Integral Electric Current Method in 3-D Electromagnetic Modeling for Large Conductivity Contrast

Michael S. Zhdanov, Vladimir I. Dmitriev, and Alexander V. Gribenko

***Abstract*—We introduce a new approach to 3-D electromagnetic (EM) modeling for models with large conductivity contrast. It is based on the equations for integral current within the cells of the discretization grid, instead of the electric field or electric current themselves, which are used in the conventional integral-equation method. We obtain these integral currents by integrating the current density over each cell. The integral currents can be found accurately for the bodies with any conductivity. As a result, the method can be applied, in principle, for the models with high-conductivity contrast. At the same time, knowing the integral currents inside the anomalous domain allows us to compute the EM field components in the receivers using the standard integral representations of the Maxwell's equations. We call this technique an integral-electric-current method. The method is carefully tested by comparison with an analytical solution for a model of a sphere with large conductivity embedded in the homogenous whole space.**

***Index Terms*—Electromagnetic (EM) modeling, high conductivity contrast, integral equations.**

I. Introduction

One of the difficult problems in electromagnetic (EM) modeling is accurate numerical solution for models with large conductivity contrast. This problem appears, for example, in modeling EM data for mineral exploration when we have a conductive target embedded in relatively resistive host rocks. The study of the topography effect on EM data requires the solution of a similar problem, because the contrast in conductivity between the conductive earth and nonconductive air can be as large as 10^8-10^{10} times. Well-logging is another area where one should take into account a strong contrast between the cased borehole, for example, and surrounding rock formations.

In this paper, we introduce a new approach to the solution of this problem based on the integral-equation (IE) method. The basic principles of the IE method were outlined in the pioneer papers [8], [9], [12], [17], [18], and [22]. Over decades, these methods were further developed and improved in a large number of publications (see, for example, [2], [6], [7], [11], [14], [19], [21], [23], [24], and [26]). However, most existing IE methods fail for large conductivity contrast, because they use the boxcar basis functions to approximate the electric field within the conductive body [10], [11], [16], [19]. The development of accurate EM modeling methods for the models with large conductivity contrast is considered one of the most challenging problems in EM geophysics.

The conventional IE algorithms are usually written for the electric field or electric current components within the domain with anomalous conductivity. This domain is divided in the number of cells, which are selected to be so small that the field components vary slowly within the cell. If the conductivity of the body and/or frequency are high, it is difficult to satisfy this condition. The EM field varies extremely fast within a good conductor, which may result in errors of numerical modeling. In order to overcome this difficulty, Newman and Hohman used a special grouping of the boxcar basis functions to form current loops within the conductor [16]. Farquharson and Oldenburg implemented the more sophisticated edge element basis functions to avoid the inaccuracy of the conventional boxcar basis function approach [10].

In this paper, we consider a novel approach for solving this problem. We develop a new form of the IE method, which is based on the equations for integral current within the cells, instead of the electric field or electric current themselves. We obtain these integral currents by integrating the current density over each cell. The integral currents can be found accurately for a body with any conductivity. We do not use anymore the requirements that the field varies slowly inside the cell, because we deal with the integral of this field. As a result, the method can be applied, in principle, for models with arbitrary conductivity contrast. At the same time, knowing the integral currents inside the anomalous domain allows us to compute the EM field components in the receivers using the standard integral representations of Maxwell's equations. We call this technique an integral-electric-current (IEC) method. We will present below the detailed description of the IEC method and will illustrate it by numerical modeling results.

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II. Formulation of the IE Method

We consider, first, the basic IEs of 3-D EM forward modeling, written for the total electric $\mathbf{E}$ and magnetic $\mathbf{H}$ fields

$$\mathbf{E}(\mathbf{r}') - \iiint_D \widehat{\mathbf{G}}_E(\mathbf{r}'|\mathbf{r}) \cdot [\Delta\widetilde{\sigma}(\mathbf{r})\mathbf{E}(\mathbf{r})]\, dv + \mathbf{E}^{\mathrm{b}}(\mathbf{r}') = \mathbf{G}_E[\Delta\widetilde{\sigma}(\mathbf{r})\mathbf{E}(\mathbf{r})] + \mathbf{E}^{\mathrm{b}}(\mathbf{r}') \quad (1)$$

$$\mathbf{H}(\mathbf{r}') = \iiint_D \widehat{\mathbf{G}}_H(\mathbf{r}'|\mathbf{r}) \cdot [\Delta\widetilde{\sigma}(\mathbf{r})\mathbf{E}(\mathbf{r})]\, dv + \mathbf{H}^{\mathrm{b}}(\mathbf{r}') - \mathbf{G}_H[\Delta\widetilde{\sigma}(\mathbf{r})\mathbf{E}(\mathbf{r})] + \mathbf{H}^{\mathrm{b}}(\mathbf{r}') \quad (2)$$

where $\widehat{\mathbf{G}}_E(\mathbf{r}_j|\mathbf{r})$ and $\widehat{\mathbf{G}}_H(\mathbf{r}_j|\mathbf{r})$ are the electric and magnetic Green's tensors defined for an unbounded conductive medium with the complex background conductivity $\widetilde{\sigma}_{\mathrm{b}} = \sigma - i\omega\epsilon$; $\mathbf{G}_E$ and $\mathbf{G}_H$ are corresponding Green's linear operators; and $\mathbf{E}^{\mathrm{b}}$, $\mathbf{H}^{\mathrm{b}}$ are the background electric and magnetic fields; domain D corresponds to the volume with the anomalous conductivity distribution $\widetilde{\sigma}(\mathbf{r}) = \widetilde{\sigma}_{\mathrm{b}} + \Delta\widetilde{\sigma}(\mathbf{r})$, $\mathbf{r} \in D$.

Equation (1) written for the points $\mathbf{r}'$ located inside domain D, $\mathbf{r}' \in D$, gives us an IE with respect to electric field $\mathbf{E}(\mathbf{r})$. The main problem is to solve this IE.

The conventional approach to discretization of the integral (1) is based on dividing domain D into N elementary cells, D_n, formed by some rectangular grid in the domain $D - \cup_{n-1}^{N} D_n$, and assuming that $\Delta\widetilde{\sigma}(\mathbf{r})$ has the constant value $\Delta\widetilde{\sigma}_n$ within the cell. Note that the coefficients $\Delta\widetilde{\sigma}_n$ can be represented as the components of a vector $\boldsymbol{\sigma}$ of the order N

$$\sigma = [\Delta\widetilde{\sigma}_1, \Delta\widetilde{\sigma}_2, \ldots, \Delta\widetilde{\sigma}_N]^{\mathrm{T}}$$

where superscript "T" denotes transposition.

We also assume that each cell D_n is so small that the electric field is approximately constant within the cell, $\mathbf{E}(\mathbf{r}) \approx \mathbf{E}(\mathbf{r}_n)$, where $\mathbf{r}_n$ is a center point of rectangular cell D_n. Under this condition, (1) takes the form

$$\mathbf{E}(\mathbf{r}_p) = \sum_{n=1}^{N} \iiint_{D_n} \widehat{\mathbf{G}}_E(\mathbf{r}_p|\mathbf{r})dv \cdot \Delta\widetilde{\sigma}_{\mathbf{n}}\mathbf{E}(\mathbf{r}_n) + \mathbf{E}^{\mathrm{b}}(\mathbf{r}_p), \quad p - 1, 2, \ldots N. \quad (3)$$

Thus, inside the anomalous domain D, the discrete analog of (1) can be written as [27]

$$\mathbf{e}_D = \widehat{\mathbf{G}}_D\widehat{\sigma}\mathbf{e}_D + \mathbf{e}_D^{\mathrm{b}} \quad (4)$$

where $\widehat{\sigma}$ is a $(3N \times 3N)$ diagonal matrix of anomalous conductivities

$$\widehat{\sigma} - \mathrm{diag}(\Delta\widetilde{\sigma}_1, \ldots, \Delta\widetilde{\sigma}_N, \widetilde{\sigma}_1, \ldots, \Delta\widetilde{\sigma}_N, \widetilde{\sigma}_1, \ldots, \Delta\widetilde{\sigma}_N) \quad (5)$$

$\mathbf{e}_D$ and $\mathbf{e}_D^{\mathrm{b}}$ are the vectors of the total and background electric fields formed by the x, y, and z components of these fields at the centers of the cells D_n of the anomalous domain D

$$\mathbf{c}_D = [E_x^1, E_x^2, \ldots E_x^N, E_y^1, E_y^2, \ldots E_y^N, E_z^1, E_z^2, \ldots E_z^N]^{\mathrm{T}}$$

$$\mathbf{e}_D^{\mathrm{b}} = [E_x^{b,1}, E_x^{b,2}, \ldots E_x^{b,N}, E_y^{b,1}, E_y^{b,2}, \ldots E_y^{b,N} E_z^{b,1}, E_z^{b,2}, \ldots E_z^{b,N}]^{\mathrm{T}}.$$

These vectors have the order $3N$.

The $3N \times 3N$ matrix $\widehat{\mathbf{G}}_D$ is formed by the volume integrals over the elementary cells D_n of the components of the corresponding electric Green's tensor $\widehat{\mathbf{G}}_E$, acting inside domain D

$$\widehat{\mathbf{G}}_D = \left[G_{\alpha\beta}^{pn}\right] \quad (6)$$

where

$$G_{\alpha\beta}^{pn} = \iiint_{D_n} G_{E\alpha\beta}(\mathbf{r}_p|\mathbf{r})dv, \quad \alpha, \beta = x, y, z; p, n = 1, 2, \ldots N.$$

Note that (3) or equivalent matrix (4) provides an adequate approximation of the original IE, if the following conditions hold.

1) The linear size h of elementary cell D_n is much smaller than the wave length λ_{b} of the EM field in the background medium

$$h \ll \lambda_{\mathrm{b}}. \quad (7)$$

2) h is much smaller than the wave length λ_{a} of the EM field in a medium with anomalous conductivity

$$h \ll \lambda_{\mathrm{a}}. \quad (8)$$

The first condition (7) usually holds for typical geophysical EM modeling problems. The second condition may fail in the case of high anomalous conductivity, which is the subject of this paper.

III. IEs for Integral Currents

Our goal is to construct a discrete analog of integral (1), which would provide an accurate approximation only under condition (7). We will consider, first, (1) and (2), written for calculation of the EM field in the receivers located outside domain D. Let us denote by R the minimal distance from the receivers to domain D with the anomalous conductivity: $R = \min_{\mathbf{r} \in D} |\mathbf{r}' - \mathbf{r}|$, where $\mathbf{r}'$ is the observation point. We assume that the linear size h of elementary cell D_n is much smaller than the distance to the observation point $\mathbf{r}'$

$$h \ll R \quad (9)$$

and that it is also much smaller than the wave length λ_{b} in the background medium [condition (7)]. Under these conditions the Green's tensor $\widehat{\mathbf{G}}_E(\mathbf{r}'|\mathbf{r})$ slowly varies inside cell D_n, if point $\mathbf{r}$ moves within this cell, and the observation point $\mathbf{r}'$ is far away from D_n. It is possible, therefore, to write the integral representations (1) and (2) in the form

$$\mathbf{E}(\mathbf{r}') \approx \sum_{n=1}^{N} \left[\widehat{\mathbf{G}}_E(\mathbf{r}'|\mathbf{r}_n) \cdot \iiint_{D_n} \mathbf{j}(\mathbf{r})dv \right] + \mathbf{E}^{\mathrm{b}}(\mathbf{r}') \quad (10)$$

$$\mathbf{H}(\mathbf{r}') \approx \sum_{n=1}^{N} \left[\widehat{\mathbf{G}}_H(\mathbf{r}'|\mathbf{r}_n) \cdot \iiint_{D_n} \mathbf{j}(\mathbf{r})dv \right] + \mathbf{H}^{\mathrm{b}}(\mathbf{r}') \quad (11)$$

where $\mathbf{r}_n$ is the center point of rectangular cell D_n, and $\mathbf{j}(\mathbf{r})$ is the excess electric current

$$\mathbf{j}(\mathbf{r}) = \Delta\widetilde{\sigma}(\mathbf{r})\mathbf{E}(\mathbf{r}). \quad (12)$$

Note that integral

$$\iiint_{D_n} \mathbf{j}(\mathbf{r}')dv' = \mathbf{I}_n \quad (13)$$

is the IEC within elementary cell D_n. Substituting (13) into (10) and (11), we obtain

$$\mathbf{E}(\mathbf{r}') = \sum_{n=1}^{N} \widehat{\mathbf{G}}_E(\mathbf{r}'|\mathbf{r}_n) \cdot \mathbf{I}_n + \mathbf{E}^{\mathrm{b}}(\mathbf{r}') \quad (14)$$

$$\mathbf{H}(\mathbf{r}') = \sum_{n=1}^{N} \widehat{\mathbf{G}}_H(\mathbf{r}'|\mathbf{r}_n) \cdot \mathbf{I}_n + \mathbf{H}^{\mathrm{b}}(\mathbf{r}'). \quad (15)$$

Thus, the EM field components can be calculated in the receivers, if we know the integral currents within the cells of the grid. Note also that we obtained formulas (14) and (15) without imposing any restriction on the behavior of the electric field or electric current within the elementary cell of the grid. In other words, we do not use condition (8), and therefore the conductivity of the anomaly can be arbitrarily high. This result indicates that we can develop the IE method for the models with high-conductivity contrast, if the corresponding equations are written not for the electric field $\mathbf{E}(\mathbf{r}_n)$ but for the integral currents $\mathbf{I}_n$.

In order to obtain a system of linear equations with respect to electric currents, let us multiply both sides of (1) by $\Delta\widetilde{\sigma}(\mathbf{r}')$, assuming that $\Delta\widetilde{\sigma}(\mathbf{r}') \neq 0$ within domain D.

As a result, we have

$$\mathbf{j}(\mathbf{r}') = \Delta\widetilde{\sigma}(\mathbf{r}') \iiint_{D} \widehat{\mathbf{G}}_E(\mathbf{r}'|\mathbf{r}) \cdot \mathbf{j}(\mathbf{r})dv + \mathbf{j}^{\mathrm{b}}(\mathbf{r}') \quad (16)$$

where

$$\mathbf{j}^{\mathrm{b}}(\mathbf{r}') = \Delta\widetilde{\sigma}(\mathbf{r}')\mathbf{E}^{\mathrm{b}}(\mathbf{r}'), \qquad \mathbf{r}' \in D$$

is the induced current due to background field $\mathbf{E}^{\mathrm{b}}$.

We should note that integral (16) with respect to the electric currents, in principle, equivalent to the original integral (1) with respect to the electric field. These equations were analyzed in many publications (e.g., [1], [5], [18], and [22]). The discretization of (16), similar to (1) written for the points $\mathbf{r}'$ located inside domain D, $\mathbf{r}' \in D$, requires holding condition (8), which may fail in the case of high anomalous conductivity. That is why we need to modify (16). At the same time, we should emphasize that in the case of the domain (16) and (1), we do not need to impose condition (9), which was used only in developing formulas (14) and (15) for calculation of the EM field in the receivers located outside domain D from the IEC given inside domain D.

In order to obtain an equation with respect to integral currents, we integrate both sides of (16) over elementary cell D_p and assume that anomalous conductivity is constant within the cell D_p, $\Delta\widetilde{\sigma} = \Delta\widetilde{\sigma}_p$

$$\mathbf{I}_p = \Delta\widetilde{\sigma}_p \sum_{n=1}^{N} \iiint_{D_n} \left[\iiint_{D_p} \widehat{\mathbf{G}}_E(\mathbf{r}'|\mathbf{r})dv' \right] \cdot \mathbf{j}(\mathbf{r})dv + \mathbf{I}_p^{\mathrm{b}} \quad (17)$$

where $\mathbf{I}_p^{\mathrm{b}}$ is the integral current in the cell D_p due to background field $\mathbf{E}^{\mathrm{b}}$

$$\mathbf{I}_p^{\mathrm{b}} = \iiint_{D_p} \mathbf{j}^{\mathrm{b}}(\mathbf{r}')dv' = \iiint_{D_p} \Delta\widetilde{\sigma}(\mathbf{r}')\mathbf{E}^{\mathrm{b}}(\mathbf{r}')dv'. \quad (18)$$

Let us introduce the notation

$$\iiint_{D_p} \widehat{\mathbf{G}}_E(\mathbf{r}'|\mathbf{r})dv' = \widehat{\overline{\mathbf{G}}}_{E,p}(\mathbf{r}). \quad (19)$$

Note that integral $\widehat{\overline{\mathbf{G}}}_{E,p}(\mathbf{r})$ represents smoothing of the Green's tensor, and it is a relatively slow varying function. Therefore, we can take this expression outside of the integral over D_n in formula (17)

$$\mathbf{I}_p \approx \Delta\widetilde{\sigma}_p \sum_{n=1}^{N} \widehat{\overline{\mathbf{G}}}_{E,p}(\mathbf{r}_n) \cdot \mathbf{I}_n + \mathbf{I}_p^{\mathrm{b}} \quad (20)$$

where in the case of a slow varying field $\widehat{\overline{\mathbf{G}}}_{E,p}(\mathbf{r})$ and small cells D_n, one can assign the points $\mathbf{r}_n$ to the centers of the cells. Thus, we have arrived at a system of linear equations with respect to the integral currents within each cell, which, using matrix notation, can be written in the form

$$\mathbf{I}_D = \widehat{\sigma}\widetilde{\widehat{\mathbf{G}}}_D\mathbf{I}_D + \mathbf{I}_D^{\mathrm{b}} \quad (21)$$

where $\widehat{\sigma}$ is a $(3N \times 3N)$ diagonal matrix of anomalous conductivities, $\mathbf{I}_D$ and $\mathbf{I}_D^{\mathrm{b}}$ are the vectors of the total and background electric field intensities formed by the x, y, and z components of these fields at the centers of the cells D_n of the anomalous domain D

$$\mathbf{I}_D = \left[I_x^1, I_x^2, \ldots I_x^N, I_y^1, I_y^2, \ldots I_y^N, I_z^1, I_z^2, \ldots I_z^N\right]^{\mathrm{T}}$$

$$\mathbf{I}_D^{\mathrm{b}} = \left[I_x^{b,1}, I_x^{b,2}, \ldots I_x^{b,N}, I_y^{b,1}, I_y^{b,2}, \ldots I_y^{b,N}, I_z^{b,1}, I_z^{b,2}, \ldots I_z^{b,N}\right]^{\mathrm{T}}.$$

These vectors have the order $3N$. Note that the background IEC can be found by the corresponding numerical integration according to (18).

The $3N \times 3N$ matrix $\widehat{\widehat{\mathbf{G}}}_D$ is formed by the volume integrals over the elementary cells D_n of the components of the corresponding electric Green's tensor $\widehat{\mathbf{G}}_E$, acting inside domain D. Due to the reciprocity principle [25], elements of this matrix, $\widetilde{G}^{pn}_{\alpha\beta}$, can be written as

$$\widetilde{G}^{pn}_{\alpha\beta} = \iiint_{D_p} G_{E\alpha\beta}(\mathbf{r}|\mathbf{r}_n)dv$$

$$= \iiint_{D_p} G_{E\beta\alpha}(\mathbf{r}_n|\mathbf{r})dv$$

$$= G^{np}_{\beta\alpha}, \qquad \alpha, \beta = x, y, z;\ p, n = 1, 2, \dots N$$

where $G^{pn}_{\beta\alpha}$ are the elements of the corresponding matrix (6) of the conventional integral (3) for the electric field. In other words, matrix $\widehat{\widehat{\mathbf{G}}}_D$ is a transposed matrix of the original linear system (4), $\widehat{\mathbf{G}}_D$, for the vector of electric field $\mathbf{e}_D$ (which justifies the notation we use for $\widehat{\widehat{\mathbf{G}}}_D$).

Thus, forward EM modeling based on the IE method is reduced to the solution of the matrix (21) for the unknown vector $\mathbf{I}_D$ of IEC components inside domain D. The equation is a $3N \times 3N$ linear system

$$\widehat{\mathbf{B}}\mathbf{I}_D = \mathbf{I}^{\mathrm{b}}_D \tag{22}$$

where

$$\widehat{\mathbf{B}} = \widehat{\mathbf{1}} - \widehat{\sigma}\widehat{\widehat{\mathbf{G}}}_D \tag{23}$$

and $\widehat{\mathbf{1}}$ is identity tensor.

We have reduced the EM forward modeling problem to the solution of the matrix (22) with respect to the IEC. The use of the integral current instead of the conventional current density constitutes the key new idea of our method. The integral currents can be found accurately for a body with any conductivity. We do not use anymore the traditional for IE method requirements that the electric field (or electric current) varies slowly inside the cell, because we deal with the integral of this field (or of this current) over the entire cell. As a result, the method can be applied, in principle, for models with arbitrary conductivity contrast.

Matrix $\widehat{\mathbf{B}}$ is a $3N \times 3N$ dense matrix. We use the contraction IE method to precondition matrix (22) [13], [27]

$$\widehat{\mathbf{M}}_1\widehat{\mathbf{B}}\widehat{\mathbf{M}}_2\widetilde{\mathbf{I}}_D = \widehat{\mathbf{M}}^{\mathrm{b}}_1\widetilde{\mathbf{I}}_D \tag{24}$$

where $\widehat{\mathbf{M}}_1$ is the $3N \times 3N$ diagonal matrix of the square root of the background conductivity, similar to matrix (5)

$$\widehat{\mathbf{M}}_1 = \operatorname{diag}\Big(\sqrt{\sigma^{\mathrm{b}}_1}, \sqrt{\sigma^{\mathrm{b}}_2}, \dots, \sqrt{\sigma^{\mathrm{b}}_N}, \sqrt{\sigma^{\mathrm{b}}_1}, \sqrt{\sigma^{\mathrm{b}}_2} \dots$$
$$\sqrt{\sigma^{\mathrm{b}}_N}, \sqrt{\sigma^{\mathrm{b}}_1}, \sqrt{\sigma^{\mathrm{b}}_2} \dots, \sqrt{\sigma^{\mathrm{b}}_N}\Big)$$

and where

$$\widetilde{\mathbf{I}}_D = \widehat{\mathbf{M}}_2^{-1}\mathbf{I}_D. \tag{25}$$

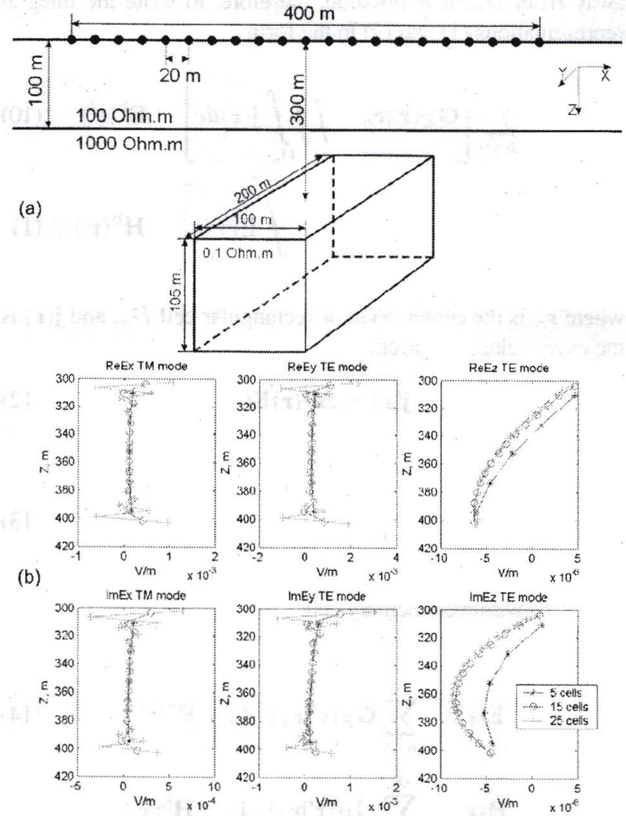

Fig. 1. Panel (a) shows a model of a prismatic conductive body with a resistivity of $0.1\ \Omega \cdot \mathrm{m}$ embedded in the two-layered background. Panel (b) presents the vertical distribution of the electric field within a conductive prism computed using three different discretizations in the vertical direction: 5 cells (stars), 15 cells (circles), and 25 cells (crosses).

with $\widehat{\mathbf{M}}_2$ defined as the diagonal matrix

$$\widehat{\mathbf{M}}_2 = \operatorname{diag}\left(a_1^{-1}, a_2^{-1}, \dots, a_N^{-1}, a_1^{-1}, a_2^{-1}, \dots\right.$$
$$\left. a_N^{-1}, a_1^{-1}, a_2^{-1}, \dots, a_N^{-1}\right) \tag{26}$$

$$a_i = \frac{2\sigma_{bi} + \Delta\widetilde{\sigma}_i}{2\sqrt{\sigma_{bi}}}, \qquad i = 1, 2, \dots, N.$$

One can use different types of iterative methods for the solution of this problem. Detailed analysis of the different solvers is given in [13]. After determining the integral current, we can find the components of the EM field by substituting this current in (14) and (15).

IV. Numerical Analysis of the Electric Current Distribution Inside the Conductive Body

Consider a model of a prismatic conductive body with a resistivity of $0.1\ \Omega \cdot \mathrm{m}$ embedded in a two-layered background (Fig. 1). The resistivity contrast between the second layer and the prism is 10^4. The incident field is a vertically propagated plane EM wave at the 25-Hz frequency, containing both the TM and TE modes. We investigated the effect of different vertical discretizations of the prismatic body on the electric current calculations. Three different discretizations were used in the vertical direction: 5, 15, and 25 cells. The discretizations in

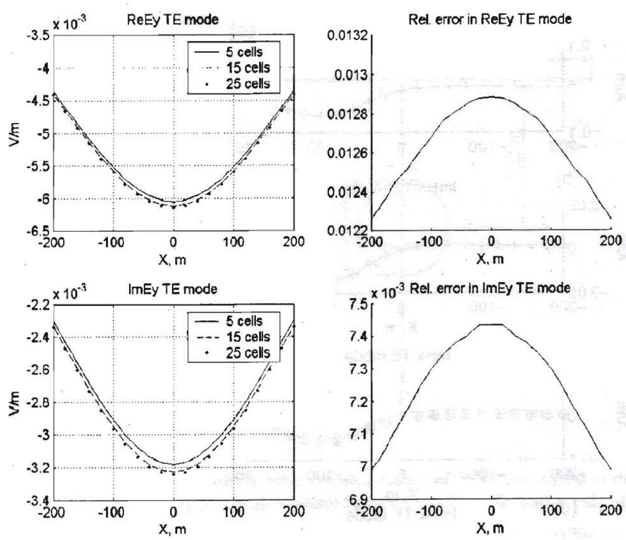

Fig. 2. y component of the electric field E_y obtained using three discretizations in the vertical direction, 5, 15, and 25 cells, respectively.

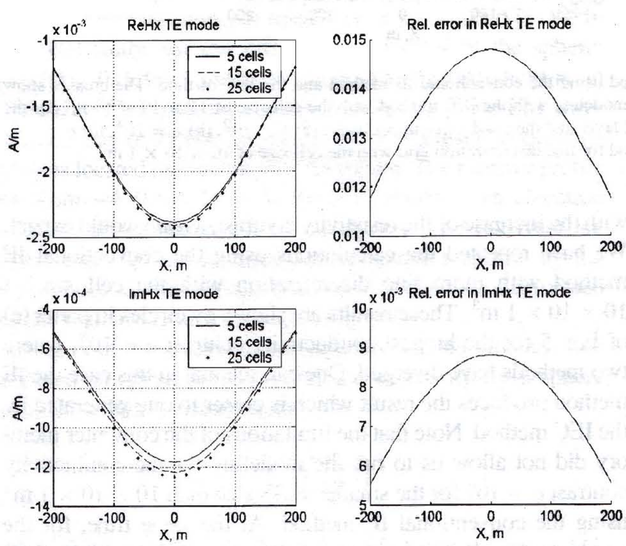

Fig. 3. Two left panels show the real part (top) and imaginary part (bottom) of the x component of the magnetic field H_x obtained using three discretizations in the vertical direction, 5, 15, and 25 cells, respectively. Two right panels show the relative errors in real (top) and imaginary (bottom) parts of the component.

the x and y directions remained the same: 10 and 20 cells, respectively. The horizontal components of the TE mode electric and magnetic fields obtained using all three discretizations are shown in the left panels of Figs. 2 and 3, respectively. The right panels in these figures present the relative errors, ε_{E_y} and ε_{H_x}, in the real (top) and imaginary (bottom) parts of the corresponding components computed as the difference between the field for the finest discretization (25) and the field, obtained for the coarsest discretization (5), normalized by the field at the finest discretization (25)

$$\varepsilon_{E_y} = \frac{E_y^{(25)} - E_y^{(5)}}{E_y^{(25)}} \qquad \varepsilon_{H_x} = \frac{H_x^{(25)} - H_x^{(5)}}{H_x^{(25)}}.$$

One can see that these errors do not exceed 1.5%.

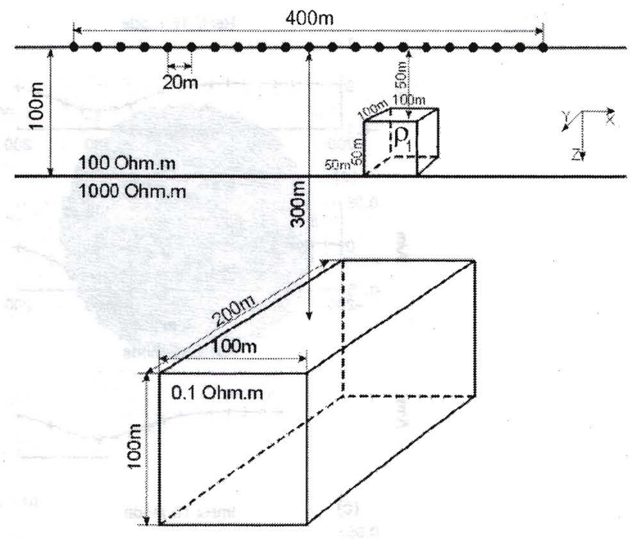

Fig. 4. Model of two conductive bodies embedded within different layers of two-layered horizontally homogeneous background medium.

Fig. 1, panel (b), presents the vertical distribution of the electric field within the conductive prism computed using different vertical disretizations. To produce these plots, we selected a central vertical column of the cells within the prism for each discretization. The electric field, $\mathbf{E}(\mathbf{r}_n)$, was calculated in the center of each elementary cell from this column according to the following formula based on expressions (13) and (12):

$$\mathbf{E}(\mathbf{r}_n) = \mathbf{I}_n / (\Delta\widetilde{\sigma}_n D_n) \tag{27}$$

using the IEC, $\mathbf{I}_n$, computed for this cell with the IEC method (where D_n and $\Delta\widetilde{\sigma}_n$ are the volume and the anomalous conductivity of the corresponding elementary cell, respectively). Fig. 1, panel (b), shows that the electric field computed for the finest discretization (25 cells in the vertical direction) describes well the skin effect within the conductive body, while the field on the coursed discretization of five vertical cells is practically insensitive to the skin effect. At the same time, the difference between the observed EM field components at the surface is within just 1.5% (Figs. 2 and 3). This remarkable property of the IEC solution is related to the main principle of the IEC method, which is based on computing the IEC, $\mathbf{I}_n$, within every cell. In this case, the electric field computed according to formula (27) should also describe the averaged electric field within the cell, which corresponds well to the plots shown in Fig. 1, panel b. One can see that the plots of the horizontal electric field components for the coarser discretization describe the average values of the same plot for the finer discretization. The plots of the vertical component of the electric field behaves a little bit differently, because the vertical field is 10^3 times smaller than the horizontal fields. Nevertheless, the plots for 15-cell and 25-cell discretizations practically coincide, which is a clear manifestation that we reached the optimal level of discretization at 25 cells in the vertical direction. The solution will not change if we will use

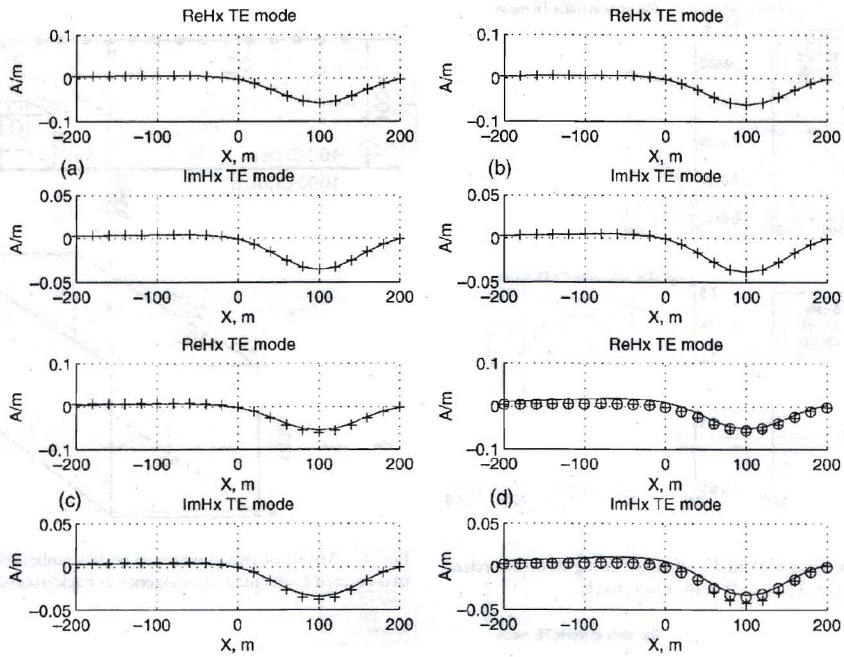

Fig. 5. Plots of the real and imaginary parts of the magnetic field H_x, computed using the conventional IE method and the IEC method. The crosses show the data obtained by the IE, while the solid lines present the results of numerical modeling with the IEC method with the cell size of $10 \times 10 \times 10$ m^3 for the following resistivity contrasts, $c = \rho_n/\rho_1$, between the homogeneous background layer and the conductive upper body: (a) $c = 10^2$, (b) $c = 10^4$, (c) $c = 10^5$, and (d) $c = 10^6$. Circles show the results obtained using the conventional IE method for fine discretization grid with the cell size of $10 \times 10 \times 1$ m^3.

the finer discretization. Thus, another important property of the new IEC method is that it does not require a very fine discretization to produce an accurate result, because it does not operate with the discretized electric field but with the integral currents, instead.

V. Comparison Between the IEC and the Conventional IE Method

In this section, we compare the conventional IE method with the new IEC algorithm. We consider a model shown in Fig. 4. The model consists of two conductive bodies embedded within different layers of two-layered horizontally homogeneous background with the resistivity of the first and second layers equal to 100 and 1000 $\Omega \cdot$ m, respectively, and with a thickness of the first layer equal to 100 m. In our numerical experiments, the resistivity of the lower body stays constant at 0.1 $\Omega \cdot$ m, while the resistivity of the upper body changes: $\rho_1 = 1$, 0.01, 0.001, and 0.0001 $\Omega \cdot$ m. The incident field is an E-polarized (TE mode) vertically propagated plane EM wave at a frequency of 25 Hz. We use the same discretization of the conductive bodies for both the conventional IE and the IEC methods with the cell size of $10 \times 10 \times 10$ m^3.

Fig. 5 presents the real and imaginary parts of the magnetic field component H_x computed using the conventional IE method (crosses) and the IEC method (solid lines), for the resistivity contrast between the homogeneous background layer and the conductive upper body, $c = \rho_n/\rho_1$, equal to 10^2, 10^4, 10^5, and, 10^6, respectively. One can see that for this model, the two methods generate practically the same result for low-resistivity contrast. However, they produce different results with the increase of the resistivity contrast, as one would expect. We have repeated the calculations using the conventional IE method with more fine discretization with the cell size of $10 \times 10 \times 1$ m^3. These results are shown by circles in panel (c) of Fig. 5 for the highest conductivity contrast $c = 10^6$, where two methods have diverged. One can see that in this case the IE method produces the result which is closer to one generated by the IEC method. Note that the limitations of the computer memory did not allow us to run the modeling for the conductivity contrast $c = 10^6$ for the smaller cell's size than $10 \times 10 \times 1$ m^3 using the conventional IE method. At the same time, for the conductivity contrasts up to $c = 10^5$, the results obtained by the conventional IE method with the fine discretization and the results of IEC modeling on the relatively coarse grid are practically the same. This example shows that, the conventional method requires more cells than the IEC method to get the same accuracy.

VI. Comparison Between the IEC Method and Analytical Solution for a Conductive Sphere

In order to check the accuracy of the new IEC method, we apply this technique to model a response of the conductive sphere excited by the vertically propagated plane EM wave. This problem represents one of a few EM problems which allow for an analytical solution. The mathematical solution of this problem has been described in several publications (see, for example, [3], [4], [15], and [20]). This problem is usually solved by means of the Debye potentials. We compare this analytical solution with numerical modeling using the IEC method.

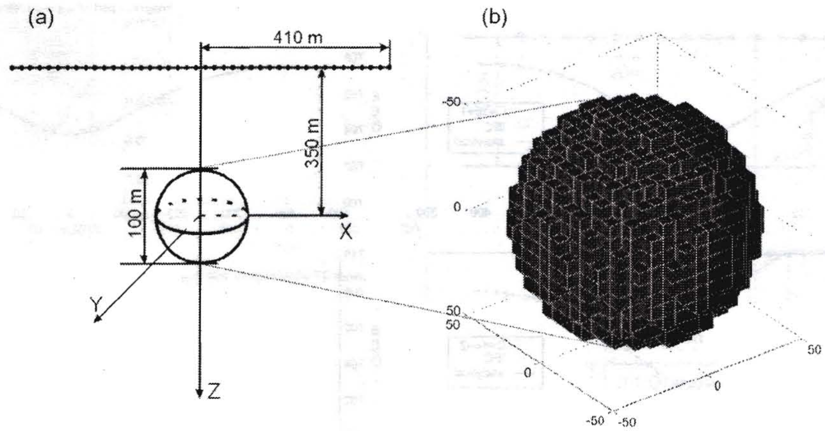

Fig. 6. (a) Model of a conductive sphere with a radius of 50 m embedded in the homogeneous whole space with a background resistivity of 1000 $\Omega \cdot$ m; and (b) approximation of the sphere with a model formed by cubic cells with a side of 6.25 m.

Fig. 6(a) shows a model of the conductive sphere with a radius of 50 m embedded in the homogeneous whole space with a background (normal) resistivity of $\rho_n = 1000\ \Omega \cdot$ m. In the model study, we use different resistivities of the sphere: $\rho_d = 100, 10, 1, 0.1,$ and $0.01\ \Omega \cdot$ m. The incident field is an E-polarized (TE mode) vertically propagated plane EM wave at a frequency of 25 Hz. The origin of the Cartesian coordinate system is located in the center of the sphere. The receiver profile runs from −410 to 410 m in the x direction at an elevation of 350 m above the center of the sphere. The receivers are located every 20 m. To calculate the sphere response by the IEC method, we approximated the sphere with a model formed by cells with a side of 6.25 m [see Fig. 6(b)].

Using both the analytical solution and the IEC method, we computed an apparent magnetotelluric resistivity for a sphere model according to the formula

$$\rho_a = \frac{1}{\omega \mu_0} \left(\frac{E_y}{H_x} \right)^2 .$$

Note that, according to the method of Debye potentials [4], the EM field components are represented in the form of series. Therefore, the result may depend on the number of the terms kept in these series in calculations. However, these series converge extremely fast. Fig. 7 represents the plot of the maximum value of the apparent resistivity versus the number of terms used in the series in analytical calculations for the model with maximum conductivity contrast $(1e + 5)$. One can see that the result practically does not change after adding the third term.

Fig. 8 shows the plots of the real and imaginary parts of the apparent resistivity, $\mathrm{Re}\rho_a$ and $\mathrm{Im}\rho_a$, for the different resistivity contrasts between the homogeneous background and the conductive sphere, $c = \rho_n/\rho_d$, equal to 10, 10^2, 10^3, 10^4, and 10^5, respectively. The solid lines correspond to the analytical solution, while the dashed lines present the numerical IEC results. One can see that the difference between the analytical and the numerical IEC solutions does not exceed 0.15% at the extremum value of the apparent resistivity for the highest conductivity contrast of 10^5. This result demonstrates that the developed new method of integral current equations produces an accurate result even for the models with high-conductivity contrast.

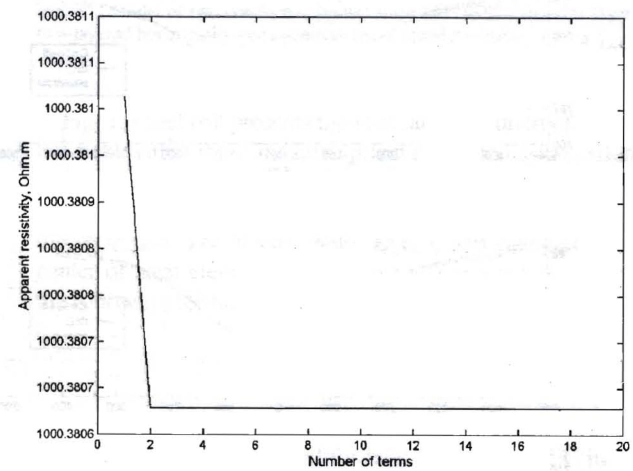

Fig. 7. Plot of the maximum value of the apparent resistivity versus the number of terms used in the series in analytical calculations for the model with maximum conductivity contrast $(1e + 5)$.

VII. Conclusion

For a long time the main limitation of the IE method was modeling the EM field for models with high-conductivity contrast. In this paper, we have developed a new approach to the construction of the IE method. It is based on using IECs, calculated over the elementary cells of the discretization grid, instead of the electric field itself within the cells, as is commonly used in the conventional IE method. As a result, the method is capable of modeling the EM response in geoelectrical structures with high contrast of conductivity.

The method was carefully tested. We compared the numerical modeling results with the exact analytical solution for a model of a conductive sphere. Future work will be directed to application of the new method for examining the complex models of geological targets with the large conductivity contrast, typical for mineral exploration.

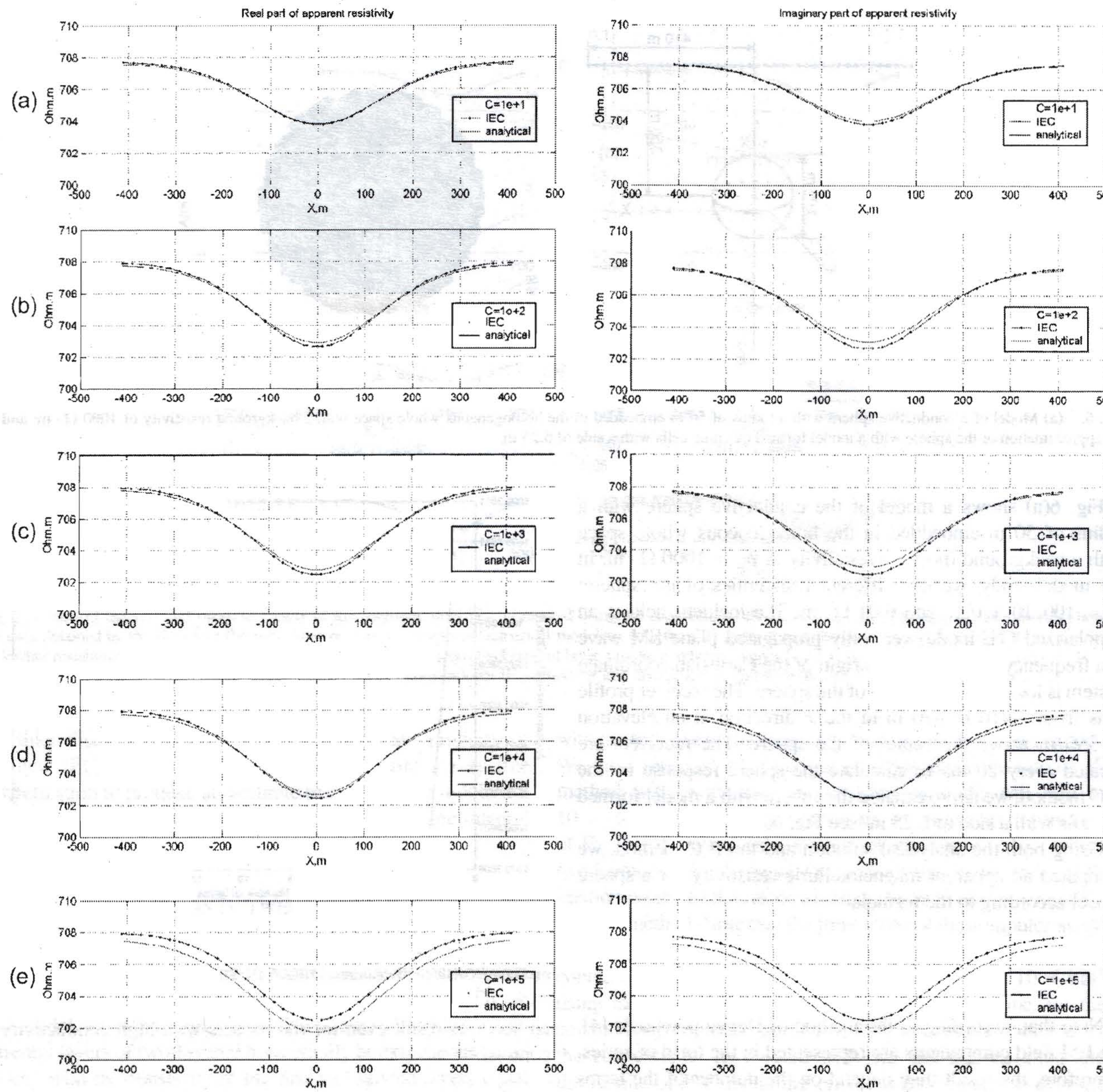

Fig. 8. Plots of the real and imaginary parts of the apparent resistivity $\mathrm{Re}\rho_a$ and $\mathrm{Im}\rho_a$ computed using the analytical solution and the IEC method. The solid lines show the data obtained by the analytical solution, while the dashed lines present the results of numerical modeling with the IEC method for the following resistivity contrasts $c = \rho_n/\rho_d$ between the homogeneous background and the conductive sphere: (a) $c = 10$, (b) $c = 10^2$, (c) $c = 10^3$, (d) $c = 10^4$, and (e) $c = 10^5$.

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