Advanced modeling and inversion technologies for high-resolution electromagnetic methods

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Abstract

One of the most challenging problems in electromagnetic (EM) geophysical methods is developing fast and stable methods of EM modeling and inversion for 3-D inhomogeneous geoelectrical structures. In this paper I present an overview of the research conducted by the Consortium for Electromagnetic Modeling and Inversion (CEMI) at the University of Utah on developing fast forward modeling and inversion technologies for high-resolution EM methods. We have recently developed a novel approach to 3-D EM modeling based on linearization of the integral equations for scattered EM fields (quasi-linear, QL, approximation). We have also developed a new approach to the solution of 3-D EM inverse problem based on QL approximation. The main advantage of the QL inversion is that we reduce the original nonlinear inverse problem to a set of linear inverse problems to obtain a rapid 3-D EM inversion. The inversion of the real data indicates that 3-D quasi-linear EM inversion is fast and stable and provides reasonable recovery of the models and real geological features. Thus, new technologies for high-resolution EM methods include fast and accurate forward modeling solution and effective quasi-linear 3-D inversion scheme.

Introduction

At the University of Utah Consortium for Electromagnetic Modeling and Inversion (CEMI) we have recently developed a novel approach to 3-D EM modeling based on linearization of the integral equations for scattered EM fields. We call this approach a quasi-linear (QL) approximation (Zhdanov and Fang, 1996a). It is based on the assumption that the anomalous field is linearly related to the background (normal) field in the inhomogeneous domain by an electrical reflectivity tensor. The reflectivity tensor inside inhomogeneities is approximated by slow varying functions which are determined numerically by a simple optimization technique.

We have demonstrated also that there is a possibility to increase the accuracy of the QL approximation by constructing QL approximations of a higher order (Zhdanov and Fang, 1997). The QL series make it possible to estimate the accuracy of the original QL approximation without direct comparison with the rigorous full integral equation (IE) solution and to construct a fast and accurate iterative method for 3-D forward modeling. The important practical result is that the accuracy of the numerical solution can be found from the value of the numerical solution itself.

3-D EM inversion continues to be a challenging problem in electrical exploration. We have developed a new approach to the solution of this problem based on QL approximation of forward modeling operator (Zhdanov and Fang, 1996b). The new inversion scheme reduces the original nonlinear inverse problem to a set of linear inverse problems.

The developed new technologies have been realized in the numerical codes which have been tested on synthetic 3-D EM data. The case histories include interpretation of 3-D magnetotelluric (MT) survey conducted by the New Energy and Industrial Technology Development Organization (NEDO) in the Minamikayabe area located in the southern part of Hokkaido, Japan (Takasugi et al., 1992), and 3-D inversion of the tensor controlled source audio magnetotelluric (CSAMT) data collected over the Sulphur Springs thermal area, Valles Caldera, New Mexico, U.S.A. (Wannamaker, 1997b).

In this paper I present an overview of the research conducted by the Consortium for Electromagnetic Modeling and Inversion at the University of Utah on developing fast quasi-linear forward modeling and inversion codes.

The quasi-linear approximation and quasi-linear series in 3-D EM modeling

Consider a 3-D geoelectrical model with the normal (background) conductivity σ_b and local inhomogeneity D with the arbitrarily varying conductivity $\sigma = \sigma_b + \Delta\sigma$. We confine ourselves to consideration of non-magnetic media and, hence, assume that $\mu = \mu_0 = 4\pi \times 10^{-7} H/m$, where μ_0 is the free-space magnetic permeability. The model is excited by an EM field generated by an arbitrary source. This field is time harmonic as $e^{-i\omega t}$.

The EM fields in this model can be presented as a sum of background (normal) and anomalous fields:

$$\mathbf{E} = \mathbf{E}^b + \mathbf{E}^a, \ \mathbf{H} = \mathbf{H}^b + \mathbf{H}^a, \tag{1}$$

where the background field is a field generated by the given sources in the model with the normal distribution of conductivity σ_b, and the anomalous field is presented as an integral over the excess currents in the inhomogeneous domain D:

$$\mathbf{E}^a(\mathbf{r}_j) = \int_D \widehat{\mathbf{G}}^b(\mathbf{r}_j \mid \mathbf{r})\, \Delta\sigma(\mathbf{r}) \left[\mathbf{E}^b(\mathbf{r}) + \mathbf{E}^a(\mathbf{r})\right] dv, \tag{2}$$

where $\widehat{\mathbf{G}}^b(\mathbf{r}_j \mid \mathbf{r})$ is the Green's tensor.

The conventional Born approximation $\mathbf{E}^B(\mathbf{r}_j)$ for the anomalous field can be obtained from (2) if we assume that the anomalous field is negligibly small inside D in comparison with the normal field. In this case it can be ignored in comparison with the normal field:

$$\mathbf{E}^B(\mathbf{r}_j) = \int_D \widehat{\mathbf{G}}^b(\mathbf{r}_j \mid \mathbf{r})\,\Delta\sigma(\mathbf{r})\,\mathbf{E}^b(\mathbf{r})\,dv = \mathbf{G}_b\left(\Delta\sigma(\mathbf{r})\,\mathbf{E}^b(\mathbf{r})\right), \tag{3}$$

where $\mathbf{G}_b$ is a corresponding Green's linear operator. Approximation (3) works reasonably well only for small conductivity contrasts between background media, a relatively small inhomogeneity, and low frequencies.

Quasi-linear (QL) approximation is based on the assumption that the anomalous field $\mathbf{E}^a$ inside the inhomogeneous domain is linearly proportional to the background field $\mathbf{E}^b$ by some tensor $\widehat{\lambda}$ (electrical reflectivity tensor, (Zhdanov and Fang, 1996a)):

$$\mathbf{E}^a(\mathbf{r}) \approx \widehat{\lambda}(\mathbf{r})\,\mathbf{E}^b(\mathbf{r}). \tag{4}$$

Substituting this formula into (2) we obtain

$$\mathbf{E}^a \approx \mathbf{E}_{ql}^{a(1)} = \int_D \widehat{\mathbf{G}}^b(\mathbf{r}_j \mid \mathbf{r})\,\Delta\sigma(\mathbf{r})\left[\widehat{\mathbf{I}}+\widehat{\lambda}(\mathbf{r})\right]\mathbf{E}^b(\mathbf{r})\,dv. \tag{5}$$

Following our paper (Zhdanov and Fang, 1997) we can extend our approach to computing the QL approximations of the higher orders. In this case we obtain a complete analog of Born series. However, to make these approximations to be converged we have to modify Green's operator according to the formula (Zhdanov and Fang, 1997):

$$\mathbf{G}_b^m\left(\Delta\sigma(\mathbf{r})\,\mathbf{E}^b(\mathbf{r})\right) = \sqrt{\sigma_b}\,\mathbf{G}_b\left(2\sqrt{\sigma_b}\,\Delta\sigma(\mathbf{r})\,\mathbf{E}^b(\mathbf{r})\right) + \Delta\sigma(\dot{\mathbf{r}})\,\mathbf{E}^b(\mathbf{r}). \tag{6}$$

Using the modified Green's operator we can rewrite the integral equation for the anomalous field (2) as follows:

$$a\mathbf{E}^a + b\mathbf{E}^b = \mathbf{G}_b^m\left[b\left(\mathbf{E}^a + \mathbf{E}^b\right)\right]. \tag{7}$$

The first order QL approximation can be also written using $\mathbf{G}_b^m$ in a form:

$$\mathbf{E}_{ql}^{a(1)} = \frac{1}{a}\mathbf{G}_b^m\left(b\left(1+\widehat{\lambda}\right)\mathbf{E}^b\right) - \beta\mathbf{E}^b. \tag{8}$$

where

$$a = \frac{2\sigma_b + \Delta\sigma}{2\sqrt{\sigma_b}},\ b = \frac{\Delta\sigma}{2\sqrt{\sigma_b}},\ \beta = \frac{b}{a}. \tag{9}$$

Note that simple calculations show that (Pankratov et al., 1995):

$$\beta^2 = \frac{|\Delta\sigma|^2}{|2\sigma_b + \Delta\sigma|^2} = 1 - \frac{4\sigma\sigma_b}{|\sigma - \sigma_b|^2 + 4\sigma\sigma_b} < 1$$

under the natural condition that

$$0 < \sigma_{\min} \le \sigma \le \sigma_{\max} < \infty,$$

$$0 < \sigma_{b\min} \le \sigma_b \le \sigma_{b\max} < \infty.$$

Thus,

$$\|\beta\|_\infty = \max\left|\frac{b(r)}{a(r)}\right| < 1. \tag{10}$$

Now, we can substitute the first order QL approximation in the right hand part of the modified integral equation (7) to obtain the second order QL approximation, etc. Finally, the $N-$th order QL approximation can be treated as the sum of N terms of the QL series:

$$a\mathbf{E}_{ql}^{a(N)} = \sum_{k=0}^{N-1}\left(\mathbf{G}_b^m\beta\right)^k\left(a\mathbf{E}^{Bm}\right) + \left(\mathbf{G}_b^m\beta\right)^N\left(a\widehat{\lambda}\mathbf{E}^b\right), \tag{11}$$

where according to (Zhdanov and Fang, 1997) $\mathbf{E}^{Bm}$ is the modified Born approximation:

$$\mathbf{E}^{Bm} = \frac{2\sigma_b}{2\sigma_b + \Delta\sigma}\mathbf{E}^B.$$

Accuracy estimation

The accuracy of the QL approximation of the N-th order can be tested by comparing it with the QL approximation of the $(N-1)$-th order (Zhdanov and Fang, 1997):

$$\varepsilon_N = \frac{\left\|a\mathbf{E}^a - a\mathbf{E}_{ql}^{a(N)}\right\|}{\left\|a\mathbf{E}_{ql}^{a(N)}\right\|} \le \frac{\|\beta\|_\infty}{1-\|\beta\|_\infty} r_N, \tag{12}$$

where $\mathbf{E}_{ql}^{a(0)} = \widehat{\lambda}\mathbf{E}^b$, and r_N is the relative convergence rate of the QL approximations:

$$r_N = \frac{\left\|a\mathbf{E}_{ql}^{a(N)} - a\mathbf{E}_{ql}^{a(N-1)}\right\|}{\left\|a\mathbf{E}_{ql}^{a(N)}\right\|}. \tag{13}$$

The important result is that formulae (12) and (13) make it possible to obtain a quantitative estimation of the QL approximation accuracy without direct comparison with the rigorous full IE forward modeling solution.

Let us analyze the accuracy of the QL series approximation for a typical geoelectrical model containing high resistivity contrasts. The model which we used to test the estimation accuracy of the QL series approximation is shown in Figure 1. The model consists of a homogeneous half-space (with the resistivity 100 Ohm-m) and a conductive rectangular inclusion with the resistivity 0.01 Ohm-m. So, the conductivity contrast is 10^4. The EM field in the model is excited by a horizontal rectangular loop, located 50 m to the left of the model, with the loop 10 m on a side and the current at 1 A.

We applied the QL series approximation to compute the EM field for this model. The QL approximations of ten different orders (from one to ten) were analyzed.

As we discussed above, the accuracy estimation of QL series can be expressed using formula (12). Table 1 presents the results of the accuracy estimation ε_N computed for this model.

Table 1
Comparison of the accuracy estimation computed by formula (12)

Frequency	QL appr	5th order QL	10th order QL
10000 Hz	0.360	0.187	0.071
1000 Hz	0.358	0.182	0,062
100 Hz	0.358	0.183	0.061
10 Hz	0.358	0.183	0.063
1 Hz	0.358	0.182	0,062

We can see from this table that the expression (12) produces a rather conservative estimation of the accuracy. The approximation errors go down as the order increases for every frequency (Table 1). At the same time, expression (12) can be used as a tool to control the order of the QL approximations which would satisfy to the required accuracy. For example, calculations show that it takes ten iterations for this model to reach the accuracy estimation less than 6%.

Figure 2 presents the profiles of the full (rigorous) integral equation solutions obtained by SYSEM code (Xiong, 1992), and QL solution with the required accuracy less than 5 %. It is important to compare the time of numerical modeling needed for full IE solution and QL series. The results of comparison obtained for the model shown in Figure 1 are presented in Table 2. We present here the CPU time for forward modeling using different number of cells within the conductive body: 400 cells, 800 cells, and 1200 cells correspondingly.

Table 2
Comparison of the CPU time (in hours, minutes and seconds) for EM modeling using full integral equation solution (SYSEM code) and QL approximations of the different orders.

Cell number	SYSEM	QL appr	5th order QL	10th order QL
400 cells	19:33 Min	1:48 Min	2:34 Min	3:32 Min
800 cells	1:28:20 H.	4:41 Min	7:44 Min	11:46 Min
1200 cells	5:55:77 H.	6:20 Min	13:05 Min	21:33 Min

One can see from this table that in the models with 800 cells and 1200 cells in anomalous domain, the QL approximation is an order of magnitude faster than the full IE solution. It takes only about three times as much time to compute the QL approximation of the 10th order than the first order QL approximation.

The important practical result is that the accuracy of the numerical solution can be found from the value of the numerical solution itself. Moreover, accuracy can be made as high as necessary by selecting corresponding number N of the QL approximation, and by choosing a fine enough grid for the vector function's discretization. In other words, the QL series provide the method for rigorous forward modeling solution.

In the same time this method is much faster than the computer codes based on the full IE solution. Thus it opens a new possibility for fast and accurate 3-D EM inversion.

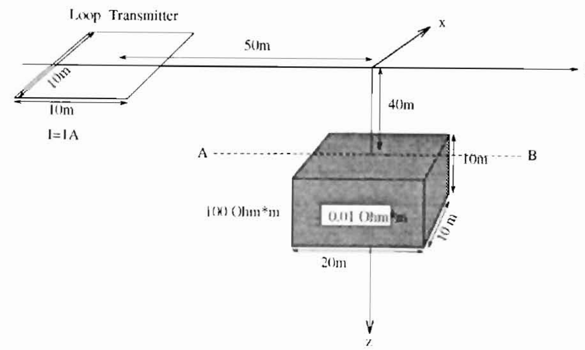

Figure 1: 3-D model of a rectangular conductive structure embedded in a homogeneous medium excited by a rectangular loop.

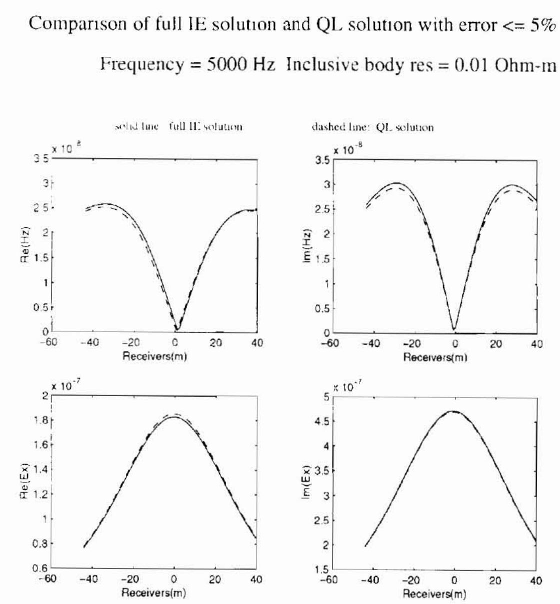

Figure 2: Numerical comparison of the full integral equation and QL series solution of the anomalous vertical magnetic and horizontal electric components for the synthetic model shown in Figure 1.

Quasi-linear inversion

Consider the same 3-D geoelectric model which we described in the section for forward modeling.

We introduce a new tensor function in equation (5):

$$\widehat{\mathbf{m}}\left(\mathbf{r}\right) = \Delta\sigma\left(\mathbf{r}\right)\left[\widehat{\mathbf{I}}+\widehat{\lambda}\left(\mathbf{r}\right)\right], \tag{14}$$

which we call *a modified material property tensor*. Equation (14) can be treated as a modification of Ohm's law:

$$\mathbf{j}^{D} = \Delta\sigma\mathbf{E} = \Delta\sigma\left(\mathbf{E}^{b}+\mathbf{E}^{a}\right) = \Delta\sigma\left[\widehat{\mathbf{I}}+\widehat{\lambda}\left(\mathbf{r}\right)\right]\mathbf{E}^{b} \tag{15}$$

Finally, equation (5) takes the form:

$$\mathbf{E}^{a}\left(\mathbf{r}_{j}\right) \approx \int_{D} \widehat{\mathbf{G}}^{F}\left(\mathbf{r}_{j} \mid \mathbf{r}\right)\widehat{\mathbf{m}}\left(\mathbf{r}\right)\mathbf{E}^{b}\left(\mathbf{r}\right)dv. \tag{16}$$

The last equation is a linear one with respect to $\widehat{\mathbf{m}}\left(\mathbf{r}\right)$. In our algorithm we use the conjugate gradient method for determining the modified material property tensor.

The electrical reflectivity tensor $\widehat{\lambda}$ can be determined from the following linear equation inside the inhomogeneous domain D, as long as we know $\widehat{\mathbf{m}}$:

$$\mathbf{E}^{a}\left(\mathbf{r}_{j}\right) \approx \int_{D} \widehat{\mathbf{G}}^{E}\left(\mathbf{r}_{j} \mid \mathbf{r}\right)\widehat{\mathbf{m}}\left(\mathbf{r}\right)\mathbf{E}^{b}\left(\mathbf{r}\right)dv \approx \widehat{\lambda}\left(\mathbf{r}_{j}\right)\mathbf{E}^{b}\left(\mathbf{r}_{j}\right). \tag{17}$$

After determining $\widehat{\mathbf{m}}$ and $\widehat{\lambda}$ it is possible to evaluate the anomalous conductivity $\Delta\sigma$ from equation (14).

This inversion scheme reduces the original nonlinear inverse problem to three linear inverse problems: the first (the quasi-Born inversion) for the parameter $\widehat{\mathbf{m}}$, another one for the parameter $\widehat{\lambda}$, and the third one (correction of the result of the quasi-Born inversion) for the conductivity $\Delta\sigma$. It is based on a QL approximation, that is why we call this approach *a QL inversion.*

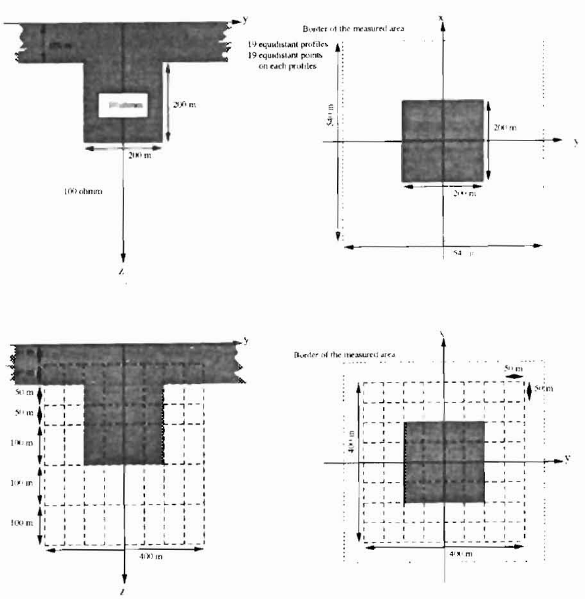

Figure 3: 3-D model of a syncline structure (graben) in the near surface conductive layer, excited by plane wave (top) and its division into substructures used for inversion (bottom).

We solve the linear inverse problem by minimizing the parametric functional:

$$P^{\alpha}(\mathbf{m}) = \phi(\mathbf{m}) + \alpha\, S(\mathbf{m}), \tag{18}$$

where misfit functional is specified as

$$\phi(\mathbf{m}) = \left\|\mathbf{W}_{d}(\mathbf{G}^{F}\,\mathbf{m} - \mathbf{F})\right\|^{2} + \left\|\mathbf{W}_{m}(\mathbf{m} - \Delta\sigma\left[\mathbf{I} + \lambda\right])\right\|^{2},$$

and $\mathbf{W}_{d}$ and $\mathbf{W}_{m}$ are the data and model parameters weighting matrices.

The stabilizer is selected to be equal to

$$S(\mathbf{m}) = \|\ \mathbf{W}_{m}(\mathbf{m} - \mathbf{m}_{apr})\ \|^{2}.$$

The a priori model $\mathbf{m}_{apr}$ is some reference model, selected on the basis of all available geological and geophysical information about the area under investigation. The scalar multiplier α is a regularization parameter.

Model study of 3-D quasi-linear inversion method

The model represents a syncline structure (graben) in the near-surface conductive layer with 10 Ohm-m resistivity. The horizontal and vertical cross-sections of the model are shown in the top panel of Figure 3. The model was excited by a plane EM wave. The theoretical observed field was calculated in the nodes of a square grid on the surface using integral equation forward modeling code SYSEM (Xiong, 1992). The distance between the observation points is 30 m in x and y direction. In each points the synthetic magnetotelluric apparent resistivities and phases were calculated for TE and TM modes.

We defined an inverted area as the rectangular body with the sides of 400 m in both horizontal directions, and of 450 m in vertical direction. It was divided into substructures with the constant but unknown resistivities. The boundaries of the substructures used for inversion are shown by dashed lines in the bottom panel of Figure 3. Figure 4 shows 3-D inversion image which corresponds well to the original model.

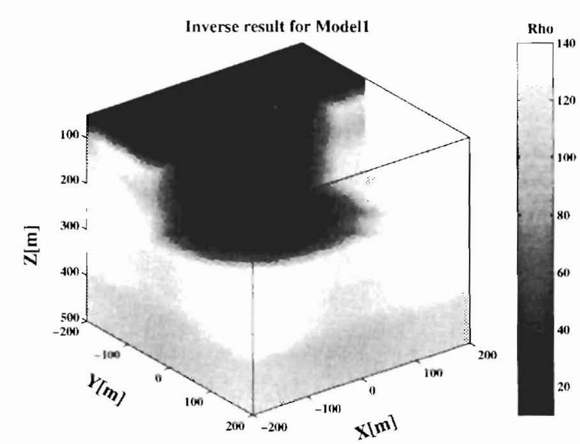

Figure 4: Volume image of the inversion result for the graben model.

3-D inversion of the tensor CSAMT data collected over the Sulphur Springs thermal area, Valles Caldera, New Mexico, U.S.A.

The Valles caldera, New Mexico, U.S.A. is one of the Quaternary rhyolite systems of North America. To examine its volcanic history an extensive CSAMT survey was carried out over the Sulphur Springs geothermal area in the caldera (Wannamaker, 1997a). The detailed geology of the measured area and the receiver system is shown in Figure 5. Two independent electric bipoles were used as a transmitter at 13 km distance from the measured area. Frequency soundings were carried out along four profiles at 45 receiver points.

The boundary of the inverted area with respect to the measuring system is marked by the bold solid line in Figure 5. We divided the area into 9 by 4 substructures horizontally corresponding to the profiles. Each cells are 500 m in x and y directions. We used 7 different depth levels. The vertical sizes of the substructures increase with depth.

The inverse results can be seen in Figure 6 which presents the volume image of the inverse model. The result of 3-D inversion is consistent with the previous interpretation results (Wannamaker, 1997a).

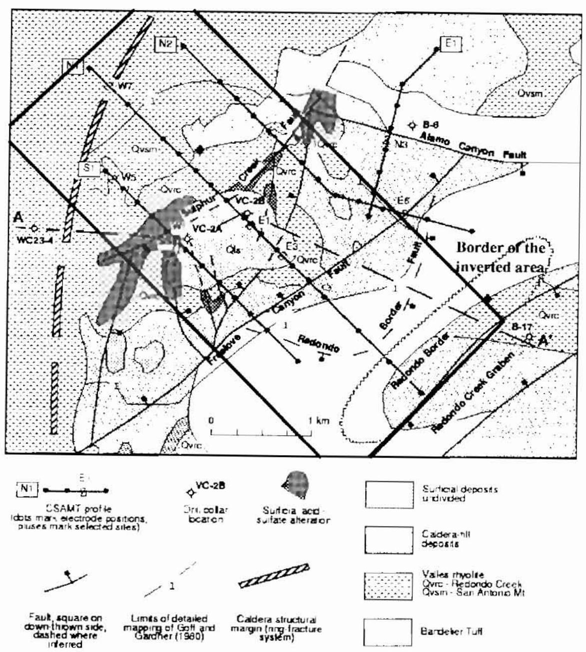

Figure 5: Detailed geological map of the Sulphur Springs Area in the Valles Caldera (after Wannamaker, 1997a) with the receiver profiles and the border of the area used for the inversion.

A case study in the Minamikayabe area in Japan

The New Energy and Industrial Technology Development Organization (NEDO) has been conducting a "Geothermal Development Promotion Survey" in the Minamikayabe area located in the southern part of Hokkaido, Japan (Takasugi et al., 1992). This area is particularly interesting because of its geothermal potential. In 1988, MT sounding and AMT sounding was conducted using the high accuracy MT system. Figure 7 shows the location of the MT measurement sites. The mesh size is 100 m. The frequency range of MT data is from 1 Hz to 130 Hz. Because the survey area at Minamikayabe is located very close to the Uchiura Bay on the northeastern side, low frequency data are strongly affected by the coast of the sea (Takasugi et al., 1992). Also we are interesting in the shallow geological structure in this area. Therefore, we process the data with the frequencies higher than 1 Hz, so that the coast effect can be neglected. We use both TE mode and TM mode data for total 161 MT soundings in the inversion. In this area we expect there are geothermal high conductivity zone in the intrusive resistive rock. So the background model is represented by a high resistive half space. The computation time was about 45 minutes on Sun Sparc 10 workstation. Figure 8 shows the inverse results. The 3-D inversion is consistent with the 2-D interpretation results included in the paper (Takasugi et al., 1992).

Conclusion

This paper presents an overview of the research conducted by the Consortium for Electromagnetic Modeling and Inversion at the University of Utah on developing advanced modeling and inversion technologies for high-resolution electromagnetic methods. Our inversion algorithms are based on the QL approximation of the forward modeling. The main advantage of the QL inversion is that we reduce the original nonlinear inverse problem to a set of linear inverse problems to obtain a rapid 3-D EM inversion. The

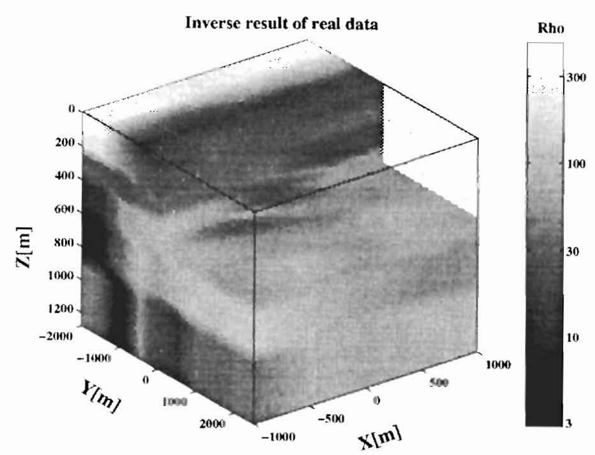

Figure 6: Volume image of the predicted model obtained by 3-D quasi-linear inversion.

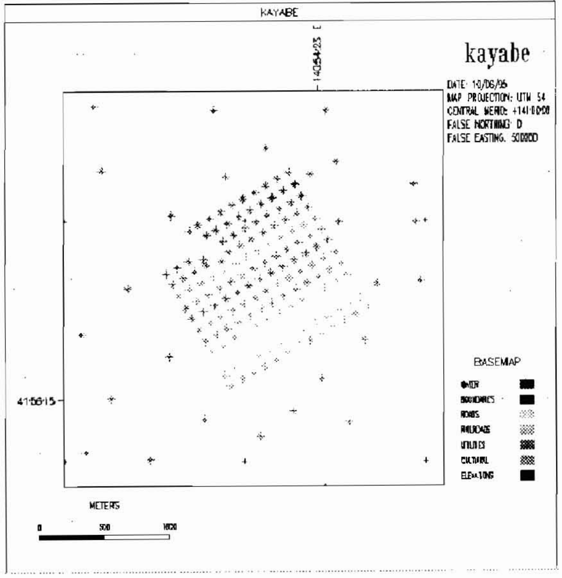

Figure 7: The distribution of MT sounding sites in the Minamikayabe area.

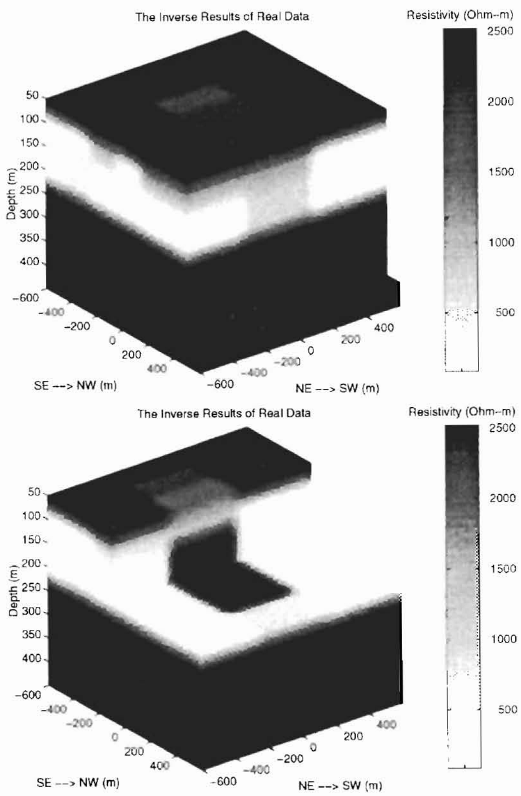

Figure 8: Volume image of the inverse results.

QL inverse problem is solved by a regularized conjugate gradient method which ensures stability and rapid convergence. The inversion of the real data indicates that 3-D QL EM inversion is fast and stable. The inverse results provide reasonable recovery of the models and real geological features.

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