PETROPHYSICS, VOL. 42, NO. 3 (MAY-JUNE 2001); P. 227–236; 8 FIGURES

Induction Logging with Directional Coil Polarizations: Modeling and Resolution Analysis

Arvidas Cheryauka[1], Michael Zhdanov[1], and Motoyuki Sato[2]

ABSTRACT

We present a novel method of formation conductivity estimation based on an induction logging tool with multicomponent transmitters and receivers. We describe a modeling study of the tool prototype with tilted transmitting and receiving coils. Theoretically, this induction system can be considered as a triaxial multisensor array of controlled polarization. The sensitivity analysis and induction tool response simulations are based on dyadic Green's function calculations in the layered media and the integral equation method. We also consider the spatial resolution of induction logging with tilted coils and its ability to delineate formation properties and geometry.

INTRODUCTION

The conventional induction logging method uses a set of transmitter and receiver coils oriented along the borehole axis (Doll, 1949; Moran and Kunz, 1962; Chew and Kleinberg, 1988). Directional electromagnetic scattering with the use of non-axially oriented source and receiving coils can improve the spatial resolution of formation properties and therefore provide an electromagnetic (EM) response suitable for quantitative interpretation. Kaufman and Kagansky in their pioneering paper (1971) theoretically investigated the characteristics of the induction tool response in a borehole with the coils oriented perpendicular to the borehole axis. Sato et al. (1994) studied the ability to determine the dip angle and azimuth mapping in dipping formations. Nekut (1994) proposed multicomponent transient measurements for apparent anisotropy definition. Analysis of directional sensitivities of three-component magnetic data in boreholes within a uniform isotropic medium was conducted recently by Portniaguine and Zhdanov (1999) and by Alumbaugh and Wilt (1999). The foundations of tensor induction logging of anisotropic geological formations using triaxial transmitter and receiver induction tools were outlined in a recent paper by Zhdanov et al. (2001). Kriegshauser et al. (2000) presented an intensive study on the evaluation of parameters of anisotropic formations with a set of mutually orthogonal transmitters and receivers. Cheryauka and Sato (2001) studied the magnetic susceptibility effect using induction measurements with tilted coils. However, substantial work remains to examine the spatial resolution of induction logging with tilted coils and its ability to infer formation properties and geometry.

In the past, directional direct current (DC) tools were employed for resistivity measurements in well logging operations (Davies et al., 1992). There were also some attempts to construct multicomponent induction tools (Sato et al., 1994; Alayrac et al., 1999). During the last decade, a few geophysical equipment companies (Baker Atlas Inc., EMI Inc., IRIS Instruments Inc.) manufactured borehole induction tools with axial transmitters and three-component receivers.

The goal of this paper is to extend existing theoretical and modeling studies for logging tools using a triaxial induction system. We conduct the sensitivity and resolution analysis for a set of typical formation models with horizontal and vertical boundaries. The modified modeling methods for a multidimensional environment combine the simplicity of a conventional Born approximation and the performance of pre-computed analytical steps. For practical situations in well logging, we show that the

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[1]University of Utah, Department of Geology and Geophysics
[2]Tohoku University, Center for Northeast Asian Studies

use of controlled nonsymmetrical transmitting and receiving coil systems increases the resolution of the boundary locations and of the electrical properties of dipping formations.

FORMULATION OF THE EM INDUCTION PROBLEM WITH TILTED COILS

We first consider the basic ideas of tilted coil measurements (Figure 1). In single borehole operations there is of course a limitation in the spatial distribution of the instrument's coils. Locations of induction coils are fixed, and the sensor system can be expanded only along the borehole axis. In this situation, variation of the coil orientation is a powerful method to improve spatial resolution of induction logging in complicated formations.

We study an induction instrument comprising transmitter and receiver coils tilted with respect to the sonde axis (Sato et al., 1994) (Figure 2). Because of the superposition principle, the instrument's response in a multidimensional environment can be represented as a superposition of fields generated by a set of transmitting coils oriented along the basis vectors $\mathbf{i}^1$, $\mathbf{i}^2$, $\mathbf{i}^3$, of Cartesian coordinate system (x_1, x_2, x_3).

We denote as $H_q^p(\mathbf{r}_s|\mathbf{r})$ a magnetic field component of the $\mathbf{i}^q$ polarization ($q = 1,2,3$), measured at the point $\mathbf{r}$ and induced by a magnetic dipole source of the unit moment polarized in the $\mathbf{i}^p$ direction ($p = 1,2,3$), located at the point $\mathbf{r}_s$. Using these notations, a magnetic field generated by the magnetic dipole source of unit moment polarized in the $\mathbf{i}^p$ direction can be written as

$$\begin{aligned}\mathbf{H}^p(\mathbf{r}_s|\mathbf{r}) &= H_1^p(\mathbf{r}_s|\mathbf{r})\mathbf{i}^1 + H_2^p(\mathbf{r}_s|\mathbf{r})\mathbf{i}^2 + H_3^p(\mathbf{r}_s|\mathbf{r})\mathbf{i}^3 \\ &= H_q^p(\mathbf{r}_s|\mathbf{r})\mathbf{i}^q,\end{aligned} \tag{1}$$

where $q = 1,2,3$. Note that in all equations with summations we use the Einstein convention: a twice recurring index indicates summation over this index.

Using the superposition principle, one can express the magnetic field $\mathbf{H}(\mathbf{r}_s|\mathbf{r})$ generated by a combination of magnetic dipoles with different moments, M_1, M_2, and M_3, or

$$\mathbf{M} = M_1\mathbf{i}^1 + M_2\mathbf{i}^2 + M_3\mathbf{i}^3 = M_p\mathbf{i}^p$$

as

$$\begin{aligned}\mathbf{H}(\mathbf{r}_s|\mathbf{r}) &= \mathbf{H}^1(\mathbf{r}_s|\mathbf{r})M_1 + \mathbf{H}^2(\mathbf{r}_s|\mathbf{r})M_2 + \mathbf{H}^3(\mathbf{r}_s|\mathbf{r})M_3 \\ &= \mathbf{H}^p(\mathbf{r}_s|\mathbf{r})\, M_p,\end{aligned} \tag{2}$$

where $p = 1,2,3$. The last formula can also be written as

$$\mathbf{H}(\mathbf{r}_s|\mathbf{r}) = H_q^p(\mathbf{r}_s|\mathbf{r})\mathbf{i}^q\, M_p = H_q(\mathbf{r}_s|\mathbf{r})\mathbf{i}^q, \tag{3}$$

where $H_q(\mathbf{r}_s|\mathbf{r})$ is a magnetic field component of the $\mathbf{i}^q$ polarization ($q = 1,2,3$) generated by the magnetic dipole

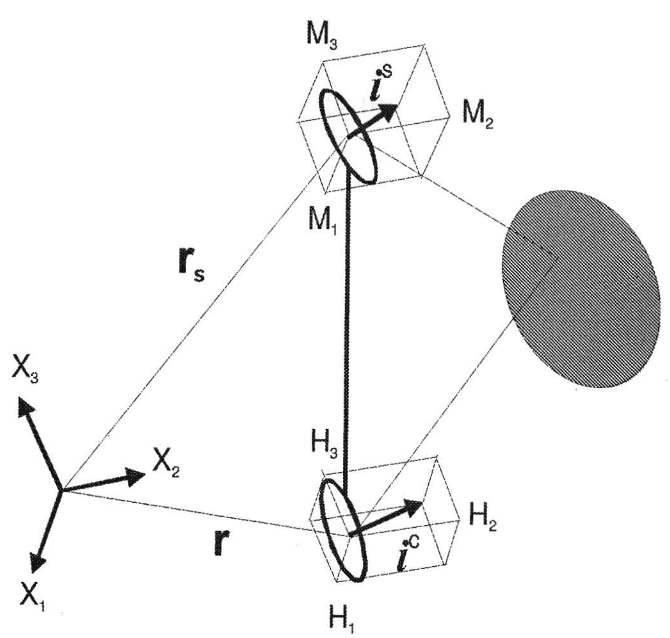

FIG. 1 Directional induction tool with tilted transmitter and receiver coils.

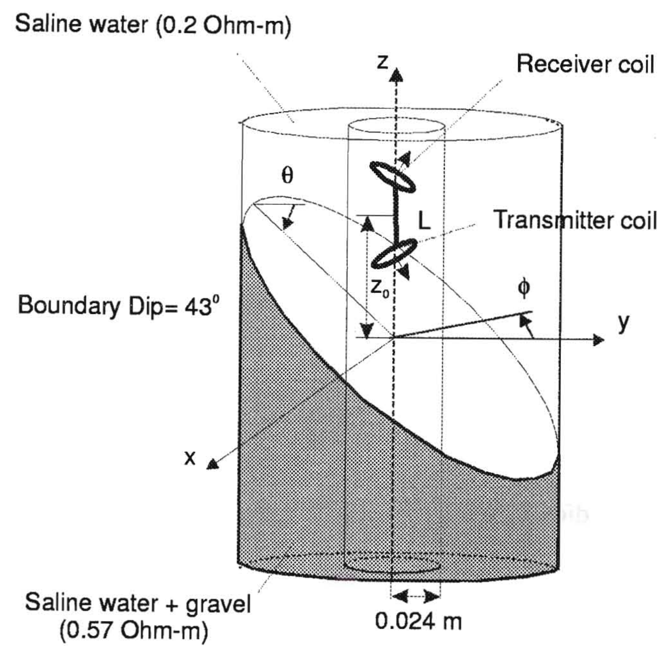

FIG. 2 Two-layer model in a water tank used for laboratory measurements (Sato et al., 1994). The boundary between two layers is dipping at an angle $\theta = 43°$. Azimuthal angle ϕ is changed in horizontal plane. Tool spacing is $L = 0.2$ m, and its midbase location is z_0.

source of the moment $\mathbf{M} = M\mathbf{i}^s$, polarized in the direction of a unit vector is

$$H_q(\mathbf{r}_s|\mathbf{r}) = H_q^p(\mathbf{r}_s|\mathbf{r})\ M_p, \quad p=1,2,3.$$

The matrix formed by the scalar components $H_q^p = H_q^p(\mathbf{r}_s|\mathbf{r})$ can be designated the magnetic induction tensor (or induction matrix), $\hat{\mathbf{H}}(\mathbf{r}_s|\mathbf{r})$, (Zhdanov et al., 2001); H_q^p is independent of the source moment.

Using matrix notation, the vector of the magnetic field can be represented as the multiplication of the magnetic moment $\mathbf{M}$ times the induction matrix. If $\mathbf{M} = M\mathbf{i}^s$ and

$$\hat{\mathbf{H}}(\mathbf{r}_s|\mathbf{r}) = \left[H_q^p(\mathbf{r}_s|\mathbf{r})\right] = \begin{bmatrix} H_1^1 & H_1^2 & H_1^3 \\ H_2^1 & H_2^2 & H_2^3 \\ H_3^1 & H_3^2 & H_3^3 \end{bmatrix} \quad (4)$$

then

$$\mathbf{H}(\mathbf{r}_s|\mathbf{r}) = \hat{\mathbf{H}}(\mathbf{r}_s|\mathbf{r}) \cdot \mathbf{M} = M\hat{\mathbf{H}}(\mathbf{r}_s|\mathbf{r}) \cdot \mathbf{i}^s.$$

Similarly, for the magnetic field component $H^c(\mathbf{r}_s|\mathbf{r})$ of $\mathbf{i}^c$ polarization, measured by the corresponding induction coil, we have

$$H^c(\mathbf{r}_s|\mathbf{r}) = \mathbf{i}^c \cdot \mathbf{H}(\mathbf{r}_s|\mathbf{r}) = M\mathbf{i}^c \cdot \hat{\mathbf{H}}(\mathbf{r}_s|\mathbf{r}) \cdot \mathbf{i}^s. \quad (5)$$

Thus, according to this formal presentation, the solution of the induction problem with tilted coils is based on a multicomponent approach. For instance, to describe multicomponent measurement for a model having two-dimensional axial symmetry, we need four sets of tool signals recorded with orthogonal transmitting and receiving coils. Note that this information can also be obtained by mechanical rotation of the tilted coils as it was presented by Sato et al. (1994).

THEORETICAL SOLUTION IN A LAYERED MODEL

Layered media of plane and cylindrical symmetry are fundamental interpretation models in formation conductivity estimation. Usually, an induction coil can be treated as a point magnetic dipole.

Consider a three-dimensional (3-D) model with a layered distribution of complex electrical conductivity $\tilde{\sigma}_b(\mathbf{r}) = \sigma_b(\mathbf{r}) - i\omega\epsilon_b(\mathbf{r})$ and constant magnetic permeability μ. Let the electromagnetic field, $\mathbf{E}^p$, $\mathbf{H}^p$, in this model be excited by the magnetic dipole source of the unit moment polarized in the $\mathbf{i}^p$ direction and located at a point with the radius vector $\mathbf{r}_s$. Time dependence of the field is $e^{-i\omega t}$. The EM fields in the model satisfy Maxwell's equations

$$\nabla \times \mathbf{H}^p = \tilde{\sigma}_b\, \mathbf{E}^p,$$
$$\nabla \times \mathbf{E}^p = i\omega\mu\, \mathbf{H}^p + i\omega\mu \mathbf{i}^p\, \delta(\mathbf{r} - \mathbf{r}_s). \quad (6)$$

A solution of these equations in integral form is

$$\mathbf{E}^p(\mathbf{r}_s|\mathbf{r}) = i\omega\mu \int_{V\infty} \hat{G}^{ME}(\mathbf{r}'|\mathbf{r}) \cdot \mathbf{i}^p\, \delta(\mathbf{r}' - \mathbf{r}_s)\, d\mathbf{r}'$$
$$= i\omega\mu\, \hat{\mathbf{G}}^{ME}(\mathbf{r}_s|\mathbf{r}) \cdot \mathbf{i}^p,$$
$$\mathbf{H}^p(\mathbf{r}_s|\mathbf{r}) = i\omega\mu \int_{V\infty} \hat{\mathbf{G}}^{MH}(\mathbf{r}'|\mathbf{r}) \cdot \mathbf{i}^p\, \delta(\mathbf{r}' - \mathbf{r}_s)\, d\mathbf{r}'$$
$$= i\omega\mu\, \hat{\mathbf{G}}^{MH}(\mathbf{r}_s|\mathbf{r}) \cdot \mathbf{i}^p, \quad (7)$$

where the magnetic tensor Green's functions of electrical and magnetic types $\mathbf{G}^{ME}$, $\mathbf{G}^{MH}$ satisfy the equations (Zhdanov, 1988, pp. 351–352)

$$\nabla \times \nabla \times \hat{\mathbf{G}}^{ME} - k_b^2 \hat{\mathbf{G}}^{ME} = \nabla \times \hat{\mathbf{I}}\, \delta(\mathbf{r} - \mathbf{r}'),$$
$$\hat{\mathbf{G}}^{MH} = (i\omega\mu)^{-1}\left(\nabla \times \hat{\mathbf{G}}^{ME} - \hat{\mathbf{I}}\, \delta(\mathbf{r} - \mathbf{r}')\right),$$

and where k_b is a wave number, $k_b^2 = i\omega\mu\tilde{\sigma}_b(\mathbf{r})$, and $\hat{\mathbf{I}}$ is a unit tensor.

The tensor Green's functions can also be presented in a dyadic form as

$$\hat{\mathbf{G}}^{ME}(\mathbf{r}_s|\mathbf{r}) = \mathbf{G}^{q^{ME}}(\mathbf{r}_s|\mathbf{r})\, \mathbf{i}^q = G_p^{q^{ME}}(\mathbf{r}_s|\mathbf{r})\, \mathbf{i}^p\, \mathbf{i}^q,$$
$$\hat{\mathbf{G}}^{MH}(\mathbf{r}_s|\mathbf{r}) = \mathbf{G}^{q^{MH}}(\mathbf{r}_s|\mathbf{r})\, \mathbf{i}^q = G_p^{q^{MH}}(\mathbf{r}_s|\mathbf{r})\, \mathbf{i}^p\, \mathbf{i}^q,$$

for $p,q = 1,2,3$.

Thus, the induction matrix (4) can be replaced with the appropriate matrix of the tensor Green's functions

$$\hat{\mathbf{H}}(\mathbf{r}_s|\mathbf{r}) = i\omega\mu\left[G_p^{q^{MH}}(\mathbf{r}_s|\mathbf{r})\right] = \begin{bmatrix} G_1^{1^{MH}} & G_1^{2^{MH}} & G_1^{3^{MH}} \\ G_2^{1^{MH}} & G_2^{2^{MH}} & G_2^{3^{MH}} \\ G_3^{1^{MH}} & G_3^{2^{MH}} & G_3^{3^{MH}} \end{bmatrix}. \quad (8)$$

The theoretical solution for the scalar components of the Green's functions in a piecewise uniform layered medium is well known. Here we just briefly summarize the basic ideas of the numerical solution. Each of the components $G_p^{q^{MH}}(\mathbf{r}_s|\mathbf{r})$ can be determined by the variable separation method and presented in the closed integral form of a generalized Laplace-type transform (Bleistein, 1984)

$$G_p^{q^{MH}}(\mathbf{r}_s|\mathbf{r}) = \int_C U_p^{q^{MH}}(x_1,\lambda)\ V_p^{q^{MH}}(\lambda,x_2,x_3)\ d\lambda, \quad (9)$$

where $\mathbf{r}_s - \mathbf{r} = (x_1, x_2, x_3)$, λ is a separation variable, and C is a contour in a complex plane of variable λ. An optimal contour for fast computation of the integral in equation (9) can be designed by analyzing the analytical properties of the kernel-functions $U_p^{q^{MH}}$ and $V_p^{q^{MH}}$ in a Riemann complex multi-sheet space. Details of these special techniques can be found in the mathematical literature (Brekhovskikh, 1960; Bleistein, 1984).

The solution expressed by equation (9) forms a basis for numerous physical and computation interpretations and also is a subject for approximations using various asymptotic expansion methods.

Following conventional practice, we evaluate a dyadic Green's function for a horizontally layered model in the form of a Bessel-Hankel transform in the cylindrical coordinate system (ρ, ϕ, z)

$$G_p^{q^{MH}}(\rho,\phi,z) = \int_0^\infty [A_p^q(\phi)\, U_p^{q^{MH}}(z,\lambda)\; J_0(\lambda\rho) + B_p^q(\phi)\; V_p^{q^{MH}}(z,\lambda)\; J_1(\lambda\rho)]\, d\lambda. \qquad (10)$$

Here $U_p^{q^{MH}}(z,\lambda)$ and $V_p^{q^{MH}}(z,\lambda)$ are so-called "layered model" functions, J_0 and J_1 are Bessel functions of the orders 0 and 1, and A_p^q and B_p^q are azimuthal coefficients. Computations of the integral in (10) can be done with several elegant numerical techniques using the fast Hankel transform (Anderson, 1979; Christensen, 1990; Mohsen and Hashish, 1994). One of the original codes by Hardman and Shen (1986) can be used for calculation of a conventional induction logging tool signal in a dipping formation. We have extended this code for full electrical and magnetic Green's tensor function computations so that it can be implemented for tilted coil tool response calculation and sensitivity analysis. Also this code was modified to treat the response due to a magnetic anomaly, whereas the original code could calculate only the electrical anomaly response.

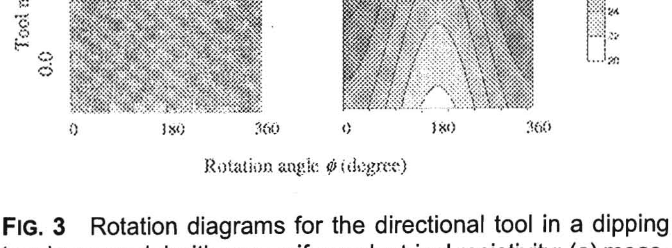

FIG. 3 Rotation diagrams for the directional tool in a dipping two-layer model with non-uniform electrical resistivity: (a) measured, (b) theoretical. Tool specifications: 50 kHz, current – 15 mA, spacing – 0.2 m, coil radius – 0.02 m, number of coil turns – 100.

LABORATORY AND SYNTHETIC DATA

In an earlier paper Sato et al. (1994) proposed a new induction logging tool with increased spatial resolution and based on nonsymmetrical sensors. A prototype tool was fabricated and tested in a scale model formation model. Figure 2 shows the two-layered model, made in a water tank, with the different resistivities of the layers and a dipping bed.

In Figure 3 the measured and theoretical R-responses (the real part of the voltage induced in a sensor coil) are presented in the form of a rotation diagram. The theoretical response is calculated for a one-dimensional (1-D) model using the Bessel-Hankel transform algorithm mentioned earlier. The measured signal agrees well with the theoretical one. Sato et al. (1994) found that an induction tool with tilted coils can evaluate resistivity properties of formations and recognize the orientation of dipping beds. We will study the sensitivity of this tool in the next section.

SENSITIVITY ANALYSIS

In this section we will study the sensitivity of the tilted induction tool to the conductivity anomalies in the medium. Following the monograph (Zhdanov and Keller, 1994, pp. 670–671), a generalized sensitivity problem in three dimensions can be examined based on perturbation of the Maxwell's equations (6)

$$\begin{aligned} \nabla \times \delta\mathbf{H}^p &= \tilde{\sigma}_b\, \delta\mathbf{E}^p + \delta\tilde{\sigma}_b\, \mathbf{E}^p, \\ \nabla \times \delta\mathbf{E}^p &= i\omega\mu\, \delta\mathbf{H}^p \end{aligned} \qquad (11)$$

where δ denotes the variational (perturbation) operator. We consider the perturbation of complex electrical conductivity $\tilde{\sigma}_b(\mathbf{r})$ within some volume δV_σ of the background medium. Magnetic permeability μ is not perturbed.

On the basis of the integral equation approach, we can represent the variations of electromagnetic fields $\delta\mathbf{E}^p$, $\delta\mathbf{H}^p$ as follows

$$\begin{aligned} \delta\mathbf{E}^p(\mathbf{r}_s|\,\mathbf{r}) &= \int_{\delta V_\sigma} \hat{\mathbf{G}}^{JE}(\mathbf{r}'|\,\mathbf{r}) \cdot \delta\tilde{\sigma}_b(\mathbf{r}')\; \mathbf{E}^p(\mathbf{r}_s|\,\mathbf{r}')\, d\mathbf{r}' \\ \delta\mathbf{H}^p(\mathbf{r}_s|\,\mathbf{r}) &= \int_{\delta V_\sigma} \hat{\mathbf{G}}^{JH}(\mathbf{r}'|\,\mathbf{r}) \cdot \delta\tilde{\sigma}_b(\mathbf{r}')\; \mathbf{E}^p(\mathbf{r}_s|\,\mathbf{r}')\, d\mathbf{r}', \end{aligned} \qquad (12)$$

where electric tensor Green's functions of electrical and magnetic types $\hat{\mathbf{G}}^{JE}$, $\hat{\mathbf{G}}^{JH}$ satisfy the equations (Zhdanov, 1988, pp. 351–352)

$$\begin{aligned} \nabla \times \nabla \times \hat{\mathbf{G}}^{JE} - k_b^2 \hat{\mathbf{G}}^{JE} &= i\omega\mu \hat{\mathbf{I}}\, \delta(\mathbf{r} - \mathbf{r}') \\ \hat{\mathbf{G}}^{JH} &= (i\omega\mu)^{-1}\, \nabla \times \hat{\mathbf{G}}^{JE}\,. \end{aligned}$$

According to formulae (12), (2) and (5), the variation of the magnetic response measured by an induction sensor (located at a point $\mathbf{r}$ and oriented in the direction $\mathbf{i}^c$ with respect to conductivity perturbation within the volume δV_σ is

$$\delta H^c(\mathbf{r}_s|\mathbf{r}) = \mathbf{i}^c \cdot \delta \mathbf{H}^p(\mathbf{r}_s|\mathbf{r})\, M_p = \mathbf{i}^c \cdot M^p \int_{\delta V_\sigma} \hat{\mathbf{G}}^{JH}(\mathbf{r}'|\mathbf{r}) \cdot \delta\tilde{\sigma}_b(\mathbf{r}')\, \mathbf{E}^p(\mathbf{r}_s|\mathbf{r}')\, d\mathbf{r}'. \tag{13}$$

The electrical field $\mathbf{E}^p(\mathbf{r}_s|\mathbf{r}')$ generated by a magnetic dipole, according to equation (7), is the product of a magnetic dyadic Green's function and a unit vector $\mathbf{i}^p$

$$\mathbf{E}^p(\mathbf{r}_s|\mathbf{r}') = i\omega\mu\, \hat{\mathbf{G}}^{ME}(\mathbf{r}_s|\mathbf{r}') \cdot \mathbf{i}^p. \tag{14}$$

Using reciprocity relations between the tensor Green's functions, we obtain

$$\hat{\mathbf{G}}^{JH}(\mathbf{r}_1|\mathbf{r}_2) = -(\hat{\mathbf{G}}^{ME}(\mathbf{r}_2|\mathbf{r}_1))^{\mathrm{T}}, \tag{15}$$

where the superscript T denotes the operation of transposition.

Now, according to equations (13) and (15), we can calculate the Frechet derivative at the arbitrary inner point $\mathbf{r}' \in \delta V_\sigma$

$$\begin{aligned} \frac{\partial H^c(\mathbf{r}_s|\mathbf{r})}{\partial \tilde{\sigma}_b(\mathbf{r}')} &= \mathbf{i}^c \cdot M_p \hat{\mathbf{G}}^{JH}(\mathbf{r}'|\mathbf{r}) \cdot \mathbf{E}^p(\mathbf{r}_s|\mathbf{r}') \\ &= -i\omega\mu \mathbf{i}^c \cdot M_p (\hat{\mathbf{G}}^{ME}(\mathbf{r}|\mathbf{r}'))^{\mathrm{T}} \cdot (\hat{\mathbf{G}}^{ME}(\mathbf{r}_s|\mathbf{r}') \cdot \mathbf{i}^p) \\ &= -i\omega\mu M_p (\hat{\mathbf{G}}^{ME}(\mathbf{r}_s|\mathbf{r}') \cdot \mathbf{i}^p) \cdot (\hat{\mathbf{G}}^{ME}(\mathbf{r}|\mathbf{r}') \cdot \mathbf{i}^c), \end{aligned} \tag{16}$$

where we use the tensor identity $\mathbf{a} \cdot (\hat{\mathbf{B}} \cdot \mathbf{c}) = \mathbf{c} \cdot (\hat{\mathbf{B}}^{\mathrm{T}} \cdot \mathbf{a})$.

Formula (16) gives a general solution for the sensitivity study of the triaxial induction tool in the layered model.

We present here the sensitivity plots of a two-coil induction probe response in a two-layer medium and two particular cases of a well deviation: vertical and horizontal holes. The presence of a borehole space is ignored. For the vertical borehole (Figure 4) and axial symmetry of the formation, we consider the distribution of axial sensitivity of an imaginary magnetic field component

$$\rho' \int_0^{2\pi} \frac{\partial \mathrm{Im}\,[H^c(\phi')]}{\partial \sigma}\, d\phi'.$$

The two-coil probe of 1 m spacing moves through the horizontal interface. We calculate the sensitivity plots at three midbase positions: – 0.75 m (i), 0.0 m (ii), and + 0.75 m (iii). Transmitter and receiver axes are tilted at 0° (vertical magnetic dipole–VMD), 45°, and 90° (horizontal magnetic dipole–HMD). The medium comprises an upper half-space ($z < 0$ m) with a resistivity of 20 ohm-m and a lower half-space ($z > 0$ m) with a resistivity of 1 ohm-m. The operating frequency is 100 kHz. Despite a resistivity contrast of 20, the VMD array has very smooth sensitivity with respect to profiling through the horizontal interface (Figure 4a), because it collects signals from circular currents only (induction mode). Arrays with tilted coils including HMD generate both induction and galvanic modes which produce the discontinuous vertical electrical field component and generate surface charges on the boundary (Figure 4b-c). The sensitivity plots for these arrays show discontinuous behavior in the vicinity of the point $z = 0$ m (at the depth of the boundary). This result demonstrates that induction logging with tilted coils is more sensitive to the bed position than conventional axial dipole instrumentation.

Figures 5a-c show the $(x,0,z)$ cross-sections of spatial sensitivity plots for the same formation model considered above but with a horizontal borehole. The tool axis is parallel to the horizontal interface ($z = 0$ m). Induction arrays are located at three depths: $z = -0.5$ m (i), $z = -0.01$ m (ii), and $z = +0.5$ m (iii). In this situation the

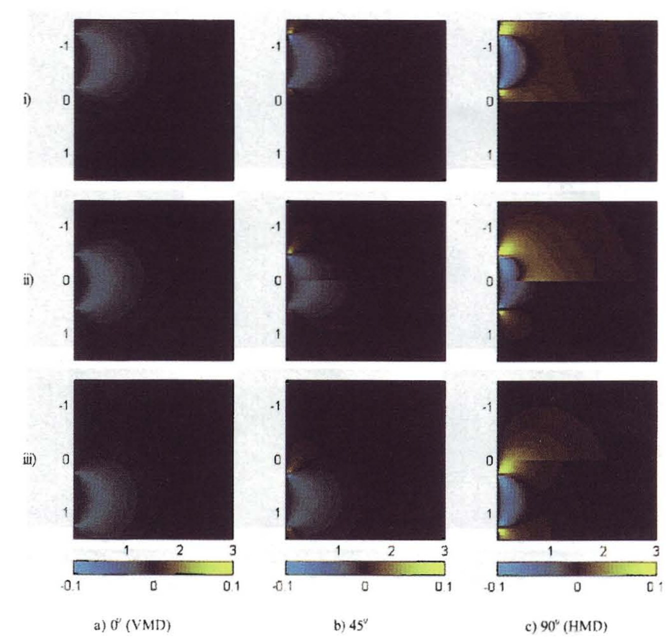

FIG. 4 Axial sensitivity plots (ρ, 0, z plane subsection, m) of two-coil induction probe response (imaginary part, A/m^2) in the two-layer model: upper halfspace ($z < 0$ m) of 20 ohm-m, lower halfspace ($z > 0$ m) of 1 ohm-m, boundary at $z = 0$ m. In vertical borehole, $r = 0$ m, tool midbase is located at z-points –0.75 m (i), 0.0 m (ii), and 0.75 m (iii). Transmitter and receiver coil axes are tilted at 0° (a), 45° (b) and 90° (c). Operating frequency is 100 kHz.

HMD array (Figure 5c) only excites an eddy current and becomes nonsensitive to the properties of the lower half-space and the boundary location. At the same time, the arrays with tilted coils with respect to horizontal boundary (Figure 5a-b) have a nonsymmetrical spatial field distribution and show higher resolution ability. Here all sensitivity plots have continuous distributions due to the presense of a tangential electrical field only in the $(x,0,z)$ plane.

THIN BED RESOLUTION

In this section we consider synthetic induction logs through a three-layered formation and study the thin bed resolution. A model of the layered formation (Figure 6a) represents a single hydrocarbon-saturated resistive layer of 20 ohm-m resistivity within a uniform conductive host (2 ohm-m). We vary the thickness of the resistsive layer from 2 m to 0.5 m and study the behavior of the synthetic logs. The induction tool comprises a conventional three-coil configuration. The 2-coil sub-arrays have sets of spacing and moments of 1.5 m, 1 A-m^2 and 1 m, – 0.2963 A-m^2, and operate at a frequency of 50 kHz. In the case of resistive borehole mud, we can ignore the borehole without great loss of accuracy. Figures 6(b,c,d) show the log profiles measured by the induction tool with tilted coils at 0, 45 and 90 degrees correspondingly. The transmitter is placed above the receivers. The depth log is measured with respect to midpoint position of the receiver coils. Curves of the same structure are related to the specific thickness of the resistive layer: solid line⇔2 m layer, dashed line⇔1 m and dotted line⇔0.5 m. The location of the top interface at $z = 0$ m can be recognized by all three types of arrays. At the same time, the bottom of the layer, especially for thin bed cases of 1 and 0.5 m, is clearly evidenced in the responses of the tilted coil induction array only.

MODELING IN INVADED FORMATIONS

The penetration of the borehole mud into a porous formation leads to the appearance of an invaded zone. The resistivity value of the invaded region with mixed mud and formation fluid content is a function of their resistivities and other factors. We simulate induction logging using tilted sensors in a vertical borehole for a model with axial symmetry of the electrical property distribution.

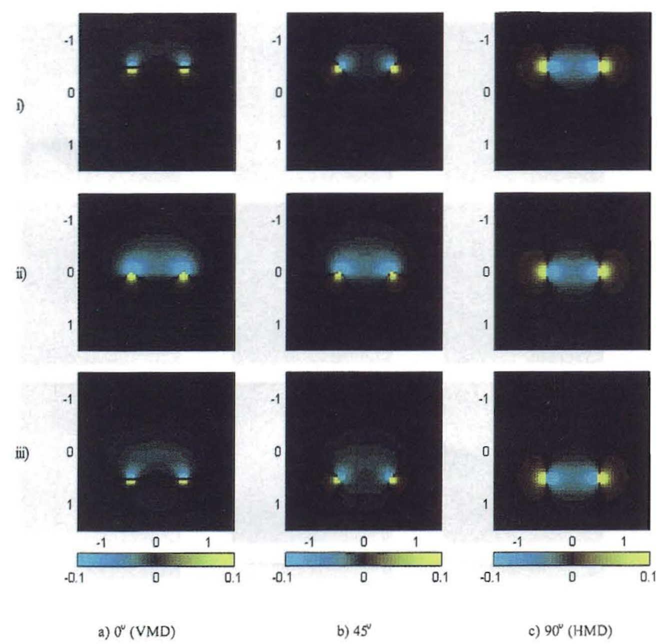

FIG. 5 Sensitivity plots (x,0,z plane section, m) of two-coil induction probe response (imaginery part, A/m^2) in the two layer model: upper half-space ($z < 0$ m) of 20 ohm-m, lower halfspace ($z > 0$ m) of 1 ohm-m, boundary at $z = 0$ m. In horizontal borehole, tool midbase is located at $x = 0$ m and at z–points – 0.5 m (i), – 0.01 m (ii) and + 0.5 m (iii). Their axes are tilted at 0° (a), 45° (b) and 90° (c). Operating frequency is 100 kHz.

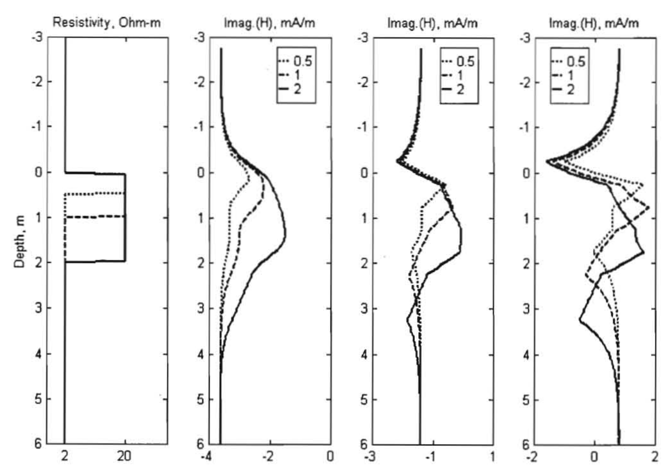

FIG. 6 Three-coil tool response in a uniform conductive host (2 ohm-m) with a single layer (20 ohm-m). The position of the upper boundary is fixed ($z = 0$ m) while the lower boundary of the layer is varied from 2 m to 0.5 m. Two-coil subarray spacings and dipole moments are: 1.5 m, 1 A-m^2 and 1 m, – 0.2963 A-m^2. Frequency is 50 kHz. Curves in (b,c,d) plots correspond to tool responses with the coils oriented at 0, 45 and 90 degrees. Log depth is at the midpoint location of the tool base. Curves of the same structure correspond to the specific layer thickness: solid line for 2 m, dashed line for 1 m and dotted line for 0.5 m.

Our goal is to study the resolution of three-coil tool responses with respect to the lateral changes in the formation and the vertical boundaries. Modeling for this 2.5-dimensional situation (2-D axial model and tilted magnetic dipoles) is based on the use of an integral equation method. Full integral equation solution is a rigorous method, but it requires significant computational resources.

We implemented an approximate approach by using the conventional Born technique and a complicated (1-D layered) background model. The linear Born approximation gives a good result when a resisitivity contrast is low. For high formation-invaded zone contrasts nonlinear approximations can be used for efficient simulations of an induction log. We address an interested reader to the papers observing various nonlinear estimations, for instance, by Zhdanov et al., (2000) and Cheryauka and Zhdanov, (2001). The analytical solutions for the Born approximations are actually provided by equation (12). We assume also that we have resistive mud, a large ratio between sub-array spacings and the borehole radius, and low resistivity contrast at invaded zones. The first and second assumptions effectively allow avoidance of the borehole influence on the response of the instrument.

Using equations (12), (14), (15), and (16), the linear combination of the magnetic fields measured by the three-coil array with one transmitter and two receivers (with the induction coils oriented in the same direction, $\mathbf{i}^c$, and with the moments C_1 and C_2) in the axially symmetrical model can be represented as

$$H^c(z_s|z_1,z_2) = H_b^c(z_s|z_1,z_2) - i\omega\mu \int_{\delta V_\sigma} \delta\tilde{\sigma}(\rho',z')\, F^c(z_s,z_1,z_2,\rho',z')\, \rho' d\rho'\, dz'$$

where

$$F^c = M_p \int_0^{2\pi} \left(\hat{\mathbf{G}}_b^{ME}(z_s|\rho',z',\phi')\cdot \mathbf{i}^p\right)\cdot \left([C_1 \hat{\mathbf{G}}_b^{ME}(z_1|\rho',z',\phi') + C_2 \hat{\mathbf{G}}_b^{ME}(z_2|\rho',z',\phi')]\cdot \mathbf{i}^c\right) d\phi'. \quad (17)$$

Function F^c can be evaluated analytically because the Green's tensor function $\hat{\mathbf{G}}^{ME}$ for the one-dimensional layered background has explicit dependence on the azimuthal angle ϕ.

Figure 7(a-c) shows the typical invaded formation model and modeling results calculated for induction arrays with tilted coils with 0 (VMD) and 90 (HMD) degrees. This benchmark model (Figure 7a) consists of a conductive host of 2 ohm-m and three productive layers of 20 ohm-m and thicknesses of 2, 1, and 0.5 m (solid line), which have invaded zones of 4 ohm-m (dashed line). Horizontal boundaries are located at depths of $z =$ 0, 2, 4, 5, 6 and 6.5 m. The invaded zone radius is equal to 0.4 m. Tool parameters are the same as in the previous paragraph. Figure 7b-c shows the difference between responses in the non-invaded background formation (solid line) and the invaded one (dashed line). For testing we also show the accurate and independent result calculated with a finite difference algorithm (Dr. S. Martakov, personal communication) ('+' markers). Our approximate solution coincides, within graphical accuracy, with the finite difference result. As can be seen from the modeling results, the VMD tool signal does not show a visible difference indicating the existence of invaded zones in the layers (Figure 7b). In contrast to conventional logs, the tool with horizontally tilted sensors can exhibit a significant difference between responses over non-invaded and invaded formations.

Figure 8 presents synthetic logs for a set of tilted coil tools over the model shown in Figure 7. The curves with different markers correspond to tilted angles of 0,15,30, 45,60,75, and 90 degrees appropriately. The sharpness of the logs gradually increases from the VMD tool response to the HMD tool. Taking into account the resolution and complexity of interpretation processing, we may note that the optimal angle for tilting of transmitting and receiving coils lies in the 30–45 degree range.

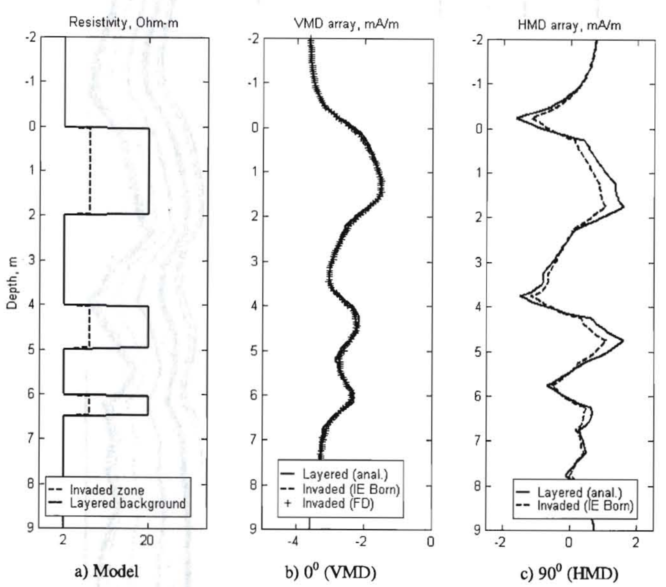

FIG. 7 Comparison of the synthetic logs for an invaded formation model. This model (a) consists of a conductive host (2 ohm-m) and three productive layers (20 ohm-m). Layers are 2, 1, 0.5 m in thickness (solid line) and have invaded zones with a resistivity of 4 ohm-m and radius at 0.4 m (dashed line). Horizontal boundaries are located at the depths z = 0, 2, 4, 5, 6, and 6.5 m. Logs are calculated for arrays with the coils at 0 (b) and 90 (c) degrees. Tool parameters are the same as in Figure 6.

DISCUSSION

Instrument responses, in general, are functions of the joint influence of the induction mode (eddy currents) and the galvanic mode (electrical charges). Only the field generated by eddy currents can be satisfactorily simulated using conventional computational techniques widely used in induction logging and based on geometrical factor. It can provide very fast computations for a wide variety of formation and tool system parameters. However, as we have shown here for the vertical borehole, the induction field component is not highly sensitive to the location of a horizontal boundary nor to the position of the invaded zone boundary. In contrast to the induction mode, tool responses generated by the galvanic mode are much more sensitive to changes in formation properties. Electrical charge distribution on formation interfaces (due to a discontinuous normal field) and within a volume (due to formation macroanisotropy) has a complicated structure. It is difficult to calculate these electromagnetic phenomena with sufficient accuracy using approximate methods. In this paper we did not consider or numerically evaluate the influence of a borehole space on an induction tool response. The three-coil or dual frequency array designs can be used to cancel the borehole effect as well as distortions from the borehole rugosity and displacement of the tool from the borehole axis. This is a common feature of existing induction tools. Obviously, these problems and, also, the development of efficient approximation techniques are the subjects of future research.

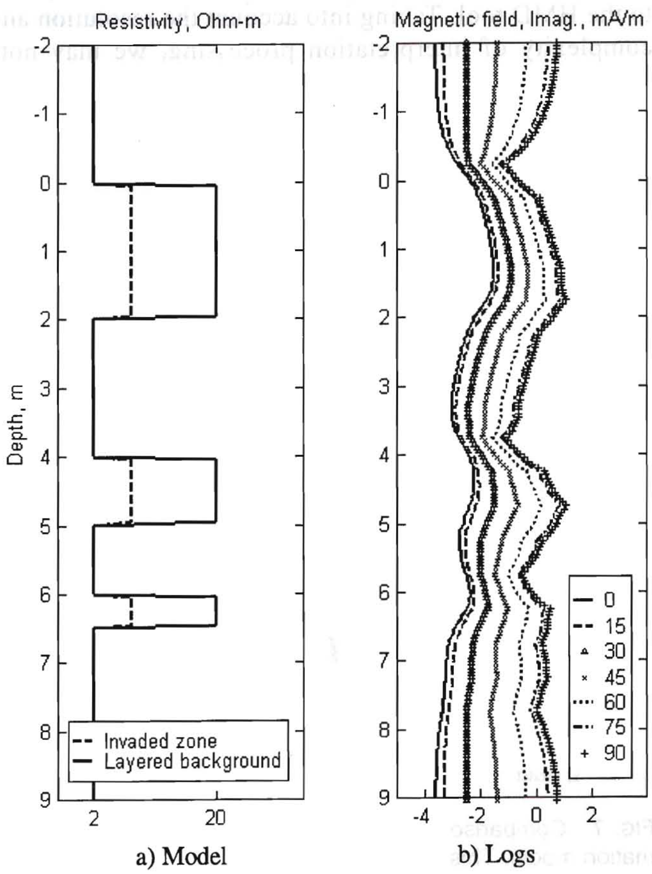

FIG. 8 Synthetic logs simulated by three-coil tools with tilted sensors in the invaded formation. Curves of the same structure correspond to the specific tilted coil angle, which varies from 0 to 90 degrees. The model and tool parameters are the same as in Figures 6 and 7.

CONCLUSIONS

We have conducted a numerical study of electromagnetic logging with tilted coils to consider the sensitivity and spatial resolution properties of this new tool. The tilted coil tool response is modeled as a multicomponent transmitter-receiver induction system.

We have considered the response characteristics of tilted coil tools in a set of standard logging models which include horizontally layered formations and axially invaded formations. Results of the sensitivity analysis can be widely used for understanding the physics of induction in inhomogeneous environments and for designing effective focusing or compensated tools. Sensitivity analysis with respect to dielectric permittivity and magnetic permeability can also be of importance in well logging operations.

Based on the integral equation approach, we introduced a fast numerical implementation of the Born approximation method for the models with axial symmetry, which reduces significantly the computational time in the 2.5-D axial symmetrical excitation case. We plan to apply this algorithm feature to the numerical simulation of dipping circular borehole and azimuthally uniform invaded zone models.

Resolution study of the tool responses in inhomogeneous formations has shown the limitations of conventional induction devices in recognizing boundary location and electrical property changes. We showed that the use of a multicomponent induction system can improve the resolution of the induction method in finely layered formations as well as in formations with deeply invaded zones.

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ABOUT THE AUTHORS

Arvidas B. Cheryauka received his MS degree in 1986 from Novosibirsk State University, the PhD degree in 1989 from Institute of Geology and Geophysics, Novosibirsk Scientific Center, Russia, both in geophysics.

He joined the Institute of Geology and Geophysics in 1989, where he was senior research scientist. During 1998-1999, he was as guest researcher with Tohoku University in Sendai, Japan. Currently, he is research associate at the Consortium for Electromagnetic Modeling and Inversion, University of Utah. His research interests include electromagnetic diffusion and wave propagation in inhomogeneous media, numerical analysis and inversion for subsurface sensing, tool design and interpretation.

Dr. Cheryauka was awarded Outstanding Young Scientist's Award in 1988 and Applied Technology Project's Award in 1992, both from Russian Academy of Sciences.

Michael S. Zhdanov is Professor of Geophysics, Director of the Consortium for Electromagnetic Modeling and Inversion (CEMI). He received his MS in geophysics from Moscow Gubkin State University of Oil and Gas in 1968, MS in mathematics in 1969 and PhD in physics and mathematics in 1970 both from Moscow State University.

He was previously the Director of the Institute for Electromagnetic Research Studies, Moscow. Dr. Zhdanov is Honorary Gauss Professor, Gettingen Academy of Sciences, Germany, Full Member of Russian Academy of Natural Sciences, Honorary Professor of China National Center of Geological Exploration Technology.

His main research interest is developing methods for the solution of forward and inverse problems in geophysics, new techniques for electromagnetic imaging of the earth's interior. His approach includes theoretical study of model data and analysis of actual geophysical data, numerical modeling and inversion.

Motoyuki Sato received his BS, MS and Dr.Eng. degrees in information engineering from Tohoku University, Sendai, Japan, in 1980, 1982 and 1985, respectively.

He has been with the Department of Resource Engineering, Faculty of Engineering, Tohoku University, since 1985. Currently he is professor at the Center for Northeast Asian Studies, Tohoku University. During 1988-1989, he was with the Federal Institute for Geoscience and Natural Resources (BGR) in Hannover, Germany as a visiting researcher. His current interests include transient electromagnetics and antennas, radar polarimetry, borehole radar, electromagnetic logging, multi component induction logging, seismic and electromagnetic field analysis and signal processing.

Dr. Sato was awarded "Best Poster" of the SPWLA 1994 Tulsa Symposium, Tulsa, OK, and the best paper of the SEG of Japan in 1999. He served as Technical Chairman of the 6th International Conference on Ground Penetrating Radar (GPR96).