	—	−0.079	−0.090	—	−0.335	−0.379	—	−0.181
22 Canada	−0.076	−0.109	−0.171	−0.068	−0.081	−0.005	−0.367	−0.417	−0.590	−0.138
25 Russia	−0.030	−0.098	−0.141	−0.035	−0.048	0.009	−0.372	−0.414	—	−0.138
26 Russia	−0.084	−0.127	−0.145	−0.075	−0.084	−0.038	−0.378	−0.451	—	−0.149
Average (0)	−0.069	−0.105	−0.163	−0.065	−0.077	−0.010	−0.348	−0.410	−0.566	−0.174
St. dev. (0)	0.018	0.014	0.016	0.013	0.014	0.016	0.067	0.071	0.024	0.017
Average (1)	−0.072	−0.109	−0.163	−0.068	−0.080	−0.010	−0.368	−0.429	−0.566	−0.178
St. dev. (1)	0.013	0.009	0.016	0.009	0.010	0.016	0.014	0.036	0.024	0.013

Table B.23. Model 2D-3A (HP, T = 100.0, z = 0)

Participant/x =	−20.0	−20.0	−20.0	0.0	0.0	0.0	20.0	20.0	20.0	25.0
Re E_x										
2 Canada	1.134	—	0.113	0.043	—	4.342	4.376	—	0.438	0.705
3 Canada	1.134	—	0.113	0.043	—	4.322	4.370	—	0.437	0.706
6 Poland	1.132	—	—	—	4.290	—	4.260	—	—	0.669
8 Uzbekistan	1.150	—	0.115	0.052	—	5.200	4.300	—	0.430	0.715
10 Russia	—	0.620	—	—	2.170	—	—	2.360	—	0.700
12 Czech	—	—	—	—	—	—	—	—	—	0.708
17 USA	—	—	0.116	—	—	4.306	—	—	0.428	0.712
18 Germany	—	0.629	—	—	2.190	—	—	2.353	—	0.704
22 Canada	1.138	—	0.114	0.043	—	4.324	4.321	—	0.432	0.720
25 Russia	1.070	—	—	0.063	—	—	—	—	0.471	0.679
26 Russia	1.046	—	—	0.074	—	—	—	—	0.483	0.629
Average (0)	1.115	0.625	0.114	0.053	2.883	4.499	4.325	2.357	0.446	0.695
St. dev. (0)	0.040	0.006	0.001	0.013	1.218	0.392	0.049	0.005	0.022	0.027
Average (1)	1.115	0.625	0.114	0.053	2.883	4.499	4.325	2.357	0.446	0.702
St. dev. (1)	0.040	0.006	0.001	0.013	1.218	0.392	0.049	0.005	0.022	0.016
Im E_x										
2 Canada	−0.174	—	−0.017	−0.005	—	−0.461	−0.580	—	−0.058	−0.185
3 Canada	−0.174	—	−0.017	−0.005	—	−0.459	−0.577	—	−0.058	−0.185
6 Poland	−0.154	—	—	—	−0.435	—	−0.534	—	—	−0.171
8 Uzbekistan	−0.179	—	−0.018	−0.016	—	−1.600	−0.548	—	−0.055	−0.187
10 Russia	—	−0.070	—	—	−0.120	—	—	−0.190	—	−0.160
12 Czech	—	—	—	—	—	—	—	—	—	−0.183
17 USA	—	—	−0.018	—	—	−0.474	—	—	−0.056	−0.187
18 Germany	—	−0.109	—	—	−0.227	—	—	−0.281	—	−0.181
22 Canada	−0.171	—	−0.017	−0.005	—	−0.472	−0.590	—	−0.059	−0.188
25 Russia	−0.141	—	—	0.009	—	—	—	—	−0.039	−0.138
26 Russia	−0.145	—	—	−0.038	—	—	—	—	−0.097	−0.149
Average (0)	−0.163	−0.089	−0.017	−0.010	−0.261	−0.693	−0.566	−0.236	−0.060	−0.174
St. dev. (0)	0.016	0.028	0.001	0.016	0.160	0.507	0.024	0.064	0.018	0.017
Average (1)	−0.163	−0.089	−0.017	−0.010	−0.261	−0.693	−0.566	−0.236	−0.054	−0.178
St. dev. (1)	0.016	0.028	0.001	0.016	0.160	0.507	0.024	0.064	0.008	0.013

Table B.24. Model 2D-3A (T = 100.0, z = 0)

Participant/x =	−40.0	−25.0	−15.0	−5.0	5.0	15.0	25.0
ρ_a (EP)							
2 Canada	13.50	6.76	1.00	1.10	10.76	21.46	17.62
3 Canada	13.50	6.78	1.00	1.11	10.80	21.50	17.60
4 Finland	14.00	7.40	1.00	1.10	10.90	19.10	17.20
6 Poland	13.90	6.80	1.10	1.20	10.60	21.60	18.00
8 Uzbekistan	13.55	6.64	1.06	1.17	10.24	20.72	17.34
11 Russia	13.60	6.72	1.01	1.11	10.60	21.50	17.50
12 Czech	14.06	6.81	1.11	1.23	10.55	21.48	18.25
15 Russia	13.74	7.05	0.97	1.05	11.49	22.73	17.55
16 Russia	14.23	7.40	0.96	1.02	11.76	22.90	17.29
17 USA	13.61	6.72	0.99	1.09	10.67	21.68	17.54
18 Germany	13.55	6.64	1.00	1.11	10.55	21.29	17.42
22 Canada	13.59	6.90	0.99	1.08	10.79	21.63	17.50
25 Russia	12.29	4.07	1.41	1.20	10.09	18.96	16.79
26 Russia	11.20	4.18	1.14	1.50	9.10	16.10	17.90
Average (0)	13.45	6.49	1.05	1.15	10.64	20.90	17.54
St. dev. (0)	0.79	1.03	0.12	0.12	0.62	1.77	0.36
Average (1)	13.62	6.89	1.02	1.12	10.75	21.27	17.54
St. dev. (1)	0.47	0.26	0.06	0.06	0.45	1.14	0.23
ρ_a (HP)							
2 Canada	15.50	15.70	1.17	1.08	253.00	249.00	8.21
3 Canada	15.50	15.70	1.17	1.08	252.00	249.00	8.23
4 Finland	15.50	15.80	1.10	1.10	251.20	257.90	7.50
6 Poland	15.10	15.44	1.04	0.95	248.12	243.18	8.01
8 Uzbekistan	15.61	15.96	1.06	0.96	272.58	266.08	8.44
11 Russia	15.50	15.80	1.16	1.07	253.00	248.00	8.35
12 Czech	15.51	15.91	1.15	1.06	253.60	247.70	8.28
15 Russia	15.59	15.89	1.16	1.07	252.70	247.60	8.48
16 Russia	15.03	16.21	1.13	1.06	247.30	252.00	8.57
17 USA	15.46	15.78	1.18	1.09	249.40	244.50	8.36
18 Germany	15.16	15.55	1.16	1.07	248.60	241.75	8.17
20 Russia	15.20	15.65	1.14	1.05	240.00	235.00	8.10
22 Canada	15.47	15.67	1.17	1.10	248.90	243.91	8.55
25 Russia	14.74	15.30	0.96	0.89	283.49	278.73	7.43
26 Russia	15.50	16.20	1.16	1.07	290.00	275.00	6.47
Average (0)	15.36	15.77	1.13	1.05	256.26	251.96	8.08
St. dev. (0)	0.25	0.25	0.06	0.06	14.15	12.37	0.56
Average (1)	15.40	15.77	1.14	1.06	253.85	250.04	8.19
St. dev. (1)	0.19	0.25	0.04	0.05	11.03	10.28	0.35

Table B.25. Model 2D-3A (EP, T = 1000.0, z = 0)

Participant/x =	−40.0	−25.0	−20.0	−15.0	−5.0	0.0	5.0	15.0	20.0	25.0
Re E_y										
3 Canada	0.919	0.874	0.848	0.827	0.829	0.854	0.880	0.914	0.925	0.935
6 Poland	0.921	0.875	0.849	0.827	0.830	0.855	0.881	0.916	0.927	0.936
8 Uzbekistan	0.919	0.873	0.848	0.826	0.828	0.853	0.879	0.913	0.925	0.934
10 Russia	0.920	0.870	0.850	0.820	0.830	0.860	0.880	0.920	0.930	0.940
11 Russia	0.920	0.873	0.848	0.826	0.829	0.853	0.880	0.915	0.926	0.935
12 Czech	0.923	0.877	0.851	0.829	0.832	0.857	0.883	0.918	0.929	0.938
13 Germany	0.914	0.871	0.846	0.825	0.828	0.853	0.879	0.912	0.922	0.930
15 Russia	0.937	0.890	0.865	0.844	0.846	0.871	0.897	0.932	0.942	0.951
16 Russia	0.944	0.902	—	0.858	0.862	0.886	0.911	0.942	0.952	0.959
17 USA	0.922	0.876	0.851	0.829	0.832	0.856	0.883	0.917	0.928	0.937
18 Germany	0.921	0.875	0.850	0.828	0.831	0.855	0.882	0.916	0.927	0.936
19 Germany	0.915	0.866	0.839	0.816	0.819	0.845	0.873	0.910	0.922	0.932
21 Canada	0.924	0.864	0.815	0.764	0.768	0.823	0.873	0.917	0.931	0.941
22 Canada	0.920	0.874	0.848	0.825	0.828	0.853	0.880	0.915	0.926	0.936
25 Russia	0.931	0.892	0.874	0.855	0.853	0.873	0.897	0.929	0.940	0.948
26 Russia	0.920	0.891	0.881	0.869	0.872	0.886	0.902	0.925	0.933	0.939
Average (0)	0.923	0.878	0.851	0.829	0.832	0.858	0.885	0.919	0.930	0.939
St. dev. (0)	0.008	0.010	0.015	0.023	0.022	0.015	0.011	0.008	0.008	0.008
Average (1)	0.922	0.876	0.851	0.834	0.837	0.861	0.883	0.918	0.929	0.938
St. dev. (1)	0.006	0.008	0.009	0.015	0.015	0.012	0.009	0.006	0.006	0.006
Im E_y										
3 Canada	0.096	0.178	0.231	0.275	0.268	0.216	0.156	0.090	0.074	0.064
6 Poland	0.093	0.177	0.229	0.273	0.266	0.214	0.154	0.088	0.072	0.061
8 Uzbekistan	0.091	0.175	0.228	0.272	0.265	0.213	0.153	0.087	0.071	0.060
10 Russia	0.090	0.170	0.230	0.280	0.270	0.210	0.150	0.080	0.070	0.050
11 Russia	0.094	0.178	0.231	0.274	0.267	0.215	0.156	0.089	0.073	0.062
12 Czech	0.093	0.177	0.230	0.274	0.267	0.214	0.155	0.088	0.072	0.061
13 Germany	0.092	0.170	0.226	0.270	0.263	0.211	0.151	0.085	0.070	0.059
15 Russia	0.094	0.180	0.233	0.278	0.270	0.218	0.157	0.088	0.071	0.060
16 Russia	0.078	0.165	—	0.265	0.258	0.205	0.143	0.075	0.059	0.048
17 USA	0.094	0.177	0.230	0.273	0.266	0.214	0.155	0.089	0.073	0.063
18 Germany	0.096	0.180	0.233	0.277	0.270	0.218	0.158	0.091	0.074	0.064
19 Germany	0.095	0.179	0.232	0.276	0.269	0.216	0.156	0.089	0.072	0.062
21 Canada	0.079	0.177	0.273	0.365	0.357	0.250	0.141	0.061	0.049	0.044
22 Canada	0.094	0.177	0.230	0.276	0.269	0.216	0.155	0.089	0.073	0.062
25 Russia	0.100	0.178	0.218	0.258	0.260	0.215	0.156	0.090	0.073	0.063
26 Russia	0.121	0.189	0.211	0.243	0.237	0.199	0.158	0.102	0.084	0.075
Average (0)	0.094	0.177	0.231	0.277	0.270	0.215	0.153	0.086	0.071	0.060
St. dev. (0)	0.009	0.005	0.013	0.025	0.025	0.010	0.005	0.009	0.007	0.007
Average (1)	0.092	0.177	0.228	0.271	0.264	0.213	0.155	0.088	0.072	0.060
St. dev. (1)	0.006	0.003	0.006	0.009	0.008	0.005	0.002	0.006	0.005	0.005

Table B.25 (continued)

Participant/x =	−40.0	−25.0	−20.0	−15.0	−5.0	0.0	5.0	15.0	20.0	25.0
Re H_z										
3 Canada	−0.098	−0.204	−0.280	−0.130	0.158	0.308	0.217	0.113	0.079	0.067
6 Poland	−0.096	−0.203	−0.273	−0.130	0.159	0.300	0.217	0.112	0.079	0.066
8 Uzbekistan	−0.093	−0.201	−0.277	−0.129	0.157	0.304	0.212	0.111	0.079	0.067
8.4 Russia	—	—	−0.269	—	—	0.295	—	—	0.079	—
8.5 Russia	—	—	−0.282	—	—	0.309	—	—	0.080	—
10 Russia	−0.100	−0.210	−0.300	−0.130	0.170	0.330	0.230	0.130	0.080	0.070
12 Czech	−0.097	−0.204	−0.265	−0.130	0.159	0.292	0.218	0.113	0.081	0.067
13 Germany	−0.086	−0.186	−0.251	−0.112	0.151	0.272	0.209	0.112	0.075	0.057
17 USA	−0.096	−0.202	−0.272	−0.129	0.157	0.300	0.215	0.111	0.078	0.066
18 Germany	−0.096	−0.212	−0.239	−0.140	0.171	0.265	0.226	0.112	0.082	0.066
19 Germany	−0.100	−0.211	−0.273	−0.134	0.165	0.302	0.226	0.118	0.084	0.070
21 Canada	−0.078	−0.217	−0.546	−0.164	0.194	0.587	0.227	0.091	0.059	0.046
22 Canada	−0.099	−0.208	−0.275	−0.134	0.161	0.304	0.219	0.113	0.080	0.067
25 Russia	−0.080	−0.173	−0.173	−0.179	0.060	0.195	0.173	0.094	0.065	0.065
26 Russia	−0.073	−0.095	−0.095	−0.106	0.084	0.164	0.137	0.085	0.050	0.050
Average (0)	−0.092	−0.194	−0.271	−0.134	0.150	0.302	0.210	0.109	0.075	0.063
St. dev. (0)	0.009	0.032	0.092	0.019	0.036	0.091	0.026	0.012	0.010	0.008
Average (1)	−0.093	−0.203	−0.252	−0.131	0.157	0.281	0.216	0.109	0.077	0.065
St. dev. (1)	0.008	0.012	0.054	0.014	0.026	0.046	0.015	0.012	0.007	0.006
Im H_z										
3 Canada	0.093	229	0.344	0.137	−0.176	−0.397	−0.257	−0.107	−0.051	−0.045
6 Poland	0.090	0.225	0.325	0.135	−0.175	−0.378	−0.255	−0.106	−0.054	−0.044
8 Uzbekistan	0.086	0.223	0.336	0.132	−0.170	−0.388	−0.250	−0.106	−0.053	−0.045
8.4 Russia	—	—	0.320	—	—	−0.371	—	—	−0.054	—
8.5 Russia	—	—	0.337	—	—	−0.390	—	—	−0.052	—
10 Russia	0.090	0.230	0.350	0.130	−0.170	−0.400	−0.260	−0.120	−0.050	−0.050
12 Czech	0.091	0.227	0.315	0.137	−0.178	−0.366	−0.258	−0.107	−0.057	−0.045
13 Germany	0.085	0.211	0.301	0.120	−0.167	−0.339	−0.251	−0.112	−0.056	−0.043
17 USA	0.090	0.224	0.326	0.136	−0.176	−0.379	−0.254	−0.105	−0.053	−0.045
18 Germany	0.091	0.243	0.275	0.149	−0.192	−0.322	−0.276	−0.107	−0.061	−0.044
19 Germany	0.089	0.224	0.309	0.134	−0.175	−0.362	−0.255	−0.106	−0.055	−0.044
21 Canada	0.060	0.213	0.598	0.125	−0.160	−0.687	−0.245	−0.064	−0.017	−0.010
22 Canada	0.093	0.231	0.325	0.139	−0.178	−0.379	−0.257	−0.106	−0.054	−0.045
25 Russia	0.080	0.208	0.208	0.215	−0.071	−0.241	−0.207	−0.087	−0.048	−0.048
26 Russia	0.083	0.120	0.120	0.144	−0.112	−0.232	−0.185	−0.099	−0.048	−0.048
Average (0)	0.086	0.216	0.319	0.141	−0.162	−0.375	−0.247	−0.102	−0.051	−0.043
St. dev. (0)	0.009	0.030	0.098	0.023	0.033	0.101	0.024	0.014	0.010	0.010
Average (1)	0.088	0.224	0.313	0.135	−0.169	−0.353	−0.252	−0.106	−0.053	−0.046
St. dev. (1)	0.004	0.010	0.037	0.008	0.020	0.054	0.016	0.008	0.003	0.002

Table B.26. Model 2D-3A (T = 1000.0, z = 0)

Participant/x =	−40.0	−25.0	−15.0	−5.0	5.0	15.0	25.0
ρ_a (EP)							
3 Canada	6.82	5.63	3.09	3.14	5.97	6.93	7.14
4 Finland	6.60	4.90	3.10	3.70	6.10	6.90	7.10
6 Poland	6.80	5.60	3.20	3.20	5.90	6.90	7.20
8 Uzbekistan	6.88	5.65	3.24	3.29	5.97	6.96	7.18
11 Russia	6.81	5.58	3.09	3.13	5.92	6.92	7.13
12 Czech	6.84	5.60	3.27	3.32	5.92	6.91	7.15
13 Germany	6.51	5.37	3.00	3.04	5.69	6.62	6.81
15 Russia	7.04	5.84	3.04	3.08	6.22	7.19	7.33
16 Russia	6.98	5.84	2.96	2.99	6.22	7.12	7.24
17 USA	6.88	5.62	3.07	3.12	5.97	6.99	7.20
18 Germany	6.84	5.56	3.16	3.21	5.91	6.94	7.16
21 Canada	3.50	3.20	1.30	1.30	3.30	3.60	3.60
22 Canada	6.83	5.64	3.09	3.13	5.96	6.94	7.16
25 Russia	6.87	5.44	3.18	3.36	6.30	6.91	7.13
26 Russia	6.20	5.53	5.34	4.23	4.85	5.92	6.58
Average (0)	6.56	5.40	3.14	3.15	5.75	6.65	6.87
St. dev. (0)	0.87	0.65	0.77	0.60	0.75	0.89	0.92
Average (1)	6.78	5.56	3.11	3.28	5.92	6.87	7.11
St. dev. (1)	0.21	0.23	0.09	0.32	0.35	0.30	0.19

Table B.26 (continued)

Participant/x =	−40.0	−25.0	−15.0	−5.0	5.0	15.0	25.0
ρ_a (HP)							
3 Canada	5.83	5.49	0.30	0.25	87.10	84.40	2.46
4 Finland	5.80	5.60	0.30	0.20	90.00	86.30	2.40
6 Poland	5.75	5.48	0.28	0.22	85.49	82.81	2.41
8 Uzbekistan	5.95	5.65	0.28	0.22	93.19	89.78	2.53
11 Russia	5.85	5.56	0.30	0.25	86.60	83.70	2.48
12 Czech	5.86	5.58	0.30	0.24	87.01	83.99	2.46
13 Germany	6.82	6.55	0.32	0.26	107.80	102.50	2.89
15 Russia	5.95	5.65	0.30	0.25	88.06	85.27	2.56
16 Russia	7.76	8.25	0.36	0.30	151.10	150.06	3.97
17 USA	5.86	5.57	0.30	0.25	86.02	83.37	2.49
18 Germany	5.79	5.51	0.29	0.24	88.11	84.47	2.47
20 Russia	5.85	5.63	0.30	0.24	86.00	84.50	2.55
21 Canada	2.90	2.60	0.00	0.00	60.60	52.80	1.40
22 Canada	5.85	5.52	0.30	0.25	85.68	82.92	2.55
25 Russia	5.30	5.18	0.19	0.15	86.70	84.99	1.92
26 Russia	5.07	4.66	0.27	0.24	116.00	112.00	1.87
Average (0)	5.76	5.53	0.27	0.22	92.84	89.62	2.46
St. dev. (0)	0.97	1.09	0.08	0.07	19.30	20.01	0.53
Average (1)	5.82	5.55	0.29	0.24	88.96	85.59	2.36
St. dev. (1)	0.38	0.39	0.03	0.03	11.86	12.27	0.36

Table B.27. Model 2D-3B (EP, T = 100.0, z = 0)

Participant/x =	−40.0	−25.0	−20.0	−15.0	−5.0	0.0	5.0	15.0	20.0	25.0
Re E_y										
3 Canada	0.921	0.689	0.551	0.436	0.506	0.667	0.813	1.015	1.083	1.137
8 Uzbekistan	0.924	0.689	0.549	0.434	0.502	0.663	0.808	1.012	1.080	1.133
17 USA	0.917	0.686	0.549	0.435	0.504	0.663	0.808	1.010	1.077	1.130
18 Germany	0.903	0.680	0.546	0.433	0.505	0.669	0.811	1.013	1.081	1.135
22 Canada	0.921	0.691	0.550	0.432	0.499	0.663	0.810	1.014	1.081	1.135
25 Russia	0.891	0.667	0.545	0.441	0.511	0.659	0.792	0.988	1.055	1.108
26 Russia	0.903	0.662	0.544	0.435	0.505	0.663	0.780	0.999	1.066	1.109
Average (0)	0.911	0.681	0.548	0.435	0.505	0.664	0.803	1.007	1.075	1.127
St. dev. (0)	0.012	0.012	0.003	0.003	0.004	0.003	0.012	0.010	0.010	0.013
Average (1)	0.911	0.681	0.548	0.434	0.505	0.664	0.803	1.007	1.075	1.127
St. dev. (1)	0.012	0.012	0.003	0.001	0.004	0.003	0.012	0.010	0.010	0.013
Im E_y										
3 Canada	−0.147	−0.104	−0.003	0.065	0.020	−0.166	−0.433	−0.747	−0.857	−0.946
8 Uzbekistan	−0.148	−0.103	−0.002	0.063	0.019	−0.163	−0.430	−0.746	−0.856	−0.945
17 USA	−0.146	−0.104	−0.003	0.058	0.013	−0.168	−0.429	−0.740	−0.847	−0.934
18 Germany	−0.141	−0.097	0.007	0.071	0.027	−0.155	−0.426	−0.748	−0.860	−0.952
22 Canada	−0.148	−0.108	−0.006	0.059	0.017	−0.167	−0.436	−0.750	−0.859	−0.949
25 Russia	−0.132	−0.080	0.011.	0.077	0.026	−0.134	−0.389	−0.694	−0.801	−0.885
26 Russia	−0.145	−0.075	−0.003	0.066	0.016	−0.169	−0.414	−0.733	−0.842	−0.912
Average (0)	−0.144	−0.096	0.000	0.066	0.020	−0.160	−0.422	−0.737	−0.846	−0.932
St. dev. (0)	0.006	0.013	0.006	0.007	0.005	0.013	0.016	0.020	0.021	0.025
Average (1)	−0.146	−0.096	0.000	0.066	0.020	−0.165	−0.428	−0.744	−0.853	−0.932
St. dev. (1)	0.003	0.013	0.006	0.007	0.005	0.005	0.008	0.006	0.007	0.025
Re H_x										
3 Canada	0.996	1.059	1.407	1.727	1.816	1.406	0.964	0.837	0.823	0.819
8 Uzbekistan	0.993	1.054	1.414	1.743	1.838	1.409	0.961	0.832	0.819	0.816
17 USA	0.989	1.059	1.439	1.731	1.820	1.443	0.964	0.828	0.814	0.811
18 Germany	0.990	1.043	1.431	1.716	1.810	1.439	0.963	0.828	0.811	0.798
22 Canada	0.995	1.054	1.415	1.720	1.816	1.442	0.963	0.835	0.820	0.816
25 Russia	0.996	1.097	1.214	1.596	1.745	1.143	0.954	0.878	0.820	0.820
26 Russia	1.023	1.262	1.262	1.484	1.619	1.519	1.151	0.891	0.886	0.886
Average (0)	0.997	1.090	1.369	1.674	1.781	1.400	0.989	0.847	0.828	0.824
St. dev. (0)	0.012	0.078	0.091	0.097	0.077	0.119	0.072	0.026	0.026	0.028
Average (1)	0.993	1.061	1.369	1.674	1.807	1.443	0.962	0.847	0.818	0.813
St. dev. (1)	0.003	0.019	0.091	0.097	0.032	0.041	0.004	0.026	0.004	0.008

Table B.27 (continued)

Participant/x =	−40.0	−25.0	−20.0	−15.0	−5.0	0.0	5.0	15.0	20.0	25.0
Im H_x										
3 Canada	−0.059	0.053	0.072	0.167	−0.015	−0.104	0.062	0.043	0.045	0.052
8 Uzbekistan	−0.062	0.053	0.083	0.181	−0.007	−0.087	0.062	0.043	0.047	0.052
17 USA	−0.057	0.057	0.067	0.180	−0.004	−0.122	0.064	0.045	0.048	0.054
18 Germany	−0.054	0.053	0.046	0.156	−0.026	−0.153	0.062	0.042	0.046	0.054
22 Canada	−0.059	0.048	0.067	0.176	−0.004	−0.138	0.060	0.042	0.046	0.052
25 Russia	−0.033	0.069	0.080	0.129	0.057	−0.008	0.056	0.056	0.061	0.061
26 Russia	−0.017	0.066	0.066	0.095	0.084	−0.076	0.029	0.055	0.053	0.053
Average (0)	−0.049	0.057	0.069	0.155	0.012	−0.098	0.056	0.047	0.049	0.054
St. dev. (0)	0.017	0.008	0.012	0.032	0.041	0.048	0.012	0.006	0.006	0.003
Average (1)	−0.049	0.057	0.069	0.155	0.012	−0.098	0.061	0.047	0.047	0.053
St. dev. (1)	0.017	0.008	0.012	0.032	0.041	0.048	0.003	0.006	0.003	0.001
Re H_z										
3 Canada	−0.067	−0.349	−0.624	−0.142	0.412	1.023	0.706	0.404	0.325	0.262
8 Uzbekistan	−0.057	−0.337	−0.609	−0.118	0.378	1.004	0.685	0.397	0.323	0.259
17 USA	−0.066	−0.338	−0.578	−0.143	0.410	0.970	0.688	0.393	0.316	0.256
18 Germany	−0.086	−0.368	−0.550	−0.154	0.381	0.907	0.680	0.383	0.301	0.231
22 Canada	−0.072	−0.354	−0.574	−0.143	0.410	0.979	0.701	0.399	0.322	0.262
25 Russia	−0.066	−0.337	−0.448	−0.326	0.233	0.791	0.568	0.352	0.276	0.276
26 Russia	−0.075	−0.363	−0.363	−0.244	0.333	0.823	0.633	0.377	0.229	0.229
Average (0)	−0.070	−0.349	−0.535	−0.181	0.365	0.928	0.666	0.386	0.299	0.254
St. dev. (0)	0.009	0.013	0.095	0.075	0.065	0.091	0.049	0.018	0.035	0.017
Average (1)	−0.070	−0.349	−0.535	−0.181	0.387	0.928	0.666	0.386	0.299	0.254
St. dev. (1)	0.009	0.013	0.095	0.075	0.031	0.091	0.049	0.018	0.035	0.017
Im H_z										
3 Canada	−0.114	−0.156	−0.035	−0.125	0.151	−0.269	−0.089	−0.039	−0.046	−0.030
8 Uzbekistan	−0.114	−0.168	−0.062	−0.143	0.188	−0.234	−0.075	−0.036	−0.044	−0.029
17 USA	−0.114	−0.162	−0.078	−0.123	0.143	−0.203	−0.078	−0.034	−0.046	−0.028
18 Germany	−0.093	−0.138	−0.090	−0.114	0.177	−0.133	−0.079	−0.030	−0.034	−0.019
22 Canada	−0.114	−0.160	−0.095	−0.129	0.150	−0.186	−0.082	−0.037	−0.042	−0.030
25 Russia	−0.101	−0.143	−0.094	−0.116	0.108	−0.147	−0.057	−0.036	−0.029	−0.029
26 Russia	−0.115	−0.136	−0.136	−0.131	0.134	−0.140	−0.074	−0.041	−0.026	−0.026
Average (0)	−0.109	−0.152	−0.084	−0.126	0.150	−0.187	−0.076	−0.036	−0.038	−0.027
St. dev. (0)	0.009	0.013	0.031	0.010	0.027	0.052	0.010	0.004	0.008	0.004
Average (1)	−0.109	−0.152	−0.084	−0.126	0.150	−0.187	−0.076	−0.036	−0.038	−0.029
St. dev. (1)	0.009	0.013	0.031	0.010	0.027	0.052	0.010	0.004	0.008	0.002

Table B.28. Model 2D-3B (HP, T = 100.0, z = 0)

Participant/x =	−40.0	−25.0	−20.0	−15.0	−5.0	0.0	5.0	15.0	20.0	25.0
Re E_x										
3 Canada	0.980	0.960	1.093	0.266	0.248	0.030	2.538	1.338	0.436	3.300
8 Russia	0.994	0.966	1.111	0.268	0.250	0.039	2.565	1.363	0.438	3.297
17 USA	0.980	0.965	—	0.269	0.251	—	2.554	1.361	—	3.300
18 Germany	0.963	0.951	—	0.262	0.244	—	2.551	1.357	—	3.322
22 Canada	0.981	0.961	1.098	0.269	0.253	0.030	2.561	1.372	0.426	3.246
25 Russia	0.957	0.932	1.045	0.234	0.213	0.063	2.537	1.343	—	3.518
26 Russia	0.995	0.986	1.032	0.266	0.248	0.059	2.678	1.371	—	3.536
Average (0)	0.979	0.960	1.076	0.262	0.244	0.044	2.569	1.358	0.433	3.360
St. dev. (0)	0.014	0.016	0.035	0.013	0.014	0.016	0.049	0.013	0.006	0.117
Average (1)	0.979	0.960	1.076	0.267	0.249	0.044	2.551	1.358	0.433	3.360
St. dev. (1)	0.014	0.016	0.035	0.003	0.003	0.016	0.012	0.013	0.006	0.117

Table B.28 (continued)

Participant/x =	−40.0	−25.0	−20.0	−15.0	−5.0	0.0	5.0	15.0	20.0	25.0
Im E_x										
3 Canada	−0.131	−0.181	−0.247	−0.083	−0.096	−0.009	−0.800	−0.603	−0.236	−1.917
8 Uzbekistan	−0.239	−0.178	−0.252	−0.080	−0.094	−0.020	−0.804	−0.610	−0.242	−1.905
17 USA	−0.128	−0.177	—	−0.078	−0.092	—	−0.808	−0.612	—	−1.929
18 Germany	−0.121	−0.176	—	−0.088	−0.100	—	−0.787	−0.595	—	−1.771
22 Canada	−0.128	−0.178	−0.246	−0.077	−0.091	−0.009	−0.804	−0.618	−0.231	−1.885
25 Russia	−0.053	−0.104	−0.160	−0.011	−0.020	0.019	−0.717	−0.515	—	−1.913
26 Russia	−0.117	−0.179	−0.209	−0.081	−0.093	−0.043	−0.815	−0.608	—	−2.031
Average (0)	−0.131	−0.168	−0.223	−0.071	−0.084	−0.012	−0.791	−0.594	−0.236	−1.907
St. dev. (0)	0.055	0.028	0.039	0.027	0.028	0.022	0.034	0.036	0.006	0.076
Average (1)	−0.131	−0.178	−0.223	−0.081	−0.094	−0.012	−0.803	−0.608	−0.236	−1.907
St. dev. (1)	0.055	0.002	0.039	0.004	0.003	0.022	0.009	0.008	0.006	0.076

Table B.29. Model 2D-3B (HP, T = 100.0, z = 0)

Participant/x =	−20.0	−20.0	−20.0	0.0	0.0	0.0	20.0	20.0	20.0	25.0
Re E_x										
3 Canada	1.093	—	0.109	0.030	—	2.982	0.436	—	4.358	3.300
8 Uzbekistan	1.111	—	0.011	0.039	—	3.900	0.438	—	4.380	3.297
17 USA	—	—	0.112	—	—	2.994	—	—	4.292	3.300
18 Germany	—	0.605	—	—	1.526	—	—	2.395	—	3.322
22 Canada	1.098	—	0.110	0.030	—	3.023	0.426	—	4.263	3.246
25 Russia	1.045	—	—	0.063	—	—	—	—	4.563	3.518
26 Russia	1.032	—	—	0.059	—	—	—	—	4.121	3.536
Average (0)	1.076	0.605	0.086	0.044	1.526	3.225	0.433	2.395	4.330	3.360
St. dev. (0)	0.035	0.000	0.050	0.016	0.000	0.450	0.006	0.000	0.146	0.117
Average (1)	1.076	0.605	0.086	0.044	1.526	3.225	0.433	2.395	4.330	3.360
St. dev. (1)	0.035	0.000	0.050	0.016	0.000	0.450	0.006	0.000	0.146	0.117
Im E_x										
3 Canada	−0.247	—	−0.025	−0.009	—	−0.922	−0.236	—	−2.365	−1.917
8 Uzbekistan	−0.252	—	−0.025	−0.020	—	−2.000	−0.242	—	−2.420	−1.905
17 USA	—	—	−0.026	—	—	−0.946	—	—	−2.350	−1.929
18 Germany	—	−0.148	—	—	−0.474	—	—	−1.213	—	−1.771
22 Canada	−0.246	—	−0.025	−0.009	—	−0.947	−0.231	—	−2.309	−1.885
25 Russia	−0.160	—	—	0.019	—	—	—	—	−2.283	−1.913
26 Russia	−0.209	—	—	−0.043	—	—	—	—	−2.145	−2.031
Average (0)	−0.223	−0.148	−0.025	−0.012	−0.474	−1.204	−0.236	−1.213	−2.312	−1.907
St. dev. (0)	0.039	0.000	0.001	0.022	0.000	0.531	0.006	0.000	0.095	0.076
Average (1)	−0.223	−0.148	−0.025	−0.012	−0.474	−1.204	−0.236	−1.213	−2.312	−1.907
St. dev. (1)	0.039	0.000	0.001	0.022	0.000	0.531	0.006	0.000	0.095	0.076

Table B.30. Model 2D-3B (T = 100.0, z = 0)

Participant/x =	−40.0	−25.0	−15.0	−5.0	5.0	15.025.0	
ρ_a (EP)							
3 Canada	13.50	6.67	1.00	1.20	14.10	35.00	50.20
8 Uzbekistan	13.68	6.73	0.97	1.16	13.97	35.22	50.30
17 USA	13.59	6.62	0.98	1.19	13.87	35.21	50.35
18 Germany	13.11	6.70	1.00	1.21	13.94	35.61	53.09
22 Canada	13.55	6.80	0.98	1.17	14.05	35.16	50.55
25 Russia	12.61	5.77	1.21	1.33	13.17	29.11	45.96
26 Russia	12.40	4.30	1.36	1.50	9.46	29.80	40.40
Average (0)	13.21	6.23	1.07	1.25	13.22	33.59	48.69
St. dev. (0)	0.51	0.92	0.15	0.12	1.69	2.83	4.22
Average (1)	13.21	6.55	1.07	1.21	13.85	33.59	48.69
St. dev. (1)	0.51	0.39	0.15	0.06	0.34	2.83	4.22

Table B.30 (continued)

Participant/x =	−40.0	−25.0	−15.0	−5.0	5.0	15.025.0	
ρ_a (HP)							
3 Canada	15.10	14.80	1.20	1.09	110.00	33.30	252.00
8 Uzbekistan	15.10	14.87	1.21	1.10	111.65	34.44	224.15
17 USA	15.10	14.87	1.21	1.10	110.90	34.40	225.80
18 Germany	14.57	14.47	1.18	1.08	110.18	33.91	219.06
20 Russia	14.63	14.68	1.18	1.06	106.50	33.20	215.00
22 Canada	15.12	14.75	1.21	1.12	111.34	35.00	217.83
25 Russia	14.21	13.59	0.85	0.71	107.48	31.99	247.87
26 Russia	15.50	15.50	1.19	1.09	121.00	34.70	257.00
Average (0)	14.92	14.69	1.15	1.04	111.13	33.87	232.34
St. dev. (0)	0.41	0.53	0.12	0.14	0.39	0.99	17.04
Average (1)	14.92	14.85	1.20	1.09	109.72	33.87	232.34
St. dev. (1)	0.41	0.32	0.01	0.02	1.98	0.99	17.04

Table B.31. Model 2D-3B (EP, T = 1000.0, z = 0)

Participant/x =	−40.0	−25.0	−20.0	−15.0	−5.0	0.0	5.0	15.0	20.0	25.0
Re E_y										
3 Canada	0.932	0.891	0.868	0.848	0.853	0.876	0.901	0.935	0.947	0.956
8 Uzbekistan	0.931	0.890	0.867	0.847	0.852	0.875	0.900	0.933	0.945	0.955
17 USA	0.934	0.892	0.869	0.850	0.854	0.878	0.903	0.936	0.948	0.957
18 Germany	0.933	0.892	0.869	0.849	0.854	0.878	0.903	0.936	0.948	0.958
21 Canada	0.933	0.876	0.831	0.782	0.788	0.842	0.889	0.933	0.947	0.958
22 Canada	0.933	0.891	0.867	0.847	0.851	0.875	0.901	0.935	0.947	0.957
25 Russia	0.945	0.910	0.894	0.878	0.878	0.897	0.919	0.949	0.960	0.969
26 Russia	0.934	0.910	0.902	0.892	0.896	0.910	0.924	0.947	0.956	0.961
Average (0)	0.934	0.894	0.871	0.849	0.853	0.879	0.905	0.938	0.950	0.959
St. dev. (0)	0.004	0.011	0.021	0.032	0.031	0.020	0.011	0.006	0.005	0.004
Average (1)	0.933	0.894	0.871	0.859	0.863	0.879	0.905	0.938	0.950	0.957
St. dev. (1)	0.001	0.011	0.021	0.018	0.018	0.020	0.011	0.006	0.005	0.002
Im E_y										
3 Canada	0.076	0.154	0.206	0.249	0.234	0.175	0.107	0.021	−0.008	−0.032
8 Uzbekistan	0.071	0.152	0.204	0.246	0.231	0.172	0.104	0.018	−0.011	−0.035
17 USA	0.075	0.155	0.206	0.248	0.233	0.174	0.107	0.021	−0.008	−0.031
18 Germany	0.077	0.157	0.209	0.251	0.236	0.177	0.108	0.021	−0.008	−0.032
21 Canada	0.065	0.161	0.257	0.349	0.333	0.220	0.101	0.000	−0.031	−0.056
22 Canada	0.074	0.154	0.206	0.250	0.235	0.175	0.106	0.020	−0.009	−0.033
25 Russia	0.076	0.150	0.188	0.227	0.222	0.170	0.103	0.018	−0.012	−0.034
26 Russia	0.098	0.162	0.183	0.212	0.197	0.152	0.103	0.030	0.001	−0.017
Average (0)	0.077	0.156	0.207	0.254	0.240	0.177	0.105	0.019	−0.011	−0.034
St. dev. (0)	0.010	0.004	0.022	0.041	0.040	0.019	0.002	0.008	0.009	0.011
Average (1)	0.073	0.156	0.200	0.240	0.227	0.171	0.105	0.021	−0.008	−0.031
St. dev. (1)	0.004	0.004	0.010	0.015	0.014	0.009	0.002	0.004	0.004	0.006
Re H_z										
3 Canada	−0.089	−0.190	−0.262	−0.114	0.167	0.308	0.225	0.134	0.108	0.089
8 Uzbekistan	−0.085	−0.187	−0.259	−0.114	0.165	0.304	0.220	0.131	0.107	0.087
17 USA	−0.088	−0.188	−0.255	−0.114	0.166	0.301	0.223	0.132	0.106	0.087
18 Germany	−0.081	−0.198	−0.223	−0.126	0.179	0.268	0.232	0.132	0.106	0.086
21 Canada	−0.074	−0.208	−0.531	−0.154	0.202	0.579	0.233	0.113	0.087	0.068
22 Canada	−0.090	−0.194	−0.257	−0.119	0.170	0.305	0.227	0.134	0.109	0.089
25 Russia	−0.072	−0.159	−0.159	−0.163	0.070	0.199	0.182	0.115	0.087	0.087
26 Russia	−0.064	−0.084	−0.084	−0.089	0.094	0.170	0.146	0.103	0.065	0.065
Average (0)	−0.080	−0.176	−0.254	−0.124	0.152	0.304	0.211	0.124	0.097	0.082
St. dev. (0)	0.009	0.040	0.129	0.024	0.045	0.123	0.031	0.012	0.016	0.010
Average (1)	−0.080	−0.189	−0.214	−0.124	0.152	0.265	0.220	0.124	0.101	0.082
St. dev. (1)	0.009	0.015	0.068	0.024	0.045	0.057	0.018	0.012	0.010	0.010

Table B.31 (continued)

Participant/x =	−40.0	−25.0	−20.0	−15.0	−5.0	0.0	5.0	15.0	20.0	25.0
Im H_z										
3 Canada	0.088	0.226	0.344	0.125	−0.212	−0.449	−0.311	−0.179	−0.144	−0.116
8 Uzbekistan	0.082	0.220	0.335	0.121	−0.205	−0.440	−0.303	−0.176	−0.143	−0.114
17 USA	0.086	0.221	0.325	0.124	−0.211	−0.429	−0.307	−0.175	−0.141	−0.113
18 Germany	0.079	0.242	0.274	0.141	−0.226	−0.368	−0.328	−0.177	−0.141	−0.112
21 Canada	0.058	0.213	0.610	0.122	−0.183	−0.737	−0.283	−0.129	−0.098	−0.075
22 Canada	0.088	0.228	0.324	0.128	−0.213	−0.431	−0.311	−0.177	−0.143	−0.116
25 Russia	0.075	0.203	0.203	0.209	−0.098	−0.282	−0.263	−0.163	−0.121	−0.121
26 Russia	0.079	0.116	0.116	0.129	−0.145	−0.278	−0.235	−0.162	−0.101	−0.101
Average (0)	0.079	0.209	0.316	0.137	−0.187	−0.427	−0.293	−0.167	−0.129	−0.108
St. dev. (0)	0.010	0.039	0.143	0.030	0.044	0.143	0.031	0.017	0.020	0.015
Average (1)	0.082	0.222	0.274	0.127	−0.199	−0.382	−0.293	−0.173	−0.129	−0.113
St. dev. (1)	0.005	0.012	0.085	0.007	0.027	0.075	0.031	0.007	0.020	0.006

Table B.32. Model 2D-3B (T = 1000.0, z = 0)

Participant/x =	−40.0	−25.0	−15.0	−5.0	5.0	15.0	25.0
ρ_a (EP)							
3 Canada	6.86	5.68	3.13	3.18	6.04	7.07	7.57
8 Uzbekistan	6.86	5.67	3.09	3.14	6.02	7.06	7.54
17 USA	6.89	5.65	3.11	3.16	6.02	7.10	7.59
18 Germany	7.03	5.64	3.21	3.26	6.00	7.12	7.63
21 Canada	3.50	3.20	1.30	1.30	3.40	3.60	3.80
22 Canada	6.87	5.69	3.12	3.17	6.03	7.08	7.59
25 Russia	6.93	5.50	3.22	3.41	6.40	7.14	7.49
26 Russia	6.28	5.63	5.42	4.30	4.94	6.03	6.89
Average (0)	6.40	5.33	3.20	3.12	5.61	6.53	7.01
St. dev. (0)	1.19	0.86	1.11	0.83	0.99	1.24	1.32
Average (1)	6.82	5.64	2.88	3.37	5.92	6.94	7.47
St. dev. (1)	0.24	0.06	0.70	0.42	0.45	0.40	0.26
ρ_a (HP)							
3 Canada	4.96	4.35	0.27	0.22	28.30	7.49	43.70
8 Uzbekistan	5.02	4.44	0.27	0.21	28.67	7.83	44.00
17 USA	4.99	4.42	0.27	0.22	28.41	7.82	44.23
18 Germany	4.90	4.35	0.26	0.21	28.65	7.61	44.81
20 Russia	5.19	4.51	0.26	0.21	28.10	7.82	43.00
21 Canada	2.30	1.90	0.00	0.00	22.70	8.00	17.10
22 Canada	4.98	4.37	0.27	0.22	28.46	7.90	42.27
25 Russia	4.48	4.10	0.16	0.12	27.53	6.94	45.36
26 Russia	4.30	3.78	0.24	0.20	30.00	7.43	62.70
Average (0)	4.57	4.02	0.22	0.18	27.87	7.65	43.02
St. dev. (0)	0.90	0.83	0.09	0.07	2.05	0.33	11.57
Average (1)	4.85	4.29	0.25	0.20	28.51	7.74	46.26
St. dev. (1)	0.30	0.24	0.04	0.03	0.70	0.20	6.71

Table B.33. Model 2D-4 ($T = 1.0$, $z = 0$)

Participant/x =	−10.0	−7.0	−6.0	−5.0	2.0	3.5	5.0
ρ_a (EP)							
3 Canada	12.70	12.00	8.80	6.84	6.67	—	6.25
4 Finland	12.80	11.50	8.30	6.70	6.30	5.90	6.00
6 Poland	12.80	—	8.90	6.90	6.70	6.30	6.40
8 Uzbekistan	12.67	12.12	8.72	6.72	6.66	6.24	6.16
11 Russia	12.60	11.90	8.95	6.92	6.64	6.24	6.06
12 Czech	12.77	11.75	8.47	6.79	6.51	—	6.32
14 Russia	12.65	—	8.74	—	6.64	—	6.35
14.1 Russia	12.30	10.80	7.62	6.40	5.98	5.78	5.74
16 Russia	13.02	—	8.23	—	6.30	—	6.26
18 Germany	12.59	12.08	8.55	6.56	6.39	—	5.92
25 Russia	12.67	11.07	9.09	7.09	6.24	—	6.20
26 Russia	12 20	9.63	8.64	7.83	6.21	—	6.12
27 Russia	12.66	12.10	8.72	6.71	6.65	6.21	6.15
Average (0)	12.65	11.49	8.59	6.86	6.45	6.11	6.15
St. dev. (0)	0.21	0.80	0.38	0.37	0.23	0.22	0.18
Average (1)	12.69	11.70	8.68	6.76	6.49	6.11	6.18
St. dev. (1)	0.17	0.48	0.26	0.19	0.19	0.22	0.14
ρ_a (HP)							
3 Canada	11.40	11.50	9.03	6.78	6.80	—	5.71
4 Finland	11.50	11.50	9.20	7.20	7.10	6.50	6.40
6 Poland	10.71	10.88	8.35	6.16	6.17	5.75	5.29
8 Uzbekistan	11.37	11.52	9.05	6.85	6.83	6.46	5.75
11 Russia	11.30	11.40	9.58	6.82	6.80	6.48	5.74
12 Czech	11.55	11.56	9.23	7.04	6.89	—	5.85
14 Russia	11.37	—	9.04	—	6.77	—	5.80
15 Russia	11.44	—	9.04	—	6.78	—	5.80
16 Russia	11.55	—	8.27	—	7.05	—	6.48
18 Germany	11.28	11.33	9.11	7.12	7.06	—	6.10
25 Russia	11.02	10.54	8.75	6.21	6.53	—	5.28
26 Russia	11.80	10.10	8.87	7.89	6.72	—	5.84
27 Russia	11.37	11.51	9.06	6.88	6.85	6.48	5.82
Average (0)	11.36	11.18	8.97	6.89	6.80	6.33	5.84
St. dev. (0)	0.26	0.51	0.35	0.49	0.24	0.33	0.35
Average (1)	11.41	11.30	8.97	6.78	6.85	6.33	5.84
St. dev. (1)	0.19	0.35	0.35	0.37	0.16	0.33	0.35

Table B.34. Model 2D-4 ($T = 9.0$, $z = 0$)

Participant/x =	−10.0	−7.0	−6.0	−5.0	2.0	3.5	5.0
ρ_a (EP)							
3 Canada	24.60	10.70	6.66	4.35	3.28	—	7.98
4 Finland	20.80	10.30	6.40	4.30	3.30	4.90	8.30
6 Poland	21.80	—	6.70	4.10	3.40	4.90	8.90
8 Uzbekistan	21.88	10.78	6.64	4.30	3.27	4.67	7.86
11 Russia	22.10	10.70	6.85	4.42	3.27	4.64	7.72
12 Czech	21.36	10.47	6.55	4.30	3.52	—	8.84
13 Germany	22.13	10.82	6.73	4.41	3.44	5.02	8.38
14 Russia	21.45	—	6.57	—	3.46	—	8.79
14.1 Russia	21.60	10.60	6.32	4.36	3.18	4.59	7.64
18 Germany	21.18	11.77	7.19	4.59	3.29	—	7.83
25 Russia	18.62	9.98	6.98	4.95	3.46	—	7.30
26 Russia	10.20	7.18	6.52	6.18	6.16	—	7.59
27 Russia	21.86	10.78	6.64	4.30	3.27	4.67	7.86
Average (0)	20.74	10.37	6.67	4.55	3.56	4.77	8.08
St. dev. (0)	3.41	1.15	0.23	0.55	0.79	0.17	0.52
Average (1)	21.62	10.69	6.63	4.40	3.34	4.77	8.08
St. dev. (1)	1.33	0.46	0.18	0.22	0.11	0.17	0.52

Table B.34 (continued)

Participant/x =	− 10.0	− 7.0	− 6.0	− 5.0	2.0	3.5	5.0
ρ_a (HP)							
3 Canada	38.30	36.20	17.40	4.20	3.55	—	18.30
4 Finland	38.00	35.00	17.60	4.63	3.72	7.37	21.60
6 Poland	38.15	36.20	16.90	3.99	3.46	5.44	20.81
8 Uzbekistan	39.22	36.94	17.40	4.06	3.54	5.15	17.69
11 Russia	38.30	35.50	19.80	4.28	3.52	4.86	16.90
12 Czech	38.68	35.38	17.61	4.59	3.95	—	21.88
13 Germany	37.80	34.95	17.21	4.46	3.63	5.52	18.37
14 Russia	38.41	—	17.23	—	3.70	—	21.59
15 Russia	38.01	—	17.47	—	3.59	—	21.95
14.1 Russia	37.10	34.80	16.50	4.03	3.52	6.37	20.70
18 Germany	38.35	36.57	17.71	4.47	3.68	—	21.39
20 Russia	38.40	35.50	18.48	4.37	3.48	—	17.01
25 Russia	38.49	29.36	14.25	8.63	3.42	—	15.01
26 Russia	30.90	21.10	17.90	13.10	8.37	—	14.00
27 Russia	39.41	37.10	17.52	4.08	3.55	5.18	17.91
Average (0)	37.83	34.20	17.40	5.30	3.91	5.70	19.01
St. dev. (0)	1.99	4.39	1.15	2.64	1.24	0.88	2.61
Average (1)	38.33	35.29	17.46	4.65	3.59	5.70	19.01
St. dev. (1)	0.57	2.02	0.47	1.27	0.14	0.88	2.61

Table B.35. Model 2D-4 (EP, T = 100.0, z = 0)

Participant/x =	− 10.0	− 7.0	− 6.0	− 5.0	0.0	2.0	3.5	5.0	8.0	16.0
Re E_y										
3 Canada	0.834	0.812	0.804	0.796	0.780	0.787	—	0.806	0.824	0.850
6 Poland	0.838	—	0.808	0.800	0.785	0.792	0.801	0.812	—	—
8 Uzbekistan	0.816	0.794	0.786	0.778	0.764	0.770	0.778	0.788	0.806	0.836
10 Russia	0.950	—	0.926	—	0.920	0.920	—	0.930	0.940	0.960
11 Russia	0.841	0.819	0.810	0.803	0.788	0.794	0.803	0.813	0.831	—
12 Czech	0.842	0.821	0.813	0.805	0.791	0.798	—	0.817	0.834	0.862
13 Germany	0.870	0.851	0.843	0.836	0.825	0.832	0.840	0.850	0.866	0.890
16 Russia	0.893	—	0.875	—	0.857	0.864	—	0.882	0.903	0.934
18 Germany	0.839	0.817	0.809	0.801	0.787	0.794	—	0.816	0.830	0.860
22 Canada	0.834	0.812	0.804	0.795	0.780	0.786	—	0.806	0.825	0.857
25 Russia	0.876	0.860	0.854	0.850	0.839	0.843	—	0.856	0.872	0.905
26 Russia	0.857	0.853	0.852	0.851	0.850	0.850	—	0.851	0.853	0.862
27 Russia	0.815	0.793	0.785	0.777	0.762	0.768	0.777	0.787	0.805	0.837
Average (0)	0.854	0.823	0.828	0.808	0.810	0.815	0.800	0.832	0.849	0.878
St. dev. (0)	0.037	0.024	0.040	0.026	0.046	0.044	0.026	0.041	0.040	0.040
Average (1)	0.846	0.823	0.820	0.808	0.801	0.806	0.800	0.824	0.841	0.869
St. dev. (1)	0.024	0.024	0.029	0.026	0.033	0.032	0.026	0.029	0.030	0.031
Im E_y										
3 Canada	0.177	0.219	0.236	0.253	0.281	0.270	—	0.235	0.200	0.151
6 Poland	0.175	—	0.234	0.251	0.279	0.268	0.251	0.228	—	—
8 Uzbekistan	0.173	0.213	0.230	0.245	0.273	0.263	0.247	0.228	0.194	0.146
10 Russia	0.090	—	0.168	—	0.210	0.200	—	0.160	0.120	0.074
11 Russia	0.173	0.215	0.233	0.249	0.277	0.265	0.249	0.229	0.195	—
12 Czech	0.178	0.219	0.236	0.252	0.278	0.265	—	0.229	0.197	0.150
13 Germany	0.181	0.225	0.242	0.259	0.287	0.274	0.257	0.236	0.202	0.152
16 Russia	0.193	—	0.227	—	0.271	0.256	—	0.218	0.169	0.109
18 Germany	0.177	0.224	0.236	0.250	0.280	0.269	—	0.234	0.200	0.150
22 Canada	0.177	0.219	0.237	0.254	0.283	0.272	—	0.235	0.200	0.149
25 Russia	0.194	0.229	0.242	0.251	0.278	0.270	—	0.245	0.213	0.152
26 Russia	0.192	0.200	0.202	0.204	0.205	0.204	—	0.202	0.196	0.176
27 Russia	0.180	0.221	0.238	0.253	0.281	0.270	0.255	0.236	0.202	0.154
Average (0)	0.174	0.218	0.228	0.247	0.268	0.257	0.252	0.224	0.191	0.142
St. dev. (0)	0.026	0.008	0.021	0.015	0.027	0.025	0.004	0.022	0.024	0.027
Average (1)	0.181	0.220	0.233	0.252	0.279	0.267	0.252	0.230	0.197	0.149
St. dev. (1)	0.008	0.005	0.011	0.004	0.004	0.005	0.004	0.011	0.011	0.016

Table B.35 (continued)

Participant/x =	−10.0	−7.0	−6.0	−5.0	0.0	2.0	3.5	5.0	8.0	16.0
Re H_x										
3 Canada	1.050	1.190	1.310	1.430	1.550	1.480	—	1.220	1.070	1.000
6 Poland	1.050	—	1.310	1.440	1.560	1.480	1.360	1.190	—	—
8 Uzbekistan	1.050	1.190	1.310	1.440	1.550	1.480	1.370	1.220	1.070	1.010
10 Russia	0.990	—	1.290	—	1.530	1.440	—	1.170	1.030	0.990
11 Russia	1.040	1.190	1.300	1.420	1.540	1.460	1.350	1.210	1.070	—
12 Czech	1.050	1.200	1.320	1.440	1.540	1.460	—	1.190	1.070	1.010
13 Germany	1.070	1.220	1.350	1.480	1.600	1.510	1.390	1.240	1.100	1.030
18 Germany	1.040	1.210	1.300	1.390	1.520	1.440	—	1.220	1.060	1.010
22 Canada	1.042	1.184	1.309	1.436	1.550	1.476	—	1.213	1.069	1.002
25 Russia	1.140	1.230	1.350	1.350	1.370	1.370	—	1.190	1.110	0.991
26 Russia	1.110	1.110	1.110	1.120	1.120	1.120	—	1.110	1.110	1.080
27 Russia	1.050	1.200	1.320	1.450	1.560	1.490	1.380	1.230	1.080	1.010
Average (0)	1.057	1.192	1.298	1.400	1.499	1.434	1.370	1.200	1.076	1.013
St. dev. (0)	0.037	0.032	0.062	0.099	0.132	0.105	0.016	0.035	0.023	0.026
Average (1)	1.049	1.202	1.315	1.428	1.534	1.462	1.370	1.208	1.076	1.006
St. dev. (1)	0.028	0.016	0.019	0.035	0.058	0.037	0.016	0.021	0.023	0.012
Im H_x										
3 Canada	0.015	−0.078	−0.166	−0.250	−0.308	−0.269	—	−0.105	−0.009	0.021
6 Poland	0.015	—	−0.167	−0.251	−0.312	−0.269	−0.198	−0.085	—	—
8 Uzbekistan	0.023	−0.068	−0.157	−0.243	−0.301	−0.262	−0.196	−0.097	0.002	0.028
10 Russia	0.030	—	−0.250	—	−0.440	−0.380	—	−0.150	−0.020	0.010
11 Russia	0.015	−0.085	−0.165	−0.250	−0.308	−0.263	−0.196	−0.104	−0.007	—
12 Czech	0.009	−0.082	−0.168	−0.250	−0.306	−0.256	—	−0.084	−0.005	0.018
13 Germany	0.004	−0.097	−0.189	−0.278	−0.345	−0.296	−0.221	−0.118	−0.019	0.013
18 Germany	0.020	−0.099	−0.164	−0.221	−0.297	−0.250	—	−0.116	−0.004	0.018
22 Canada	0.017	−0.075	−0.165	−0.250	−0.306	−0.268	—	−0.102	−0.007	0.022
25 Russia	−0.067	−0.132	−0.218	−0.218	−0.233	−0.233	—	−0.114	−0.066	0.010
26 Russia	−0.057	−0.057	−0.057	−0.065	−0.062	−0.062	—	−0.054	−0.054	−0.031
27 Russia	0.022	−0.068	−0.156	−0.241	−0.297	−0.257	−0.194	−0.096	0.001	0.026
Average (0)	0.004	−0.084	−0.169	−0.229	−0.293	−0.255	−0.201	−0.102	−0.017	0.014
St. dev. (0)	0.032	0.021	0.045	0.057	0.087	0.071	0.011	0.023	0.022	0.017
Average (1)	0.010	−0.079	−0.179	−0.245	−0.314	−0.273	−0.201	−0.102	−0.012	0.018
St. dev. (1)	0.023	0.014	0.030	0.017	0.049	0.039	0.011	0.012	0.016	0.007
Re H_z										
3 Canada	−0.308	−0.415	−0.420	−0.368	0.067	0.225	—	0.339	0.251	0.127
6 Poland	−0.308	—	−0.434	−0.372	0.073	0.235	0.327	0.340	—	—
8 Uzbekistan	−0.296	−0.406	−0.421	−0.361	0.067	0.221	0.312	0.337	0.241	0.118
10 Russia	−0.169	—	−0.380	—	0.070	0.220	—	0.300	0.190	0.040
11 Russia	−0.306	−0.413	−0.423	−0.367	0.076	0.241	0.315	0.338	0.246	—
12 Czech	−0.302	−0.409	−0.413	−0.358	0.086	0.242	—	0.326	0.234	0.118
13 Germany	−0.275	−0.392	−0.409	−0.355	0.088	0.241	0.322	0.329	0.225	0.100
18 Germany	−0.306	−0.396	−0.409	−0.346	0.080	0.229	—	0.316	0.262	0.123
22 Canada	−0.313	−0.426	−0.434	−0.380	0.070	0.233	—	0.349	0.255	0.130
25 Russia	−0.277	−0.298	−0.226	−0.226	0.103	0.103	—	0.274	0.247	0.134
26 Russia	−0.063	−0.063	−0.063	−0.011	0.012	0.012	—	0.051	0.051	0.065
27 Russia	−0.302	−0.409	−0.423	−0.363	0.066	0.222	0.312	0.340	0.248	0.129
Average (0)	−0.269	−0.363	−0.371	−0.319	0.072	0.202	0.318	0.303	0.223	0.108
St. dev. (0)	0.076	0.111	0.112	0.110	0.022	0.071	0.007	0.082	0.060	0.031
Average (1)	−0.287	−0.396	−0.399	−0.350	0.077	0.219	0.318	0.326	0.240	0.116
St. dev. (1)	0.041	0.038	0.059	0.044	0.012	0.039	0.007	0.022	0.020	0.022

Table B.35 (continued)

Participant/x =	−10.0	−7.0	−6.0	−5.0	0.0	2.0	3.5	5.0	8.0	16.0
Im H_z										
3 Canada	0.187	0.269	0.274	0.235	−0.037	−0.132	—	−0.211	−0.146	−0.058
6 Poland	0.187	—	0.286	0.238	−0.041	−0.140	−0.202	−0.212	—	—
8 Uzbekistan	0.178	0.263	0.276	0.230	−0.036	−0.129	−0.190	−0.210	−0.140	−0.053
10 Russia	0.160	—	0.360	—	−0.060	−0.190	—	−0.280	−0.170	−0.040
11 Russia	0.188	0.272	0.280	0.237	−0.045	−0.141	−0.196	−0.212	−0.143	—
12 Czech	0.186	0.266	0.271	0.231	−0.051	−0.148	—	−0.207	−0.136	−0.057
13 Germany	0.194	0.283	0.296	0.254	−0.045	−0.152	−0.216	−0.232	−0.155	−0.063
18 Germany	0.188	0.258	0.267	0.221	−0.043	−0.133	—	−0.194	−0.157	−0.057
22 Canada	0.189	0.275	0.281	0.241	−0.038	−0.136	—	−0.216	−0.147	−0.059
25 Russia	0.187	0.206	0.161	0.161	−0.059	−0.059	—	−0.173	−0.155	−0.077
26 Russia	0.042	0.042	0.042	0.006	−0.012	−0.012	—	−0.038	−0.038	−0.044
27 Russia	0.175	0.256	0.268	0.224	−0.036	−0.127	−0.186	−0.206	−0.139	−0.056
Average (0)	0.172	0.239	0.255	0.207	−0.042	−0.125	−0.198	−0.199	−0.139	−0.056
St. dev. (0)	0.042	0.072	0.080	0.071	0.013	0.046	0.012	0.057	0.035	0.010
Average (1)	0.184	0.261	0.275	0.227	−0.045	−0.135	−0.198	−0.214	−0.149	−0.054
St. dev. (1)	0.009	0.022	0.046	0.025	0.009	0.031	0.012	0.026	0.010	0.007

Table B.36. Model 2D-4 (HP, T = 100.0, z = 0)

Participant/x =	−10.0	−7.0	−6.0	−5.0	0.0	2.0	3.5	5.0	8.0	16.0
Re E_x										
3 Canada	1.140	1.090	0.755	0.319	0.224	0.269	—	0.871	0.910	0.912
6 Poland	1.140	1.110	0.738	0.299	0.218	0.259	0.400	0.860	0.907	0.909
8 Uzbekistan	1.150	1.120	0.749	0.302	0.222	0.262	0.387	0.790	0.920	0.923
10 Russia	1.100	—	0.700	—	0.210	0.260	—	0.770	0.820	0.820
12 Czech	1.150	1.090	0.756	0.326	0.230	0.300	—	0.882	0.913	0.915
13 Germany	1.140	1.100	0.752	0.320	0.220	0.267	0.405	0.810	0.836	0.883
18 Germany	1.130	0.988	0.755	0.504	0.234	0.305	—	0.699	0.904	0.907
20 Russia	1.095	1.050	0.680	0.295	0.211	0.245	—	0.763	0.897	0.905
22 Canada	1.139	1.098	0.754	0.307	0.222	0.263	—	0.871	0.907	0.909
25 Russia	1.120	0.907	0.683	0.460	0.288	0.351	—	0.649	0.894	0.908
26 Russia	0.910	0.783	0.738	0.692	0.619	0.637	—	0.718	0.780	0.899
27 Russia	1.140	1.110	0.746	0.301	0.221	0.262	0.385	0.789	0.917	0.920
Average (0)	1.113	1.041	0.734	0.375	0.260	0.307	0.394	0.789	0.884	0.901
St. dev. (0)	0.066	0.107	0.029	0.127	0.115	0.108	0.010	0.075	0.046	0.027
Average (1)	1.131	1.066	0.734	0.343	0.227	0.277	0.394	0.789	0.893	0.908
St. dev. (1)	0.019	0.068	0.029	0.075	0.021	0.030	0.010	0.075	0.033	0.011
Im E_x										
3 Canada	0.059	0.055	0.030	−0.006	−0.018	−0.009	—	0.051	0.055	0.057
6 Poland	0.062	0.059	0.030	−0.005	−0.017	−0.009	0.009	0.053	0.058	0.060
8 Uzbekistan	0.049	0.047	0.023	−0.009	−0.019	−0.012	0.004	0.040	0.052	0.055
10 Russia	0.050	—	0.030	—	−0.020	−0.008	—	0.040	0.046	0.046
12 Czech	0.046	0.042	0.021	−0.008	−0.019	−0.008	—	0.042	0.046	0.048
13 Germany	0.070	0.066	0.037	−0.003	−0.017	−0.008	0.010	0.051	0.058	0.058
18 Germany	0.053	0.038	0.027	0.006	−0.016	−0.004	—	0.033	0.051	0.054
20 Russia	0.044	0.043	0.019	−0.007	−0.016	−0.009	—	0.047	0.061	0.069
22 Canada	0.061	0.057	0.031	−0.006	−0.018	−0.010	—	0.053	0.057	0.059
25 Russia	0.057	0.041	0.022	0.004	−0.007	0.001	—	0.032	0.056	0.060
26 Russia	0.027	0.021	0.019	0.017	0.017	0.021	—	0.030	0.037	0.050
27 Russia	0.039	0.037	0.017	−0.010	−0.020	−0.013	0.002	0.035	0.045	0.050
Average (0)	0.051	0.046	0.026	−0.002	−0.014	−0.006	0.006	0.042	0.052	0.056
St. dev. (0)	0.012	0.013	0.006	0.008	0.010	0.009	0.004	0.009	0.007	0.006
Average (1)	0.054	0.046	0.026	−0.004	−0.017	−0.008	0.006	0.042	0.053	0.054
St. dev. (1)	0.009	0.013	0.006	0.005	0.004	0.004	0.004	0.009	0.006	0.005

Table B.37. Model 2D-4 (T = 100.0, z = 0)

Participant/x =	−10.0	−7.0	−6.0	−5.0	2.0	3.5	5.0
ρ_a (EP)							
3 Canada	37.90	28.40	22.90	18.80	17.50	—	26.90
4 Finland	34.20	27.30	20.80	17.10	17.00	20.80	27.40
6 Poland	38.20	—	23.00	18.90	17.70	21.20	28.50
8 Uzbekistan	36.17	27.27	21.90	17.88	16.75	19.94	25.72
11 Russia	38.40	28.70	23.60	19.20	17.70	21.10	27.00
12 Czech	38.12	28.49	23.10	19.04	18.47	—	28.61
13 Germany	39.05	29.25	23.55	19.29	18.37	22.13	28.49
15 Russia	37.72	—	29.16	—	15.55	—	23.01
16 Russia	37.01	—	24.58	—	19.17	—	32.87
18 Germany	39.12	28.26	23.60	20.61	18.78	—	27.20
22 Canada	38.19	28.68	22.98	18.70	17.53	—	27.10
25 Russia	35.38	29.53	23.93	23.85	23.18	—	31.88
26 Russia	35.50	35.30	35.20	34.60	34.80	—	35.60
27 Russia	35.80	26.95	21.64	17.67	16.54	19.67	25.37
Average (0)	37.20	28.92	24.28	20.47	19.22	20.81	28.26
St. dev. (0)	1.53	2.27	3.68	4.77	4.82	0.90	3.25
Average (1)	37.20	28.28	23.44	19.19	18.02	20.81	27.70
St. dev. (1)	1.53	0.86	1.99	1.81	1.84	0.90	2.57
ρ_a (HP)							
3 Canada	74.50	68.20	32.50	5.80	4.12	—	43.40
4 Finland	80.30	73.10	34.80	6.50	4.80	13.70	47.40
6 Poland	73.98	69.94	31.07	5.11	3.82	9.13	42.27
8 Uzbekistan	75.54	71.03	31.98	5.19	3.93	8.53	35.67
11 Russia	75.20	69.50	37.20	5.58	3.81	7.82	34.40
12 Czech	75.02	68.37	32.60	6.07	5.13	—	44.45
13 Germany	74.88	68.84	32.33	5.84	4.06	9.34	37.52
14 Russia	72.94	—	31.05	—	4.25	—	43.98
15 Russia	71.37	—	31.17	—	3.97	—	44.68
14.1 Russia	75.20	69.50	37.20	5.58	3.81	7.82	34.40
16 Russia	69.56	—	18.30	—	4.60	—	41.26
18 Germany	73.45	52.24	32.52	18.11	5.32	—	30.06
20 Russia	76.10	70.20	35.00	6.04	3.82	—	36.95
22 Canada	74.12	68.86	32.47	5.36	3.93	—	43.39
25 Russia	71.06	46.99	26.64	12.04	7.02	—	24.09
26 Russia	47.20	35.00	31.00	27.30	23.10	—	29.50
27 Russia	74.65	70.23	31.73	5.15	3.91	8.47	35.56
Average (0)	72.65	64.43	31.74	8.55	5.49	9.26	38.17
St. dev. (0)	6.96	11.28	4.26	6.50	4.61	2.04	6.44
Average (1)	74.24	66.69	32.58	7.11	4.39	8.52	39.06
St. dev. (1)	2.42	7.76	2.57	3.77	0.85	0.64	5.50

Table B.38. Model 2D-5 (T = 300.0, z = 0)

Participant/x =	−50.0	0.0	30.0	65.0	100.0	150.0	220.0
ρ_a (EP)							
3 Canada	147.00	71.20	37.10	6.22	57.10	38.70	317.00
4 Finland	140.50	67.50	31.70	6.40	58.10	48.10	302.60
6 Poland	155.70	77.30	39.50	6.30	58.10	38.00	320.50
8 Uzbekistan	148.54	71.87	37.89	6.19	58.79	39.57	320.64
11 Russia	—	71.10	37.60	6.22	57.10	40.70	324.00
12 Czech	151.00	73.39	37.59	6.18	56.77	40.74	318.30
17 USA	148.20	72.03	37.54	6.45	58.94	38.32	318.70
18 Germany	152.70	71.20	36.87	6.65	55.40	38.01	309.95
25 Russia	143.69	65.87	32.46	7.16	59.54	46.35	344.69
Average (0)	148.42	71.27	36.47	6.42	57.76	40.94	319.60
St. dev. (0)	4.86	3.26	2.60	0.32	1.29	3.74	11.41
Average (1)	148.42	71.27	36.47	6.33	57.76	40.94	316.46
St. dev. (1)	4.86	3.26	2.60	0.16	1.29	3.74	6.91

Table B.38 (continued)

Participant/x =	−50.0	0.0	30.0	65.0	100.0	150.0	220.0
ρ_a (HP)							
3 Canada	173.00	41.80	28.80	110.00	570.00	4.83	456.00
4 Finland	177.60	42.40	27.50	119.80	589.10	5.60	513.80
6 Poland	172.78	41.69	28.96	109.55	571.91	3.67	469.40
8 Uzbekistan	174.95	42.45	30.10	111.96	574.90	4.88	461.66
11 Russia	—	41.20	28.60	108.00	556.00	5.84	564.00
12 Czech	172.50	41.46	28.49	112.60	566.60	4.80	464.50
17 USA	175.00	42.73	30.24	122.50	586.70	4.31	480.60
18 Germany	159.78	35.85	18.81	121.48	565.63	4.17	370.15
25 Russia	172.01	40.62	27.72	115.39	562.78	3.90	468.54
Average (0)	172.20	41.13	27.69	114.59	571.51	4.67	472.07
St. dev. (0)	5.34	2.09	3.46	5.46	10.78	0.73	51.35
Average (1)	173.98	41.79	28.80	114.59	571.51	4.67	472.07
St. dev. (1)	1.98	0.71	0.98	5.46	10.78	0.73	51.35

Table B.39. Model 2D-5 (EP, T = 3600.0, z = 0)

Participant/x =	−50.0	0.0	30.0	50.0	65.0	80.0	100.0	130.0	150.0	180.0
Re E_y										
3 Canada	0.923	0.904	0.886	0.869	0.863	0.872	0.890	0.909	0.918	0.932
6 Poland	0.937	0.912	0.894	0.878	0.872	0.880	0.899	0.918	0.927	0.941
8 Uzbekistan	0.925	0.906	0.888	0.872	0.866	0.910	0.893	0.911	0.920	0.934
11 Russia	—	0.908	0.891	0.875	0.868	0.877	0.895	0.914	0.923	0.937
12 Czech	0.960	0.941	0.922	0.905	0.899	0.907	0.926	0.944	0.952	0.965
17 USA	0.932	0.913	0.895	0.879	0.873	0.881	0.899	0.918	0.926	0.940
18 Germany	0.935	0.915	0.897	0.880	0.874	0.882	0.901	0.920	0.928	0.942
25 Russia	1.010	0.999	0.987	0.978	0.980	0.984	0.993	1.010	1.020	1.030
Average (0)	0.946	0.925	0.908	0.892	0.887	0.899	0.912	0.931	0.939	0.953
St. dev. (0)	0.031	0.032	0.034	0.036	0.039	0.037	0.035	0.034	0.034	0.033
Average (1)	0.935	0.914	0.896	0.880	0.874	0.887	0.900	0.919	0.928	0.942
St. dev. (1)	0.013	0.012	0.012	0.012	0.012	0.015	0.012	0.012	0.011	0.011
Im E_y										
3 Canada	0.095	0.142	0.184	0.221	0.236	0.214	0.169	0.129	0.114	0.070
6 Poland	0.084	0.131	0.173	0.211	0.226	0.205	0.158	0.118	0.104	0.059
8 Uzbekistan	0.088	0.135	0.177	0.114	0.229	0.186	0.162	0.121	0.106	0.063
11 Russia	—	0.138	0.179	0.216	0.231	0.210	0.164	0.123	0.108	0.064
12 Czech	0.079	0.128	0.171	0.210	0.226	0.205	0.157	0.115	0.101	0.057
17 USA	0.089	0.136	0.178	0.215	0.229	0.209	0.163	0.122	0.108	0.064
18 Germany	0.096	0.145	0.187	0.225	0.238	0.220	0.174	0.134	0.120	0.076
25 Russia	0.024	0.071	0.115	0.149	0.144	0.131	0.100	0.055	0.038	−0.007
Average (0)	0.079	0.128	0.171	0.195	0.220	0.198	0.156	0.115	0.100	0.056
St. dev. (0)	0.025	0.024	0.023	0.041	0.031	0.029	0.023	0.025	0.026	0.026
Average (1)	0.089	0.136	0.178	0.195	0.231	0.207	0.164	0.123	0.109	0.065
St. dev. (1)	0.006	0.006	0.006	0.041	0.005	0.011	0.006	0.006	0.006	0.006
Re H_x										
3 Canada	1.020	1.110	1.270	1.670	1.970	1.670	1.200	1.090	1.210	1.000
6 Poland	1.020	1.110	1.260	1.670	1.980	1.680	1.200	1.090	1.210	1.010
8 Uzbekistan	1.030	1.120	1.270	1.680	1.990	1.680	1.200	1.090	1.210	1.010
11 Russia	—	1.110	1.260	1.660	1.970	1.660	1.190	1.080	1.200	1.000
12 Czech	1.050	1.140	1.300	1.710	2.020	1.710	1.230	1.120	1.230	1.030
17 USA	1.020	1.110	1.270	1.670	1.960	1.680	1.190	1.080	1.210	1.000
18 Germany	0.995	1.110	1.260	1.660	1.920	1.660	1.190	1.080	1.210	1.000
25 Russia	1.110	1.250	1.370	1.780	1.740	1.650	1.440	1.210	1.270	1.100
Average (0)	1.035	1.132	1.282	1.687	1.944	1.674	1.230	1.105	1.219	1.019
St. dev. (0)	0.037	0.049	0.038	0.041	0.087	0.018	0.086	0.044	0.022	0.034
Average (1)	1.022	1.116	1.270	1.674	1.973	1.674	1.200	1.090	1.211	1.007
St. dev. (1)	0.018	0.011	0.014	0.017	0.030	0.018	0.014	0.014	0.009	0.011

Table B.39 (continued)

Participant/x =	−50.0	0.0	30.0	50.0	65.0	80.0	100.0	130.0	150.0	180.0
Im H_x										
3 Canada	0.011	−0.064	−0.166	−0.423	−0.628	−0.431	−0.098	−0.042	−0.197	0.022
6 Poland	0.017	−0.054	−0.158	−0.434	−0.650	−0.440	−0.099	−0.040	−0.203	0.023
8 Uzbekistan	0.015	−0.062	−0.160	−0.425	−0.642	−0.434	−0.093	−0.034	−0.193	0.027
11 Russia	—	−0.065	−0.163	−0.427	−0.639	−0.434	−0.101	−0.041	−0.192	0.022
12 Czech	−0.021	−0.104	−0.212	−0.484	−0.703	−0.487	−0.144	−0.078	−0.230	−0.010
17 USA	0.092	−0.067	−0.173	−0.437	−0.641	−0.442	−0.098	−0.039	−0.206	0.021
18 Germany	0.028	−0.060	−0.161	−0.421	−0.597	−0.425	−0.093	−0.037	−0.194	0.026
25 Russia	−0.102	−0.230	−0.339	−0.683	−0.643	−0.568	−0.387	−0.210	−0.307	−0.088
Average (0)	0.006	−0.088	−0.192	−0.467	−0.643	−0.458	−0.139	−0.065	−0.215	0.005
St. dev. (0)	0.058	0.059	0.062	0.090	0.029	0.049	0.102	0.060	0.039	0.040
Average (1)	0.006	−0.068	−0.170	−0.436	−0.634	−0.442	−0.104	−0.044	−0.202	0.019
St. dev. (1)	0.058	0.016	0.019	0.022	0.018	0.021	0.018	0.015	0.013	0.013
Re H_z										
3 Canada	−0.166	−0.260	−0.402	−0.414	0.039	0.502	0.451	0.217	0.261	0.272
6 Poland	−0.165	−0.260	−0.404	−0.428	−0.040	0.530	0.454	0.221	0.261	0.283
8 Uzbekistan	−0.163	−0.248	−0.403	−0.436	0.040	0.544	0.453	0.228	0.265	0.259
11 Russia	—	−0.235	−0.369	−0.405	0.042	0.590	0.456	0.249	0.321	0.334
12 Czech	−0.169	−0.270	−0.419	−0.428	0.036	0.511	0.457	0.214	0.250	0.270
17 USA	−0.172	−0.263	−0.404	−0.429	0.025	0.519	0.453	0.219	0.258	0.264
18 Germany	−0.172	−0.273	−0.402	−0.448	0.025	0.516	0.452	0.206	0.234	0.273
25 Russia	−0.125	−0.267	−0.340	−0.199	0.159	0.202	0.381	0.214	0.257	0.239
Average (0)	−0.162	−0.260	−0.393	−0.398	0.041	0.489	0.445	0.221	0.263	0.274
St. dev. (0)	0.017	0.013	0.026	0.082	0.055	0.119	0.026	0.013	0.025	0.027
Average (1)	−0.168	−0.260	−0.400	−0.427	0.024	0.530	0.454	0.217	0.255	0.266
St. dev. (1)	0.004	0.013	0.015	0.014	0.029	0.030	0.002	0.007	0.010	0.014
Im H_z										
3 Canada	0.109	0.170	0.263	0.269	0.028	−0.342	−0.300	−0.105	−0.176	−0.217
6 Poland	0.112	0.174	0.271	0.287	−0.030	−0.371	−0.307	−0.111	−0.179	−0.222
8 Uzbekistan	0.109	0.178	0.266	0.291	−0.030	−0.379	−0.307	−0.120	−0.185	−0.207
11 Russia	—	0.207	0.305	0.293	−0.024	−0.091	−0.295	−0.010	−0.030	−0.075
12 Czech	0.117	0.180	0.274	0.283	−0.030	−0.357	−0.316	−0.117	−0.179	−0.223
17 USA	0.118	0.174	0.268	0.286	−0.028	−0.363	−0.306	−0.111	−0.178	−0.212
18 Germany	0.107	0.176	0.258	0.294	−0.020	−0.353	−0.302	−0.096	−0.148	−0.220
25 Russia	0.124	0.242	0.301	0.175	−0.125	−0.161	−0.316	−0.138	−0.226	−0.233
Average (0)	0.114	0.188	0.276	0.272	−0.039	−0.302	−0.306	−0.101	−0.163	−0.201
St. dev. (0)	0.006	0.025	0.018	0.040	0.035	0.111	0.007	0.039	0.058	0.052
Average (1)	0.114	0.180	0.276	0.286	−0.027	−0.302	−0.306	−0.114	−0.182	−0.219
St. dev. (1)	0.006	0.012	0.018	0.009	0.004	0.111	0.007	0.013	0.023	0.008

Table B.40. Model 2D-5 (EP, T = 3600.0, z = 0)

Participant/x =	−50.0	0.0	30.0	50.0	65.0	80.0	100.0	130.0	150.0	180.0
Re E_x										
3 Canada	0.842	0.403	0.330	4.930	0.599	0.972	1.390	0.475	0.115	0.877
6 Poland	0.849	0.407	0.335	4.960	0.597	0.959	1.410	0.485	0.104	0.875
8 Uzbekistan	0.863	0.414	0.344	4.970	0.619	0.993	1.450	0.492	0.119	0.908
11 Russia	—	0.400	0.328	4.810	0.592	0.991	1.370	0.470	0.129	0.838
12 Czech	0.849	0.405	0.330	5.030	0.618	1.010	1.390	0.480	0.116	0.893
17 USA	0.854	0.412	0.341	4.700	0.644	0.973	1.430	0.516	0.109	0.870
18 Germany	0.809	0.373	0.265	3.380	0.631	0.978	1.380	0.487	0.105	0.918
25 Russia	0.814	0.379	0.294	5.370	0.991	1.170	1.230	0.447	0.097	0.828
Average (0)	0.840	0.399	0.321	4.769	0.661	1.006	1.381	0.482	0.112	0.876
St. dev. (0)	0.021	0.015	0.027	0.594	0.134	0.068	0.067	0.020	0.010	0.031
Average (1)	0.840	0.399	0.329	4.967	0.614	0.982	1.403	0.482	0.112	0.876
St. dev. (1)	0.021	0.015	0.016	0.210	0.019	0.017	0.029	0.020	0.010	0.031

Table B.40 (continued)

Participant/x =	−50.0	0.0	30.0	50.0	65.0	80.0	100.0	130.0	150.0	180.0
Im E_x										
3 Canada	−0.030	−0.018	−0.016	−0.230	−0.048	−0.066	−0.086	−0.036	−0.012	−0.064
6 Poland	−0.018	−0.012	−0.011	−0.161	−0.040	−0.056	−0.067	−0.029	−0.010	−0.051
8 Uzbekistan	−0.015	−0.011	−0.011	−0.144	−0.038	−0.051	−0.076	−0.029	−0.011	−0.050
11 Russia	—	−0.016	−0.014	−0.202	−0.045	−0.062	−0.079	−0.033	−0.012	−0.058
12 Czech	−0.025	−0.017	−0.016	−0.195	−0.038	−0.066	−0.085	−0.035	−0.012	−0.061
17 USA	−0.025	−0.015	−0.014	−0.186	−0.045	−0.059	−0.078	−0.035	−0.012	−0.057
18 Germany	−0.024	−0.014	−0.012	−0.141	−0.045	−0.060	−0.078	−0.034	−0.012	−0.061
25 Russia	−0.032	−0.018	−0.015	−0.300	−0.062	−0.078	−0.089	−0.036	−0.012	−0.064
Average (0)	−0.024	−0.015	−0.014	−0.195	−0.045	−0.062	−0.080	−0.033	−0.012	−0.058
St. dev. (0)	0.006	0.003	0.002	0.052	0.008	0.008	0.007	0.003	0.001	0.005
Average (1)	−0.024	−0.015	−0.014	−0.180	−0.043	−0.062	−0.080	−0.033	−0.012	−0.058
St. dev. (1)	0.006	0.003	0.002	0.033	0.004	0.008	0.007	0.003	0.000	0.005

Table B.41. Model 2D-5 (T = 3600.0, z = 0)

Participant/x =	−50.0	0.0	30.0	65.0	100.0	150.0	220.0
ρ_a (EP)							
33 Canada	120.00	97.80	72.40	27.10	82.50	82.80	135.00
4 Finland	106.40	85.90	60.30	25.20	81.60	80.30	121.70
6 Poland	121.30	100.10	74.20	26.90	83.00	82.80	135.50
8 Uzbekistan	117.73	96.57	72.24	26.52	81.80	86.44	132.58
11 Russia	—	98.80	74.10	27.20	83.30	84.90	137.00
12 Czech	122.40	99.88	73.35	27.16	82.65	83.82	134.00
17 USA	122.20	99.50	73.48	27.59	84.51	83.50	137.30
18 Germany	128.89	101.05	75.06	29.28	85.23	84.75	140.12
25 Russia	118.29	90.21	71.40	41.33	64.68	86.99	137.13
Average (0)	119.65	96.65	71.84	28.70	81.03	84.03	134.48
St. dev. (0)	6.36	5.16	4.47	4.85	6.24	2.03	5.26
Average (1)	121.54	97.99	73.28	27.12	83.07	84.03	136.08
St. dev. (1)	3.71	3.44	1.20	1.13	1.26	2.03	2.32
ρ_a (HP)							
3 Canada	103.00	23.50	15.70	52.10	280.00	1.94	195.00
4 Finland	116.20	26.40	16.60	63.40	318.60	2.50	241.20
6 Poland	104.09	23.95	16.18	51.60	287.00	1.58	205.17
8 Uzbekistan	107.60	24.82	17.09	55.48	293.96	2.04	205.34
11 Russia	—	23.10	15.60	50.90	272.00	2.41	240.00
12 Czech	104.30	23.68	15.76	55.31	282.10	1.95	202.60
17 USA	105.40	24.52	16.87	60.10	295.10	1.73	210.40
18 Germany	94.63	20.11	10.14	57.50	276.07	1.62	159.04
25 Russia	95.93	20.75	12.50	142.32	219.88	1.38	194.00
Average (0)	103.89	23.43	15.16	65.41	280.52	1.91	205.86
St. dev. (0)	6.74	1.96	2.32	29.13	26.59	0.37	24.74
Average (1)	103.89	23.43	15.79	55.80	288.10	1.91	205.86
St. dev. (1)	6.74	1.96	1.44	4.39	14.74	0.37	24.74

Appendix C

Diagrams for the 2D results are presented in Figs. 6–25.

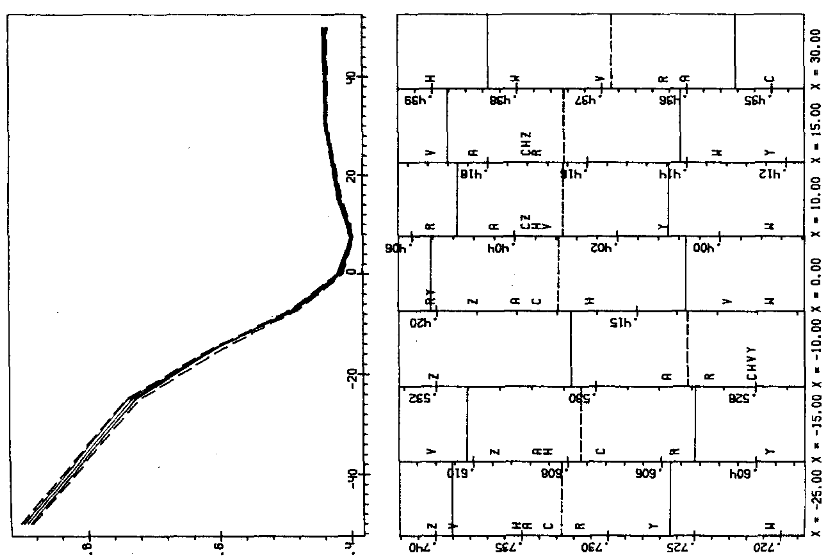


Fig. 6. Model 2D-0. Electric field E_y(EP), $z = 0$, $T = 300$.

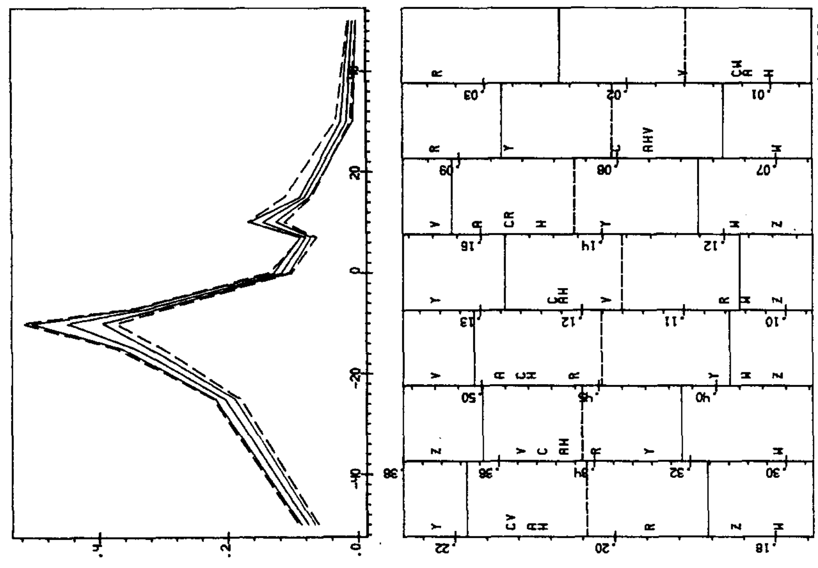


Fig. 7. Model 2D-0. Magnetic field H_z(EP), $z = 0$, $T = 300$.

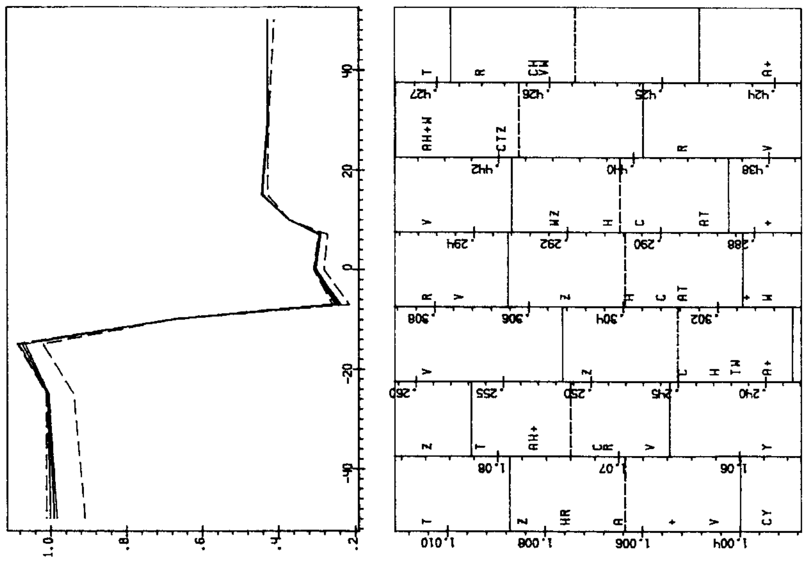


Fig. 9. Model 2D-0. Electric field E_x(HP), $z = 0$, $T = 300$.

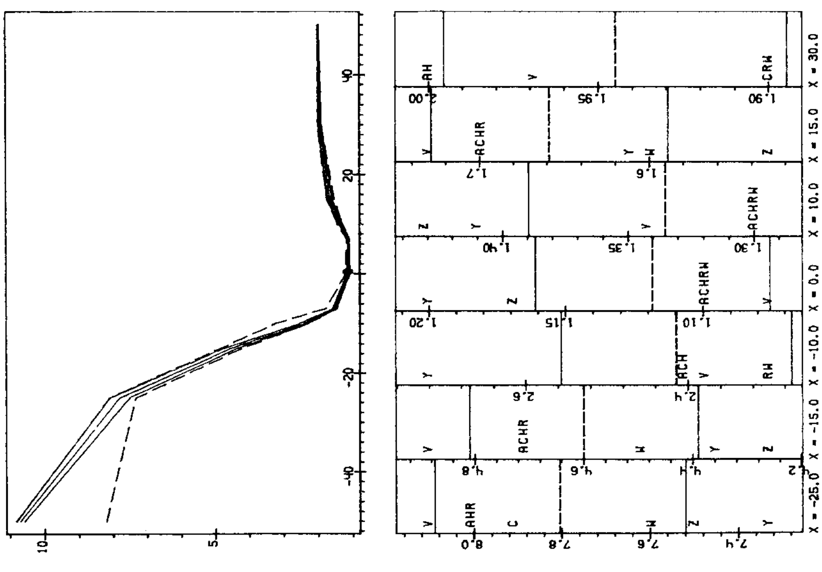


Fig. 8. Model 2D-0. Apparent resistivity ρ_a(EP), $z = 0$, $T = 300$.

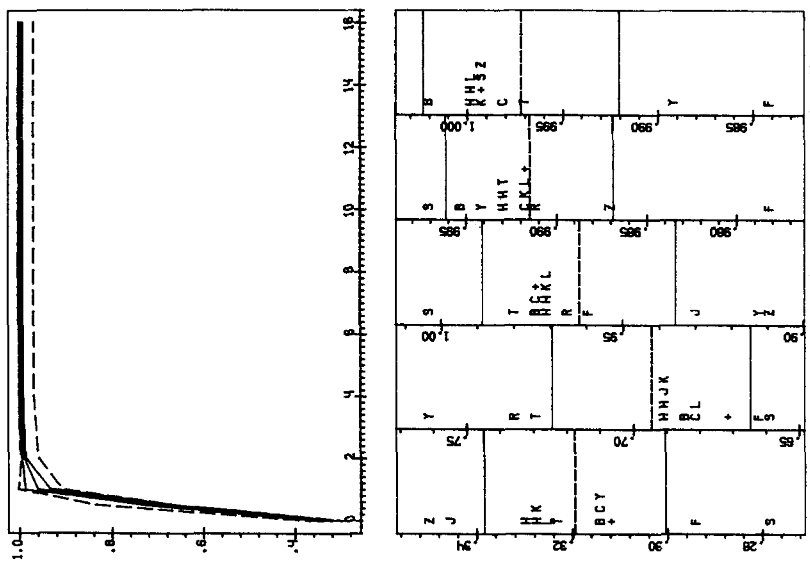


Fig. 11. Model 2D-1. Electric field E_x(HP), $z = 0$, $T = 0.1$.

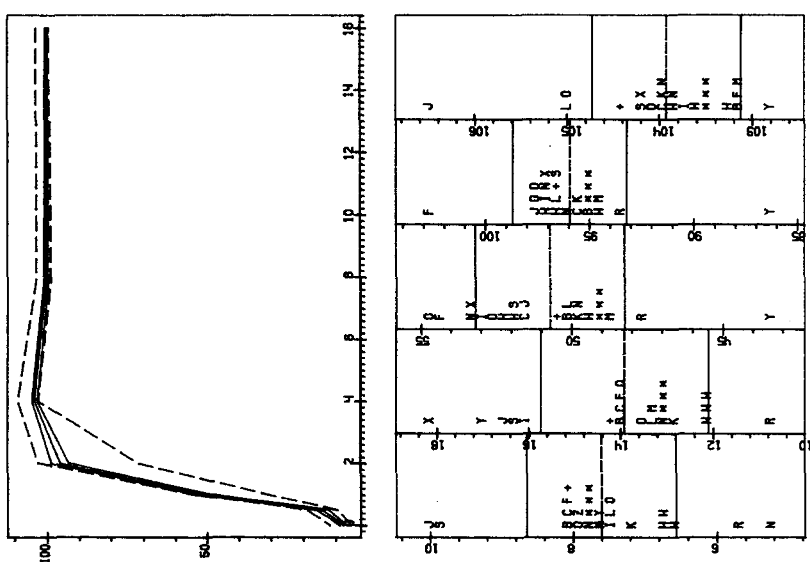


Fig. 10. Model 2D-1. Apparent resistivity ρ_a(EP), $z = 0$, $T = 0.1$.

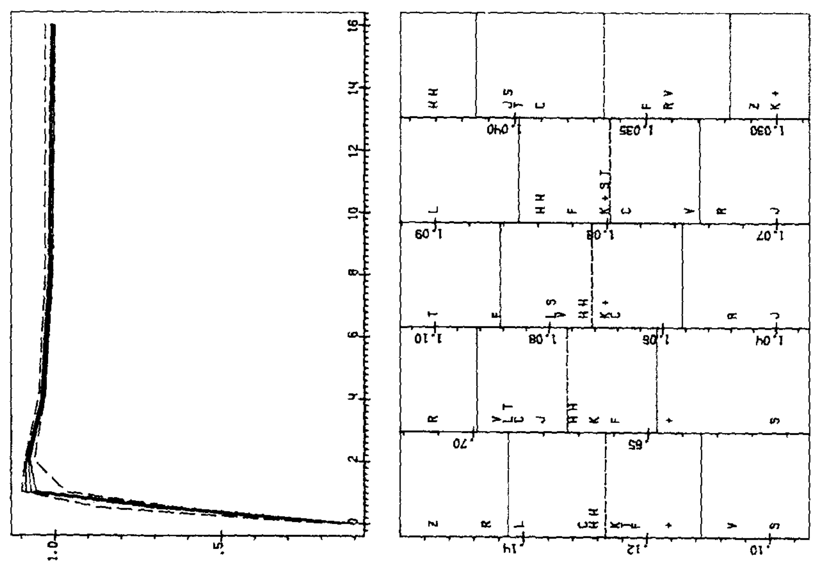


Fig. 13. Model 2D-1. Electric field E_x(HP), $z = 0$, $T = 10$.

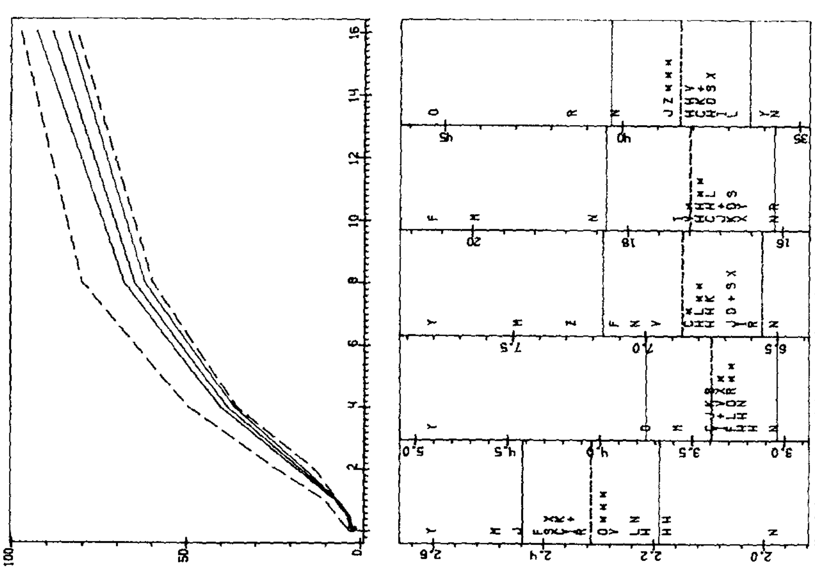


Fig. 12. Model 2D-1. Apparent resistivity ρ_a(EP), $z = 0$, $T = 10$.

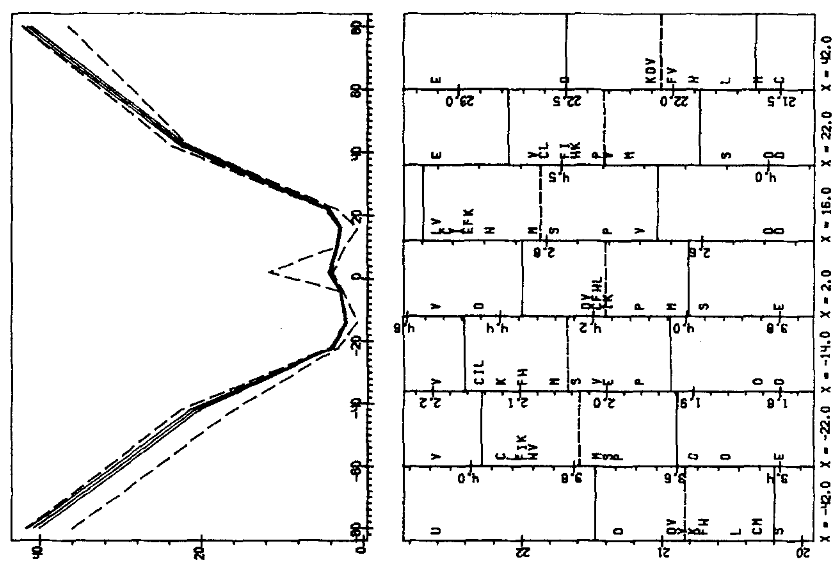


Fig. 14. Model 2D-2. Apparent resistivity ρ_a(EP), $z = 0$, $T = 1000$.

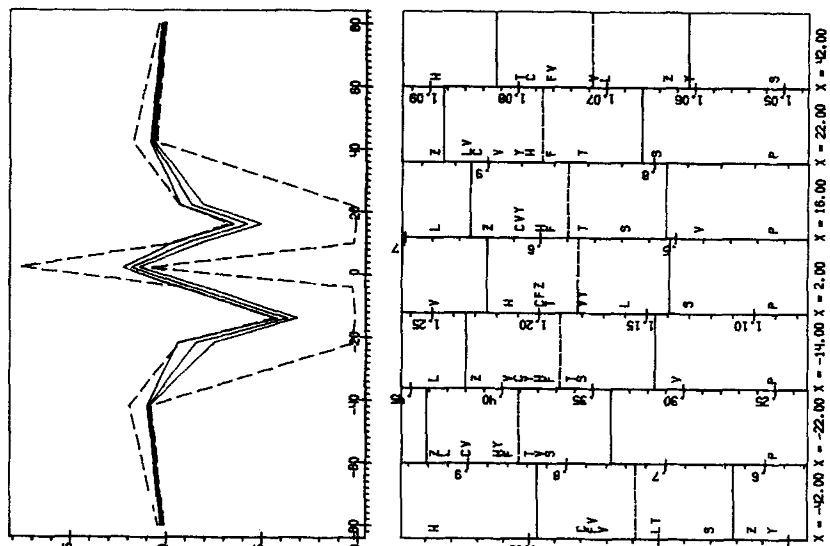


Fig. 15. Model 2D-2. Electric field E_x(HP), $z = 0$, $T = 1000$.

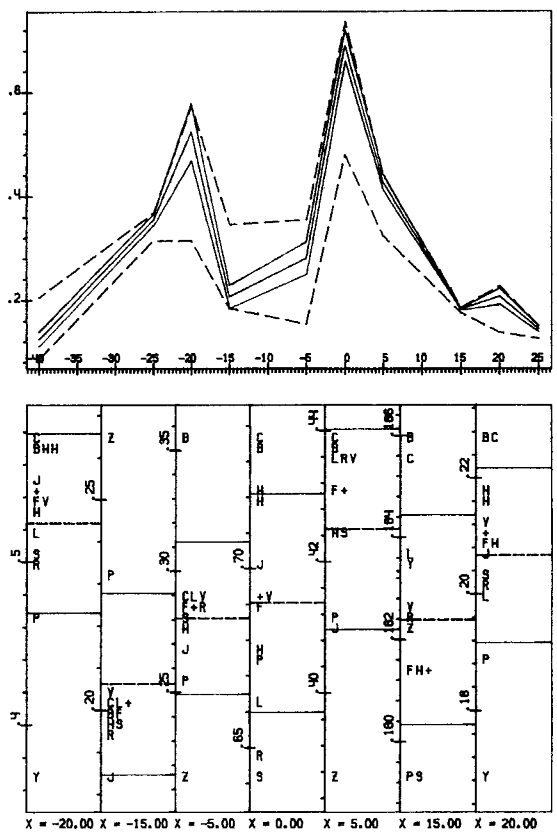


Fig. 16. Model 2D-3a. Magnetic field H_z(EP), $z = 0$, $T = 100$, $\rho = 10$.

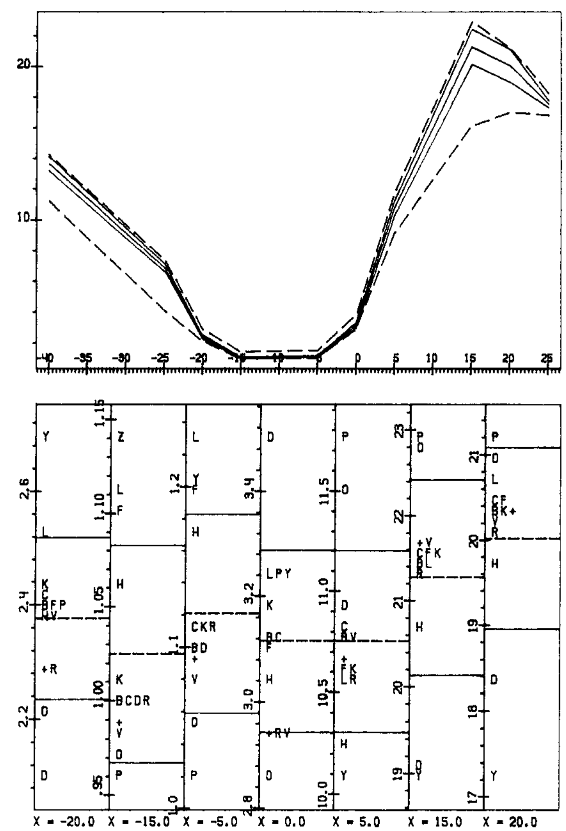


Fig. 17. Model 2D-3a. Apparent resistivity ρ_a(EP), $z = 0$, $T = 100$, $\rho = 10$.

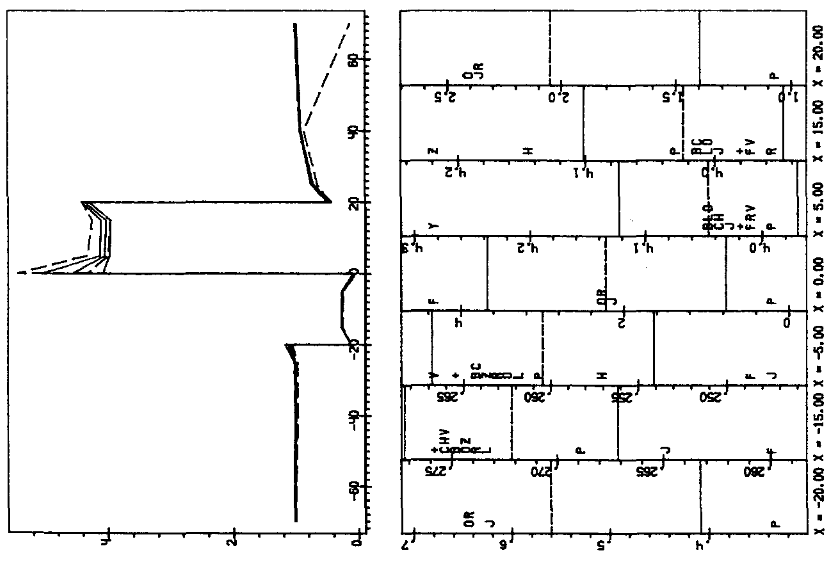


Fig. 18. Model 2D-3a. Electric field E_x(HP), $z=0$, $T=100$, $\rho=10$.

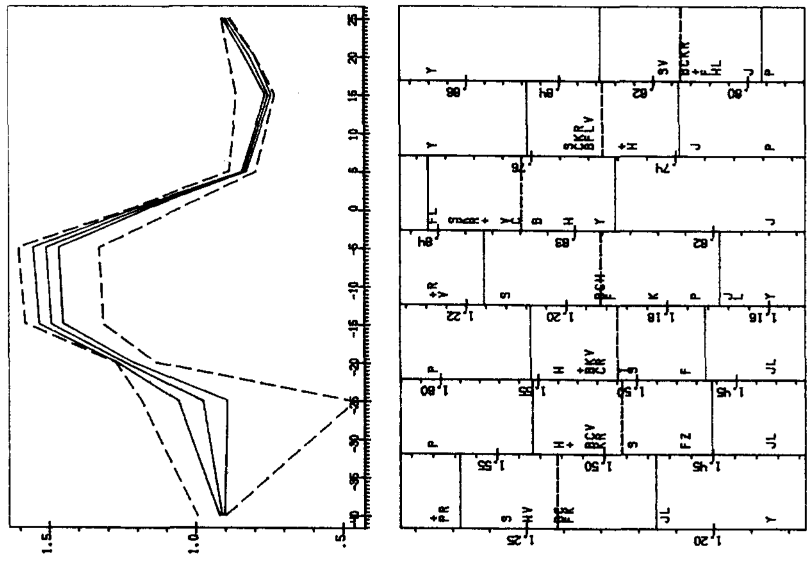


Fig. 19. Model 2D-3a. Magnetic field H_x(EP), $z=0$, $T=100$.

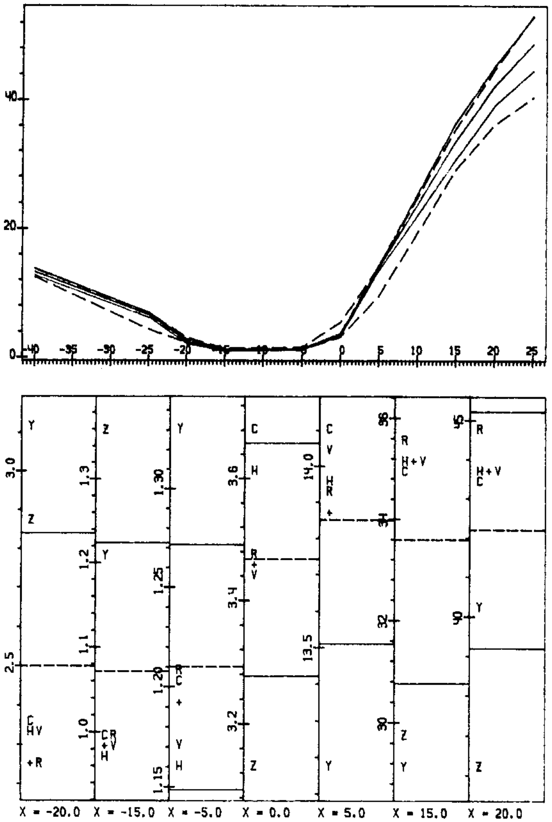


Fig. 20. Model 2D-3b. Apparent resistivity ρ_a(EP), $z = 0$, $T = 300$, $\rho = 1000$.

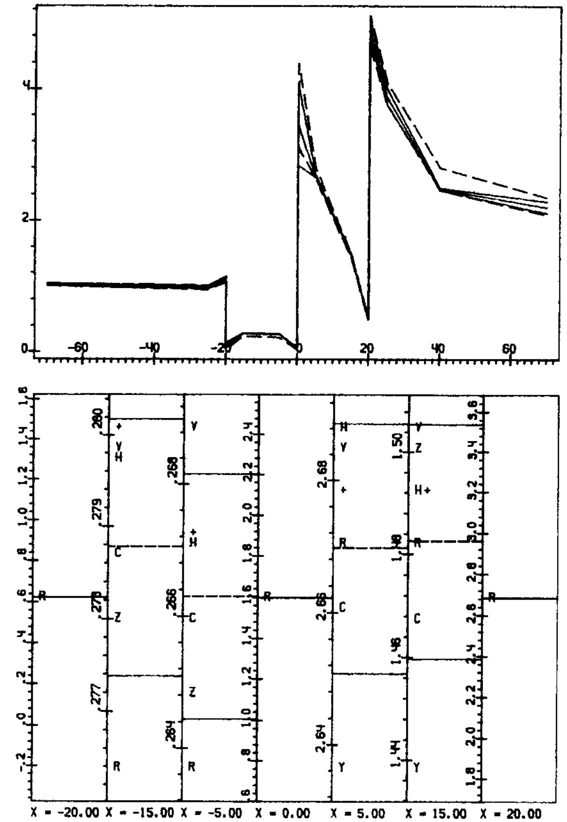


Fig. 21. Model 2D-3b. Electric field E_x(HP), $z = 0$, $T = 100$, $\rho = 1000$.

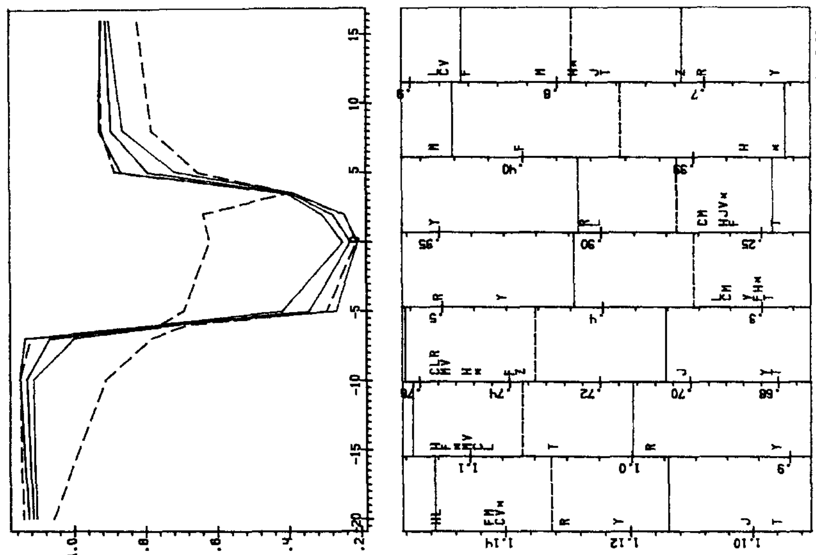


Fig. 23. Model 2D-4. Electric field E_x(HP), $z = 0$, $T = 100$.

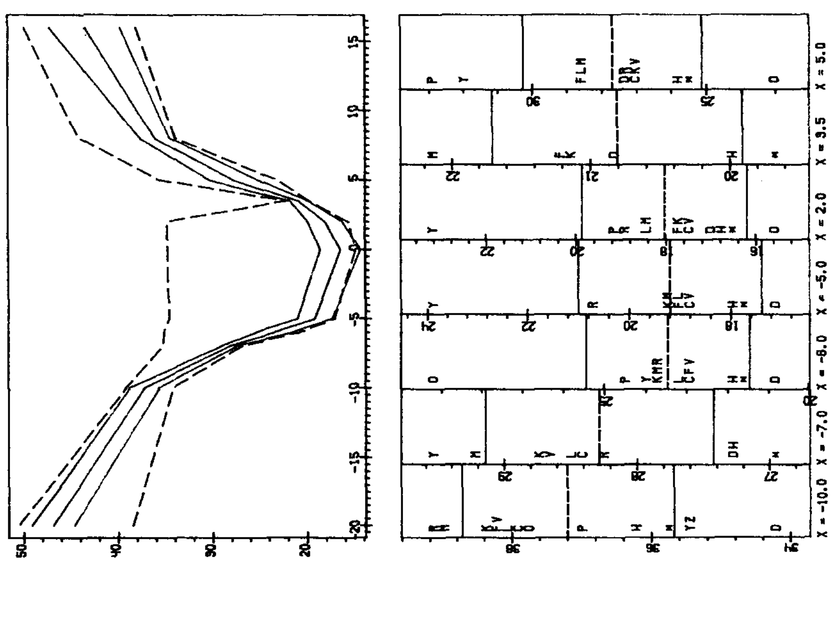


Fig. 22. Model 2D-4. Apparent resistivity ρ_a(EP), $z = 0$, $T = 100$.

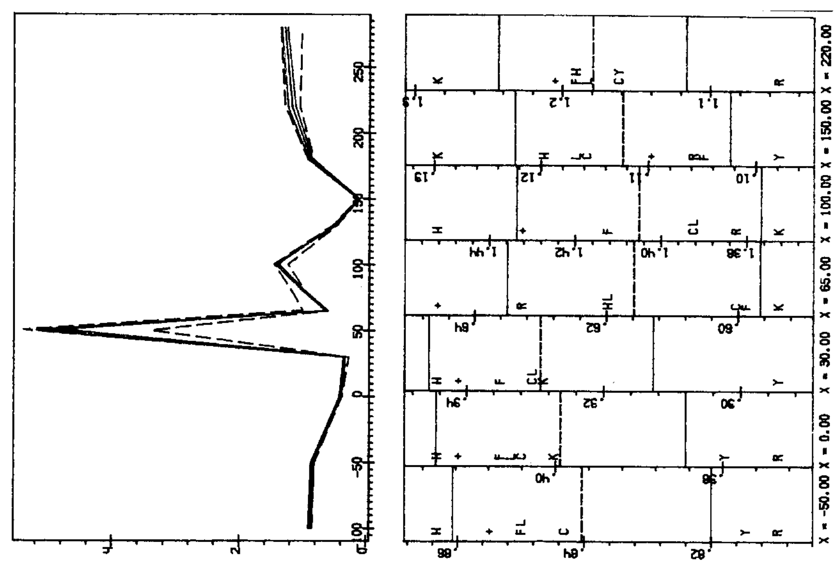


Fig. 25. Model 2D-5. Electric field E_x(HP), $z = 0$, $T = 300$.

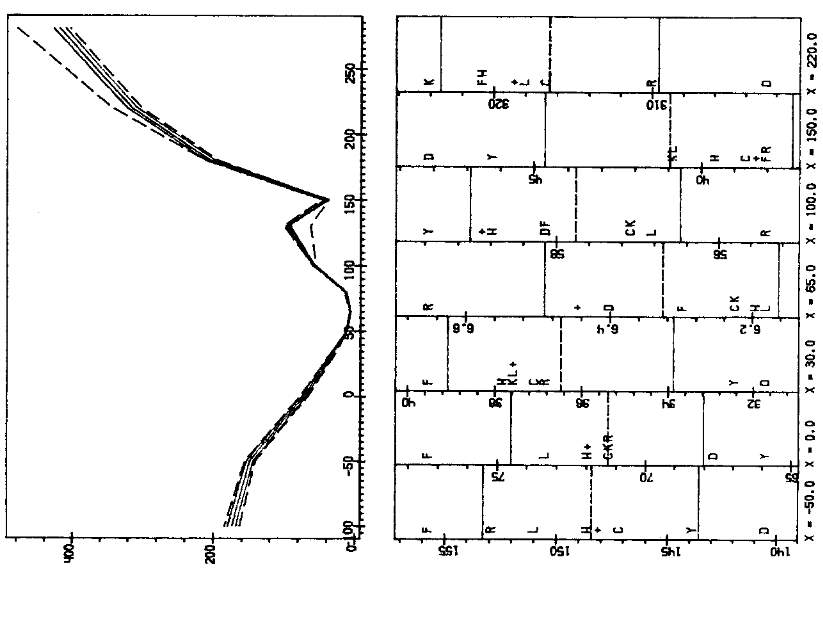


Fig. 24. Model 2D-5. Apparent resistivity ρ_a(EP), $z = 0$, $T = 300$.

Appendix D. Results for three-dimensional models

Table D.1. Model 3D-1A (EXN, $T = 0.1$, $z = 0$)

Participant/ $x =$	0.0	0.25	0.5	0.75	1.0	1.5	2.0	4.0
Re E_x								
1 Russia–Czech	0.390	0.460	0.720	1.020	1.060	1.040	1.020	1.000
1.1 Russia–Czech	0.440	0.500	0.730	1.000	1.050	1.030	1.020	1.000
4 Russia	0.310	0.310	0.340	1.040	1.080	1.060	1.020	—
6 Russia	0.440	0.500	0.740	1.000	1.060	1.040	1.020	1.000
7 USA	0.350	0.425	0.761	1.025	1.062	1.037	1.017	1.001
7.1 USA	0.360	0.410	0.740	1.040	1.080	1.040	1.020	1.000
9 Germany	0.395	0.472	0.735	0.994	1.049	1.034	1.016	—
Average (0)	0.384	0.440	0.681	1.017	1.063	1.040	1.019	1.000
St. dev. (0)	0.048	0.067	0.151	0.019	0.013	0.010	0.002	0.000
Average (1)	0.384	0.440	0.738	1.017	1.063	1.037	1.019	1.000
St. dev. (1)	0.048	0.067	0.014	0.019	0.013	0.004	0.002	0.000
Im E_x								
1 Russia–Czech	−0.120	−0.110	−0.060	0.040	0.060	0.050	0.030	0.000
1.1 Russia–Czech	−0.120	−0.120	−0.060	0.030	0.060	0.050	0.030	0.000
4 Russia	−0.110	−0.110	−0.110	0.030	0.050	0.040	0.020	—
6 Russia	−0.130	−0.120	−0.070	0.030	0.060	0.050	0.030	0.000
7 USA	−0.187	−0.155	−0.025	0.059	0.071	0.052	0.033	0.005
7.1 USA	−0.200	−0.180	−0.040	0.060	0.070	0.060	0.040	0.010
9 Germany	−0.137	−0.123	−0.056	0.033	0.057	0.046	0.046	—
Average (0)	−0.143	−0.131	−0.060	0.040	0.061	0.050	0.033	0.003
St. dev. (0)	0.035	0.026	0.027	0.014	0.007	0.006	0.008	0.004
Re H_y								
1 Russia–Czech	1.090	1.080	1.070	1.060	1.050	1.030	1.020	1.000
1.1 Russia–Czech	1.080	1.080	1.070	1.060	1.050	1.030	1.020	1.000
6 Russia	1.080	1.080	1.070	1.060	1.050	1.030	1.020	1.000
7 USA	1.076	1.074	1.067	1.059	1.050	1.033	1.021	1.004
7.1 USA	1.070	1.070	1.070	1.060	1.050	1.030	1.020	1.000
9 Germany	1.071	1.070	1.066	1.058	1.049	1.032	1.020	—
Average (0)	1.078	1.076	1.069	1.059	1.050	1.031	1.020	1.001
St. dev. (0)	0.007	0.005	0.002	0.001	0.000	0.001	0.000	0.002
Im H_y								
1 Russia–Czech	0.020	0.020	0.020	0.020	0.020	0.010	0.010	0.000
1.1 Russia–Czech	0.020	0.020	0.020	0.020	0.020	0.010	0.010	0.000
6 Russia	0.020	0.020	0.020	0.020	0.020	0.010	0.010	0.000
7 USA	0.024	0.022	0.019	0.017	0.015	0.013	0.010	0.004
7.1 USA	0.020	0.020	0.020	0.020	0.020	0.010	0.010	0.010
9 Germany	0.025	0.024	0.022	0.020	0.018	0.015	0.012	—
Average (0)	0.021	0.021	0.020	0.019	0.019	0.011	0.010	0.003
St. dev. (0)	0.002	0.002	0.001	0.001	0.002	0.002	0.001	0.004
Average (1)	0.021	0.021	0.020	0.020	0.019	0.011	0.010	0.003
St. dev. (1)	0.002	0.002	0.001	0.000	0.002	0.002	0.000	0.004

Table D.2. Model 3D-1A (EYN, $T = 0.1$, $z = 0$)

Participant/ $x =$	0.0	0.25	0.5	0.75	1.0	1.5	2.0	4.0
Re E_y								
1 Russia–Czech	0.360	0.380	0.460	0.580	0.690	0.830	0.910	0.990
1.1 Russia–Czech	0.380	0.410	0.480	0.590	0.700	0.840	0.910	0.990
3 Russia	0.360	0.370	0.410	0.540	0.650	0.760	0.890	—
4 Russia	0.350	0.350	0.410	0.530	0.630	0.820	0.890	—
5 Russia	0.320	0.340	0.390	0.510	0.640	0.790	0.900	1.000
6 Russia	0.400	0.420	0.490	0.600	0.700	0.840	0.910	0.990
7 USA	0.386	0.405	0.470	0.581	0.689	0.834	0.909	0.989
7.1 USA	0.380	0.400	0.470	0.580	0.680	0.830	0.910	0.990
9 Germany	0.373	0.397	0.465	0.573	0.684	0.831	0.917	—
10 Russia	0.300	0.320	0.390	0.520	0.650	0.820	0.920	—
Average (0)	0.361	0.379	0.443	0.560	0.671	0.819	0.907	0.991
St. dev. (0)	0.031	0.033	0.039	0.032	0.026	0.025	0.010	0.004
Average (1)	0.361	0.379	0.443	0.560	0.671	0.826	0.907	0.990
St. dev. (1)	0.031	0.033	0.039	0.032	0.026	0.015	0.010	0.000
Im E_y								
1 Russia–Czech	−0.160	−0.160	−0.170	−0.160	−0.140	−0.100	−0.070	−0.020
1.1 Russia–Czech	−0.170	−0.170	−0.170	−0.160	−0.140	−0.100	−0.070	−0.020
3 Russia	−0.160	−0.160	−0.150	−0.100	−0.090	−0.090	−0.070	—
4 Russia	−0.100	−0.110	−0.120	−0.140	−0.140	−0.120	−0.090	—
5 Russia	−0.110	−0.110	−0.120	−0.140	−0.150	−0.130	−0.110	−0.020
6 Russia	−0.170	−0.170	−0.180	−0.170	−0.150	−0.100	−0.070	−0.020
7 USA	−0.186	−0.191	−0.191	−0.177	−0.150	−0.102	−0.070	−0.019
7.1 USA	−0.170	−0.180	−0.190	−0.180	−0.150	−0.110	−0.070	−0.020
9 Germany	−0.174	−0.174	−0.179	−0.172	−0.149	−0.104	−0.072	—
10 Russia	−0.120	−0.120	−0.120	−0.110	−0.090	−0.050	−0.030	—
Average (0)	−0.152	−0.155	−0.159	−0.151	−0.135	−0.101	−0.072	−0.020
St. dev. (0)	0.030	0.030	0.029	0.028	0.024	0.021	0.020	0.000
Average (1)	−0.152	−0.155	−0.159	−0.151	−0.135	−0.106	−0.077	−0.020
St. dev. (1)	0.030	0.030	0.029	0.028	0.024	0.012	0.014	0.000
Re H_x								
1 Russia–Czech	1.300	1.280	1.160	1.010	0.950	0.930	0.950	0.990
1.1 Russia–Czech	1.270	1.250	1.150	1.020	0.960	0.940	0.950	0.990
3 Russia	1.270	1.270	1.200	1.040	0.970	0.890	0.920	0.970
5 Russia	1.390	1.380	1.290	1.120	1.000	0.910	0.910	1.000
6 Russia	1.270	1.250	1.150	1.020	0.960	0.940	0.950	0.990
7 USA	1.281	1.272	1.166	1.016	0.950	0.933	0.947	—
9 Germany	1.264	1.259	1.168	1.025	0.956	0.934	0.947	—
10 Russia	1.330	1.330	1.200	1.030	0.950	0.920	0.930	—
Average (0)	1.297	1.286	1.186	1.035	0.962	0.925	0.938	0.988
St. dev. (0)	0.044	0.046	0.047	0.035	0.017	0.017	0.016	0.010
Average (1)	1.284	1.273	1.171	1.023	0.957	0.930	0.938	0.988
St. dev. (1)	0.024	0.028	0.021	0.010	0.007	0.011	0.016	0.010
Im H_x								
1 Russia–Czech	0.070	0.050	0.040	0.040	0.030	0.010	−0.010	−0.010
1.1 Russia–Czech	0.080	0.060	0.040	0.040	0.030	0.000	−0.010	−0.010
3 Russia	0.210	0.200	0.120	0.040	0.010	−0.050	−0.050	−0.030
5 Russia	0.030	0.030	0.030	0.020	0.020	0.020	0.020	0.020
6 Russia	0.070	0.060	0.040	0.040	0.030	0.000	−0.010	−0.010
7 USA	0.077	0.056	0.035	0.041	0.030	0.005	−0.007	−0.010
9 Germany	0.089	0.071	0.042	0.035	0.024	0.003	−0.008	—
10 Russia	0.040	0.000	0.040	0.000	0.000	0.000	0.000	—
Average (0)	0.083	0.066	0.048	0.032	0.022	−0.002	−0.009	−0.008
St. dev. (0)	0.055	0.059	0.029	0.015	0.011	0.021	0.019	0.016
Average (1)	0.065	0.047	0.038	0.037	0.022	0.005	−0.004	−0.008
St. dev. (1)	0.022	0.024	0.004	0.008	0.011	0.007	0.011	0.016

Table D.2 (continued)

Participant/ $x =$	0.0	0.25	0.5	0.75	1.0	1.5	2.0	4.0
Re H_z								
1 Russia–Czech		0.150	0.280	0.270	0.200	0.100	0.050	0.000
1.1 Russia–Czech		0.140	0.250	0.250	0.190	0.100	0.050	0.000
3 Russia		0.150	0.180	0.200	0.200	0.110	0.050	0.010
4 Russia		—	0.090	0.180	0.150	0.090	0.040	—
5 Russia		0.140	0.350	0.260	0.160	0.080	0.030	0.010
6 Russia		0.130	0.250	0.240	0.190	0.100	0.050	0.000
7 USA		0.134	0.273	0.269	0.201	0.101	0.051	0.002
7.1 USA		0.130	0.280	0.280	0.210	0.100	0.050	0.000
9 Germany		0.120	0.254	0.261	0.200	0.102	0.052	—
10 Russia		0.130	0.280	0.310	0.250	0.170	0.080	—
Average (0)		0.136	0.249	0.252	0.195	0.105	0.050	0.003
St. dev. (0)		0.010	0.070	0.038	0.027	0.024	0.012	0.005
Average (1)		0.136	0.266	0.252	0.189	0.098	0.047	0.003
St. dev. (1)		0.010	0.044	0.038	0.020	0.008	0.007	0.005
Im H_z								
1 Russia–Czech		0.030	0.030	0.040	0.050	0.050	0.040	0.010
1.1 Russia–Czech		0.030	0.030	0.040	0.050	0.050	0.040	0.010
3 Russia		0.080	0.150	0.150	0.100	0.080	0.070	0.030
4 Russia		—	0.090	0.180	0.150	0.090	0.040	—
5 Russia		0.010	0.010	0.000	0.000	0.000	0.000	0.000
6 Russia		0.030	0.030	0.040	0.050	0.050	0.040	0.010
7 USA		0.036	0.029	0.038	0.051	0.050	0.039	0.010
7.1 USA		0.040	0.020	0.030	0.050	0.050	0.040	0.010
9 Germany		0.043	0.049	0.051	0.057	0.052	0.041	—
10 Russia		−0.030	0.000	0.030	0.010	−0.020	−0.010	—
Average (0)		0.030	0.044	0.060	0.057	0.045	0.034	0.011
St. dev. (0)		0.029	0.045	0.057	0.042	0.033	0.023	0.009
Average (1)		0.037	0.032	0.047	0.046	0.045	0.034	0.008
St. dev. (1)		0.020	0.026	0.041	0.029	0.033	0.023	0.004

Table D.3. Model 3D-1A (EXN, $T = 0.1$, $z = 0$)

Participant/ $x =$	0.5	0.75	1.0	1.25	1.5	2.0
Re H_y						
1 Russia–Czech	1.090	1.090	1.040	0.980	0.960	0.960
1.1 Russia–Czech	1.090	1.080	1.040	0.990	0.970	0.970
6 Russia	1.090	1.080	1.040	0.990	0.970	0.970
7 USA	1.089	1.098	1.050	0.988	0.964	0.963
7.1 USA	1.090	1.100	1.050	0.990	0.970	0.960
9 Germany	1.090	1.095	1.050	0.982	0.964	0.963
Average (0)	1.090	1.090	1.045	0.987	0.966	0.964
St. dev. (0)	0.000	0.009	0.005	0.005	0.004	0.005
Im H_y						
1 Russia–Czech	0.010	0.000	0.000	0.020	0.020	0.010
1.1 Russia–Czech	0.010	0.000	0.000	0.010	0.010	0.000
6 Russia	0.010	0.000	0.000	0.010	0.010	0.000
7 USA	0.014	−0.003	−0.002	0.015	0.015	0.006
7.1 USA	0.010	0.010	0.000	0.020	0.020	0.010
9 Germany	0.018	0.004	0.001	0.013	0.013	0.004
Average (0)	0.012	0.002	0.000	0.015	0.015	0.005
St. dev. (0)	0.003	0.005	0.001	0.005	0.005	0.005

Table D.3 (continued)

Participant / x =	0.5	0.75	1.0	1.25	1.5	2.0
Re E_x						
1 Russia–Czech	0.400	0.440	0.530	0.680	0.800	0.910
1.1 Russia–Czech	0.450	0.490	0.570	0.700	0.810	0.910
4 Russia	0.320	0.350	0.450	0.570	0.720	—
6 Russia	0.450	0.490	0.570	0.700	0.810	0.910
7 USA	0.362	0.387	0.490	0.652	0.783	0.905
7.1 USA	0.360	0.380	0.480	0.640	0.780	0.900
9 Germany	0.410	0.447	0.532	0.671	0.788	0.905
Average (0)	0.393	0.426	0.517	0.659	0.784	0.907
St. dev. (0)	0.049	0.055	0.046	0.045	0.031	0.004
Average (1)	0.393	0.426	0.517	0.659	0.795	0.907
St. dev. (1)	0.049	0.055	0.046	0.045	0.013	0.004
Im E_x						
1 Russia–Czech	−0.120	−0.120	−0.120	−0.100	−0.070	−0.040
1.1 Russia–Czech	−0.120	−0.130	−0.120	−0.100	−0.080	−0.040
4 Russia	−0.110	−0.110	−0.130	−0.140	−0.140	—
6 Russia	−0.130	−0.130	−0.130	−0.100	−0.080	−0.040
7 USA	−0.189	−0.192	−0.161	−0.119	−0.081	−0.044
7.1 USA	−0.200	−0.200	−0.180	−0.140	−0.090	−0.050
9 Germany	−0.136	−0.135	−0.132	−0.109	−0.080	−0.046
Average (0)	−0.144	−0.145	−0.139	−0.115	−0.089	−0.043
St. dev. (0)	0.036	0.036	0.023	0.018	0.023	0.004
Average (1)	−0.144	−0.145	−0.139	−0.115	−0.080	−0.043
St. dev. (1)	0.036	0.036	0.023	0.018	0.006	0.004
Re H_z						
1 Russia–Czech	0.060	0.110	0.150	0.140	0.100	0.050
1.1 Russia–Czech	0.050	0.100	0.140	0.120	0.090	0.050
4 Russia	0.040	0.090	—	0.110	0.100	0.060
6 Russia	0.060	0.100	0.140	0.120	0.090	0.050
7 USA	0.042	0.093	0.152	0.140	0.102	0.050
7.1 USA	0.040	0.090	0.150	0.140	0.100	0.050
9 Germany	0.039	0.092	0.149	0.139	0.102	0.050
Average (0)	0.047	0.096	0.147	0.130	0.098	0.051
St. dev. (0)	0.009	0.007	0.005	0.013	0.005	0.004
Average (1)	0.047	0.096	0.147	0.130	0.098	0.050
St. dev. (1)	0.009	0.007	0.005	0.013	0.005	0.000
Im H_z						
1 Russia–Czech	0.020	0.010	−0.010	0.000	0.010	0.020
1.1 Russia–Czech	0.020	0.010	0.000	0.000	0.010	0.020
4 Russia	0.040	0.090	—	0.110	0.100	0.060
6 Russia	0.020	0.010	0.000	0.000	0.010	0.020
7 USA	0.020	0.016	−0.006	−0.004	0.008	0.016
7.1 USA	0.020	0.010	−0.020	−0.010	0.000	0.020
9 Germany	0.019	0.020	0.003	0.003	0.012	0.018
Average (0)	0.023	0.024	−0.005	0.014	0.021	0.025
St. dev. (0)	0.008	0.029	0.009	0.042	0.035	0.016
Average (1)	0.020	0.013	−0.005	−0.002	0.008	0.019
St. dev. (1)	0.000	0.004	0.009	0.005	0.004	0.002

Table D.4. Model 3D-1A (EYN, $T = 0.1$, $z = 0$)

Participant/ $x =$	0.5	0.75	1.0	1.25	1.5	2.0
Re E_y						
1 Russia–Czech	0.410	0.520	0.810	1.120	1.130	1.050
1.1 Russia–Czech	0.440	0.560	0.810	1.090	1.110	1.050
3 Russia	0.370	0.390	0.440	1.060	1.050	1.010
4 Russia	0.350	0.350	—	1.140	1.130	1.070
5 Russia	0.320	0.300	0.370	1.160	1.140	1.080
6 Russia	0.450	0.560	0.820	1.100	1.110	1.050
7 USA	0.396	0.482	0.842	1.120	1.119	1.051
7.1 USA	0.390	0.460	0.810	1.130	1.130	1.060
9 Germany	0.414	0.533	0.824	1.085	1.105	1.047
10 Russia	0.330	0.400	0.850	1.180	1.200	1.150
Average (0)	0.387	0.456	0.731	1.118	1.122	1.062
St. dev. (0)	0.044	0.092	0.186	0.036	0.037	0.036
Average (1)	0.387	0.456	0.731	1.118	1.114	1.052
St. dev. (1)	0.044	0.092	0.186	0.036	0.027	0.019
Im E_y						
1 Russia–Czech	−0.140	−0.100	−0.010	0.130	0.130	0.080
1.1 Russia–Czech	−0.140	−0.110	−0.020	0.120	0.130	0.080
3 Russia	−0.160	−0.100	−0.020	0.200	0.160	0.080
4 Russia	−0.110	−0.110	—	0.070	0.060	0.050
5 Russia	−0.110	−0.110	−0.080	0.050	0.050	0.040
6 Russia	−0.150	−0.110	−0.020	0.120	0.130	0.080
7 USA	−0.179	−0.122	0.050	0.157	0.140	0.079
7.1 USA	−0.180	−0.140	0.040	0.160	0.150	0.080
9 Germany	−0.152	−0.112	−0.003	0.119	0.126	0.077
10 Russia	−0.010	−0.020	0.090	0.200	0.200	0.150
Average (0)	−0.133	−0.103	0.003	0.133	0.128	0.080
St. dev. (0)	0.049	0.031	0.050	0.049	0.044	0.029
Average (1)	−0.147	−0.113	0.003	0.133	0.128	0.072
St. dev. (1)	0.025	0.012	0.050	0.049	0.044	0.015
Re H_x						
1 Russia–Czech	1.260	1.220	1.170	1.120	1.090	1.050
1.1 Russia–Czech	1.240	1.200	1.160	1.120	1.080	1.050
3 Russia	1.260	1.250	1.210	1.120	1.080	1.060
5 Russia	1.390	1.400	1.460	0.940	0.960	0.980
6 Russia	1.240	1.200	1.150	1.110	1.080	1.040
7 USA	1.248	1.209	1.163	1.120	1.087	1.047
9 Germany	1.237	1.203	1.160	1.120	1.087	1.047
10 Russia	1.300	1.250	1.200	1.150	1.110	1.070
Average (0)	1.272	1.241	1.209	1.100	1.072	1.043
St. dev. (0)	0.052	0.067	0.104	0.066	0.046	0.027
Average (1)	1.255	1.219	1.173	1.123	1.088	1.052
St. dev. (1)	0.022	0.022	0.023	0.013	0.011	0.010
Im H_x						
1 Russia–Czech	0.060	0.050	0.050	0.040	0.040	0.030
1.1 Russia–Czech	0.070	0.060	0.050	0.040	0.040	0.030
3 Russia	0.190	0.150	0.120	0.040	0.040	0.020
5 Russia	0.030	0.030	0.040	0.020	0.020	0.020
6 Russia	0.060	0.060	0.050	0.040	0.040	0.030
7 USA	0.069	0.060	0.050	0.042	0.037	0.027
9 Germany	0.078	0.067	0.055	0.046	0.040	0.029
10 Russia	0.020	0.020	0.010	0.000	0.000	0.000
Average (0)	0.072	0.062	0.053	0.034	0.032	0.023
St. dev. (0)	0.052	0.039	0.031	0.016	0.015	0.010
Average (1)	0.055	0.050	0.044	0.038	0.037	0.027
St. dev. (1)	0.022	0.018	0.015	0.008	0.007	0.005

Table D.5. Model 3D-1A (EXN, $T = 10.0$, $z = 0$)

Participant/ $x =$	0.0	0.25	0.5	0.75	1.0	1.5	2.0	4.0
Re E_x								
1 Russia–Czech	0.160	0.270	0.700	1.090	1.170	1.150	1.100	1.030
1.1 Russia–Czech	0.180	0.290	0.710	1.080	1.160	1.140	1.100	1.030
1.2 Russia–Czech	0.110	0.230	0.700	1.100	1.170	1.150	1.110	—
2 Russia–Czech	0.100	0.240	0.740	1.090	1.160	1.150	1.110	1.030
6 Russia	0.180	0.290	0.710	1.080	1.160	1.140	1.100	1.030
7 USA	0.109	0.245	0.752	1.110	1.176	1.145	1.101	1.026
7.1 USA	0.080	0.190	0.680	1.080	1.170	1.160	1.120	1.030
9 Germany	0.164	0.268	0.704	1.085	1.169	1.146	1.103	—
11 Russia	0.140	0.270	0.730	1.090	1.170	1.140	1.100	1.030
Average (0)	0.136	0.255	0.714	1.089	1.167	1.147	1.105	1.029
St. dev. (0)	0.037	0.032	0.023	0.010	0.006	0.007	0.007	0.002
Average (1)	0.136	0.263	0.714	1.087	1.167	1.145	1.103	1.030
St. dev. (1)	0.037	0.022	0.023	0.007	0.006	0.005	0.004	0.000
Im E_x								
1 Russia–Czech	−0.050	−0.030	0.000	0.000	0.000	0.010	0.010	0.000
1.1 Russia–Czech	−0.050	−0.040	0.000	0.010	0.010	0.010	0.010	0.000
1.2 Russia–Czech	0.000	0.000	0.000	0.000	0.000	0.000	0.000	—
2 Russia–Czech	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6 Russia	−0.050	−0.040	0.000	0.010	0.010	0.010	0.010	0.000
7 USA	−0.024	−0.021	−0.011	−0.002	0.003	0.006	0.006	0.003
7.1 USA	−0.030	−0.020	−0.010	−0.010	0.000	0.010	0.010	0.000
9 Germany	−0.053	−0.041	−0.003	0.007	0.007	0.008	0.007	—
11 Russia	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Average (0)	−0.029	−0.021	−0.003	0.002	0.003	0.006	0.006	0.000
St. dev. (0)	0.024	0.018	0.005	0.006	0.004	0.005	0.005	0.001
Average (1)	−0.029	−0.021	−0.003	0.002	0.003	0.006	0.006	0.000
St. dev. (1)	0.024	0.018	0.005	0.006	0.004	0.005	0.005	0.000
Re H_y								
1 Russia–Czech	1.010	1.010	1.010	1.010	1.010	1.010	1.000	1.000
1.1 Russia–Czech	1.010	1.010	1.010	1.010	1.010	1.010	1.000	1.000
6 Russia	1.010	1.010	1.010	1.010	1.010	1.010	1.000	1.000
7 USA	1.013	1.012	1.011	1.009	1.008	1.006	1.005	1.002
7.1 USA	1.010	1.010	1.010	1.010	1.010	1.010	1.010	1.000
9 Germany	1.012	1.012	1.010	1.009	1.008	1.006	1.005	—
Average (0)	1.011	1.011	1.010	1.010	1.009	1.009	1.003	1.000
St. dev. (0)	0.001	0.001	0.000	0.001	0.001	0.002	0.004	0.001
Im H_y								
1 Russia–Czech	−0.010	−0.010	−0.010	−0.010	−0.010	−0.010	0.000	0.000
1.1 Russia–Czech	−0.010	−0.010	−0.010	−0.010	−0.010	−0.010	0.000	0.000
6 Russia	−0.010	−0.010	−0.010	−0.010	−0.010	−0.010	0.000	0.000
7 USA	−0.013	−0.012	−0.010	−0.009	−0.008	−0.005	0.004	0.001
7.1 USA	−0.010	−0.010	−0.010	−0.010	−0.010	−0.010	0.000	0.000
9 Germany	−0.013	−0.012	−0.011	−0.010	−0.008	−0.006	−0.004	—
Average (0)	−0.011	−0.011	−0.010	−0.010	−0.009	−0.009	0.000	0.000
St. dev. (0)	0.002	0.001	0.000	0.000	0.001	0.002	0.003	0.000

Table D.6. Model 3D-1A (EYN, $T = 10.0$, $z = 0$)

Participant/ $x =$	0.0	0.25	0.5	0.75	1.0		1.5	2.0	4.0
Re E_y									
1 Russia–Czech	0.120	0.130	0.210	0.360	0.510		0.730	0.840	0.970
1.1 Russia–Czech	0.130	0.140	0.210	0.360	0.510		0.730	0.840	0.970
1.2 Russia–Czech	0.060	0.070	0.150	0.310	0.480		0.710	0.830	—
2 Russia–Czech	0.050	0.070	0.150	0.320	0.480		0.700	0.830	0.970
6 Russia	0.130	0.150	0.220	0.360	0.510		0.730	0.840	0.970
7 USA	0.069	0.095	0.183	0.339	0.503		0.726	0.842	0.969
7.1 USA	0.060	0.080	0.160	0.300	0.470		0.700	0.820	0.970
9 Germany	0.123	0.137	0.199	0.337	0.492		0.715	0.833	—
10 Russia	0.120	0.150	0.180	0.360	0.500		0.750	0.900	—
11 Russia	0.160	0.180	0.260	0.400	0.540		0.740	0.850	—
Average (0)	0.102	0.120	0.192	0.345	0.500		0.723	0.843	0.970
St. dev. (0)	0.039	0.039	0.035	0.030	0.020		0.017	0.022	0.000
Average (1)	0.102	0.120	0.192	0.345	0.495		0.723	0.836	0.970
St. dev. (1)	0.039	0.039	0.035	0.030	0.015		0.017	0.009	0.000
Im E_y									
1 Russia–Czech	−0.050	−0.050	−0.050	−0.040	−0.030		−0.020	−0.010	0.000
1.1 Russia–Czech	−0.060	−0.060	−0.050	−0.040	−0.030		−0.020	−0.010	0.000
1.2 Russia–Czech	0.000	0.000	0.000	0.000	0.000		0.000	0.000	—
2 Russia–Czech	0.000	0.000	0.000	0.000	0.000		0.000	0.000	0.000
6 Russia	−0.060	−0.060	−0.050	−0.040	−0.030		−0.020	−0.010	0.000
7 USA	−0.032	−0.031	−0.030	−0.027	−0.022		−0.013	−0.008	−0.001
7.1 USA	−0.030	−0.030	−0.030	−0.030	−0.030		−0.020	−0.010	0.000
9 Germany	−0.062	−0.061	−0.052	−0.039	−0.029		−0.017	−0.010	—
10 Russia	−0.030	−0.030	−0.020	−0.010	0.000		0.000	0.000	—
11 Russia	0.000	0.000	0.000	0.000	0.000		0.000	0.000	—
Average (0)	−0.032	−0.032	−0.028	−0.023	−0.017		−0.011	−0.006	0.000
St. dev. (0)	0.026	0.025	0.022	0.018	0.015		0.010	0.005	0.000
Re H_x									
1 Russia–Czech	1.040	1.030	1.020	1.010	1.000		1.000	1.000	1.000
1.1 Russia–Czech	1.030	1.030	1.020	1.010	1.000		1.000	1.000	1.000
6 Russia	1.040	1.030	1.020	1.010	1.000		1.000	1.000	1.000
7 USA	1.041	1.037	1.024	1.011	1.003		0.997	0.996	0.998
9 Germany	1.038	1.034	1.023	1.012	1.004		0.998	0.997	—
Average (0)	1.038	1.032	1.021	1.011	1.001		0.999	0.999	1.000
St. dev. (0)	0.004	0.003	0.002	0.001	0.002		0.001	0.002	0.001
Im H_x									
1 Russia–Czech	−0.040	−0.040	−0.020	−0.010	0.000		0.000	0.000	0.000
1.1 Russia–Czech	−0.040	−0.040	−0.020	−0.010	0.000		0.000	0.000	0.000
6 Russia	−0.040	−0.040	−0.020	−0.010	0.000		0.000	0.000	0.000
7 USA	−0.046	−0.041	−0.026	−0.009	0.000		0.005	0.005	0.003
9 Germany	−0.048	−0.042	−0.027	−0.010	−0.001		0.005	0.006	—
Average (0)	−0.043	−0.041	−0.023	−0.010	0.000		0.002	0.002	0.001
St. dev. (0)	0.004	0.001	0.004	0.000	0.000		0.003	0.003	0.002
Re H_z									
1 Russia–Czech		0.020	0.030	0.030	0.020	0.020		0.010	0.000
1.1 Russia–Czech		0.010	0.030	0.030	0.020	0.020		0.010	0.010
6 Russia		0.010	0.030	0.030	0.020	0.020		0.010	0.010
7 USA		0.018	0.031	0.032	0.027	0.019		0.013	0.004
7.1 USA		0.020	0.030	0.030	0.030	0.020		0.010	0.010
9 Germany		0.016	0.027	0.029	0.026	0.018		0.013	—
Average (0)		0.016	0.030	0.030	0.024	0.019		0.011	0.007
St. dev. (0)		0.005	0.001	0.001	0.004	0.001		0.002	0.005

Table D.6 (continued)

Participant/ $x=$	0.0	0.25	0.5	0.75	1.0		1.5	2.0	4.0
Im H_z									
1 Russia–Czech		−0.020	−0.030	−0.040	−0.030	−0.020		−0.010	0.000
1.1 Russia–Czech		−0.020	−0.030	−0.030	−0.030	−0.020		−0.010	0.000
6 Russia		−0.020	−0.030	−0.030	−0.030	−0.020		−0.010	0.000
7 USA		−0.021	−0.036	−0.036	−0.030	−0.019		−0.013	−0.004
7.1 USA		−0.020	−0.030	−0.030	−0.030	−0.020		−0.010	0.000
9 Germany		−0.022	−0.037	−0.038	−0.032	−0.020		−0.014	—
Average (0)		−0.020	−0.032	−0.034	−0.030	−0.020		−0.011	−0.001
St. dev. (0)		0.001	0.003	0.005	0.001	0.000		0.002	0.002
Average (1)		−0.020	−0.032	−0.034	−0.030	−0.020		−0.011	−0.001
St. dev. (1)		0.001	0.003	0.005	0.000	0.000		0.002	0.002

Table D.7. Model 3D-1A (EXN, $T=10.0$, $z=0$)

Participant/ $x=$	0.5	0.75	1.0	1.25	1.5	2.0
Re E_x						
1 Russia–Czech	0.170	0.200	0.320	0.540	0.710	0.870
1.1 Russia–Czech	0.190	0.220	0.340	0.550	0.710	0.870
1.2 Russia–Czech	0.120	0.160	0.280	0.510	0.690	0.860
2 Russia–Czech	0.110	0.150	0.290	0.520	0.690	0.860
6 Russia	0.190	0.220	0.340	0.550	0.710	0.870
7 USA	0.121	0.166	0.314	0.529	0.705	0.870
7.1 USA	0.090	0.130	0.280	0.480	0.670	0.850
9 Germany	0.175	0.205	0.314	0.521	0.693	0.861
11 Russia	0.170	0.220	0.360	0.560	0.720	0.870
Average (0)	0.148	0.186	0.315	0.529	0.700	0.865
St. dev. (0)	0.038	0.035	0.028	0.025	0.015	0.007
Average (1)	0.148	0.186	0.315	0.529	0.700	0.866
St. dev. (1)	0.038	0.035	0.028	0.025	0.015	0.005
Im E_x						
1 Russia–Czech	−0.050	−0.040	−0.040	−0.020	−0.010	−0.010
1.1 Russia–Czech	−0.050	−0.050	−0.040	−0.020	−0.020	−0.010
1.2 Russia–Czech	0.000	0.000	0.000	0.000	0.000	0.000
2 Russia–Czech	0.000	0.000	0.000	0.000	0.000	0.000
6 Russia	−0.050	−0.050	−0.040	−0.020	−0.020	−0.010
7 USA	−0.024	−0.022	−0.018	−0.015	−0.011	−0.005
7.1 USA	−0.030	−0.020	−0.020	−0.020	−0.010	−0.010
9 Germany	−0.053	−0.050	−0.040	−0.024	−0.015	−0.007
11 Russia	0.000	0.000	0.000	0.000	0.000	0.000
Average (0)	−0.029	−0.026	−0.022	−0.013	−0.010	−0.006
St. dev. (0)	0.024	0.022	0.019	0.010	0.008	0.005
Re H_z						
1 Russia–Czech	0.010	0.010	0.010	0.010	0.010	0.010
1.1 Russia–Czech	0.010	0.010	0.010	0.010	0.010	0.010
6 Russia	0.010	0.010	0.010	0.010	0.010	0.010
7 USA	0.008	0.012	0.015	0.014	0.012	0.008
7.1 USA	0.010	0.010	0.010	0.010	0.010	0.010
9 Germany	0.007	0.011	0.014	0.013	0.011	0.008
Average (0)	0.009	0.010	0.012	0.011	0.010	0.009
St. dev. (0)	0.001	0.001	0.002	0.002	0.001	0.001
Im H_z						
1 Russia–Czech	−0.010	−0.010	−0.020	−0.010	−0.010	−0.010
1.1 Russia–Czech	−0.010	−0.010	−0.010	−0.010	−0.010	−0.010
6 Russia	−0.010	−0.010	−0.010	−0.010	−0.010	−0.010
7 USA	−0.008	−0.012	−0.016	−0.014	−0.011	−0.007
7.1 USA	−0.010	−0.010	−0.010	−0.010	−0.010	−0.010
9 Germany	−0.008	−0.013	−0.017	−0.015	−0.012	−0.008
Average (0)	−0.009	−0.011	−0.014	−0.011	−0.011	−0.009
St. dev. (0)	0.001	0.001	0.004	0.002	0.001	0.001

Table D.7 (continued)

Participant/ $x =$	0.5	0.75	1.0	1.25	1.5	2.0
Re H_y						
1 Russia–Czech	1.010	1.010	1.010	1.000	1.000	1.000
1.1 Russia–Czech	1.010	1.010	1.010	1.000	1.000	1.000
6 Russia	1.010	1.010	1.010	1.000	1.000	1.000
7 USA	1.012	1.010	1.006	1.001	0.999	0.998
7.1 USA	1.010	1.010	1.010	1.000	1.000	1.000
9 Germany	1.011	1.010	1.006	1.002	1.000	0.998
Average (0)	1.010	1.010	1.009	1.000	1.000	0.999
St. dev. (0)	0.001	0.000	0.002	0.001	0.000	0.001
Im H_y						
1 Russia–Czech	−0.010	−0.010	−0.010	0.000	0.000	0.000
1.1 Russia–Czech	−0.010	−0.010	−0.010	0.000	0.000	0.000
6 Russia	−0.010	−0.010	−0.010	0.000	0.00	0.000
7 USA	−0.011	−0.010	−0.005	0.000	0.002	0.002
7.1 USA	−0.010	−0.010	−0.010	0.000	0.000	0.000
9 Germany	−0.012	−0.010	−0.006	−0.001	0.002	0.003
Average (0)	−0.011	−0.010	−0.009	0.000	0.001	0.001
St. dev. (0)	0.001	0.000	0.002	0.000	0.001	0.001

Table D.8. Model 3D-1A (EYN, $T = 10.0$, $z = 0$)

Participant/ $x =$	0.5	0.75	1.0	1.25	1.5	2.0
Re H_x						
1 Russia–Czech	1.030	1.030	1.020	1.020	1.020	1.010
1.1 Russia–Czech	1.030	1.030	1.020	1.020	1.020	1.010
6 Russia	1.030	1.030	1.020	1.020	1.020	1.010
7 USA	1.037	1.032	1.026	1.021	1.017	1.011
9 Germany	1.034	1.030	1.025	1.020	1.017	1.012
Average (0)	1.032	1.030	1.022	1.020	1.019	1.011
St. dev. (0)	0.003	0.001	0.003	0.000	0.002	0.001
Im H_x						
1 Russia–Czech	−0.040	−0.030	−0.030	−0.020	−0.020	−0.010
1.1 Russia–Czech	−0.040	−0.030	−0.020	−0.020	−0.020	−0.010
6 Russia	−0.040	−0.030	−0.020	−0.020	−0.020	−0.010
7 USA	−0.041	−0.035	−0.028	−0.021	−0.017	−0.011
9 Germany	−0.042	−0.036	−0.030	−0.023	−0.018	−0.012
Average (0)	−0.041	−0.032	−0.026	−0.021	−0.019	−0.011
St. dev. (0)	0.001	0.003	0.005	0.001	0.001	0.001
Re E_y						
1 Russia–Czech	0.170	0.350	0.920	1.410	1.420	1.270
1.1 Russia–Czech	0.180	0.370	0.940	1.390	1.400	1.260
1.2 Russia–Czech	0.120	0.320	0.950	1.420	1.430	1.280
2 Russia–Czech	0.090	0.330	1.030	1.400	1.390	1.270
6 Russia	0.180	0.370	0.940	1.390	1.400	1.260
7 USA	0.125	0.350	1.041	1.440	1.422	1.264
7.1 USA	0.080	0.280	0.970	1.420	1.430	1.290
9 Germany	0.169	0.348	0.934	1.392	1.412	1.270
10 Russia	0.160	0.300	0.900	1.450	1.450	1.220
11 Russia	0.220	0.410	0.990	1.370	1.370	1.240
Average (0)	0.149	0.343	0.962	1.408	1.412	1.262
St. dev. (0)	0.044	0.037	0.046	0.025	0.023	0.020
Average (1)	0.149	0.343	0.962	1.408	1.412	1.267
St. dev. (1)	0.044	0.037	0.046	0.025	0.023	0.014

Table D.8 (continued)

Participant/ $x=$	0.5	0.75	1.0	1.25	1.5	2.0
Im E_y						
1 Russia–Czech	−0.050	−0.030	0.020	0.020	0.020	0.020
1.1 Russia–Czech	−0.050	−0.030	0.030	0.030	0.020	0.020
1.2 Russia–Czech	0.000	0.000	0.000	0.000	0.000	0.000
2 Russia–Czech	0.000	0.000	0.000	0.000	0.000	0.000
6 Russia	−0.050	−0.030	0.030	0.030	0.020	0.020
7 USA	−0.031	−0.023	0.000	0.010	0.015	0.015
7.1 USA	−0.030	−0.030	0.000	0.010	0.010	0.020
9 Germany	−0.056	−0.034	0.023	0.030	0.024	0.019
10 Russia	−0.020	−0.010	0.010	0.030	0.030	0.030
11 Russia	0.000	0.000	0.000	0.000	0.000	0.000
Average (0)	−0.029	−0.019	0.011	0.016	0.014	0.014
St. dev. (0)	0.023	0.014	0.013	0.013	0.011	0.011
Average (1)	−0.029	−0.019	0.011	0.016	0.014	0.014
St. dev. (1)	0.023	0.014	0.013	0.013	0.011	0.011

Table D.9. Model 3D-1A ($T=0.1$, $z=0$)

Participant/ $x=$	0.0	0.25	0.5	0.75	1.0	1.5	2.0
ρ_a^{xy}							
1 Russia–Czech	13.90	18.90	45.30	92.40	103.00	101.30	99.50
1.1 Russia–Czech	17.70	23.10	47.60	89.40	101.40	100.90	99.40
4 Russia	5.90	5.80	7.60	134.00	134.00	114.00	98.00
6 Russia	17.50	23.00	48.00	90.10	101.80	101.20	99.50
7 USA	13.60	17.74	50.86	94.02	102.80	101.20	99.43
7.1 USA	14.82	17.58	47.84	96.65	105.80	102.10	99.56
9 Germany	15.25	20.77	47.80	88.36	100.30	100.70	99.30
Average (0)	14.10	18.13	42.14	97.85	107.01	103.06	99.24
St. dev. (0)	3.96	5.89	15.32	16.20	12.02	4.85	0.55
Average (1)	15.46	20.18	47.90	91.82	102.52	101.23	99.45
St. dev. (1)	1.76	2.50	1.77	3.14	1.89	0.48	0.09
ρ_a^{yx}							
1 Russia–Czech	9.30	10.60	17.70	35.20	55.00	81.30	92.70
1.1 Russia–Czech	10.80	12.30	19.60	36.50	55.50	81.40	92.80
4 Russia	6.60	7.00	10.80	—	63.60	97.50	—
5 Russia	5.80	6.60	10.20	23.00	42.80	75.00	98.80
6 Russia	11.50	13.10	20.50	37.20	56.20	81.90	93.10
7 USA	11.12	12.36	18.93	35.68	55.00	81.15	92.71
7.1 USA	10.70	11.75	18.31	35.15	54.75	81.30	92.97
9 Germany	10.55	11.82	18.16	34.11	53.65	80.42	94.32
10 Russia	5.90	6.60	12.00	27.00	48.00	80.00	97.00
Average (0)	9.14	10.24	16.24	32.98	53.83	82.22	94.30
St. dev. (0)	2.37	2.71	4.04	5.12	5.74	6.10	2.33
Average (1)	9.14	10.24	16.24	32.98	53.83	80.31	94.30
St. dev. (1)	2.37	2.71	4.04	5.12	5.74	2.23	2.33

Table D.10. Model 3D-1A ($T=0.1$, $z=0$)

Participant/ $x=$	0.5	0.75	1.0	1.25	1.5	2.0
ρ_a^{xy}						
1 Russia–Czech	14.80	17.60	27.60	49.10	69.20	89.20
1.1 Russia–Czech	18.80	22.00	32.00	51.80	70.50	89.00
4 Russia	5.90	6.30	14.00	35.00	61.70	94.20
6 Russia	18.70	21.90	31.80	51.60	70.40	89.60
7 USA	14.05	15.49	24.06	45.08	66.70	88.47
7.1 USA	14.42	15.65	23.61	43.67	65.43	88.23
9 Germany	15.71	18.21	27.26	47.93	67.51	88.53
Average (0)	14.63	16.74	25.76	46.31	67.35	89.60
St. dev. (0)	4.32	5.31	6.15	5.84	3.13	2.08
Average (1)	16.08	16.74	25.76	46.31	67.35	88.84
St. dev. (1)	2.14	5.31	6.15	5.84	3.13	0.52

Table D.10 (continued)

Participant/ $x =$	0.5	0.75	1.0	1.25	1.5	2.0
ρ_a^{yx}						
1 Russia–Czech	11.60	18.90	47.70	101.20	108.50	101.50
1.1 Russia–Czech	14.00	22.50	49.30	97.00	106.60	101.00
4 Russia	6.40	6.10	—	146.00	135.00	110.00
5 Russia	5.70	5.30	6.70	153.00	140.00	120.00
6 Russia	14.50	22.80	49.90	97.90	107.00	101.20
7 USA	12.07	16.88	52.56	101.80	107.50	101.30
7.1 USA	11.65	15.48	48.29	102.60	109.50	101.90
9 Germany	12.63	20.41	50.37	94.82	104.50	100.50
10 Russia	6.40	10.00	51.00	92.00	128.00	117.00
Average (0)	10.55	15.37	44.48	109.59	116.29	106.04
St. dev. (0)	3.44	6.73	15.34	22.95	13.93	7.67
Average (1)	10.55	15.37	49.87	109.59	116.29	106.04
St. dev. (1)	3.44	6.73	1.65	22.95	13.93	7.67

Table D.11. Model 3D-1A ($T = 10.0$, $z = 0$)

Participant/ $x =$	0.0	0.25	0.5	0.75	1.0	1.5	2.0
ρ_a^{xy}							
1 Russia–Czech	2.80	7.20	48.00	117.70	135.40	129.70	120.30
1.1 Russia–Czech	3.30	8.20	49.90	115.00	135.40	129.00	120.00
1.2 Russia–Czech	1.30	5.40	48.90	120.00	138.00	132.30	122.50
2 Russia–Czech	1.10	5.70	52.70	118.00	136.00	132.00	122.00
6 Russia	3.40	8.31	49.90	115.00	133.30	129.00	120.00
7 USA	1.21	5.92	55.26	120.90	136.00	129.50	120.10
7.1 USA	0.68	3.69	45.80	115.30	135.20	132.30	123.00
9 Germany	2.89	7.17	48.59	115.60	134.50	129.80	120.50
11 Russia	2.00	7.00	53.80	120.00	136.00	131.00	121.00
Average (0)	2.08	6.51	50.32	117.50	135.53	130.51	121.04
St. dev. (0)	1.04	1.48	3.03	2.38	1.27	1.40	1.16
ρ_a^{yx}							
1 Russia–Czech	1.60	2.00	4.30	12.50	25.80	52.70	70.90
1.1 Russia–Czech	1.80	2.20	4.70	12.90	26.10	52.90	71.00
1.2 Russia–Czech	0.30	0.60	2.40	9.80	22.60	49.80	68.60
2 Russia–Czech	0.30	0.50	2.30	10.00	22.60	49.60	68.40
6 Russia	1.92	2.30	4.70	12.90	26.10	52.90	71.00
7 USA	0.53	0.93	3.27	11.31	25.17	52.98	71.34
7.1 USA	0.44	0.68	2.39	9.08	21.70	49.01	68.30
9 Germany	1.74	2.10	4.02	11.22	24.11	51.33	69.81
10 Russia	1.40	2.20	3.20	13.00	25.00	56.00	81.00
11 Russia	2.70	3.30	6.50	15.70	27.10	55.00	72.00
Average (0)	1.27	1.68	3.78	11.84	24.63	52.22	71.24
St. dev. (0)	0.83	0.94	1.34	1.97	1.80	2.30	3.68
Average (1)	1.27	1.68	3.78	11.84	24.63	52.22	70.15
St. dev. (1)	0.83	0.94	1.34	1.97	1.80	2.30	1.41

Table D.12. Model 3D-1A ($T = 10.0$, $z = 0$)

Participant/ $x =$	0.5	0.75	1.0	1.25	1.5	2.0
ρ_a^{xy}						
1 Russia–Czech	3.10	4.30	10.40	28.70	49.80	75.40
1.1 Russia–Czech	3.70	4.90	11.30	29.70	50.50	75.60
1.2 Russia–Czech	1.50	2.40	8.00	26.00	47.40	73.70
2 Russia–Czech	1.40	2.30	8.40	27.00	48.00	74.00
6 Russia	3.76	4.98	11.40	29.70	50.40	75.60
7 USA	1.47	2.73	9.78	27.89	49.87	75.91
7.1 USA	0.80	1.79	7.67	22.83	44.24	71.95
9 Germany	3.24	4.36	9.93	27.14	48.02	74.43
11 Russia	2.80	4.90	12.80	31.60	51.50	75.70
Average (0)	2.42	3.63	9.96	27.84	48.86	74.70
St. dev. (0)	1.13	1.30	1.72	2.54	2.20	1.31
Average (1)	2.42	3.63	9.96	27.84	48.86	75.04
St. dev. (1)	1.13	1.30	1.72	2.54	2.20	0.86

Table D.12 (continued)

Participant/ $x =$	0.5	0.75	1.0	1.25	1.5	2.0
ρ_a^{yz} ρ_a^{yx}						
1 Russia–Czech	2.90	11.80	81.50	190.20	194.40	156.60
1.1 Russia–Czech	3.30	13.10	83.80	184.80	191.00	156.10
1.2 Russia–Czech	1.30	10.50	90.10	202.30	204.30	163.40
2 Russia–Czech	1.00	11.10	101.00	196.00	197.00	163.00
6 Russia	3.40	13.20	84.00	184.80	191.00	156.10
7 USA	1.54	11.55	102.80	198.80	195.50	156.20
7.1 USA	0.78	7.51	89.74	192.20	197.70	161.50
9 Germany	2.96	11.48	83.85	186.20	192.80	157.50
10 Russia	2.60	9.00	81.00	210.00	210.00	150.00
11 Russia	4.60	16.80	98.60	188.00	188.00	154.00
Average (0)	2.44	11.60	89.64	193.33	196.17	157.44
St. dev. (0)	1.24	2.51	8.32	8.37	6.63	4.16
Average (1)	2.44	11.03	89.64	193.33	194.63	157.44
St. dev. (1)	1.24	1.83	8.32	8.37	4.78	4.16

Table D.13. Model 3D-1B (EXN, $T = 0.1$, $z = 0$)

Participant/ $x =$	0.0	0.25	0.5	0.75	1.0	1.5	2.0	4.0
Re E_x								
1 Russia–Czech	0.410	0.460	0.670	0.930	0.990	1.000	1.000	1.000
6 Russia	0.400	0.460	0.690	0.940	1.000	1.000	1.000	1.000
7 USA	0.315	0.389	0.708	0.965	1.010	1.007	1.002	1.001
2D	0.290	0.360	0.690	0.920	0.970	0.990	0.990	1.000
Average (0)	0.354	0.417	0.690	0.939	0.993	0.999	0.998	1.000
St. dev. (0)	0.060	0.051	0.016	0.019	0.017	0.007	0.005	0.000
Im E_x								
1 Russia–Czech	−0.110	−0.130	−0.090	−0.010	0.020	0.020	0.010	0.000
6 Russia	−0.130	−0.130	−0.090	−0.020	0.010	0.020	0.010	0.000
7 USA	−0.169	−0.164	−0.085	−0.022	−0.001	0.009	0.009	0.004
2D	−0.160	−0.140	−0.060	−0.010	0.000	0.010	0.000	0.000
Average (0)	−0.142	−0.141	−0.081	−0.015	0.007	0.015	0.007	0.001
St. dev. (0)	0.027	0.016	0.014	0.006	0.010	0.006	0.005	0.002
Re H_y								
1 Russia–Czech	1.010	1.010	1.010	1.010	1.010	1.000	1.000	1.000
6 Russia	1.010	1.010	1.010	1.010	1.010	1.000	1.000	1.000
7 USA	0.999	0.998	0.996	0.997	0.998	1.001	1.003	1.002
Im H_y								
1 Russia–Czech	0.010	0.010	0.010	0.010	0.010	0.000	0.000	0.000
6 Russia	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.000
7 USA	−0.016	−0.016	−0.014	−0.011	−0.007	−0.001	0.003	0.004

Table D.14. Model 3D-1B (EYN, $T = 0.1$, $z = 0$)

Participant/ $x =$	0.0	0.25	0.5	0.75	1.0	1.5	2.0	4.0
Re E_y								
1 Russia–Czech	0.420	0.440	0.510	0.610	0.690	0.790	0.860	1.010
6 Russia	0.430	0.450	0.510	0.600	0.690	0.810	0.890	0.990
7 USA	0.405	0.426	0.491	0.583	0.668	0.793	0.873	0.986
2D	0.380	0.400	0.470	0.570	0.650	0.790	0.870	0.990
Average (0)	0.409	0.429	0.495	0.591	0.674	0.796	0.873	0.994
St. dev. (0)	0.022	0.022	0.019	0.018	0.019	0.010	0.012	0.011
Average (1)	0.409	0.429	0.495	0.591	0.674	0.796	0.873	0.994
St. dev. (1)	0.022	0.022	0.019	0.018	0.019	0.010	0.012	0.011

Table D.14 (continued)

Participant/ $x=$	0.0	0.25	0.5	0.75	1.0	1.5	2.0	4.0
Im E_y								
1 Russia–Czech	−0.160	−0.160	−0.170	−0.190	−0.200	−0.180	−0.160	−0.070
6 Russia	−0.170	−0.170	−0.180	−0.200	−0.200	−0.190	−0.170	−0.070
7 USA	−0.130	−0.133	−0.146	−0.168	−0.181	−0.177	−0.156	−0.064
2D	−0.100	−0.100	−0.120	−0.150	−0.160	−0.170	−0.150	−0.060
Average (0)	−0.140	−0.141	−0.154	−0.177	−0.185	−0.179	−0.159	−0.066
St. dev. (0)	0.032	0.031	0.027	0.022	0.019	0.008	0.008	0.005
Re H_x								
1 Russia–Czech	1.390	1.380	1.260	1.070	0.960	0.920	0.910	0.970
6 Russia	1.390	1.390	1.260	1.070	0.960	0.910	0.910	0.970
7 USA	1.427	1.434	1.307	1.075	0.958	0.900	0.905	0.970
2D	1.470	1.500	1.370	1.050	0.930	0.880	0.890	0.970
Average (0)	1.419	1.426	1.299	1.066	0.952	0.903	0.904	0.970
St. dev. (0)	0.038	0.055	0.052	0.011	0.015	0.017	0.009	0.000
Im H_x								
1 Russia–Czech	0.360	0.330	0.240	0.150	0.080	0.000	−0.040	−0.040
6 Russia	0.360	0.340	0.260	0.160	0.090	0.000	−0.040	−0.050
7 USA	0.349	0.318	0.230	0.147	0.082	0.004	−0.032	−0.045
2D	0.400	0.380	0.270	0.180	0.110	0.020	−0.020	−0.040
Average (0)	0.367	0.342	0.250	0.159	0.091	0.006	−0.033	−0.044
St. dev. (0)	0.022	0.027	0.018	0.015	0.014	0.010	0.009	0.005
Re H_z								
1 Russia–Czech		0.150	0.320	0.330	0.260	0.130	0.060	−0.010
6 Russia		0.140	0.320	0.330	0.250	0.130	0.060	−0.010
7 USA		0.150	0.358	0.372	0.284	0.145	0.069	−0.007
2D		0.140	0.380	0.390	0.290	0.140	0.060	−0.010
Average (0)		0.145	0.345	0.355	0.271	0.136	0.062	−0.009
St. dev. (0)		0.006	0.030	0.030	0.019	0.008	0.005	0.001
Im H_z								
1 Russia–Czech		0.130	0.210	0.230	0.220	0.180	0.140	0.040
6 Russia		0.120	0.210	0.240	0.230	0.190	0.140	0.040
7 USA		0.123	0.200	0.221	0.215	0.175	0.134	0.035
2D		0.120	0.190	0.220	0.220	0.180	0.140	0.040
Average (0)		0.123	0.202	0.228	0.221	0.181	0.138	0.039
St. dev. (0)		0.005	0.010	0.009	0.006	0.006	0.003	0.003

Table D.15. Model 3D-1B (EXN, $T=0.1$, $z=0$)

Participant/ $x=$	2.5	3.75	5.0	6.25	7.5	10.0
Re E_x						
1 Russia–Czech	0.410	0.430	0.590	0.930	0.980	1.000
6 Russia	0.400	0.420	0.560	0.930	0.980	1.000
7 USA	0.262	0.304	0.577	0.936	0.984	0.998
Im E_x						
1 Russia–Czech	−0.110	−0.130	−0.130	−0.040	−0.020	0.000
6 Russia	−0.130	−0.130	−0.130	−0.040	−0.020	−0.010
7 USA	−0.242	−0.223	−0.152	−0.043	−0.019	−0.005
Re H_y						
1 Russia–Czech	1.010	1.040	1.010	0.960	0.980	1.000
6 Russia	1.010	1.030	1.020	0.970	0.980	1.000
7 USA	1.011	1.041	1.008	0.965	0.985	0.997
Im H_y						
1 Russia–Czech	0.010	0.020	0.000	0.000	−0.010	0.000
6 Russia	0.010	0. 010	0.000	0.000	−0.010	0.000
7 USA	0.023	0.023	−0.013	−0.007	−0.009	−0.004
Re H_z						
1 Russia–Czech	0.000	0.020	0.150	0.030	0.010	0.000
6 Russia	0.000	0.010	0.130	0.030	0.010	0.000
7 USA	−0.002	0.033	0.144	0.030	0.005	0.000
Im H_z						
1 Russia–Czech	0.000	0.020	0.000	0.020	0.010	0.000
6 Russia	0.010	0.020	0.000	0.020	0.010	0.000
7 USA	−0.004	0.070	−0.001	0.018	0.010	0.002

Table D.16. Model 3D-1B (EYN, $T=0.1$, $z=0$)

Participant/ $x=$	2.5	3.75	5.0	6.25	7.5	10.0
Re E_y						
1 Russia–Czech	0.420	0.400	0.760	1.030	1.000	1.000
6 Russia	0.430	0.420	0.830	1.020	1.000	1.000
7 USA	0.399	0.419	0.838	1.034	1.002	1.001
Im E_y						
1 Russia–Czech	−0.160	−0.140	−0.050	0.090	0.030	0.000
6 Russia	−0.190	−0.240	−0.040	0.090	0.030	0.010
7 USA	−0.165	−0.090	0.054	0.097	0.026	0.005
Re H_x						
1 Russia–Czech	1.390	1.350	1.170	1.040	1.010	1.000
6 Russia	1.380	1.340	1.160	1.030	1.010	1.000
7 USA	1.423	1.368	1.169	1.036	1.009	1.001
Im H_x						
1 Russia–Czech	0.320	0.250	0.110	0.050	0.020	0.010
6 Russia	0.330	0.250	0.120	0.050	0.020	0.010
7 USA	0.302	0.218	0.102	0.044	0.020	0.006

Table D.17. Model 3D-1B (EXN, $T = 10.0$, $z = 0$)

Participant/ $x =$	0.0	0.25	0.5	0.75	1.0	1.5	2.0	4.0
Re E_x								
1 Russia–Czech	0.150	0.240	0.650	0.990	1.080	1.100	1.090	1.040
1.2 Russia–Czech	0.090	0.210	0.640	0.980	1.070	1.100	1.090	1.050
2 Russia–Czech	0.090	0.210	0.650	0.980	1.070	1.100	1.090	1.050
6 Russia	0.160	0.260	0.640	0.980	1.070	1.100	1.090	1.040
7 USA	0.108	0.220	0.663	1.004	1.085	1.101	1.080	1.043
2D	0.120	0.220	0.670	0.990	1.070	1.090	1.080	1.040
Average (0)	0.120	0.227	0.652	0.987	1.074	1.098	1.087	1.044
St. dev. (0)	0.030	0.020	0.012	0.010	0.007	0.004	0.005	0.005
Average (1)	0.120	0.227	0.652	0.987	1.074	1.100	1.087	1.044
St. dev. (1)	0.030	0.020	0.012	0.010	0.007	0.000	0.005	0.005
Im E_x								
1 Russia–Czech	−0.050	−0.040	−0.010	0.000	0.000	0.010	0.010	0.010
1.2 Russia–Czech	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2 Russia–Czech	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6 Russia	−0.050	−0.040	0.000	0.000	0.000	0.010	0.010	0.010
7 USA	−0.026	−0.022	−0.013	−0.007	−0.001	0.006	0.007	0.006
2D	−0.030	−0.030	−0.010	0.000	0.010	0.010	0.010	0.010
Average (0)	−0.026	−0.022	−0.005	−0.001	0.002	0.006	0.006	0.006
St. dev. (0)	0.022	0.018	0.006	0.003	0.004	0.005	0.005	0.005
Average (1)	−0.026	−0.022	−0.005	0.000	0.000	0.006	0.006	0.006
St. dev. (1)	0.022	0.018	0.006	0.000	0.000	0.005	0.005	0.005
Re H_y								
1 Russia–Czech	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
6 Russia	1.010	1.010	1.010	1.010	1.010	1.010	1.010	1.010
7 USA	1.005	1.005	1.005	1.005	1.004	1.004	1.004	1.003
Im H_y								
1 Russia–Czech	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6 Russia	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7 USA	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.002

Table D.18. Model 3D-1B (EYN, $T = 10.0$, $z = 0$)

Participant/ $x =$	0.0	0.25	0.5	0.75	1.0	1.5	2.0	4.0
Re E_y								
1 Russia–Czech	0.200	0.200	0.220	0.260	0.310	0.410	0.500	0.750
1.2 Russia–Czech	0.130	0.130	0.150	0.200	0.250	0.360	0.450	0.730
2 Russia–Czech	0.110	0.110	0.160	0.240	0.310	0.390	0.460	0.730
6 Russia	0.260	0.260	0.270	0.290	0.330	0.410	0.500	0.750
7 USA	0.190	0.207	0.241	0.280	0.334	0.435	0.522	0.761
2D	0.420	0.420	0.440	0.450	0.470	0.510	0.540	0.630
Average (0)	0.218	0.221	0.247	0.287	0.334	0.419	0.495	0.725
St. dev. (0)	0.112	0.112	0.105	0.086	0.073	0.051	0.035	0.048
Im E_y								
1 Russia–Czech	−0.070	−0.070	−0.070	−0.070	−0.070	−0.060	−0.050	−0.020
1.2 Russia–Czech	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2 Russia–Czech	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6 Russia	−0.050	−0.050	−0.050	−0.050	−0.050	−0.040	−0.060	−0.030
7 USA	−0.049	−0.048	−0.047	−0.048	−0.046	−0.039	−0.033	−0.011
2D	0.230	0.220	0.210	0.200	0.180	0.150	0.130	0.060
Average (0)	0.010	0.009	0.007	0.005	0.002	0.002	−0.002	0.000
St. dev. (0)	0.111	0.107	0.103	0.100	0.092	0.076	0.069	0.032
Re H_x								
1 Russia–Czech	1.300	1.280	1.220	1.160	1.110	1.050	1.020	0.980
6 Russia	1.370	1.350	1.270	1.180	1.120	1.050	1.010	0.980
7 USA	1.357	1.332	1.259	1.173	1.111	1.042	1.008	0.976
2D	3.210	3.090	2.690	2.240	1.930	1.550	1.340	1.040
Average (0)	1.809	1.763	1.610	1.438	1.318	1.173	1.095	0.994
St. dev. (0)	0.934	0.885	0.720	0.535	0.408	0.251	0.164	0.031

Table D.18 (continued)

Participant/ $x =$	0.0	0.25	0.5	0.75	1.0	1.5	2.0	4.0
Im H_x								
1 Russia–Czech	−0.230	−0.210	−0.160	−0.110	−0.070	−0.020	0.000	0.020
6 Russia	−0.310	−0.280	−0.210	−0.130	−0.070	−0.020	0.010	0.030
7 USA	−0.289	−0.267	−0.201	−0.123	−0.070	−0.016	0.007	0.023
2D	0.320	0.300	0.290	0.280	0.250	0.200	0.160	0.090
Average (0)	−0.127	−0.114	−0.070	−0.021	0.010	0.036	0.044	0.041
St. dev. (0)	0.300	0.278	0.241	0.201	0.160	0.109	0.077	0.033
Re H_z								
1 Russia–Czech		0.090	0.160	0.180	0.190	0.180	0.150	0.080
6 Russia		0.110	0.200	0.220	0.220	0.180	0.150	0.080
7 USA		0.110	0.192	0.216	0.210	0.177	0.146	0.071
2D		0.520	0.900	1.050	1.050	0.950	0.830	0.500
Average (0)		0.208	0.363	0.416	0.418	0.372	0.319	0.183
St. dev. (0)		0.209	0.358	0.423	0.422	0.386	0.341	0.212
Im H_z								
1 Russia–Czech		−0.070	−0.130	−0.150	−0.140	−0.130	−0.100	−0.050
6 Russia		−0.100	−0.180	−0.190	−0.180	−0.140	−0.110	−0.050
7 USA		−0.095	−0.167	−0.183	−0.171	−0.136	−0.108	−0.047
2D		0.040	0.060	0.090	0.110	0.130	0.140	0.110
Average (0)		−0.056	−0.104	−0.108	−0.095	−0.069	−0.044	−0.009
St. dev. (0)		0.065	0.112	0.133	0.138	0.133	0.123	0.080

Table D.19. Model 3D-1B (EXN, $T = 10.0$, $z = 0$)

Participant/ $x =$	2.5	3.75	5.0	6.25	7.5	10.0
Re E_x						
1 Russia–Czech	0.150	0.150	0.320	0.890	0.970	0.990
1.2 Russia–Czech	0.100	0.100	0.290	0.890	0.960	0.990
2 Russia–Czech	0.090	0.090	0.350	0.890	0.970	0.990
6 Russia	0.160	0.170	0.330	0.900	0.970	0.990
7 USA	0.085	0.105	0.336	0.899	0.968	0.991
Average (0)	0.117	0.123	0.325	0.894	0.968	0.990
St. dev. (0)	0.035	0.035	0.022	0.005	0.004	0.000
Im E_x						
1 Russia–Czech	−0.050	−0.050	−0.040	−0.010	0.000	0.000
1.2 Russia–Czech	0.000	0.000	0.000	0.000	0.000	0.000
2 Russia–Czech	0.000	0.000	0.000	0.000	0.000	0.000
6 Russia	−0.050	−0.050	−0.040	−0.010	0.000	0.000
7 USA	−0.009	−0.014	−0.019	−0.004	−0.006	0.000
Average (0)	−0.022	−0.023	−0.020	−0.005	−0.001	0.000
St. dev. (0)	0.026	0.025	0.020	0.005	0.003	0.000
Re H_y						
1 Russia–Czech	1.010	1.010	1.000	1.000	1.000	1.000
6 Russia	1.010	1.030	1.020	0.970	0.980	1.000
7 USA	1.006	1.007	1.002	0.996	0.997	0.999
Im H_y						
1 Russia–Czech	0.000	−0.010	0.000	0.000	0.000	0.000
6 Russia	0.000	−0.010	0.000	0.000	0.000	0.000
7 USA	−0.005	−0.006	−0.001	0.004	0.003	0.001
Re H_z						
1 Russia–Czech	0.000	0.010	0.020	0.010	0.000	0.000
6 Russia	0.000	0.010	0.020	0.010	0.010	0.000
7 USA	0.005	0.009	0.019	0.008	0.005	0.002
Im H_z						
1 Russia–Czech	0.000	−0.010	−0.020	−0.010	0.000	0.000
6 Russia	0.000	−0.010	−0.020	−0.010	0.000	0.000
7 USA	−0.004	−0.009	−0.019	−0.007	−0.004	−0.001

Table D.20. Model 3D-1B (EYN, $T = 10.0$, $z = 0$)

Participant/ $x =$	2.5	3.75	5.0	6.75	7.5	10.0
Re E_y						
1 Russia–Czech	0.190	0.190	1.560	1.900	1.420	1.130
1.2 Russia–Czech	0.130	0.130	1.690	2.010	1.480	1.160
2 Russia–Czech	0.100	0.050	1.890	1.970	1.480	1.160
6 Russia	0.180	0.200	1.680	1.890	1.410	1.140
7 USA	0.066	0.210	1.686	1.885	1.403	1.129
Average (0)	0.133	0.156	1.701	1.931	1.439	1.144
St. dev. (0)	0.053	0.067	0.119	0.056	0.038	0.015
Im E_y						
1 Russia–Czech	−0.070	−0.060	0.200	0.180	0.100	0.040
1.2 Russia–Czech	0.000	0.000	0.000	0.000	0.000	0.000
2 Russia–Czech	0.000	0.000	0.000	0.000	0.000	0.000
6 Russia	−0.060	−0.050	0.200	0.170	0.090	0.040
7 USA	−0.035	−0.071	0.171	0.155	0.087	0.036
Average (0)	−0.033	−0.036	0.114	0.101	0.055	0.023
St. dev. (0)	0.033	0.034	0.105	0.093	0.051	0.021
Re H_x						
1 Russia–Czech	1.260	1.210	1.120	1.060	1.040	1.020
6 Russia	1.320	1.250	1.140	1.070	1.040	1.020
7 USA	1.304	1.235	1.130	1.064	1.039	1.020
Im H_x						
1 Russia–Czech	−0.190	−0.150	−0.080	−0.030	−0.020	−0.010
6 Russia	−0.270	−0.200	−0.100	−0.050	−0.030	−0.010
7 USA	−0.245	−0.187	−0.097	−0.043	−0.024	−0.011

Table D.21. Model 3D-1B ($T = 0.1$, $z = 0$)

Participant/ $x =$	0.0	0.25	0.5	0.75	1.0	1.5	2.0
ρ_a^{xy}							
1 Russia–Czech	18.00	22.90	45.40	84.80	96.60	98.10	98.10
6 Russia	17.50	23.00	47.40	87.60	98.90	99.60	98.90
7 USA	12.79	17.91	51.25	93.86	102.30	101.10	99.80
2D Russia	10.80	15.00	47.90	84.30	94.30	97.90	98.60
Average (0)	14.77	19.70	47.99	87.64	98.02	99.17	98.85
St. dev. (0)	3.54	3.93	2.43	4.39	3.41	1.49	0.71
ρ_a^{yx}							
1 Russia–Czech	10.00	11.10	17.70	34 50	54.50	80.40	90.30
6 Russia	10.36	11.30	17.80	34.60	54.80	83.90	97.60
7 USA	8.37	9.22	14.90	31.29	51.84	81.64	95.87
2D Russia	6.80	7.20	12.30	30.20	52.10	83.00	96.80
Average (0)	8.88	9.70	15.68	32.65	53.31	82.24	95.14
St. dev. (0)	1.64	1.91	2.62	2.24	1.56	1.54	3.30
Average (1)	8.88	9.70	15.68	32.65	53.31	82.24	95.14
St. dev. (1)	1.64	1.91	2.62	2.24	1.56	1.54	3.30

Table D.22. Model 3D-1B ($T = 0.1$, $z = 0$)

Participant/ $x =$	2.5	3.75	5.0	6.25	7.5	10.0
ρ_a^{xy}						
1 Russia–Czech	17.90	18.20	35.30	93.50	99.50	100.00
6 Russia	17.10	17.90	31.90	93.80	99.70	100.20
7 USA	12.44	13.11	28.62	94.29	99.84	100.20
ρ_a^{yx}						
1 Russia–Czech	9.90	9.90	42.20	99.10	98.50	99.90
6 Russia	10.80	12.70	50.70	98.90	98.70	99.90
7 USA	8.81	9.59	51.14	100.30	98.60	99.91

Table D.23. Model 3D-1B (T = 10.0, z = 0)

Participant/ x =	0.0	0.25	0.5	0.75	1.0	1.5	2.0
ρ_a^{xy}							
1 Russia–Czech	2.40	6.10	41.70	96.50	114.90	119.30	117.70
1.2 Russia–Czech	0.82	4.20	41.10	96.40	115.20	121.20	119.60
2 Russia–Czech	0.80	4.40	41.70	95.70	115.00	120.00	119.00
6 Russia	2.88	6.77	40.90	95.30	114.30	119.50	117.50
7 USA	1.22	4.83	43.52	99.95	116.60	120.20	117.90
2D Russia	1.63	5.00	45.20	98.90	115.20	119.60	117.40
Average (0)	1.63	5.22	42.35	97.12	115.20	119.97	118.18
St. dev. (0)	0.86	1.01	1.67	1.87	0.76	0.69	0.90
ρ_a^{yx}							
1 Russia–Czech	2.50	2.70	3.50	5.30	8.10	15.30	24.10
1.2 Russia–Czech	1.60	1.70	2.30	3.90	6.30	12.70	20.60
2 Russia–Czech	1.20	1.30	2.60	5.80	9.20	14.80	21.60
6 Russia	3.50	3.80	4.50	6.00	8.70	15.90	24.50
7 USA	2.00	2.45	3.70	5.80	9.19	17.56	26.96
2D Russia	2.20	2.40	3.20	4.80	6.70	11.50	16.90
Average (0)	2.17	2.39	3.30	5.27	8.03	14.63	22.44
St. dev. (0)	0.80	0.87	0.79	0.80	1.26	2.20	3.53

Table D.24. Model 3D-1B (T = 10.0, z = 0)

Participant/ x =	2.5	3.75	5.0	6.25	7.5	10.0
ρ_a^{xy}						
1 Russia–Czech	2.50	2.60	10.60	80.00	93.90	98.50
1.2 Russia–Czech	0.90	1.00	8.10	78.00	93.00	98.20
2 Russia–Czech	0.80	0.90	12.20	79.90	93.20	98.20
6 Russia	2.78	3.06	10.95	80.70	93.90	98.50
7 USA	0.73	1.10	11.26	81.42	94.11	98.59
Average (0)	1.54	1.73	10.62	80.00	93.62	98.40
St. dev. (0)	1.01	1.02	1.53	1.28	0.49	0.18
ρ_a^{yx}						
1 Russia–Czech	2.60	2.70	198.00	329.00	188.00	123.70
1.2 Russia–Czech	1.60	1.70	285.00	406.00	218.00	134.50
2 Russia–Czech	1.00	1.70	356.00	388.00	217.00	133.00
6 Russia	1.90	2.60	218.50	315.30	185.70	123.80
7 USA	0.32	3.15	223.30	315.20	182.80	122.60
Average (0)	1.48	2.37	256.16	350.70	198.30	127.52
St. dev. (0)	0.87	0.65	64.58	43.11	17.63	5.73

Appendix E

Diagrams for the 2D results are presented in Figs. 26–35.

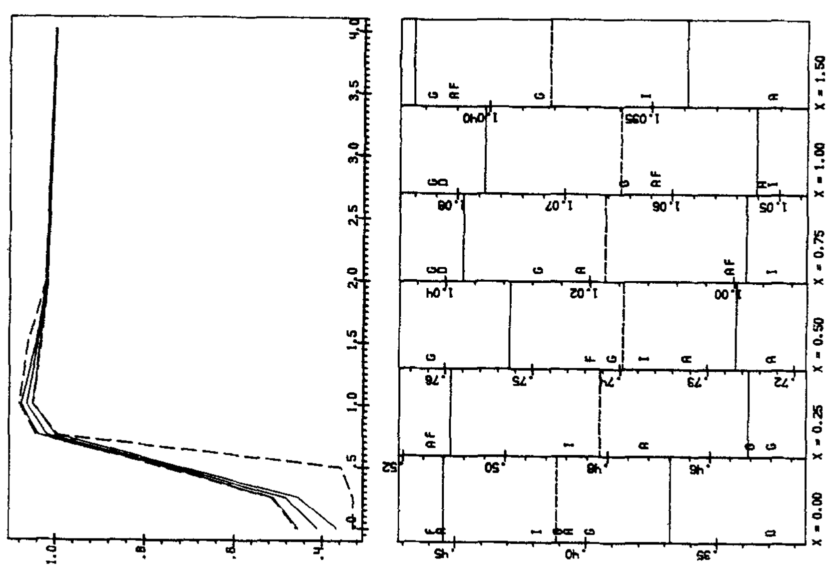


Fig. 26. Model 3D-1a. Electric field component $E_x(x)$, EX, $z=0$, $T=0.1$.

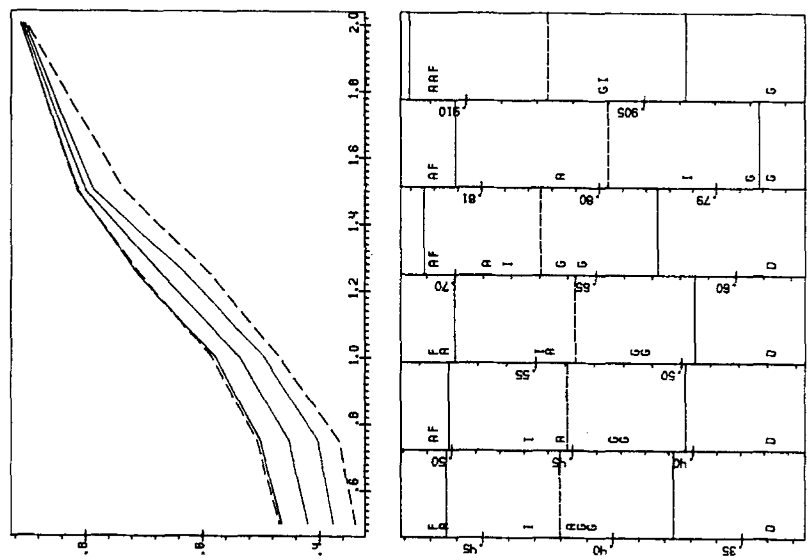


Fig. 27. Model 3D-1a. Electric field component $E_x(y)$, EX, $z=0$, $T=0.1$.

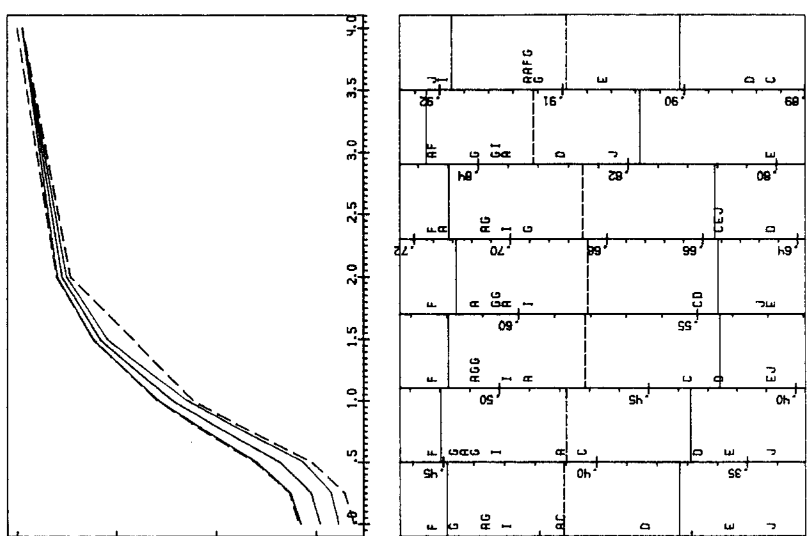


Fig. 28. Model 3D-1a. Electric field component $E_y(x)$, EY, $z = 0$, $T = 0.1$.

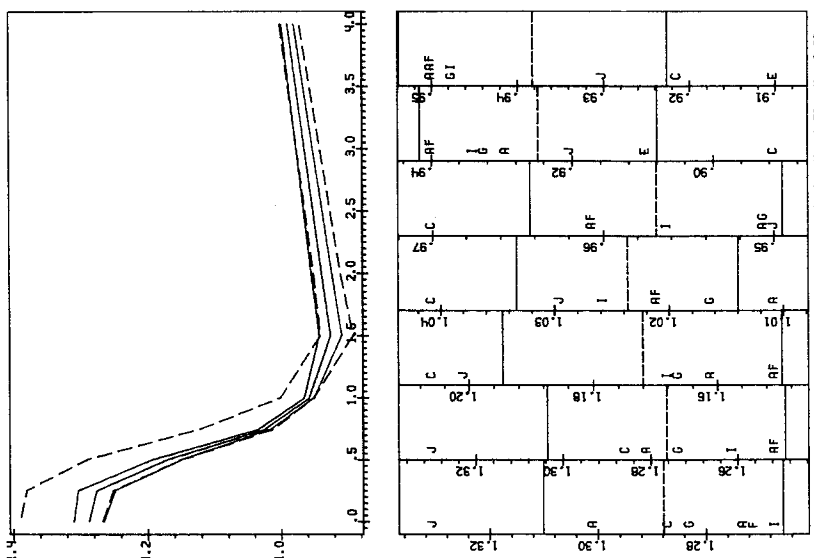


Fig. 29. Model 3D-1a. Magnetic field component $H_x(x)$, EY, $z = 0$, $T = 0.1$.

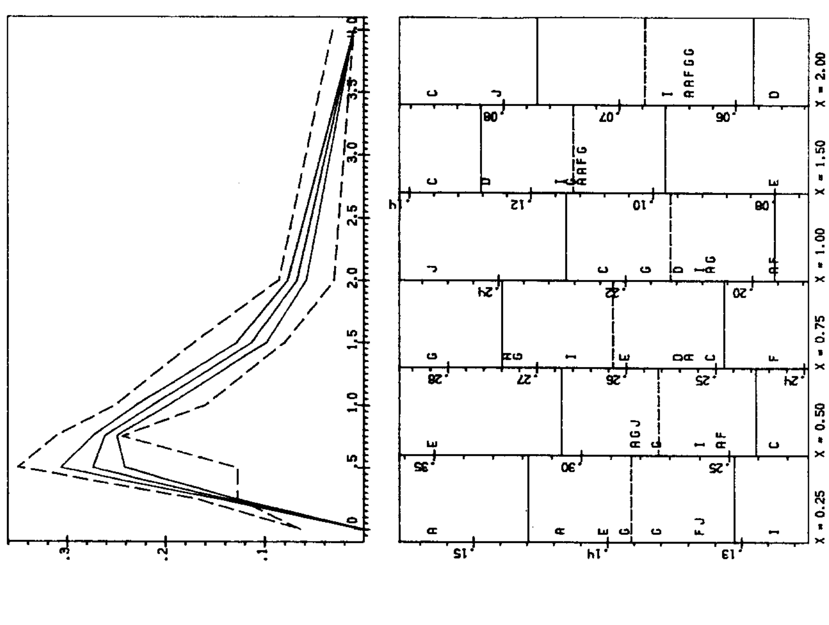


Fig. 30. Model 3D-1a. Magnetic field component $H_z(x)$, EY, $z = 0$, $T = 0.1$.

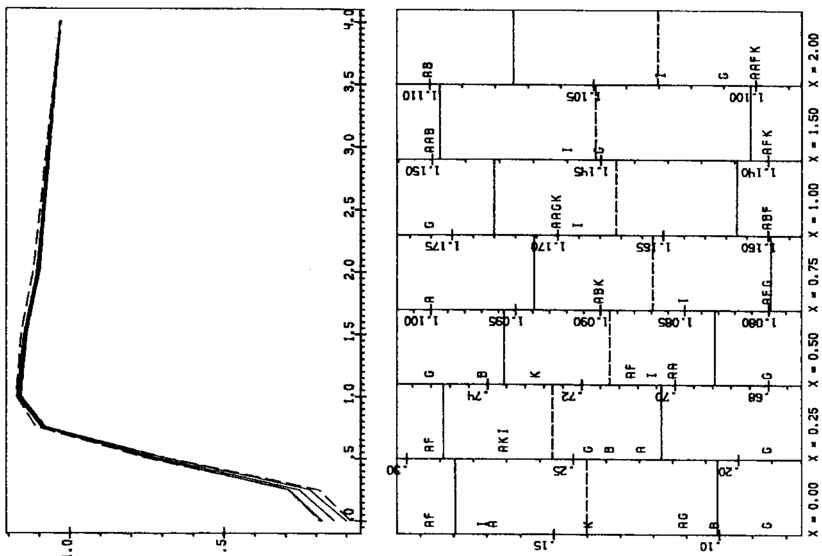


Fig. 31. Model 3D-1a. Electric field component $E_x(x)$, EX, $z = 0$, $T = 10$.

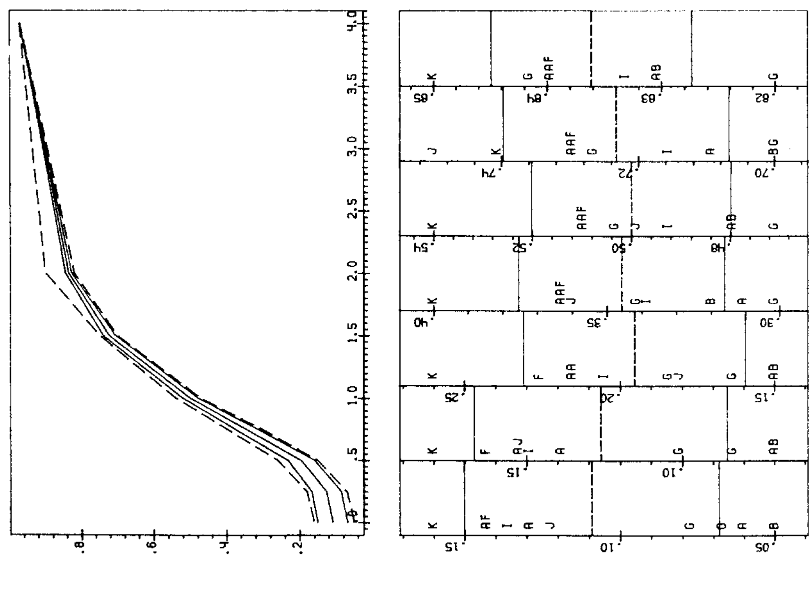


Fig. 33. Model 3D-1a. Electric field component $E_y(x)$, EY, $z = 0$, $T = 10$.

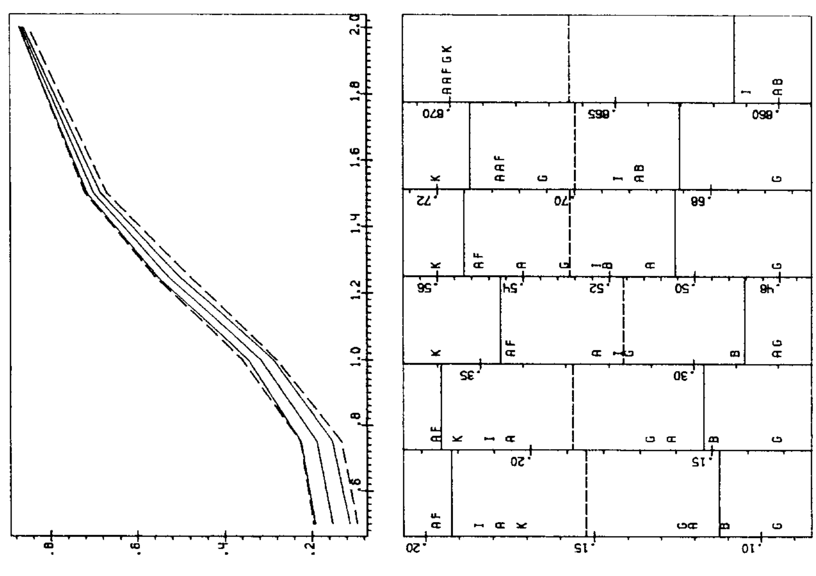


Fig. 32. Model 3D-1a. Electric field component $E_x(y)$, EY, $z = 0$, $T = 10$.

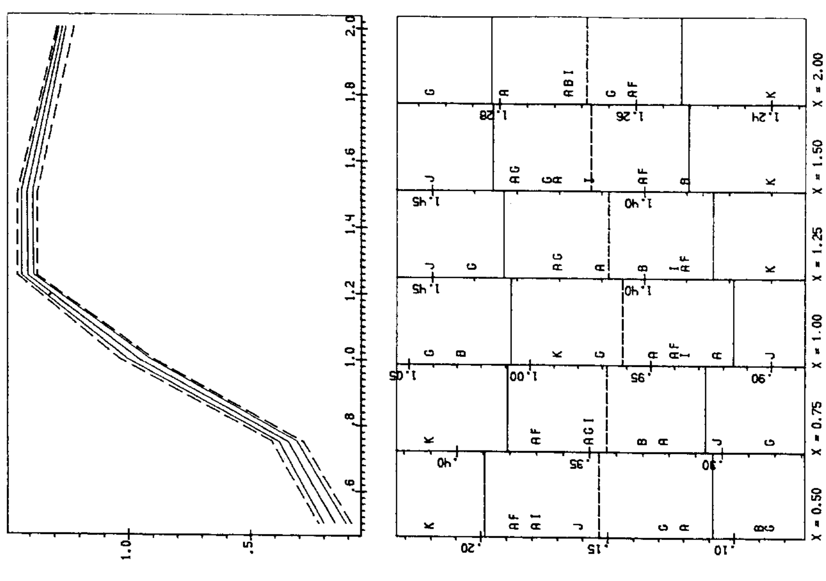


Fig. 34. Model 3D-1a. Electric field component $E_y(y)$, EX, $z = 0$, $T = 10$.

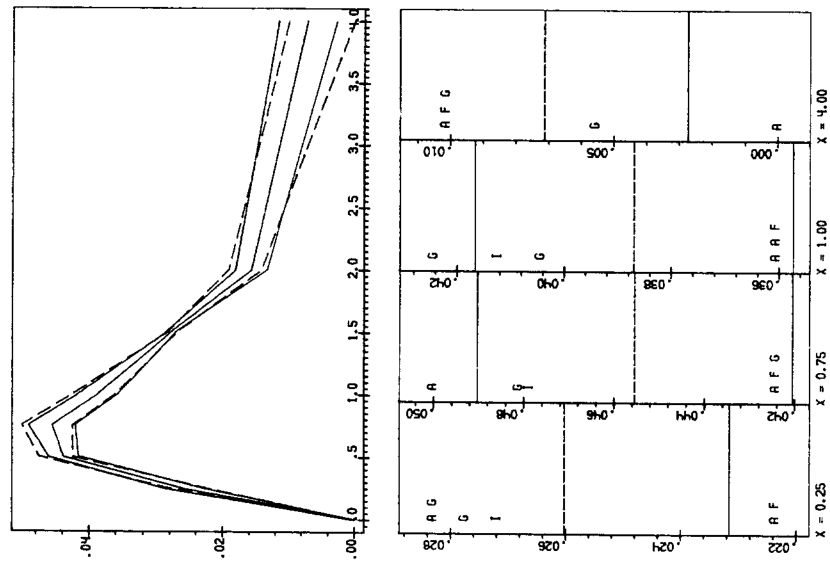


Fig. 35. Model 3D-1a. Magnetic field component $H_z(x)$, EY, $z = 0$, $T = 10$.

Appendix F. Modelling results for model 3D-2

The following modelling results submitted later than the others by Dr. Z. Xiong were computed with an integral equation code (Xiong, 1992; Xiong and Tripp, 1993a,b, 1995). The algorithm uses the method of system iteration and spatial symmetry reductions and thus allows a large number of cells to be used in the computation. The method of system iteration reduces the computation time for solving the matrix equation. With the aid of the spatial symmetry reductions, elements of the scattering impedence matrix can be re-computed in each iteration so that huge memory requirements are avoided.

Model 3D-2, with $l_y^1 = 20$ km, was computed using 16800 prismatic cells of dimensions $1000 \times 2666.667 \times 357.143$ m^3, or $20 \times 15 \times 28$ cells for each of the two blocks. The model with $l_y^1 = 100$ km was computed using 18000 prismatic cells of dimensions $2000 \times 2666.667 \times 500$ m^3, or $10 \times 15 \times 20$ cells for the conductive block with $l_y^2 = 20$ km and $10 \times 75 \times 20$ cells for the resistive block with $l_y^1 = 100$ km.

Table F.1a. Model 3D-2, $l_y^1 = 20$ km, $T = 100$ s, E_{xn}, $y = 0$

x (km)		−70	−40	−25	−20.5	−20(+)	−15	−10	−1	0(−)
E_x	Re	1.000	1.013	1.068	1.191	0.151	0.246	0.259	0.227	0.063
	Im	0.002	0.015	0.034	−0.012	−0.018	−0.047	−0.045	−0.057	−0.027
H_y	Re	1.001	1.010	1.022	1.022	1.021	1.007	0.978	0.928	0.872
	Im	0.001	0.001	0.001	0.002	0.003	0.007	0.011	0.021	0.045

Table F.1b. Model 3D-2, $l_y^1 = 20$ km, $T = 100$ s, E_{xn}, $y = 0$

x (km)		0(+)	5	10	15	20(−)	20.5	25	40	70
E_x	Re	2.774	2.392	2.192	2.299	2.655	0.344	0.599	0.882	0.986
	Im	−0.235	−0.171	−0.262	−0.289	−0.417	−0.078	−0.124	−0.082	−0.024
H_y	Re	0.872	0.846	0.837	0.837	0.848	0.849	0.872	0.944	0.989
	Im	0.045	0.057	0.061	0.060	0.053	0.052	0.037	0.006	−0.002

Table F.2a. Model 3D-2, $l_y^1 = 20$ km, $T = 100$ s, E_{xn}, $y = 30$ km

x (km)		−70	−40	−25	−20.5	−20(+)	−15	−10	−5	0(−)
E_x	Re	1.000	0.992	0.940	0.894	0.888	0.846	0.866	0.971	1.135
	Im	−0.001	−0.008	−0.033	−0.050	−0.053	−0.067	−0.061	−0.030	0.010
E_y	Re	−0.001	−0.013	−0.037	−0.022	−0.018	0.056	0.168	0.264	0.267
	Im	−0.002	−0.009	−0.009	0.004	0.006	0.040	0.084	0.114	0.107
H_x	Re	0.001	0.008	0.012	0.001	−0.002	−0.036	−0.082	−0.118	−0.118
	Im	0.001	−0.001	−0.008	−0.007	−0.007	0.002	0.017	0.032	0.034
H_y	Re	0.999	0.994	0.970	0.953	0.951	0.938	0.948	0.986	1.041
	Im	0.000	0.000	0.007	0.013	0.014	0.020	0.017	0.001	−0.024

Table F.2b. Model 3D-2, $l_y^1 = 20$ km, $T = 100$ s, E_{xn}, $y = 30$ km

x (km)		0(+)	5	10	15	20(−)	20.5	25	40	70
E_x	Re	1.135	1.246	1.274	1.256	1.199	1.191	1.118	1.006	0.996
	Im	0.010	0.030	0.030	0.023	0.014	0.013	0.003	−0.017	−0.013
E_y	Re	0.267	0.165	0.052	−0.044	−0.119	−0.124	−0.144	−0.073	−0.010
	Im	0.107	0.070	0.031	0.001	−0.023	−0.024	−0.037	−0.036	−0.011
H_x	Re	−0.118	−0.080	−0.034	0.008	0.040	0.042	0.055	0.040	0.010
	Im	0.034	0.022	0.005	−0.010	−0.021	−0.022	−0.024	−0.009	0.001
H_y	Re	1.041	1.080	1.093	1.089	1.073	1.070	1.049	1.006	0.997
	Im	−0.024	−0.045	−0.053	−0.051	−0.042	−0.041	−0.028	−0.006	−0.001

Table F.3a. Model 3D-2, $l_{y1} = 20$ km, $T = 100$ s, E_{yn}, $y = 0$

x (km)		−70	−40	−25	−20.5	−20(+)	−15	−10	−5	0(−)
E_y	Re	0.986	0.880	0.629	0.495	0.468	0.389	0.355	0.391	0.510
	Im	−0.023	−0.090	−0.134	−0.097	−0.041	0.021	0.031	0.008	−0.120
H_x	Re	0.982	0.921	0.913	1.105	1.190	1.424	1.417	1.418	1.127
	Im	−0.011	−0.004	0.050	0.027	−0.019	0.023	0.066	−0.038	−0.056
H_z	Re	−0.004	−0.072	−0.316	−0.528	−0.560	−0.175	0.030	0.254	0.651
	Im	−0.011	−0.043	−0.031	0.062	0.087	−0.077	0.008	0.061	−0.233

Table F.3b. Model 3D-2, l_{y1} = 20 km, T = 100 s, E_{yn}, y = 0

x (km)		0(+)	5	10	15	20(−)	20.5	25	40	70
E_y	Re	0.510	0.726	0.925	0.996	1.036	1.050	1.056	1.033	1.006
	Im	−0.120	−0.190	−0.210	−0.196	−0.142	−0.139	−0.094	−0.028	−0.004
H_x	Re	1.127	0.801	0.752	0.760	0.851	0.873	0.950	0.998	1.002
	Im	−0.056	0.110	0.112	0.097	0.029	0.011	−0.032	−0.026	−0.007
H_z	Re	0.651	0.365	0.183	0.039	−0.101	−0.098	−0.071	−0.025	−0.004
	Im	−0.233	−0.065	0.008	0.066	0.139	0.131	0.078	0.017	0.001

Table F.4a. Model 3D-2, l_{y1} = 20 km, T = 100 s, E_{yn}, y = 30 km

x (km)		−70	−40	−25	−20.5	−20(+)	−15	−10	−5	0(−)
E_x	Re	−0.002	−0.036	−0.091	−0.079	−0.075	−0.020	0.053	0.146	0.233
	Im	−0.015	−0.068	−0.106	−0.087	−0.084	−0.034	0.025	0.086	0.136
E_y	Re	0.992	0.975	1.034	1.086	1.092	1.137	1.154	1.139	1.050
	Im	−0.010	0.001	0.101	0.150	0.156	0.191	0.198	0.176	0.115
H_x	Re	0.991	0.989	1.039	1.070	1.073	1.098	1.105	1.090	1.044
	Im	−0.006	0.003	0.019	0.021	0.021	0.021	0.019	0.017	0.020
H_y	Re	0.008	0.039	0.063	0.052	0.050	0.017	−0.026	−0.073	−0.109
	Im	0.006	0.015	0.011	0.008	0.008	0.007	0.007	0.010	0.014
H_z	Re	−0.002	−0.021	−0.032	−0.023	−0.022	0.001	0.031	0.063	0.087
	Im	−0.007	−0.018	−0.014	−0.009	−0.008	−0.001	0.005	0.009	0.012

Table F.4b. Model 3D-2, l_{y1} = 20 km, T = 100 s, E_{yn}, y = 30 km

x (km)		0(+)	5	10	15	20(−)	20.5	25	40	70
E_x	Re	0.233	0.227	0.136	0.029	−0.042	−0.046	−0.057	−0.026	−0.004
	Im	0.136	0.144	0.116	0.080	0.049	0.046	0.027	0.004	−0.001
E_y	Re	1.050	0.911	0.827	0.823	0.877	0.884	0.938	0.998	1.002
	Im	0.115	0.034	−0.025	−0.052	−0.057	−0.057	−0.050	−0.022	−0.004
H_x	Re	1.044	0.983	0.941	0.929	0.940	0.942	0.959	0.990	1.000
	Im	0.020	0.026	0.028	0.020	0.007	0.005	−0.005	−0.012	−0.005
H_y	Re	−0.109	−0.111	−0.081	−0.042	−0.012	−0.010	0.002	0.006	0.001
	Im	0.014	0.009	−0.004	−0.018	−0.026	−0.026	−0.025	−0.011	−0.002
H_z	Re	0.087	0.087	0.063	0.032	0.007	0.006	−0.005	−0.009	−0.003
	Im	0.012	0.016	0.023	0.029	0.029	0.029	0.025	0.009	0.001

Table F.5a. Model 3D-2, l_{y1} = 20 km, T = 100 s, y = 0

x (km)	−70	−40	−25	−20.5	−20(+)	−15	−10	−5	0(−)
ρ_a^{yx}	15.602	14.263	7.646	3.222	2.410	1.158	0.974	1.177	3.300
ρ_a^{xy}	15.446	15.547	16.891	20.991	0.341	0.958	1.120	0.984	0.094

Table F.5b. Model 3D-2, l_{y1} = 20 km, T = 100 s, y = 0

x (km)	0(+)	5	10	15	20(−)	20.5	25	40	70
ρ_a^{yx}	3.300	13.307	24.066	27.102	23.332	22.732	19.253	16.568	15.952
ρ_a^{xy}	157.04	123.51	107.00	117.78	154.79	2.663	7.589	13.612	15.378

Table F.6a. Model 3D-2, l_{y1} = 20 km, T = 100 s, y = 30 km

x (km)	−70	−40	−25	−20.5	−20(+)	−15	−10	−5	0(−)
ρ_a^{yx}	15.495	15.041	15.391	16.206	16.303	17.031	17.253	16.894	15.253
ρ_a^{xy}	15.465	15.391	14.515	13.615	13.503	12.654	12.891	14.663	17.768

Table F.6b. Model 3D-2, l_{y1} = 20 km, T = 100 s, y = 30 km

x (km)	0(+)	5	10	15	20(−)	20.5	25	40	70
ρ_a^{yx}	15.253	13.000	11.910	12.206	13.485	13.632	14.754	15.672	15.526
ρ_a^{xy}	17.768	20.158	20.866	20.532	19.217	19.039	17.446	15.417	15.439

Table F.7a. Model 3D-2, l_{y1} = 20 km, T = 1000 s, E_{xn}, y = 0

x (km)		−70	−40	−25	−20.5	−20(+)	−15	−10	−5	0(−)
E_x	Re	1.001	1.023	1.084	1.183	0.142	0.209	0.215	0.181	0.048
	Im	0.000	0.002	0.002	−0.009	−0.005	−0.015	−0.017	−0.017	−0.007
H_y	Re	1.000	1.001	1.002	1.002	1.002	1.001	0.998	0.994	0.990
	Im	0.000	−0.003	−0.005	−0.006	−0.005	−0.003	0.005	0.017	0.031

Table F.7b. Model 3D-2, l_{y1} = 20 km, T = 1000 s, E_{xn}, y = 0

x (km)		0(+)	5	10	15	20(−)	20.5	25	40	70
E_x	Re	2.615	2.337	2.090	2.160	2.378	0.308	0.549	0.830	0.951
	Im	−0.040	−0.008	−0.018	−0.025	−0.059	−0.011	−0.019	−0.014	−0.006
H_y	Re	0.990	0.988	0.987	0.987	0.988	0.988	0.990	0.995	0.999
	Im	0.031	0.037	0.039	0.039	0.037	0.036	0.031	0.014	−0.003

Table F.8a. Model 3D-2, l_{y1} = 20 km, T = 1000 s, E_{xn}, y = 30 km

x (km)		−70	−40	−25	−20.5	−20(+)	−15	−10	−5	0(−)
E_x	Re	0.997	0.984	0.918	0.863	0.856	0.807	0.828	0.943	1.117
	Im	−0.001	−0.002	−0.006	−0.009	−0.010	−0.012	−0.011	−0.007	0.000
E_y	Re	−0.003	−0.018	−0.037	−0.016	−0.011	0.079	0.208	0.314	0.313
	Im	0.000	−0.001	−0.001	0.001	0.001	0.006	0.013	0.017	0.016
H_x	Re	0.000	0.000	0.000	−0.001	−0.001	−0.003	−0.007	−0.009	−0.009
	Im	0.000	−0.002	−0.002	0.000	0.001	0.009	0.020	0.028	0.028
H_y	Re	1.000	0.999	0.998	0.996	0.996	0.995	0.996	0.999	1.003
	Im	0.000	0.002	0.007	0.011	0.012	0.015	0.013	0.004	−0.010

Table F.8b. Model 3D-2, l_{y1} = 20 km, T = 1000 s, E_{xn}, y = 30 km

x (km)		0(+)	5	10	15	20(−)	20.5	25	40	70
E_x	Re	1.117	1.233	1.257	1.236	1.177	1.169	1.095	0.983	0.979
	Im	0.000	0.003	0.003	0.002	0.000	0.000	−0.002	−0.005	−0.004
E_y	Re	0.313	0.198	0.071	−0.035	−0.117	−0.123	−0.148	−0.087	−0.020
	Im	0.016	0.011	0.005	0.001	−0.003	−0.003	−0.005	−0.005	−0.002
H_x	Re.	−0.009	−0.006	−0.003	0.000	0.003	0.003	0.004	0.003	0.001
	Im	0.028	0.019	0.008	−0.002	−0.009	−0.010	−0.013	−0.009	0.003
H_y	Re	1.003	1.005	1.006	1.006	1.005	1.005	1.003	1.000	0.999
	Im	−0.010	−0.019	−0.022	−0.021	−0.017	−0.016	−0.011	−0.001	0.001

Table F.9a. Model 3D-2, l_{y1} = 20 km, T = 1000 s, E_{yn}, y = 0

x (km)		−70	−40	−25	−20.5	−20(+)	−15	−10	−5	0(−)
E_y	Re	0.967	0.830	0.552	0.424	0.437	0.420	0.411	0.411	0.442
	Im	−0.003	−0.012	−0.024	−0.023	−0.010	0.002	0.007	0.000	−0.020
H_x	Re	0.997	0.992	0.995	1.014	1.021	1.050	1.054	1.046	1.014
	Im	0.007	0.021	0.020	−0.032	−0.054	−0.127	−0.132	−0.121	−0.038
H_z	Re	−0.003	−0.013	−0.038	−0.056	−0.058	−0.027	0.003	0.033	0.061
	Im	0.005	0.027	0.094	0.152	0.161	0.061	−0.008	−0.078	−0.180

Table F.9b. Model 3D-2, l_{y1} = 20 km, T = 1000 s, E_{yn}, y = 0

x (km)		0(+)	5	10	15	20(−)	20.5	25	40	70
E_y	Re	0.442	0.679	0.895	0.969	1.012	1.018	1.020	1.007	0.999
	Im	−0.020	−0.022	−0.018	−0.014	−0.008	−0.010	−0.007	−0.002	0.000
H_x	Re	1.014	0.986	0.981	0.981	0.987	0.989	0.995	0.998	0.999
	Im	−0.038	0.049	0.062	0.061	0.037	0.031	0.012	0.003	0.001
H_z	Re	0.061	0.038	0.022	0.009	−0.002	−0.002	−0.001	0.001	0.001
	Im	−0.180	−0.102	−0.054	−0.016	0.021	0.020	0.011	0.002	0.000

Table F.10a. Model 3D-2, l_{y1} = 20 km, T = 1000 s, E_{yn}, y = 30 km

x (km)		−70	−40	−25	−20.5	−20(+)	−15	−10	−5	0(−)
E_x	Re	−0.028	−0.122	−0.207	−0.175	−0.167	−0.062	0.070	0.220	0.354
	Im	−0.006	−0.019	−0.027	−0.023	−0.022	−0.009	0.005	0.020	0.033
E_y	Re	0.990	1.017	1.188	1.290	1.301	1.381	1.404	1.365	1.220
	Im	0.001	0.010	0.036	0.048	0.049	0.058	0.060	0.054	0.040
H_x	Re	0.998	1.000	1.007	1.011	1.011	1.014	1.015	1.013	1.008
	Im	0.004	0.002	−0.015	−0.024	−0.025	−0.032	−0.034	−0.030	−0.017
H_y	Re	0.002	0.006	0.008	0.007	0.006	0.003	−0.002	−0.007	−0.011
	Im	−0.004	−0.013	−0.020	−0.016	−0.015	−0.006	0.007	0.020	0.030
H_z	Re	−0.002	−0.005	−0.005	−0.004	−0.003	0.000	0.004	0.008	0.011
	Im	0.003	0.010	0.012	0.008	0.008	0.000	−0.009	−0.019	−0.026

Table F.10b. Model 3D-2, l_{y1} = 20 km, T = 1000 s, E_{yn}, y = 30 km

x (km)		0(+)	5	10	15	20(−)	20.5	25	40	70
E_x	Re	0.354	0.358	0.248	0.117	0.024	0.019	−0.008	−0.005	0.001
	Im	0.033	0.035	0.028	0.020	0.013	0.013	0.009	0.004	0.001
E_y	Re	1.220	1.009	0.870	0.833	0.870	0.875	0.924	0.984	0.997
	Im	0.040	0.020	0.006	−0.001	−0.003	−0.003	−0.003	−0.001	0.000
H_x	Re	1.008	1.002	0.997	0.995	0.995	0.995	0.996	0.998	0.999
	Im	−0.017	0.000	0.012	0.016	0.014	0.014	0.010	0.004	0.001
H_y	Re	−0.011	−0.012	−0.009	−0.006	−0.003	−0.003	−0.002	−0.001	0.000
	Im	0.030	0.031	0.023	0.013	0.006	0.005	0.002	0.000	0.000
H_z	Re	0.011	0.011	0.010	0.007	0.005	0.004	0.003	0.001	0.001
	Im	−0.026	−0.027	−0.021	−0.013	−0.007	−0.007	−0.004	−0.001	0.000

Table F.11a. Model 3D-2, l_{y1} = 20 km, T = 1000 s, y = 0

x (km)	−70	−40	−25	−20.5	−20(+)	−15	−10	−5	0(−)
ρ_a^{yx}	7.257	5.388	2.377	1.346	1.411	1.218	1.155	1.180	1.468
ρ_a^{xy}	7.728	8.047	9.022	10.744	0.154	0.337	0.360	0.258	0.018

Table F.11b. Model 3D-2, l_{y1} = 20 km, T = 1000 s, y = 0

x (km)	0(+)	5	10	15	20(−)	20.5	25	40	70
ρ_a^{yx}	1.468	3.645	6.398	7.500	8.091	8.164	8.113	7.854	7.702
ρ_a^{xy}	53.761	43.076	34.505	36.838	44.638	0.748	2.375	5.370	7.110

Table F.12a. Model 3D-2, l_{y1} = 20 km, T = 1000 s, y = 30 km

x (km)	−70	−40	−25	−20.5	−20(+)	−15	−10	−5	0(−)
ρ_a^{yx}	7.573	7.976	10.714	12.566	12.770	14.308	14.741	13.944	11.206
ρ_a^{xy}	7.669	7.464	6.520	5.783	5.697	5.075	5.319	6.817	9.485

Table F.12b. Model 3D-2, l_{y1} = 20 km, T = 1000 s, y = 30 km

x (km)	0(+)	5	10	15	20(−)	20.5	25	40	70
ρ_a^{yx}	11.206	7.777	5.851	5.401	5.895	5.971	6.631	7.487	7.675
ρ_a^{xy}	9.485	11.518	12.008	11.640	10.574	10.436	9.190	7.442	7.390

Table F.13a. Model 3D-2, l_{y1} = 100 km, T = 100 s, E_{xn}, y = 0

x (km)		−70	−40	−25	−20.5	−20(+)	−15	−10	−1	0(−)
E_x	Re	1.003	0.998	1.041	1.195	0.165	0.249	0.263	0.227	0.098
	Im	−0.030	−0.059	−0.071	−0.139	−0.034	−0.056	−0.061	−0.070	−0.044
H_y	Re	0.994	1.003	1.021	1.027	1.027	1.030	1.027	1.017	1.004
	Im	−0.002	−0.005	−0.011	−0.011	−0.010	−0.007	−0.007	−0.010	−0.014

Table F.13b. Model 3D-2, l_{y1} = 100 km, T = 100 s, E_{xn}, y = 0

x (km)		0(+)	5	10	15	20(−)	20.5	25	40	70
E_x	Re	4.038	3.560	3.283	3.418	3.765	0.479	0.715	0.893	0.982
	Im	−0.460	−0.470	−0.564	−0.589	−0.660	−0.123	−0.162	−0.142	−0.072
H_y	Re	1.004	0.996	0.990	0.985	0.983	0.981	0.980	0.980	0.986
	Im	−0.014	−0.015	−0.014	−0.011	−0.010	−0.008	−0.007	−0.006	−0.004

Table F.14a. Model 3D-2, l_{y1} = 100 km, T = 100 s, E_{xn}, y = 30 km

x (km)		−70	−40	−25	−20.5	−20(+)	−15	−10	−5	(−)
E_x	Re	1.000	0.963	0.871	0.804	0.796	0.714	0.657	0.604	1.188
	Im	−0.033	−0.084	−0.133	−0.154	−0.157	−0.179	−0.188	−0.173	−0.410
E_y	Re	0.000	−0.020	−0.063	−0.064	−0.062	−0.015	0.060	0.129	0.182
	Im	0.000	−0.007	−0.012	−0.006	−0.005	0.014	0.039	0.058	0.050
H_x	Re	−0.002	0.009	0.023	0.020	0.019	−0.002	−0.033	−0.056	−0.049
	Im	−0.001	−0.005	−0.016	−0.017	−0.017	−0.010	0.002	0.015	0.016
H_y	Re	0.991	0.980	0.951	0.933	0.930	0.911	0.906	0.919	0.942
	Im	−0.003	−0.003	0.007	0.016	0.017	0.028	0.031	0.025	0.012

Table F.14b. Model 3D-2, l_{y1} = 100 km, T = 100 s, E_{xn}, y = 30 km

x (km)		0(+)	5	10	15	20(−)	20.5	25	40	70
E_x	Re	3.606	3.289	3.130	3.297	3.653	0.461	0.700	0.887	0.982
	Im	−0.584	−0.551	−0.587	−0.591	−0.648	−0.124	−0.162	−0.143	−0.071
E_y	Re	0.182	0.109	0.052	0.023	−0.002	−0.007	−0.008	−0.009	−0.005
	Im	0.050	0.025	0.004	−0.002	−0.004	−0.004	−0.006	−0.009	−0.007
H_x	Re	−0.049	−0.027	−0.017	−0.009	−0.003	−0.002	0.002	0.008	0.007
	Im	0.016	0.011	0.007	0.004	0.002	0.002	0.001	0.001	0.001
H_y	Re	0.942	0.955	0.960	0.963	0.964	0.964	0.966	0.973	0.985
	Im	0.012	0.004	0.000	−0.002	−0.002	−0.001	−0.002	−0.004	−0.004

Table F.15a. Model 3D-2, l_{y1} = 100 km, T = 100 s, E_{yn}, y = 0

x (km)		−70	−40	−25	−20.5	−20(+)	−15	−10	−5	0(−)
E_y	Re	0.993	0.887	0.625	0.487	0.446	0.376	0.348	0.379	0.470
	Im	−0.022	−0.095	−0.151	−0.119	−0.040	0.016	0.018	−0.010	−0.128
H_x	Re	0.988	0.926	0.905	1.084	1.164	1.386	1.366	1.340	1.060
	Im	−0.009	−0.003	0.045	0.013	−0.034	−0.013	0.027	−0.067	−0.053
H_z	Re	−0.003	−0.066	−0.303	−0.511	−0.540	−0.175	0.029	0.234	0.570
	Im	−0.007	−0.032	−0.009	0.090	0.114	−0.063	0.000	0.029	−0.259

Table F.15b. Model 3D-2, l_{y1} = 100 km, T = 100 s, E_{yn}, y = 0

x (km)		0(+)	5	10	15	20(−)	20.5	25	40	70
E_y	Re	0.470	0.674	0.858	0.944	1.014	1.040	1.050	1.044	1.017
	Im	−0.128	−0.210	−0.229	−0.214	−0.171	−0.171	−0.120	−0.038	−0.001
H_x	Re	1.060	0.774	0.735	0.750	0.844	0.866	0.945	1.006	1.012
	Im	−0.053	0.115	0.120	0.104	0.034	0.015	−0.030	−0.026	−0.005
H_z	Re	0.570	0.302	0.134	0.001	−0.130	−0.127	−0.097	−0.042	−0.008
	Im	−0.259	−0.086	−0.009	0.053	0.128	0.120	0.066	0.006	−0.005

Table F.16a. Model 3D-2, l_{y1} = 100 km, T = 100 s, E_{yn}, y = 30 km

x (km)		−70	−40	−25	−20.5	−20(+)	−15	−10	−5	0(−)
E_x	Re	−0.003	−0.052	−0.129	−0.126	−0.123	−0.082	−0.036	0.006	0.174
	Im	−0.018	−0.080	−0.126	−0.111	−0.108	−0.067	−0.025	0.008	−0.019
E_y	Re	1.000	1.004	1.097	1.167	1.175	1.240	1.284	1.321	1.249
	Im	−0.007	0.008	0.091	0.130	0.133	0.148	0.125	0.068	−0.030
H_x	Re	0.999	1.007	1.066	1.098	1.101	1.124	1.123	1.088	0.947
	Im	−0.004	0.000	0.001	−0.004	−0.005	−0.015	−0.028	−0.035	0.021
H_y	Re	0.009	0.049	0.085	0.080	0.078	0.053	0.022	−0.009	−0.032
	Im	0.006	0.014	0.006	0.003	0.002	0.000	−0.001	0.001	0.005
H_z	Re	0.000	−0.008	0.003	0.025	0.028	0.071	0.128	0.202	0.298
	Im	−0.003	−0.009	−0.004	−0.002	−0.002	−0.004	−0.016	−0.047	−0.119

Table F.16b. Model 3D-2, l_{y1} = 100 km, T = 100 s, E_{yn}, y = 30 km

x (km)		0(+)	5	10	15	20(−)	20.5	25	40	70
E_x	Re	0.454	0.372	0.263	0.227	0.215	0.022	0.023	0.009	0.001
	Im	0.210	0.159	0.090	0.069	0.063	0.015	0.012	0.007	0.001
E_y	Re	1.249	1.209	1.131	1.105	1.086	1.096	1.090	1.058	1.019
	Im	−0.030	−0.150	−0.217	−0.215	−0.179	−0.171	−0.117	−0.033	0.001
H_x	Re	0.947	0.806	0.773	0.781	0.870	0.893	0.968	1.017	1.014
	Im	0.021	0.092	0.103	0.093	0.024	0.005	−0.038	−0.028	−0.004
H_y	Re	−0.032	−0.038	−0.036	−0.031	−0.027	−0.026	−0.022	−0.010	−0.001
	Im	0.005	0.006	0.006	0.004	0.003	0.003	0.001	−0.001	−0.001
H_z	Re	0.298	0.168	0.057	−0.049	−0.168	−0.163	−0.121	−0.049	−0.009
	Im	−0.119	−0.047	0.007	0.062	0.136	0.127	0.068	0.004	−0.006

Table F.17a. Model 3D-2, l_{y1} = 100 km, T = 100 s, y = 0

x (km)	−70	−40	−25	−20.5	−20(+)	−15	−10	−5	0(−)
ρ_a^{yx}	15.622	14.346	7.787	3.310	2.290	1.141	1.007	1.236	3.282
ρ_a^{xy}	15.773	15.351	16.146	21.210	0.418	0.946	1.065	0.840	0.176

Table F.17b. Model 3D-2, l_{y1} = 100 km, T = 100 s, y = 0

x (km)	0(+)	5	10	15	20(−)	20.5	25	40	70
ρ_a^{yx}	3.282	12.561	21.945	25.263	22.933	22.893	19.313	16.660	15.614
ρ_a^{xy}	252.69	200.95	175.03	191.52	234.01	3.924	8.649	13.172	15.420

Table F.18a. Model 3D-2, l_{y1} = 100 km, T = 100 s, y = 30 km

x (km)	−70	−40	−25	−20.5	−20(+)	−15	−10	−5	0(−)
ρ_a^{yx}	15.512	15.343	16.376	17.561	17.706	19.042	20.399	22.826	26.800
ρ_a^{xy}	15.764	15.045	13.183	11.842	11.674	10.071	8.791	7.203	27.190

Table F.18b. Model 3D − 2, l_{y1} = 100 km, T = 100 s, y = 30 km

x (km)	0(+)	5	10	15	20(−)	20.5	25	40	70
ρ_a^{yx}	26.800	34.667	33.665	31.669	24.754	23.889	19.776	16.714	15.597
ρ_a^{xy}	229.44	187.37	169.59	186.86	228.68	3.796	8.566	13.167	15.418

Table F.19a. Model 3D-2, l_{y1} = 100 km, T = 1000 s, E_{xn}, y = 0

x (km)		−70	−40	−25	−20.5	−20(+)	−15	−10	−5	0(−)
E_x	Re	0.941	0.906	0.931	1.058	0.138	0.191	0.195	0.164	0.066
	Im	−0.014	−0.020	−0.025	−0.036	−0.009	−0.019	−0.021	−0.020	−0.010
H_y	Re	0.999	0.999	1.000	1.001	1.001	1.001	1.001	1.000	0.999
	Im	0.002	0.001	−0.003	−0.005	−0.005	−0.006	−0.006	−0.003	0.001

Table F.19b. Model 3D-2, l_{y1} = 100 km, T = 1000 s, E_{xn}, y = 0

x (km)		0(+)	5	10	15	20(−)	20.5	25	40	70
E_x	Re	3.417	3.049	2.765	2.849	3.059	0.372	0.601	0.762	0.882
	Im	−0.124	−0.095	−0.100	−0.110	−0.144	−0.025	−0.029	−0.030	−0.022
H_y	Re	0.999	0.998	0.997	0.997	0.997	0.997	0.997	0.997	0.998
	Im	0.001	0.003	0.004	0.005	0.006	0.006	0.006	0.006	0.005

Table F.20a. Model 3D-2, l_{y1} = 100 km, T = 1000 s, E_{xn}, y = 30 km

x (km)		−70	−40	−25	−20.5	−20(+)	−15	−10	−5	0(−)
E_x	Re	0.936	0.860	0.750	0.681	0.672	0.591	0.539	0.501	0.783
	Im	−0.014	−0.023	−0.028	−0.030	−0.030	−0.032	−0.032	−0.028	−0.095
E_y	Re	0.002	−0.018	−0.057	−0.055	−0.052	−0.003	0.074	0.141	0.180
	Im	0.000	0.000	0.000	0.000	0.000	0.002	0.004	0.005	0.003
H_x	Re	0.000	0.000	0.001	0.001	0.001	−0.001	−0.002	−0.004	−0.003
	Im	0.001	−0.002	−0.005	−0.004	−0.004	0.001	0.007	0.012	0.011
H_y	Re	0.998	0.997	0.995	0.996	0.996	0.993	0.993	0.994	0.995
	Im	0.003	0.006	0.012	0.017	0.017	0.021	0.022	0.019	0.014

Table F.20b. Model 3D-2, l_{y1} = 100 km, T = 1000 s, E_{xn}, y = 30 km

x (km)		0(+)	5	10	15	20(−)	20.5	25	40	70
E_x	Re	3.003	2.779	2.624	2.752	2.987	0.358	0.590	0.757	0.883
	Im	−0.121	−0.099	−0.100	−0.108	−0.138	−0.025	−0.029	−0.030	−0.021
E_y	Re	0.180	0.107	0.047	0.020	0.000	−0.004	−0.007	−0.014	−0.012
	Im	0.003	0.002	0.000	0.000	0.001	0.000	0.000	−0.001	−0.001
H_x	Re.	−0.003	−0.002	−0.001	−0.001	0.000	0.000	0.000	0.001	0.001
	Im	0.011	0.006	0.004	0.002	0.001	0.001	0.000	−0.002	−0.002
H_y	Re	0.995	0.995	0.996	0.996	0.996	0.996	0.996	0.996	0.998
	Im	0.014	0.012	0.010	0.010	0.010	0.010	0.009	0.008	0.005

Table F.21a. Model 3D-2, l_{y1} = 100 km, T = 1000 s, E_{yn}, y = 0

x (km)		−70	−40	−25	−20.5	−20(+)	−15	−10	−5	0(−)
E_y	Re	0.967	0.819	0.525	0.388	0.402	0.386	0.372	0.369	0.398
	Im	−0.005	−0.017	−0.029	−0.028	−0.014	−0.003	−0.001	−0.007	−0.023
H_x	Re	0.998	0.993	0.994	1.009	1.015	1.038	1.040	1.032	1.005
	Im	0.005	0.019	0.023	−0.022	−0.042	−0.105	−0.105	−0.092	−0.018
H_z	Re	−0.002	−0.010	−0.031	−0.047	−0.049	−0.023	0.002	0.025	0.046
	Im	0.003	0.022	0.083	0.137	0.145	0.056	−0.005	−0.064	−0.150

Table F.21b. Model 3D-2, l_{y1} = 100 km, T = 1000 s, E_{yn}, y = 0

x (km)		0(+)	5	10	15	20(−)	20.5	25	40	70
E_y	Re	0.398	0.603	0.791	0.875	0.945	0.956	0.968	0.989	1.001
	Im	−0.023	−0.031	−0.029	−0.027	−0.021	−0.023	−0.018	−0.010	−0.003
H_x	Re	1.005	0.984	0.980	0.980	0.987	0.988	0.994	0.998	1.000
	Im	−0.018	0.056	0.066	0.062	0.038	0.032	0.013	0.001	−0.002
H_z	Re	0.046	0.027	0.014	0.003	−0.007	−0.007	−0.005	−0.003	−0.001
	Im	−0.150	−0.079	−0.037	−0.003	0.031	0.030	0.021	0.009	0.003

Table F.22a. Model 3D-2, l_{y1} = 100 km, T = 1000 s, E_{yn}, y = 30 km

x (km)		−70	−40	−25	−20.5	−20(+)	−15	−10	−5	0(−)
E_x	Re	−0.031	−0.138	−0.243	−0.224	−0.218	−0.139	−0.055	0.015	0.006
	Im	−0.006	−0.019	−0.026	−0.023	−0.022	−0.014	−0.005	0.002	−0.055
E_y	Re	0.992	1.020	1.187	1.287	1.300	1.374	1.402	1.406	1.291
	Im	−0.002	0.004	0.021	0.029	0.030	0.034	0.031	0.023	0.009
H_x	Re	0.999	1.001	1.007	1.010	1.011	1.012	1.012	1.009	0.997
	Im	0.001	−0.002	−0.019	−0.027	−0.028	−0.034	−0.034	−0.026	0.010
H_y	Re	0.002	0.006	0.009	0.008	0.008	0.006	0.002	−0.001	−0.003
	Im	−0.004	−0.015	−0.024	−0.022	−0.021	−0.014	−0.006	0.002	0.009
H_z	Re	−0.001	−0.002	0.000	0.002	0.002	0.007	0.012	0.018	0.026
	Im	0.001	0.004	0.001	−0.005	−0.006	−0.017	−0.032	−0.051	−0.079

Table F.22b. Model 3D-2, l_{y1} = 100 km, T = 1000 s, E_{yn}, y = 30 km

x (km)		0(+)	5	10	15	20(−)	20.5	25	40	70
E_x	Re	0.572	0.484	0.317	0.260	0.249	0.050	0.032	0.016	0.002
	Im	0.026	0.031	0.018	0.014	0.018	0.010	0.003	0.002	0.000
E_y	Re	1.291	1.195	1.072	1.032	1.007	1.011	1.009	1.006	1.004
	Im	0.009	−0.008	−0.021	−0.023	−0.022	−0.022	−0.017	−0.009	−0.003
H_x	Re	0.997	0.985	0.982	0.983	0.989	0.990	0.996	0.999	1.001
	Im	0.010	0.047	0.056	0.054	0.032	0.025	0.007	−0.002	−0.003
H_y	Re	−0.003	−0.003	−0.003	−0.003	−0.003	−0.003	−0.002	−0.001	0.000
	Im	0.009	0.010	0.009	0.008	0.007	0.007	0.006	0.003	0.001
H_z	Re	0.026	0.016	0.007	−0.001	−0.010	−0.010	−0.008	−0.004	−0.001
	Im	−0.079	−0.045	−0.017	0.010	0.041	0.039	0.027	0.011	0.003

Table F.23a. Model 3D-2, l_{y1} = 100 km, T = 1000 s, y = 0

x (km)	−70	−40	−25	−20.5	−20(+)	−15	−10	−5	0(−)
ρ_a^{yx}	7.235	5.246	2.157	1.147	1.209	1.056	0.977	0.979	1.219
ρ_a^{xy}	6.839	6.339	6.691	8.626	0.147	0.283	0.296	0.209	0.034

Table F.23b. Model 3D-2, l_{y1} = 100 km, T = 1000 s, y = 0

x (km)	0(+)	5	10	15	20(−)	20.5	25	40	70
ρ_a^{yx}	1.219	2.894	5.004	6.129	7.066	7.206	7.318	7.567	7.715
ρ_a^{xy}	90.354	72.044	59.319	62.987	72.700	1.075	2.808	4.512	6.022

Table F.24a. Model 3D-2, l_{y1} = 100 km, T = 1000 s, y = 30 km

x (km)	−70	−40	−25	−20.5	−20(+)	−15	−10	−5	0(−)
ρ_a^{yx}	7.593	7.998	10.696	12.500	12.698	14.196	14.794	14.970	12.934
ρ_a^{xy}	6.779	5.732	4.377	3.616	3.526	2.740	2.283	1.964	4.850

Table F.24b. Model 3D-2, l_{y1} = 100 km, T = 1000 s, y = 30 km

x (km)	0(+)	5	10	15	20(−)	20.5	25	40	70
ρ_a^{yx}	12.934	11.306	9.153	8.482	7.997	8.034	7.910	7.809	7.765
ρ_a^{xy}	70.196	60.086	53.590	58.919	69.493	1.003	2.710	4.460	6.047

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