Deep Electromagnetic Exploration
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15. Methods for the Analysis and Interpretation of the Sea Floor Electromagnetic Fields

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1. Introduction

During the last decade sea floor electromagnetic observations have been done in several areas of the World's Oceans. The most exciting experiment has been done in the Pacific ocean close to California in the framework of the EMSLAB project (Booker et al, 1989). As a result of these observations, the sea bottom array data have been collected. It seems very important to process these data in the same way as we process the data of geomagnetic array study on the land. In the latter case, according to the main approach developed in the book by Berdichevsky and Zhdanov (1984), we use the different methods of EM-field space-time analysis, including separation of the normal and anomalous deep and surface parts of the fields and the different methods of the solution of the inverse problem.

This article develops the theory of deep electromagnetic profiling on the sea bottom, analogues to that on the land and is based on three main stages of interpretation: (i) separation of the electromagnetic field at the sea bottom into normal and anomalous parts, (ii) separation of the sea bottom electromagnetic anomalies into the close-to-bottom and deep parts and (iii) inversion of the sea-bottom data.

The main aspects of this theory have been described in our previous publications in Russian (Berdichevsky, Zhdanova and Zhdanov, 1989). It was shown that horizontal components of the magnetic field and the vertical component of the electric field appeared to be the most informative in the ocean bottom study. So, the most interesting case for the sea bottom EM profiling is the case of H-polarization. The detail theory of the inversion of the sea-bottom data for this case is described.

2. Separation of H-polarized Field, Measured at the Bottom of the Sea, into Normal and Anomalous Parts

Let us examine the model of oceanic geoelectric section shown in Fig. 1. In this model a horizontal layer of sea-water of thickness D and with

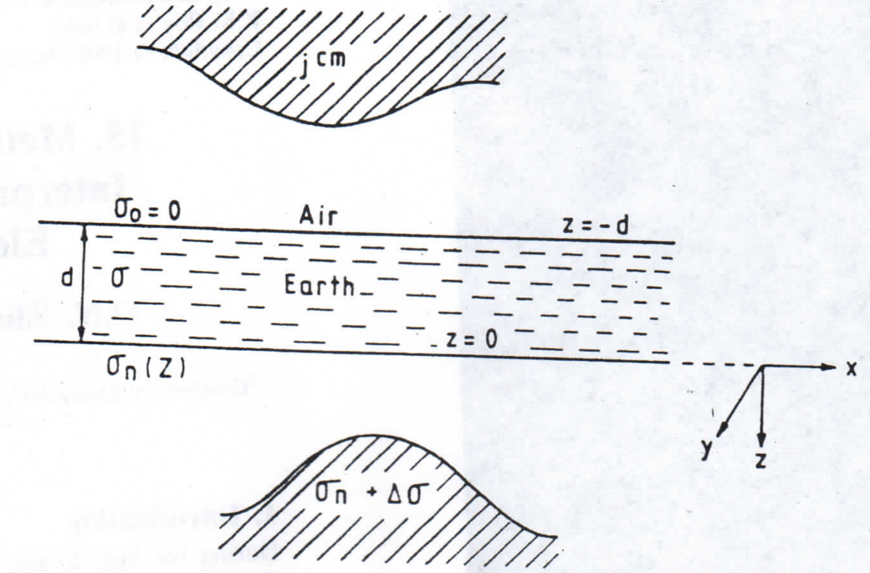

Fig. 1 Model of geoelectrical section used for the separation of EM field into normal and anomalous parts.

electrical conductivity σ separates nonconducting atmosphere ($\sigma_0 = 0$) in which extrinsic current J^{ex} is localized from nonuniform earth with arbitrary two-dimensional conductivity distribution

$$\sigma(X, Z) = \sigma_n(Z) + \Delta\sigma(X, Z)$$

where $\sigma_n(Z)$ is normal electrical conductivity (depends on Z only) and $\Delta\sigma$ is anomalous electrical conductivity that describes geoelectric heterogeneities of the ocean bottom. Plane $Z = 0$ coincides with the bottom of the sea. The problem is formulated in the following way: distribution of electromagnetic field E, H is given at the bottom of the sea (where $Z = 0$); it is necessary to separate this field into normal and anomalous parts.

In accordance with the definition given in Berdichevsky, Zhdanov (1984) normal field is a field E^h, H^h generated by extrinsic current J^{ex} in normal geoelectric section σ_n (in other words, in the absence of anomalous electric conductivity: $\Delta\sigma = 0$). Anomalous E^a, H^a is the part of a field which appears as a result of heterogeneity $\Delta\sigma$, that is, an anomalous field can be determined as the difference between complete and normal fields:

$$H^a = H - H^n; \quad E^a = E - E^n \tag{1}$$

The theory of separation of electromagnetic field into the normal and anomalous parts in three-dimensional and two-dimensional cases (the latter being for E-polarized field) has been developed in a monograph written by Berdichevsky and Zhdanov (1984). Theoretically, the anomalous general theory can also be developed for electromagnetic data detected at the bottom of the sea. In order to separate fields in three-dimensional cases, it is necessary to make array bottom measurements that are now difficult to do because of limited number of special sea instrumentation. In this connection, as it has already been mentioned above, the most interesting, from the standpoint of practical use, is the technique of separating H-polarized fields obtained while profile measurements. In case of H-polarization the task of separating fields E_z, E_x, H_y into normal and anomalous parts becomes much more simple as compared to three-dimensional problem. Indeed, it should be first mentioned that vertical component of electric field E_z is of pure anomalous character when measured at the sea bottom:

$$E_{z|z=0} - E^a_{z|z=0} \tag{2}$$

It is connected with the fact that in normal field within model for only inductional excitation of the earth vertical component of electric field is identically equal to zero everywhere in the earth

$$E^n_z = 0 \tag{3}$$

where $-d \leq Z < +\infty$.

At the same time, in case of H-polarization it is possible to restore the other two components $E^a_x|_{z=-0}$ and $H^a_y|_{z=-0}$ on the basis of one of them, namely, vertical component of anomalous electric field E^a_z, measured at the bottom of the sea, using correlations resulting from Maxwell equations.

To solve this problem let us take the first Maxwell equation written for normal and anomalous fields in the layer of sea-water:

$$\text{rot } H^n = \sigma E^n \tag{4a}$$

$$\text{rot } H^a = \sigma E^a \tag{4b}$$

As it is known, in case of H-polarization the following components of the field are equal to zero:

$$H_x = E_y = H_z = 0$$

Besides, all field components do not change along the axis Y, that is, all derivatives applying to Y are also equal to zero. Hence, equations (4a, b) bring into formulae

$$\partial H^n_y / \partial x = \sigma E^n_z = 0 \tag{5a}$$

$$\partial H^a_y / \partial x = \sigma E^a_z \tag{5b}$$

It follows from formula (5a), in particular, that

$$H^n_y|_{z=\text{const}} = \text{const} \tag{6}$$

that is, normal magnetic field does not alter in the horizontal direction. From the fourth Maxwell equation div $E^n = 0$ and condition (3) it is easily concluded that the field E_x^n is also constant at each level z = const. In other words, in case of H-polarization normal field is described by plane homogeneous electromagnetic wave. Thus, all declinations from a certain constant level of horizontal components of a field in case of H-polarization are determined by anomalous field. This significantly simplifies the task of separating fields into normal and anomalous parts provided that in the limits of observation area, fields get their normal values on the left as well as on the right side. Nevertheless the problem—formulated above—of direct calculating anomalous horizontal component of the field, using measured vertical component E_z at the bottom of the sea, is rather interesting. In order to solve this problem let us pass from electromagnetic field component E_x^a, H_y^a, E_z^a to their spatial spectra

$$h_y^a(z) = \int_{-\infty}^{\infty} H_y^a(X, Z) \exp(iK_x X)\, dx$$

$$e_x^a(z) = \int_{-\infty}^{\infty} E_x^a(X, Z) \exp(iK_x X)\, dx$$

$$e_z^a(z) = \int_{-\infty}^{\infty} E_z^a(X, Z) \exp(iK_x X)\, dx$$

where $-d \le Z$.

Then using spectra, Eq. (5b) can be written as

$$-iK_x h_y^a = \sigma e_z^a$$

Hence
$$h_y^a(-0) = i(\sigma / K_x) e_z^a(-0) \tag{7}$$

Let us now examine the fourth Maxwell equation for anomalous field in the layer of sea water:

$$\operatorname{div} E^a = 0$$

In case of H-polarization this equation according to (5) assumes the following form:

$$\partial E_x^a / \partial x + \partial E_z^a / \partial z = 0 \tag{8}$$

or, using spectra language:

$$e_x^a = -(i / K_x) e_z^{a'} \tag{9}$$

where prime means spectrum differentiation with respect to Z.

Let us calculate the value of vertical derivative of vertical component of the electric field $e_z^{a'}$. In order to do so let us write down Helmholtz equation for anomalous electric field in the layer of sea water

$$\Delta E^a + K^2 E^a = 0 \tag{10}$$

where $K^2 = i\omega\mu_0\sigma$.

In case of H-polarisation this equation is written as follows:

$$\frac{\partial^2 E_x^a}{\partial X^2} + \frac{\partial^2 E_x^a}{\partial Z^2} + K^2 E_x^a = 0 \tag{11a}$$

$$\frac{\partial^2 E_z^a}{\partial X^2} + \frac{\partial^2 E_z^a}{\partial Z^2} + K^2 E_z^a = 0 \tag{11b}$$

Passing to spatial spectra in the latter equation we obtain the following:

$$(e_x^a)'' = \eta^2 e_x^a \tag{12a}$$

$$(e_z^a)'' = \eta^2 e_z^a \tag{12b}$$

$$-d \le Z \le -0$$

where $\eta^2 = K_x^2 - K^2$.

The general solution of Eqs (12a) and (12b) can be expressed in the form of exponents. In particular, for (12b) we have

$$e_z^a(Z) = a \exp(\eta Z) + b \exp(-\eta Z) \tag{13}$$

where a and b are unknown constants that should be easily determined when the values e_z^a at the sea level (when $Z = -d$) and at the bottom (when $Z = -0$) are known

$$e_z^a(Z = -d) = 0; \quad e_z^a(Z = -0) = e_z(-0) \tag{14}$$

Introducing (13) into (14) we obtain a system of two linear equations relative to two undeterminates a and b:

$$0 = a \exp(-\eta d) + b \exp(\eta d)$$

$$e_z(-0) = a + b \tag{15}$$

Solving (15) we obtain

$$a = [\{\exp(\eta d)\}/(2sh\eta d)]e_z(-0)$$

$$b = -[\{\exp(nd)\}/(2sh\eta d)]e_z(-0) \tag{16}$$

Introducing (16) into (13) we, finally, write down the following:

$$e_z^a(Z) = e_z(-0)\,[\{sh\eta(d+z)\}/(sh\eta d)],\ -d \le z \le -0 \tag{17}$$

Differentiating (17) with respect to Z we obtain

$$(e_z^a)' = e_z(-0)\,[\{ch\eta(d+z)\}/(sh\eta d)] \tag{18}$$

In particular, in case of the sea bottom:

$$[e_z(-0)]' = [e_z^a(-0)]' = e_z^a(-0)\eta cth\eta d = e_z(-0)\eta cht\eta d \qquad (19)$$

Now, writing (9) for the case of sea bottom and taking into consideration (19) we, finally, obtain:

$$e_x^a(-0) = -(i/K_x)\, e_z^a(-0) = -ie_z(-0)\,(\eta/K_x)cth\eta d \qquad (20)$$

Thus, formulae (7) and (20) solve the problem of determining spectra of anomalous component of *H*-polarized field on the basis of measurement of vertical electrical field component. In order to determine the anomalous fields themselves it is necessary to make inverse Fourier transforms

$$H_y^a(X_1 - 0) = [1/2\tau_1)] \int_{-\infty}^{+\infty} h_y^a(-0) \exp(-iK_xX)\, dK_x \qquad (21)$$

$$E_x^a(X_1 - 0) = [1/2\tau_1)] \int_{-\infty}^{+\infty} e_x^a(-0) \exp(-iK_xX)\, dK_x \qquad (22)$$

According to (1) normal fields can be determined by simply deducting the anomalous fields from measured complete field components.

3. Separation of Electromagnetic Anomalies Detected at the Bottom of the Sea, into Surface (Near Bottom) and Deep Parts

In practice, when studying geoelectric section of the sea and ocean bottom, it is necessary to bear in mind that anomalies of conductivity usually have surface part connected with heterogeneous layer of bottom sediments and deep part conditioned by the structure of deep layers of the earth's crust and upper mantle. In this connection, at sea as on the earth, it is necessary to separate electromagnetic anomalies detected at the sea bottom into surface (near bottom) and deep parts. The solution of this problem is likely to be possible only if we have information on heterogeneities at the bottom sediment layers.

To formulate this problem mathematically let us complicate the model of geoelectric section given in Fig. 1. Let us assume that the area of anomalous conductivity $\Delta\sigma(X, Z)$ consists of two parts: (i) near bottom heterogeneous Price thin sheet with the conductivity $S(X, Y) = S_1 + \Delta S(X, Y)$ and lying on conducting bottom with normal electrical conductivity $\sigma_n(Z)$. This thin sheet approximates a heterogeneous layer of bottom sediments. (ii) The deep heterogeneity *D* with electrical conductivity $\sigma_d(X, Z) = \sigma_n(Z) + \Sigma\, \sigma(X, Z)$ (Fig. 2).

In this model an anomalous electromagnetic field according to the study (Berdichevsky and Zhdanov, 1984) can be represented as a sum of surface (near bottom) anomaly H^s, E^s (S-component) and deep anomaly H^d, E^d (d-component):

$$H^a = H^s + H^d$$

$$E^a = E^s + E^d$$

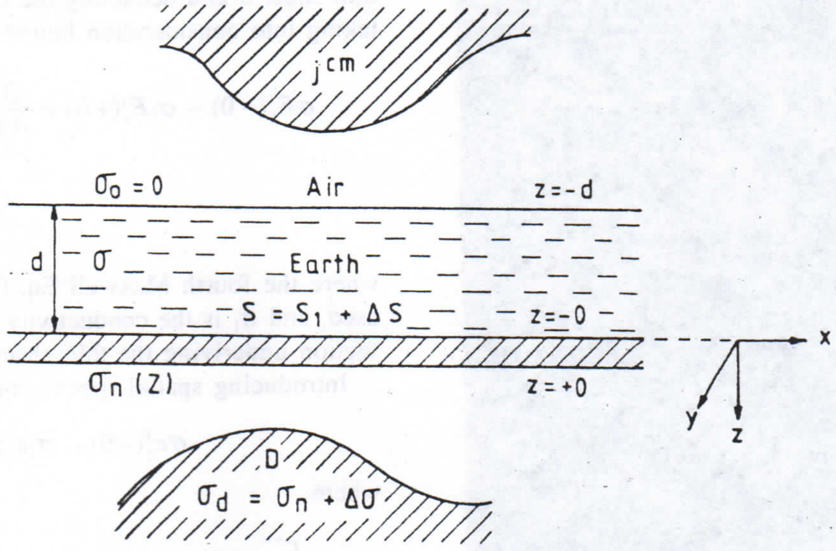

Fig. 2 Model of geoelectrical section used for the separation of the EM anomalies into the surface the deep parts

In this case *d*-component is excited by excess current which flows at $J^d = \Delta\sigma E$ density in the deep heterogeneity and S-component is excited by the excess current distributed at surface density equal to $I^s = \Delta S E_\tau$ in heterogeneous Price thin sheet (E_τ-horizontal component of complete electric field in the thin sheet, that is when $Z = 0$).

S-component everywhere outside the thin sheet satisfies equations

$$\text{rot}\, H^s = \begin{cases} 0 & Z \le -d \\ \sigma E^s & -d \le Z \le -0 \\ \sigma_n E^s & +0 \le Z \le +\infty \end{cases} \tag{22}$$

$$\text{rot}\, E^s = i\omega\mu_0 H^s$$

As it is known for H-polarized field on the thin sheet *S* the following boundary conditions are realised:

$$[E_x^s]_{z=0} = 0 \tag{23a}$$

$$[E_y^s]_{z=0} = S_1 E_x^s|_{z=0} + I_x^s \tag{23b}$$

according to which a horizontal component of electric field is continuous when crossing the thin sheet S, and a magnetic one undergoes the discontinuity that can be determined by surface current density in the thin sheet.

When crossing the thin sheet S, vertical component of magnetic field H_z^s is continuous, and that of the electric field E_z^s is subject to discontinuity, the value of which can be determined with the use of equation (5b). Writing down this equation for the upper ($Z = 0$) and lower ($Z = +0$) sides of the thin sheet S and deducting the equalities term wise one from another and taking into consideration boundary conditions (23b) we obtain:

$$\sigma E_z^s(-0) - \sigma_1 E_z^s(+0) = \frac{\partial}{\partial X}[H_y^s]_{z=0} = S_1 \left.\frac{\partial E_x^s}{\partial X}\right|_{z=0} + \frac{\partial I_x^s}{\partial X}$$

$$= -S_1 \frac{\partial E_z^s}{\partial Z} + \frac{\partial I_x^s}{\partial X} \qquad (24)$$

where the fourth Maxwell Eq. (8) for surface anomalous electric field is used, and σ_1 is the conductivity of the first homogeneous layer of normal section underlying the thin sheet S.

Introducing spatial spectra into (24) we obtain

$$\sigma e_z^s(-0) - \sigma_1 e_z^s(+0) = -S_1 e_z^{s'}\big|_{z=0} - iK_x J_x^s \qquad (25)$$

where

$$J_x^s = \int_{-\infty}^{+\infty} I_x^s(X) \exp(-iK_x X)\, dx = \int_{-\infty}^{+\infty} \Delta S(X) E_x(X) \exp(-iK_x X)\, dx \qquad (26)$$

According to a well known Lipskaya formula (Berdichevsky and Zhdanov, 1984 [8], p. 262) the specter of vertical component of surface electric field e_z^s on the lower side of the thin sheet S is expressed by its vertical derivative:

$$\sigma_1 e_z^s(+0) = (1/Z_n^{gx}) e_z^{s'}|_{z=0} \qquad (27)$$

This formula is true for S-component of electric field because this component, everywhere in the medium below the thin sheet ($Z \geq +0$), satisfies according to (22) the same equation that normal field does. In (27) Z_n^{g*} is a galvanic spectral impedance, which confirms to a normal deep section $\sigma_n(Z)$ and is determined by the following formula:

$$Z_n^{g*} = -(1/\sigma_1)[1/(\bar{R}^*)] \qquad (28)$$

where $$\bar{R}^* = {}^{cth}_{th}\{\eta_1 d_1 + {}^{\mathrm{arcth}}_{\mathrm{arcth}}[(\eta_1/\eta_2)(\sigma_2/\sigma_1) {}^{cth}_{th}\{\eta_2 d_2 + \ldots$$

$$+ {}^{\mathrm{arcth}}_{\mathrm{arcth}}(\eta_{n-1}/\eta_n)(\sigma_N/\sigma_{N-1}) \ldots \}]\} \qquad (29)$$

$$\eta_j = K_x^2 - i\omega\mu_0\sigma_j$$

$\sigma_1, d_1, \sigma_2, d_2, \ldots \sigma_n$ are the parameters of normal section $\sigma_n(Z)$ underlying the thin sheet S.

Introducing (27) into (25) we write down the following:

$$\sigma e_z^s(-0) + e_z^{s'}|_{z=0}(S_1 - 1/Z_n^{g*}) = iK_x J_x^s \tag{30}$$

It should be noted that for the surface anomalies formula (19) is true: it has been obtained for anomalous field e_z^a spectrum, because e_z^s satisfied (in the layer of sea water) the same equations as anomalous fields do:

$$e_z^{s'}|_{z=0} = e_z^s(-0)\eta cth\eta d \tag{31}$$

Introducing (31) into (30) we obtain

$$\sigma e_z^s(-0) + e_z^s(-0)\eta cth\eta d(S_1 - 1/Z_n^{g*}) = -iK_x J_x^s \tag{32}$$

and, using it, finally obtain

$$e_z^s(-0) = -iK_x J_x^s[\sigma - \{(1 - S_1 Z_n^{g*})/Z_n^{g*}\}\eta cth\eta d]^{-1} \tag{33}$$

The spectra of horizontal components of electric e_x^s and magnetic h_y^s fields are determined according to $e_x^s(-0)$ by means of correlations analogous to formulae (7) and (20) for anomalous fields because equations for anomalous field and S-components coincide everywhere above a heterogeneous thin sheet. They are

$$h_y^s(-0) = i(\sigma/K_x)\, e_x^s(-0) = \sigma J_x^s[\sigma - \{(1 - S_1 Z_n^{g*})/Z_n^{g*}\}\eta cth\eta d]^{-1} \tag{34}$$

$$e_x^s(-0) = -i(\eta/K_x)cth\eta d\, e_z^s(-0)$$

$$= -J_x^s\eta cth\eta d[\sigma - \{(1 - S_1 Z_n^{g*})/Z_n^{g*}\}\eta cth\eta d]^{-1} \tag{35}$$

In order to obtain the values of surface anomalous field themselves it is necessary to realise the Fourier inverse transformations according to formulae of (21) type.

Thus, measuring horizontal components of electric field at the bottom of the sea, the distribution of anomalous longitudinal conductivity of bottom sediments being known, it is possible to determine surface component of electromagnetic field at the bottom of the sea. Excluding this component from anomalous field we can find a deep component E_x^d, H_y^d, E_z^d, which give information on deep geoelectric heterogeneities of the ocean's bottom.

4. Solving Inverse Problem of Bottom Electromagnetic Profiling

Ratio (33) can be used for solving the inverse problem of determining the longitudinal conductivity of bottom sediments layer. Indeed, let us assume that there are no deep heterogeneities in the model of geoelectric section (Fig. 2). Then the surface anomaly coincides with complete anomalous field and, in particular, according to (2):

$$e_z^s = e_z^a = e_z \tag{36}$$

Introducing (36) into (33) we obtain

$$J_x^s = (i/K_x)\,[\sigma - \{(1 - S_1 Z_n^{g*})/Z_n^{g*}\}\eta cth\eta d]e_z(-0) \tag{37}$$

Applying the Fourier inverse transforms to the left and right parts of (37) and taking into consideration (26) we write down the following:

$$\begin{aligned}\Delta S(X) &= [2\pi E_x(X)]^{-1} \int_{-\infty}^{+\infty} (i/K_x)\,[\sigma - \{(1 - S_1 Z_n^{g*})/Z_n^{g*}\}\eta cth\eta d]e_z(-0) \\ &\quad \times \exp(-iK_xX)\,dK_x \\ &= [2\pi E_x(X)]^{-1} \int_{-\infty}^{+\infty} (i\sigma e_z(-0)/K_x]\exp(-iK_xX)\,dK_x \\ &= [2\pi E_x(X)]^{-1} \int_{-\infty}^{+\infty} [(i\eta cth\eta d)/(K_x Z_n^{g*})]e_z(-0)\exp(-iK_xX)\,dK_x \\ &\quad + S_1[2\pi E_x(X)]^{-1} \int_{-\infty}^{+\infty} [(i\eta cth\eta d)/K_x]e_z(-0)\exp(-iK_xX)\,dK_x\end{aligned} \tag{38}$$

According to the first Maxwell equation and formulae (7) and (1) we obtain

$$[i\sigma e_z(-0)]/K_x = h_y^a(-0) = H_y(-0) - h_y^n(-0) \tag{39}$$

and following the fourth Maxwell equation and formulae (19) and (1) we have:

$$[i\eta cth\eta d)/K_x]e_z(-0) = -e_x^a(-0) = -e_x(-0) + e_x^n(-0) \tag{40}$$

Introducing (39) into the first integral and (40) into third integral (38) and making the Fourier inverse transforms we obtain:

$$\begin{aligned}\Delta S(X) &= [H_y(X)]/[E_x(X) - [H_y^n|_{z=-0}]/[E_x(X)] \\ &\quad - [2\pi E_x(X)]^{-1} \int_{-\infty}^{+\infty} [(\eta cth\eta d)/(K_x Z_n^{g*})]e_z(-0) \\ &\quad \times \exp(-iK_xX)\,dK_x - S_1 + S_1[E_x^n|_{z=-0}]/[E_x(X)]\end{aligned}$$

Hence we can lay down a formula for calculating summary longitudinal conductivity $S(X)$ of the bottom sediments:

$$S(X) = S_1\Delta S(X) = [H_y(X)]/[E_x(X)] + [S_1 E_x^n - H_y^n]_{z=-0}/[E_x(X)]$$

$$- i[2\pi E_x(X)]^{-1} \int_{-\infty}^{+\infty} [(\eta cth\eta d)/K_x Z_n^{g*})]e_z(-0) \exp(-iK_x X)\, dK_x \quad (42)$$

Let us note that if electromagnetic field variations are measured in the periods' interval corresponding to interval S for normal field, then

$$H_y^n / E_x^n = S_1$$

Hence, the second term in (41) under the integral sign disappears and we have the following:

$$S(X) = [H_y(X)]/[E_x(X)]$$

$$- i[2\pi E_x(X)]^{-1} \int_{-\infty}^{+\infty} [(\eta cth\eta d)/K_x Z_n^{g*})]e_z(-0) \exp(-iK_x X)\, dK_x \quad (43)$$

Thus, the value $S(X)$ being determined by formulae (43), consists of two parts: (i) the main part having sense of the apparent admittance (this part is equal to S_1 in the Tikhonov-Kanjar model), (ii) integral correction which takes into consideration the heterogeneity of Price thin sheet.

Let us examine the integral correction structure in the model with infinite homogeneous conducting bottom with σ_1 conductivity. Then, according to (28) we have

$$Z_n^{g*} = -\eta_1/\sigma_1 \quad (44)$$

where

$$S(X) = H_y/E_x - i[2\pi E_x(X)]^{-1} \int_{-\infty}^{+\infty} [(\eta cth\eta d\sigma_1)/K_x \eta_1)]e_z(-0) \exp(-iK_x X)\, dK_x \quad (45)$$

At last, in case of nonconducting bottom ($\sigma_1 = 0$) the integrated or overall longitudinal conductivity of the bottom sediments layer is determined directly according to the admittance

$$S(X) = [H_y(X)]/[E_x(X)] \,|_{z=-0} \quad (46)$$

Generally speaking, the latter formula can be obtained directly from boundary conditions (23b) and a well known fact is the horizontal component of magnetic field on the roof of insulating bottom is reduced to zero.

$$H_y(X) \,|_{z=+0} = 0$$

Conclusion

1. For the sea bottom electromagnetic anomalies we have the situation that differs from the continental anomalies: the horizontal components of the

magnetic field and the vertical component of the electric field appear to be most informative in the oceanic bottom studies.

2. For the sea bottom data the methods of field separation and inversion could be developed similar to those of interpretation of the continental anomalies.

3. The methods of the sea-bottom data interpretation developed in this paper, are planned to be used for the analysis of the results of the EMSLAB experiment.

References

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