CHAPTER 19

Foundations of the Method of EM Field Separation into Upgoing and Downgoing Parts and its Application to MCSEM Data

Michael S. Zhdanov[a] *and* Shuming Wang[a]

Contents

1. Introduction

The problem of electromagnetic field separation into upgoing and downgoing parts is one of the oldest problems of geophysics. It was addressed

[a] Department of Geology and Geophysics, University of Utah, USA

E-mail address: mzhdanov@mines.utah.edu (M.S. Zhdanov).

Handbook of Geophysical Exploration: Seismic Exploration, Volume 40
ISSN 0950-1401, DOI: 10.1016/S0950-1401(10)04024-3

first by the great German mathematician Carl Friedrich Gauss in the middle of the 19th century in his study of the nature of the geomagnetic field and its separation into external and internal parts (Chapman and Bartels, 1940). The Gaussian method was originally applied to models with a spherical or plane earth surface. This restriction was removed by Kertz (1954) and Siebert (1962), by using the theory of analytical functions of complex variables. Both Gaussian and Kertz-Siebert methods were developed for potential magnetic field separation in nonconductive media only. In the pioneering paper by Zhdanov (1980) a generalized method of electromagnetic (EM) field separation into upgoing and downgoing parts was introduced. Later on this technique was extended to field separation into parts related to sources located in different regions of the space, including external and internal, normal and anomalous parts of a transient EM field. It was also extended to the separation of EM anomalies into the surface and deep parts and to the determination of the main part of a deep anomaly (Berdichevsky and Zhdanov, 1984; Zhdanov, 1988; Zhdanov et al., 1996). This technique was also extended to the separation of EM fields measured at the sea bottom (Zhdanova and Zhdanov, 1999). All of these publications provide a solid foundation for the electromagnetic field separation and its application in EM geophysics.

Recently, we have observed a renewed interest in the problem of upgoing/downgoing EM field separation, generated by increased research and development of marine controlled source electromagnetic (MCSEM) methods. A problem that arises in the MCSEM method is that EM energy may travel from the source to the receiver along many paths. For example, seafloor receivers of EM data measure not only a response from the sea-bottom geoelectrical formations (the "upgoing field") but they also measure a direct part of the primary field from the source, the field travelling from the source to the sea surface and reflected back to the sea bottom (a so-called airwave), and the natural magnetotelluric field. The latter parts of the total EM field form the "downgoing field" because the sources of these fields are located above the receivers (above the seafloor). The methods of EM field decomposition into upgoing and downgoing components in application to MCSEM data are discussed in the recent paper by Amundsen et al. (2006). They presented a constructive method of the decomposition of the EM field given on a horizontal plane, and provided practical examples of its application for 1D and 2D models.

The main goal of our paper is to review the basic principles of EM field separation. The most general approach to the solution of this problem is based on the formalism of the Stratton-Chu type integrals, introduced by Zhdanov (1980, 1988). We begin our paper with a brief discussion of the theory of the Stratton-Chu type integrals and its application to the field separation. This theory is used for decomposition of the EM field measured on an arbitrary surface, e.g., on a seafloor with a variable bathymetry.

In a case where the data are measured on a horizontal plane, the most effective technique for field separation can be developed using a spatial Fourier transform in the (k, ω) domain (Berdichevsky and Zhdanov, 1984; Zhdanova and Zhdanov, 1999). Finally, we introduce a new method of field separation based on using horizontal gradients of the observed EM fields. This method allows us to develop a fast and accurate separation of 3D EM fields into upgoing and downgoing fields, which can be effectively used for interpretation of MCSEM field data.

2. Integral Transforms of Electromagnetic Fields Using Stratton-Chu Type Integrals

A general integral method of EM field decomposition into upgoing and downgoing components has been developed by Zhdanov (1980, 1988). We will discuss below the basic ideas of this method as applied to MCSEM data. Our approach is based on using Stratton-Chu Type integrals. A summary of the basic definitions and properties of these integrals is provided in Appendices A and B, for convenience. We will demonstrate below that this theory provides the foundations for the solution of the problem of EM field separation into upgoing and downgoing parts.

2.1. Basic equations of upgoing and downgoing fields

Let us consider a typical MCSEM survey conducted in an area with rough bathymetry. The conductivity of seawater is known and it is equal to σ_w. Air is characterized by a complex conductivity σ_0. In fact, for the typically low frequencies used in the MCSEM method, we can neglect the displacement currents in the air and assume that $\sigma_0 = 0$, however, for completeness, we will keep this term in the equations. Sea-bottom formations are characterized by an arbitrary 3D distribution of the conductivity, $\sigma(\mathbf{r})$, where $\mathbf{r}$ is a radius vector of the given point in some Cartesian system of coordinates (x, y, z). The frequency domain EM field is generated by an electric bipole transmitter Tx located at some depth within the sea-water layer. This field is measured by a system of receivers located at surface S at a small elevation (usually a few meters) above the sea bottom. We assume that S is a smooth surface extending to infinity in the horizontal directions. The magnetic permeability is everywhere equal to the free space constant μ_0.

The electromagnetic field in this model satisfies the following equations:

$$\nabla \times \mathbf{H} = \begin{cases} \sigma_0 \mathbf{E}, & \mathbf{r} \in A, \\ \sigma_w \mathbf{E} + \mathbf{j}^e, & \mathbf{r} \in W, \\ \sigma(\mathbf{r})\mathbf{E}, & \mathbf{r} \in B, \end{cases} \quad \nabla \times \mathbf{E} = i\omega\mu_0 \mathbf{H}, \tag{1}$$

where A denotes the upper half-space occupied by the atmosphere, W represents the sea-water layer, B denotes the lower half-space formed by the sea-bottom geoelectrical formations with the conductivity $\sigma(\mathbf{r})$, and $\mathbf{j}^e$ is an extraneous current in the transmitter. Note that all components of the electromagnetic field are continuous at surface S, because it is located within the homogeneous seawater layer.

We can represent the electromagnetic field $\mathbf{E}$, $\mathbf{H}$ as a sum of the upgoing $\mathbf{E}^u$, $\mathbf{H}^u$ (internal) and downgoing $\mathbf{E}^d$, H^d (external) components:

$$\mathbf{E} = \mathbf{E}^u + \mathbf{E}^d, \quad \mathbf{H} = \mathbf{H}^u + \mathbf{H}^d. \tag{2}$$

The upgoing field $\mathbf{E}^u$, $\mathbf{H}^u$ is due to the excess electric currents, $\mathbf{j}^B$, induced within the conductive sea bottom, in the model where the conductivity of the atmosphere, the upper half-space A, is equal to the electric conductivity of the seawater. It satisfies the equations:

$$\nabla \times \mathbf{H}^u = \begin{cases} \sigma_w \mathbf{E}^u, & \mathbf{r} \in A, \\ \sigma_w \mathbf{E}^u + \mathbf{j}^e, & \mathbf{r} \in W, \\ \sigma_w \mathbf{E}^u + \mathbf{j}^B, & \mathbf{r} \in B, \end{cases} \quad \nabla \times \mathbf{E}^u = i\omega\mu_0 \mathbf{H}^u, \tag{3}$$

where

$$\mathbf{j}^B = [\sigma(\mathbf{r}) - \sigma_w]\mathbf{E}. \tag{4}$$

The downgoing field $\mathbf{E}^d$, $\mathbf{H}^d$ is produced by the current in the transmitter and the "excess" electric currents, $\mathbf{j}^A$, distributed within the atmosphere, in the model where the conductivity both in the sea-bottom formations (the lower half-space B), and in the atmosphere (the upper half-space A), is equal to the electric conductivity of the seawater. It satisfies the equations:

$$\nabla \times \mathbf{H}^d = \begin{cases} \sigma_w \mathbf{E}^d + \mathbf{j}^A, & \mathbf{r} \in A, \\ \sigma_w \mathbf{E}^d + \mathbf{j}^e, & \mathbf{r} \in W, \\ \sigma_w \mathbf{E}^d, & \mathbf{r} \in B, \end{cases} \quad \nabla \times \mathbf{E}^d = i\omega\mu_0 \mathbf{H}^d, \tag{5}$$

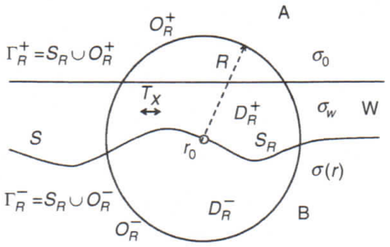

Figure 1 A scheme illustrating the application of the Stratton-Chu type integrals to the separation of the upgoing and downgoing fields.

where

$$\mathbf{j}^A = [\sigma_0 - \sigma_w]\mathbf{E}. \tag{6}$$

It is easy to verify that, by adding the left- and right-hand sides of Eqs (5) and (3), we obtain the original system of Maxwell's equations for the total field (1).

2.2. Application of the Stratton-Chu type integrals for field separation

The problem of field decomposition can be formulated as follows: find the upgoing and downgoing components at the surface of observation S from the total EM field $\mathbf{E}$, $\mathbf{H}$, observed also at S.

We will apply the theory of the Stratton-Chu type integrals to solve this problem. Let us take a point at the surface of observation, $\mathbf{r}_0 \in S$, and draw a sphere O_R of radius R with the center at $\mathbf{r}_0$ (see Figure 1). We select the radius R so large that it completely encloses the electric bipole transmitter T_x. We denote by O_R^- the part of the sphere lying below the observational surface S, and by O_R^+ the part of the sphere lying above the observational surface S within the water layer W and within the upper half-space A of the atmosphere. S_R is the part of the surface of observation found inside the sphere. We denote by Γ_R^+ a piece-wise smooth close surface formed by the semisphere O_R^+ and S_R, $\Gamma_R^+ = O_R^+ \cup S_R$ (see Figure 1). This surface Γ_R^+ is a closed boundary of the upper part, D_R^+, of the ball bounded by a sphere O_R, located above surface S. In a similar way, we denote by Γ_R^- a piece-wise smooth closed surface formed by the lower semisphere O_R^- and S_R: $\Gamma_R^- = O_R^- \cup S_R$. This surface Γ_R^- is a closed boundary of the lower part, D_R^-, of the ball located below surface S.

We can introduce now the Stratton-Chu integrals over the piece-wise smooth closed surface Γ_R^+:

$$\mathbf{C}^E_{\Gamma_R^+}(\mathbf{r}') = \iint_{\Gamma_R^+} [(\mathbf{n}\cdot\mathbf{E})\nabla G_w + (\mathbf{n}\times\mathbf{E})\times\nabla G_w + i\omega\mu(\mathbf{n}\times\mathbf{H})G_w]ds, \tag{7}$$

$$\mathbf{C}^H_{\Gamma_R^+}(\mathbf{r}') = \iint_{\Gamma_R^+} [(\mathbf{n}\cdot\mathbf{H})\nabla G_w + (\mathbf{n}\times\mathbf{H})\times\nabla G_w + \sigma_w(\mathbf{n}\times\mathbf{E})G_w]ds, \tag{8}$$

where $\mathbf{r}' \in D_R^+$, $\mathbf{n}$ is the unit vector of an inward-pointing normal to D_R^+, and G_w is the fundamental Green's function for the Helmholtz equation in a homogeneous full space with the seawater conductivity σ_w:

$$G_w(\mathbf{r}'|\mathbf{r}) = -\frac{1}{4\pi|\mathbf{r}-\mathbf{r}'|}\exp(ik_w|\mathbf{r}-\mathbf{r}'|), \quad k_w = \sqrt{i\omega\mu_0\sigma_w}, \operatorname{Re} k_w > 0. \tag{9}$$

The Stratton-Chu type integrals are linear operators with respect to the electric and magnetic fields, $\mathbf{E}$ and $\mathbf{H}$. Therefore, according to equations (2), integrals (7) and (8) can be represented as the sums of similar Stratton-Chu type integrals, calculated for the values of the upgoing and downgoing fields, respectively:

$$\mathbf{C}^E_{\Gamma_R^+}(\mathbf{r}') = \mathbf{C}^{E^u}_{\Gamma_R^+}(\mathbf{r}') + \mathbf{C}^{E^d}_{\Gamma_R^+}(\mathbf{r}'), \quad \mathbf{C}^H_{\Gamma_R^+}(\mathbf{r}') = \mathbf{C}^{H^u}_{\Gamma_R^+}(\mathbf{r}') + \mathbf{C}^{H^d}_{\Gamma_R^+}(\mathbf{r}'). \tag{10}$$

Note that the upgoing EM field, $\mathbf{E}^u$, $\mathbf{H}^u$, inside domain D_R^+ satisfies the equations:

$$\nabla\times\mathbf{H}^u = \sigma_w\mathbf{E}^u, \quad \nabla\times\mathbf{E}^u = i\omega\mu_0\mathbf{H}^u, \tag{11}$$

and vanishes at infinity. Hence, according to the property (2) of the Stratton-Chu type integrals, we have the following:

$$\mathbf{C}^{E^u}_{\Gamma_R^+}(\mathbf{r}') = \begin{cases}\mathbf{E}^u(\mathbf{r}'), & \mathbf{r}'\in D_R^+\\ 0, & \mathbf{r}'\in C\overline{D}_R^+,\end{cases} \quad \mathbf{C}^{H^u}_{\Gamma_R^+}(\mathbf{r}') = \begin{cases}\mathbf{H}^u(\mathbf{r}'), & \mathbf{r}'\in D_R^+\\ 0, & \mathbf{r}'\in C\overline{D}_R^+,\end{cases} \tag{12}$$

where D_R^+ and $C\overline{D}_R^+$ are the domains which are internal and external with respect to the closed surface Γ_R^+.

In a similar way, we introduce the Stratton-Chu integrals over a piece-wise smooth closed surface Γ_R^-, which can also be decomposed into Stratton-Chu type integrals, calculated for the values of the upgoing and downgoing fields, respectively:

$$\mathbf{C}^{E}_{\Gamma_R^-}(\mathbf{r}') = \mathbf{C}^{E^u}_{\Gamma_R^-}(\mathbf{r}') + \mathbf{C}^{E^d}_{\Gamma_R^-}(\mathbf{r}'), \quad \mathbf{C}^{H}_{\Gamma_R^-}(\mathbf{r}') = \mathbf{C}^{H^u}_{\Gamma_R^-}(\mathbf{r}') + \mathbf{C}^{H^d}_{\Gamma_R^-}(\mathbf{r}'). \quad (13)$$

We can recall, according to Eqs (5), that the downgoing EM field, $\mathbf{E}^d$, $\mathbf{H}^d$, inside domain $D_R^- \subset B$ satisfies the equations:

$$\nabla \times \mathbf{H}^d = \sigma_w \mathbf{E}^d, \quad \nabla \times \mathbf{E}^d = i\omega\mu_0 \mathbf{H}^d, \quad (14)$$

and vanishes at infinity. Hence, according to the property (2) of the Stratton-Chu type integrals, we have:

$$\mathbf{C}^{E^d}_{\Gamma_R^-}(\mathbf{r}') = \begin{cases} -\mathbf{E}^d(\mathbf{r}'), & \mathbf{r}' \in D_R^- \\ 0, & \mathbf{r}' \in C\overline{D}_R^-, \end{cases} \quad \mathbf{C}^{H^d}_{\Gamma_R^-}(\mathbf{r}') = \begin{cases} -\mathbf{H}^d(\mathbf{r}'), & \mathbf{r}' \in D_R^- \\ 0, & \mathbf{r}' \in C\overline{D}_R^-, \end{cases} \quad (15)$$

where D_R^- and $C\overline{D}_R^-$ are the domains which are internal and external with respect to the closed surface Γ_R^-, and the minus sign on the right side of Eqs (15) is due to the fact that the normal vector n is directed outward to domain D_R^- (see Figure 1).

Summing up Eqs (12) and (15), we have:

$$\begin{aligned} \mathbf{C}^{E^u}_{\Gamma_R^+}(\mathbf{r}') + \mathbf{C}^{E^d}_{\Gamma_R^-}(\mathbf{r}') &= \begin{cases} \mathbf{E}^u(\mathbf{r}'), & \mathbf{r}' \in D_R^+ \\ -\mathbf{E}^d(\mathbf{r}'), & \mathbf{r}' \in D_R^-, \end{cases} \\ \mathbf{C}^{H^u}_{\Gamma_R^+}(\mathbf{r}') + \mathbf{C}^{H^d}_{\Gamma_R^-}(\mathbf{r}') &= \begin{cases} \mathbf{H}^u(\mathbf{r}'), & \mathbf{r}' \in D_R^+ \\ -\mathbf{H}^d(\mathbf{r}'), & \mathbf{r}' \in D_R^-. \end{cases} \end{aligned} \quad (16)$$

One can readily obtain the limit values of the last equations at the surface S_R thanks to the continuity of the electric and magnetic fields at this surface:

$$\begin{aligned} \lim_{r' \to r_0^+} [\mathbf{C}^{E^u}_{\Gamma_R^-}(\mathbf{r}') + \mathbf{C}^{E^d}_{\Gamma_R^+}(\mathbf{r}')] &= \mathbf{E}^u(\mathbf{r}_0), \quad \mathbf{r}_0 \in S_R, \quad \mathbf{r}_0^+ \in \overline{D}_R^+, \\ \lim_{r' \to r_0^+} [\mathbf{C}^{H^u}_{\Gamma_R^-}(\mathbf{r}') + \mathbf{C}^{H^d}_{\Gamma_R^+}(\mathbf{r}')] &= \mathbf{H}^u(\mathbf{r}_0), \quad \mathbf{r}_0 \in S_R, \quad \mathbf{r}_0^+ \in \overline{D}_R^+, \end{aligned} \quad (17)$$

and

$$\lim_{r' \to r_0^-} [\mathbf{C}^{E^u}_{\Gamma_R^-}(\mathbf{r}') + \mathbf{C}^{E^d}_{\Gamma_R^+}(\mathbf{r}')] = -\mathbf{E}^d(\mathbf{r}_0), \quad \mathbf{r}_0 \in S_R, \quad \mathbf{r}_0^- \in \overline{D}_R^-,$$
$$\lim_{r' \to r_0^-} [\mathbf{C}^{H^u}_{\Gamma_R^-}(\mathbf{r}') + \mathbf{C}^{H^d}_{\Gamma_R^+}(\mathbf{r}')] = -\mathbf{H}^d(\mathbf{r}_0), \quad \mathbf{r}_0 \in S_R, \quad \mathbf{r}_0^- \in \overline{D}_R^-. \tag{18}$$

The limit values of the Stratton-Chu type integrals can also be defined by Eqs (88) and (89), for example:

$$\lim_{r' \to r_0^+} [\mathbf{C}^{E^u}_{\Gamma_R^-}(\mathbf{r}') + \mathbf{C}^{E^d}_{\Gamma_R^+}(\mathbf{r}')] = \mathbf{C}^{E^u}_{O_R^-}(\mathbf{r}_0) + \mathbf{C}^{E^u}_{S_R}(\mathbf{r}_0) + \frac{1}{2}\mathbf{E}^u(\mathbf{r}_0)$$
$$+ \mathbf{C}^{E^d}_{O_R^+}(\mathbf{r}_0) + \mathbf{C}^{E^d}_{S_R}(\mathbf{r}_0) + \frac{1}{2}\mathbf{E}^d(\mathbf{r}_0),$$
$$\lim_{r' \to r_0^-} [\mathbf{C}^{E^u}_{\Gamma_R^-}(\mathbf{r}') + \mathbf{C}^{E^d}_{\Gamma_R^+}(\mathbf{r}')] = \mathbf{C}^{E^u}_{O_R^-}(\mathbf{r}_0) + \mathbf{C}^{E^u}_{S_R}(\mathbf{r}_0) - \frac{1}{2}\mathbf{E}^u(\mathbf{r}_0)$$
$$+ \mathbf{C}^{E^d}_{O_R^+}(\mathbf{r}_0) + \mathbf{C}^{E^d}_{S_R}(\mathbf{r}_0) - \frac{1}{2}\mathbf{E}^d(\mathbf{r}_0), \tag{19}$$

where the singular integrals $\mathbf{C}^{E^u}_{S_R}(\mathbf{r}_0)$ and $\mathbf{C}^{E^d}_{S_R}(\mathbf{r}_0)$ are determined in terms of the Cauchy principal value. Similar equations can be written for magnetic fields as well.

Proceeding to the limit for $R \to \infty$, and equating the Eqs (17) and (19), we obtain:

$$\mathbf{E}^u(\mathbf{r}_0) = \frac{1}{2}[\mathbf{E}^u(\mathbf{r}_0) + \mathbf{E}^d(\mathbf{r}_0)] + \mathbf{C}^{E^u}_S(\mathbf{r}_0) + \mathbf{C}^{E^d}_S(\mathbf{r}_0), \tag{20}$$

and

$$\mathbf{E}^d(\mathbf{r}_0) = \frac{1}{2}[\mathbf{E}^u(\mathbf{r}_0) + \mathbf{E}^d(\mathbf{r}_0)] - \mathbf{C}^{E^u}_S(\mathbf{r}_0) - \mathbf{C}^{E^d}_S(\mathbf{r}_0), \tag{21}$$

where we take into account that, according to the radiation conditions, integrals $\mathbf{C}^{E^u}_{O_R^-}(\mathbf{r}_0)$ and $\mathbf{C}^{E^d}_{O_R^+}(\mathbf{r}_0)$ vanish when $R \to \infty$.

Finally, according to formulas (2), we have

$$\mathbf{E}^u(\mathbf{r}_0) = \frac{1}{2}\mathbf{E}(\mathbf{r}_0) + \mathbf{C}^E_S(\mathbf{r}_0), \tag{22}$$

and

$$\mathbf{E}^d(\mathbf{r}_0) = \frac{1}{2}\mathbf{E}(\mathbf{r}_0) - \mathbf{C}_S^E(\mathbf{r}_0). \tag{23}$$

Similar expressions can be derived for magnetic field components as well:

$$\mathbf{H}^u(\mathbf{r}_0) = \frac{1}{2}\mathbf{H}(\mathbf{r}_0) + \mathbf{C}_S^H(\mathbf{r}_0), \tag{24}$$

and

$$\mathbf{H}^d(\mathbf{r}_0) = \frac{1}{2}\mathbf{H}(\mathbf{r}_0) - \mathbf{C}_S^H(\mathbf{r}_0). \tag{25}$$

In formulas (22) through (25), expressions $\mathbf{C}_S^E(\mathbf{r}_0)$ and $\mathbf{C}_S^H(\mathbf{r}_0)$ stand for the Stratton-Chu type integrals over the entire observation surface S, determined at a singular point $\mathbf{r}_0 \in S$ in terms of the Cauchy principal value:

$$\mathbf{C}_S^E(\mathbf{r}_0) = \iint_S [(\mathbf{n}\cdot\mathbf{E})\nabla G(\mathbf{r}_0|\mathbf{r}) + (\mathbf{n}\times\mathbf{E})\times\nabla G(\mathbf{r}_0|\mathbf{r}) + i\omega\mu_0(\mathbf{n}\times\mathbf{H})G(\mathbf{r}_0|\mathbf{r})]ds, \tag{26}$$

$$\mathbf{C}_S^H(\mathbf{r}_0) = \iint_S [(\mathbf{n}\cdot\mathbf{H})\nabla G(\mathbf{r}_0|\mathbf{r}) + (\mathbf{n}\times\mathbf{H})\times\nabla G(\mathbf{r}_0|\mathbf{r}) + \sigma_w(\mathbf{n}\times\mathbf{E})G(\mathbf{r}_0|\mathbf{r})]ds, \tag{27}$$

and $G_w(\mathbf{r}_0|\mathbf{r})$ is the fundamental Green's function for the Helmholtz equation in a homogeneous full space with the seawater conductivity σ_w, defined by Eq. (9). Equations (22) through (25) describe a general integral transformation of the total EM field, observed on the arbitrary surface S within the water layer, into upgoing and downgoing parts. These equations serve as a theoretical foundation of the method of up/down decomposition of the EM field.

3. Spatial Fourier Transform Method of EM Field Separation into Upgoing and Downgoing Parts

Consider again a 3D geoelectrical model of a marine CSEM survey. We assume now that the background conductivity in this model is formed by a horizontally layered model consisting of nonconductive air, a conductive seawater layer with the conductivity σ_w, and the homogeneous part of a sea-bottom formation. The sea-bottom inhomogeneities are located within domain V with the conductivity $\sigma = \sigma_b + \Delta\sigma$, varying spatially. Note that the frequencies of the EM field used in the marine EM are very low (usually less than 10 Hz). Therefore we can ignore the displacement currents. In the marine CSEM method, measurements are conducted at the sea bottom, while the electric dipole transmitter is located above the sea bottom. We introduce a Cartesian coordinate system with the origin at sea level and axis z directed downward. We assume also that the seafloor is flat, and the receivers are located at the horizontal plane $z = z_0$. It was demonstrated by Berdichevsky and Zhdanov (1984) that in this situation the field decomposition can be easily conducted in the spatial frequency (k, ω) domain. A similar approach was considered recently in Amundsen et al. (2006) as well. We will show below that these spectral based decomposition formulas are equivalent to the general Stratton-Chu decomposition transforms in the case of a flat surface of observation.

3.1. Electromagnetic field in the (k, ω) domain

We begin with Maxwell's equations (78), written for low frequency (less than 10 Hz) EM fields within a homogeneous seawater layer with the conductivity σ_w:

$$\begin{aligned} \nabla \times \mathbf{H} &= \sigma_w \mathbf{E}, \quad &(a)\\ \nabla \times \mathbf{E} &= i\omega\mu\mathbf{H}, \quad &(b)\\ \nabla \cdot \mathbf{H} &= 0, \quad &(c)\\ \nabla \cdot \mathbf{E} &= 0. \quad &(d) \end{aligned} \tag{28}$$

Separating Maxwell's equations (28), we arrive at the following Helmholtz equations:

$$\begin{aligned} \nabla^2\mathbf{E} + k_w^2\mathbf{E} &= 0,\\ \nabla^2\mathbf{H} + k_w^2\mathbf{H} &= 0, \end{aligned} \tag{29}$$

where $k_w = \sqrt{i\omega\mu_0\sigma_w}$.

Let us define spatial Fourier spectrums of the electric and magnetic fields using the following expressions:

$$\mathbf{e}(k_x, k_y, z) = \int_{-\infty}^{\infty}\int \mathbf{E}(x, y, z)\exp[i(k_x x + k_y y)]dxdy, \tag{30}$$

$$\mathbf{h}(k_x, k_y, z) = \int_{-\infty}^{\infty}\int \mathbf{H}(x, y, z)\exp[i(k_x x + k_y y)]dxdy, \tag{31}$$

where k_x and k_y are spatial frequencies.

[illegible] as follows:

$$\mathbf{e}(x, y, z) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathbf{e}(k_x, k_y, z) \exp[-i(k_x x + k_y y)] dk_x dk_y \quad (32)$$

$$\mathbf{h}(x, y, z) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathbf{h}(k_x, k_y, z) \exp[-i(k_x x + k_y y)] dk_x dk_y \quad (33)$$

Applying the Fourier transform to equations (29), we obtain

$$\frac{\partial^2}{\partial z^2} \mathbf{e} = \nu^2 \mathbf{e}, \quad \text{and} \quad \frac{\partial^2}{\partial z^2} \mathbf{h} = \nu^2 \mathbf{h}, \quad z_b \geq z \geq 0, \quad (34)$$

where $\nu = (k_x^2 + k_y^2 - i\omega\mu\sigma_w)^{1/2}$, $\mathrm{Re}\,\nu > 0$, is the wave number in the (k, ω) domain, and z_b is the depth of the sea bottom.

The general solutions of the last equations have the following form:

$$\mathbf{e}(k_x, k_y, z) = \mathbf{e}^u \exp[\nu(z - z_0)] + \mathbf{e}^d \exp[-\nu(z - z_0)], \quad z_b \geq z \geq 0, \quad (35)$$

$$\mathbf{h}(k_x, k_y, z) = \mathbf{h}^u \exp[\nu(z - z_0)] + \mathbf{h}^d \exp[-\nu(z - z_0)], \quad z_b \geq z \geq 0, \quad (36)$$

where z_0 is the depth of the location of the receivers, and $\mathbf{e}^{u,d}$ and $\mathbf{h}^{u,d}$ are the spectrums of the upgoing and downgoing components of the electromagnetic field to be determined. The goal is to find the upgoing and downgoing components of the field.

3.2. Separation of the observed EM field into upgoing and downgoing components

Following Berdichevsky and Zhdanov (1984), we can develop a technique for separation of the observed EM field into upgoing and downgoing components based on Eqs (35) and (36). Indeed, direct analysis of Eqs (35) and (36) shows that this problem can be solved by differentiation of both sides of these equations with respect to the vertical coordinate z:

$$\frac{\partial}{\partial z} \mathbf{e}(k_x, k_y, z) = \nu \mathbf{e}^u \exp[\nu(z - z_0)] - \nu \mathbf{e}^d \exp[-\nu(z - z_0)], \quad z_b \geq z \geq 0, \quad (37)$$

$$\frac{\partial}{\partial z} \mathbf{h}(k_x, k_y, z) = \nu \mathbf{h}^u \exp[\nu(z - z_0)] - \nu \mathbf{h}^d \exp[-\nu(z - z_0)], \quad z_b \geq z \geq 0. \quad (38)$$

Multiplying both sides of Eq. (35) by ν and adding Eq. (37), we have

$$\nu\mathbf{e}(k_x, k_y, z) + \frac{\partial}{\partial z}\mathbf{e}(k_x, k_y, z) = 2\nu\mathbf{e}^u \exp[\nu(z - z_0)]. \tag{39}$$

A similar combination of Eqs (36) and (38) produces:

$$\nu\mathbf{h}(k_x, k_y, z) + \frac{\partial}{\partial z}\mathbf{h}(k_x, k_y, z) = 2\nu\mathbf{h}^u \exp[\nu(z - z_0)]. \tag{40}$$

From the last two expressions we find immediately that:

$$\mathbf{e}^u = \frac{1}{2}\left[\mathbf{e}(k_x, k_y, z) + \frac{1}{\nu}\frac{\partial}{\partial z}\mathbf{e}(k_x, k_y, z)\right]\exp[-\nu(z - z_0)] \tag{41}$$

and

$$\mathbf{h}^u = \frac{1}{2}\left[\mathbf{h}(k_x, k_y, z) + \frac{1}{\nu}\frac{\partial}{\partial z}\mathbf{h}(k_x, k_y, z)\right]\exp[-\nu(z - z_0)]. \tag{42}$$

Thus, the problem of field separation is reduced to computations of the vertical derivatives of the electric and magnetic fields. The last problem can be solved using the spectral form of Maxwell's equations (28).

Let us apply the Fourier transform (30) and (31) to both sides of Eqs (28):

$$\left.\begin{aligned} &\delta_z \times \mathbf{h} = \sigma_w \mathbf{e}, && \text{(a)}\\ &\delta_z \times \mathbf{e} = \mathrm{i}\omega\mu\mathbf{h}, && \text{(b)}\\ &\delta_z \cdot \mathbf{h} = 0, && \text{(c)}\\ &\delta_z \cdot \mathbf{e} = 0, && \text{(d)} \end{aligned}\right. \tag{43}$$

where δ_z stands for a symbolic vector $(-\mathrm{i}k_x, -\mathrm{i}k_y, \partial/\partial z)$.

It follows directly from the last equations that the spectrums of the vertical derivatives of the different components of the electromagnetic field can be found as a linear combination of the spectrum of the fields themselves.

For example, we can find from Eqs (43)c and (43)d that

$$\frac{\partial}{\partial z}h_z = \mathrm{i}k_x h_x + \mathrm{i}k_y h_y, \tag{44}$$

and

$$\frac{\partial}{\partial z} e_z = \mathrm{i}k_x e_x + \mathrm{i}k_y e_y. \tag{45}$$

In a similar way, we determine from Eqs (43)a and (43)b that

$$\frac{\partial}{\partial z} h_x = -\mathrm{i}k_x h_z + \sigma_w e_y,$$
$$\frac{\partial}{\partial z} h_y = -\mathrm{i}k_y h_z - \sigma_w e_x,$$
$$\frac{\partial}{\partial z} e_x = -\mathrm{i}k_x e_z + \mathrm{i}\omega\mu h_y,$$
$$\frac{\partial}{\partial z} e_y = -\mathrm{i}k_y e_z - \mathrm{i}\omega\mu h_x.$$

Combining these formulas together, we have:

$$\frac{\partial}{\partial z}\mathbf{h} = \hat{\mathbf{K}}_{xy}\mathbf{h} + \sigma_w \hat{\boldsymbol{\eta}}\mathbf{e}, \tag{46}$$
$$\frac{\partial}{\partial z}\mathbf{e} = \mathrm{i}\omega\mu\hat{\boldsymbol{\eta}}\mathbf{h} + \hat{\mathbf{K}}_{xy}\mathbf{e}, \tag{47}$$

where we use the following notations:

$$\hat{\mathbf{K}}_{xy} = \begin{bmatrix} 0 & 0 & -\mathrm{i}k_x \\ 0 & 0 & -\mathrm{i}k_y \\ \mathrm{i}k_x & \mathrm{i}k_y & 0 \end{bmatrix}, \quad \hat{\eta} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Substituting Eqs (46) and (47) into (41) and (42), we finally find that:

$$\mathbf{e}^u = \frac{1}{2}\left[\left(\hat{\mathbf{I}} + \frac{1}{\nu}\hat{\mathbf{K}}_{xy}\right)\cdot\mathbf{e} + \frac{\mathrm{i}\omega\mu}{\nu}\hat{\eta}\cdot\mathbf{h}\right]\exp[-\nu(z - z_0)] \tag{48}$$

and

$$\mathbf{h}^u = \frac{1}{2}\left[\left(\hat{\mathbf{I}} + \frac{1}{\nu}\hat{\mathbf{K}}_{xy}\right)\cdot\mathbf{h} + \frac{\sigma_w}{\nu}\hat{\eta}\cdot\mathbf{e}\right]\exp[-\nu(z - z_0)]. \tag{49}$$

We can write the last formulas using scalar notations, assuming $z = z_0$:

$$e_x^u = \frac{1}{2}e_x - \frac{\mathrm{i}k_x}{2\nu}e_z + \frac{\mathrm{i}\omega\mu}{2\nu}h_y, \tag{50}$$

$$e_y^u = \frac{1}{2}e_y - \frac{\mathrm{i}k_y}{2\nu}e_z - \frac{\mathrm{i}\omega\mu}{2\nu}h_x, \tag{51}$$

$$e_z^u = \frac{1}{2}e_z + \frac{\mathrm{i}k_x}{2\nu}e_x + \frac{\mathrm{i}k_y}{2\nu}e_y, \tag{52}$$

$$h_x^u = \frac{1}{2}h_x - \frac{\mathrm{i}k_x}{2\nu}h_z + \frac{\sigma_w}{2\nu}e_y, \tag{53}$$

$$h_y^u = \frac{1}{2}h_y - \frac{\mathrm{i}k_y}{2\nu}h_z - \frac{\sigma_w}{2\nu}e_x, \tag{54}$$

$$h_z^u = \frac{1}{2}h_z + \frac{\mathrm{i}k_x}{2\nu}h_x + \frac{\mathrm{i}k_y}{2\nu}h_y. \tag{55}$$

Note that, according to Eqs (43)a and (43)b, the vertical components of the spectrum of the electric and magnetic fields are equal to:

$$e_z = \frac{\mathrm{i}k_y h_x - \mathrm{i}k_x h_y}{\sigma_w}, \quad h_z = \frac{\mathrm{i}k_y e_x - \mathrm{i}k_x e_y}{\mathrm{i}\omega\mu}. \tag{56}$$

3.3. Convolution form of decomposition operators

Consider, as an example, the x component of the electric field. Applying the inverse Fourier transforms to Eq. (50), we have:

$$\begin{aligned} E_x^u = \frac{1}{2}E_x - \frac{1}{8\pi^2}\int_{-\infty}^{\infty}\int \frac{\mathrm{i}k_x}{\nu}e_z(k_x, k_y, z)\exp[-\mathrm{i}(k_x x + k_y y)]\mathrm{d}k_x\mathrm{d}k_y \\ + \frac{\mathrm{i}\omega\mu}{2}\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int \frac{1}{\nu}h_y(k_x, k_y, z)\exp[-\mathrm{i}(k_x x + k_y y)]\mathrm{d}k_x\mathrm{d}k_y. \end{aligned} \tag{57}$$

Using the convolution theorem (Arfken and Weber, 1995, pp. 863–865), we can write Eq. (57) in the form:

$$\begin{aligned} E_x^u(x', y', z) = &\frac{1}{2}E_x(x', y', z) \\ &+ \frac{1}{2}\int_{-\infty}^{\infty}\int \frac{\partial}{\partial x'}K(x'-x, y'-y)E_z(x, y, z)\mathrm{d}x\mathrm{d}y \\ &+ \frac{\mathrm{i}\omega\mu}{2}\int_{-\infty}^{\infty}\int K(x'-x, y'-y)H_y(x, y, z)\mathrm{d}x\mathrm{d}y, \end{aligned} \tag{58}$$

where $K(x', y')$ is the convolution kernel function for a separation operator given by the formula:

$$K(x', y') = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int \frac{1}{\nu}\exp[-\mathrm{i}(k_x x' + k_y y')]\mathrm{d}k_x\mathrm{d}k_y. \tag{59}$$

Note that both integrals in Eq. (58) are determined in the sense of the Cauchy principal value, which means that these integrals are equal to the limits of the following integrals:

$$\int_{-\infty}^{\infty}\int \frac{\partial}{\partial x'}K(x'-x,y'-y)E_z(x,y,z)\mathrm{d}x\mathrm{d}y = \lim_{\rho\to 0}\iint_{S_\rho}\frac{\partial}{\partial x'}K(x'-x,y'-y)E_z(x,y,z)\mathrm{d}x\mathrm{d}y, \tag{60}$$

$$\int_{-\infty}^{\infty}\int K(x'-x,y'-y)H_y(x,y,z)\mathrm{d}x\mathrm{d}y = \lim_{\rho\to 0}\iint_{S_\rho}K(x'-x,y'-y)H_y(x,y,z)\mathrm{d}x\mathrm{d}y, \tag{61}$$

where S_ρ is the entire horizontal plane XY with the exclusion of a circle $(x'-x)^2+(y'-y)^2 \leqslant \rho$.

The integral in Eq. (59) can be expressed using the tabulated integral $J_1(x',y',z')$ (Zhdanov and Keller, 1994):

$$J_1(x',y',z') = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int \frac{\exp[-\nu(z'-z)]}{\nu}\exp[-\mathrm{i}(k_x x'+k_y y')]\mathrm{d}k_x\mathrm{d}k_y = \frac{\exp(\mathrm{i}k_w r)}{r}, \tag{62}$$

where $r=\sqrt{x'^2+y'^2+(z'-z)}$ and $k_w=\sqrt{\mathrm{i}\omega\mu\sigma_w}$.

Comparing Eqs (59) and (62), we can see that

$$K(x',y') = \frac{1}{2\pi}J_1(x',y',z')_{z'=z_b}. \tag{63}$$

Note that function J_1 is nothing else but the fundamental Green's function (9) for the Helmholtz equation in a homogeneous full space with the sea-water conductivity σ_w, multiplied by (-4π):

$$J_1(x',y',z_b) = -4\pi G_w(\mathbf{r}'|\mathbf{r}_0), \tag{64}$$

where $\mathbf{r}'=(x',y',z_b)$ and $\mathbf{r}_0=(0,0,z)$. Thus we have:

$$K(x',y') = -2G_w(\mathbf{r}'|\mathbf{r}_0). \tag{65}$$

Therefore, integral transformation (58) can be written as:

$$E_x^u(x', y', z) = \frac{1}{2}E_x(x', y', z) - \int_{-\infty}^{\infty}\int \frac{\partial}{\partial x'}G_w(\mathbf{r}'|\mathbf{r}_0)E_z(x, y, z)\mathrm{d}x\mathrm{d}y$$
$$- \mathrm{i}\omega\mu \int_{-\infty}^{\infty}\int G_w(\mathbf{r}'|\mathbf{r}_0)H_y(x, y, z)\mathrm{d}x\mathrm{d}y. \qquad (66)$$

We can derive analogous convolution-type expressions for all other upgoing/downgoing components of the electromagnetic field observed on a horizontal plane.

Direct algebraic calculations (which we do not present here because of their length) show that similar convolution forms of the decomposition operators can be obtained directly from the Stratton-Chu decomposition transformations (22)–(25).

4. Electromagnetic Field Separation into Upgoing and Downgoing Parts Using Horizontal Gradients

We should note that the methods of EM field decomposition based on Stratton-Chu type transforms and on the Fourier transform in the (k, ω) domain require measurements of the total EM field on the entire surface of observation S. In practice, however, we measure these fields at a discrete number of observation points located within a limited part of S. That is why it is important to develop simplified methods of field decomposition which would require measurements at just a few points. The simplest way to solve this problem is based on an assumption that the observed data can be approximated by a vertically propagated plane EM wave. In this case, we can use only one spatial frequency, $k_x = k_y = 0$, and the spectral decomposition formulas (50)–(55) are reduced to the simple algebraic formulas for field separation at a given point. This approach, for example, was discussed in (Amundsen et al., 2006) and is widely used for decomposition of field MCSEM data.

However, one can produce a more accurate but still rapid decomposition method assuming just slow horizontal variations of the EM field observed within the area of observation. In this case, the EM field within the seawater can be expressed approximately as (Zhdanov et al., 1996; Zhdanov, 2002, 2009):

$$\begin{aligned}\mathbf{E}(x, y, z) &= \mathbf{Q}_E^u(x, y, z)\exp[-\mathrm{i}k(z - z_0)] + \mathbf{Q}_E^d(x, y, z)\exp[\mathrm{i}k(z - z_0)],\\ \mathbf{H}(x, y, z) &= \mathbf{Q}_H^u(x, y, z)\exp[-\mathrm{i}k(z - z_0)] + \mathbf{Q}_H^d(x, y, z)\exp[\mathrm{i}k(z - z_0)],\end{aligned} \qquad (67)$$

where $z_b \geq z \geq 0$, z_b is the depth of the sea bottom, z_0 is the depth of the location of the receivers, and $k = k_w = \sqrt{\mathrm{i}\omega\mu\sigma_w}$.

We assume that the vector functions $\mathbf{Q}_E^u$, $\mathbf{Q}_E^d$, $\mathbf{Q}_H^u$, and $\mathbf{Q}_H^d$ vary relatively slowly with the depth:

$$|\partial \mathbf{Q}_{E,H}^{u,d}/\partial z| \ll |\partial \exp[\pm ik(z - z_0)]/\partial z|. \tag{68}$$

The goal is to find the upgoing and downgoing components of the EM field. This problem can be solved by differentiation of both sides of Eq. (67) with respect to the vertical coordinate z:

$$\begin{aligned}\partial \mathbf{E}(x, y, z)/\partial z &= ik\mathbf{Q}_E^d \exp[ik(z - z_0)] - ik\mathbf{Q}_E^u \exp[-ik(z - z_0)],\\ \partial \mathbf{H}(x, y, z)/\partial z &= ik\mathbf{Q}_H^d \exp[ik(z - z_0)] - ik\mathbf{Q}_H^u \exp[-ik(z - z_0)].\end{aligned} \tag{69}$$

Multiplying both sides of Eq. (67) by ik and adding (69), we have:

$$\begin{aligned}ik\mathbf{E}(x, y, z) - \partial \mathbf{E}(x, y, z)/\partial z &= 2ik\mathbf{Q}_E^u \exp[-ik(z - z_0)],\\ ik\mathbf{H}(x, y, z) - \partial \mathbf{H}(x, y, z)/\partial z &= 2ik\mathbf{Q}_H^u \exp[-ik(z - z_0)].\end{aligned} \tag{70}$$

From the last expression we find immediately that:

$$\begin{aligned}\mathbf{Q}_E^u(x, y, z) &= \frac{1}{2}[\mathbf{E}(x, y, z) - \frac{1}{ik}\partial \mathbf{E}(x, y, z)/\partial z] \exp[ik(z - z_0)],\\ \mathbf{Q}_H^u(x, y, z) &= \frac{1}{2}[\mathbf{H}(x, y, z) - \frac{1}{ik}\partial \mathbf{H}(x, y, z)/\partial z] \exp[ik(z - z_0)].\end{aligned} \tag{71}$$

Thus the problem of field separation is reduced to computations of the vertical derivatives of the electric and magnetic fields. The last problem can be solved using Maxwell's equations.

Within the homogeneous seawater layer the EM field satisfies Maxwell's equations (28). It follows directly from Eq. (28) that the vertical derivatives of the different components of the electromagnetic field can be found as a linear combination of the fields themselves. For example, we can find from Eqs (28)c and (28)d that

$$\begin{aligned}\frac{\partial}{\partial z}H_z &= -\frac{\partial}{\partial x}H_x - \frac{\partial}{\partial y}H_y,\\ \frac{\partial}{\partial z}E_z &= -\frac{\partial}{\partial x}E_x - \frac{\partial}{\partial y}E_y.\end{aligned} \tag{72}$$

In a similar way, we determine from (28)a and (28)b that

$$\begin{aligned}
\frac{\partial}{\partial z} H_x &= \frac{\partial}{\partial x} H_z + \sigma_w E_y, \\
\frac{\partial}{\partial z} H_y &= \frac{\partial}{\partial y} H_z - \sigma_w E_x, \\
\frac{\partial}{\partial z} E_x &= \frac{\partial}{\partial x} E_z + \mathrm{i}\omega\mu H_y, \\
\frac{\partial}{\partial z} E_y &= \frac{\partial}{\partial y} E_z - \mathrm{i}\omega\mu H_x.
\end{aligned} \tag{73}$$

Substituting Eqs (73) into (71), we can write the last formulas using scalar notations, assuming $z = z_0$:

$$\begin{aligned}
Q^u_{E_x} &= \frac{1}{2} E_x - \frac{1}{2\mathrm{i}k}\frac{\partial}{\partial x} E_z - \frac{\mathrm{i}\omega\mu}{2\mathrm{i}k} H_y, \\
Q^u_{E_y} &= \frac{1}{2} E_y - \frac{1}{2\mathrm{i}k}\frac{\partial}{\partial y} E_z + \frac{\mathrm{i}\omega\mu}{2\mathrm{i}k} H_x, \\
Q^u_{E_z} &= \frac{1}{2} E_z + \frac{1}{2\mathrm{i}k}\frac{\partial}{\partial x} E_x + \frac{1}{2\mathrm{i}k}\frac{\partial}{\partial y} E_y, \\
Q^u_{H_x} &= \frac{1}{2} H_x - \frac{1}{2\mathrm{i}k}\frac{\partial}{\partial x} H_z - \frac{\sigma_w}{2\mathrm{i}k} E_y, \\
Q^u_{H_y} &= \frac{1}{2} H_y - \frac{1}{2\mathrm{i}k}\frac{\partial}{\partial y} H_z + \frac{\sigma_w}{2\mathrm{i}k} E_x, \\
Q^u_{H_z} &= \frac{1}{2} H_z + \frac{1}{2\mathrm{i}k}\frac{\partial}{\partial x} H_x + \frac{1}{2\mathrm{i}k}\frac{\partial}{\partial y} H_y,
\end{aligned} \tag{74}$$

where $Q^u_{E_x}$, $Q^u_{E_y}$, $Q^u_{E_z}$ are the scalar components of vector $\mathbf{Q}^u_E$, and $Q^u_{H_x}$, $Q^u_{H_y}$, $Q^u_{H_z}$ are the scalar components of vector $\mathbf{Q}^u_H$, respectively.

According to Eqs (28)a and (28)b, the vertical components of the electric and magnetic fields are equal to:

$$\begin{aligned}
E_z &= \frac{1}{\sigma_w}\left[\frac{\partial}{\partial x} H_y - \frac{\partial}{\partial y} H_x\right], \\
H_z &= \frac{1}{\mathrm{i}\omega\mu}\left[\frac{\partial}{\partial x} E_y - \frac{\partial}{\partial y} E_x\right].
\end{aligned} \tag{75}$$

Therefore, in principle, in order to solve the upgoing/downgoing field separation problem, one can measure the horizontal components of the EM field only.

After we determine the vector coefficients $\mathbf{Q}_E^u$, $\mathbf{Q}_E^d$, and $\mathbf{Q}_H^u$, $\mathbf{Q}_H^d$, we can find the upgoing and downgoing fields themselves, e.g.:

$$\mathbf{E}^u(x, y, z) = \mathbf{Q}_E^u \exp[-ik(z - z_0)],$$
$$\mathbf{H}^u(x, y, z) = \mathbf{Q}_H^u \exp[-ik(z - z_0)]. \quad (76)$$

Note that the expressions $\frac{\partial}{\partial x}E_x$, $\frac{\partial}{\partial y}E_y$, $\frac{\partial}{\partial x}E_z$, $\frac{\partial}{\partial y}E_z$, etc. represent the horizontal gradients of the electric and magnetic field components. These horizontal gradients can be calculated numerically using finite differences, e.g.:

$$\frac{\partial}{\partial x}E_x \approx \frac{E_x(x + \Delta x, y, z) - E_x(x, y, z)}{\Delta x}. \quad (77)$$

By substituting the finite difference expressions into formulas (74), we arrive at the finite difference expression for the upgoing and downgoing fields.

5. Numerical Examples of Marine EM Data Decomposition

In order to check the effectiveness of the separation technique developed above based on the horizontal gradients, we have conducted a number of numerical experiments with synthetic EM data. We present some of these results below.

5.1. Model 1

We begin our numerical study with Model 1, shown in Figure 2. The geoelectrical section of this model is formed by a seawater layer with a resistivity of 0.4 Ohm m and a thickness of 340 m, and conductive sea-bottom sediments with a resistivity of 1 Ohm m, respectively. A 3D reservoir with 100 Ohm m resistivity is embedded in the sea bottom at a depth of 500 m below the seafloor. The EM field in the model is generated by a horizontal electric bipole transmitter towing behind the ship at an elevation of 50 m above the seafloor. The receivers measuring all six components of the EM field (or four horizontal components only) are located at the seafloor.

As an example, we consider the field generated by one transmitter located at the point whose horizontal coordinates are $x = -10$ km and $y = 0$, where the origin of the coordinates, $x = 0$ and $y = 0$, is at the center of the reservoir. The receivers are located along several survey lines with a separation of 500 m in both x and y directions, as shown in Figure 3.

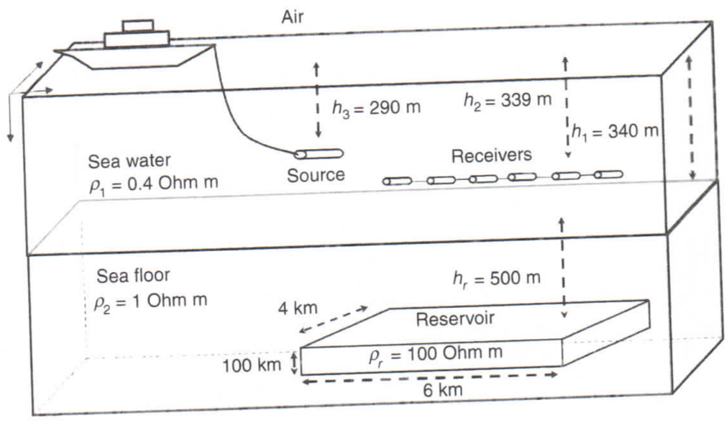

Figure 2 3D view of Model 1. The geoelectrical section of this model is formed by a seawater layer with a resistivity of 0.4 Ohm m and a thickness of 340 m, and conductive sea-bottom sediments with a resistivity of 1 Ohm m, respectively. A 3D reservoir with 100 Ohm m resistivity is embedded in the sea bottom at a depth of 500 m below the seafloor.

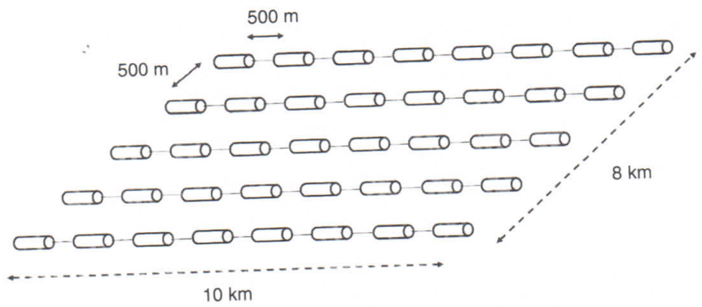

Figure 3 The array of receivers for Model 1. Each spatial step along the line of receivers is 500 meters, and the distance between two neighboring survey lines is also 500 meters.

Figures 4 and 5 show the results of the decomposition of the x component of the electric field, E_x, and the y component of the magnetic field, H_y, into upgoing and downgoing parts along one of the observational profiles. The top panels in these figures show the plots of the amplitude vs. offset curves for the total and upgoing fields, while the bottom panels present the similar plots of the phase vs. offset curves. The calculation of the upgoing field component was done using the method of horizontal gradients introduced above.

We should note that the problem of upgoing/downgoing field separation can be solved using the data along one survey line only. However, the most profound result can be seen in the maps of the upgoing and downgoing parts of the field (see Figures 6 and 7). One can clearly see in these figures

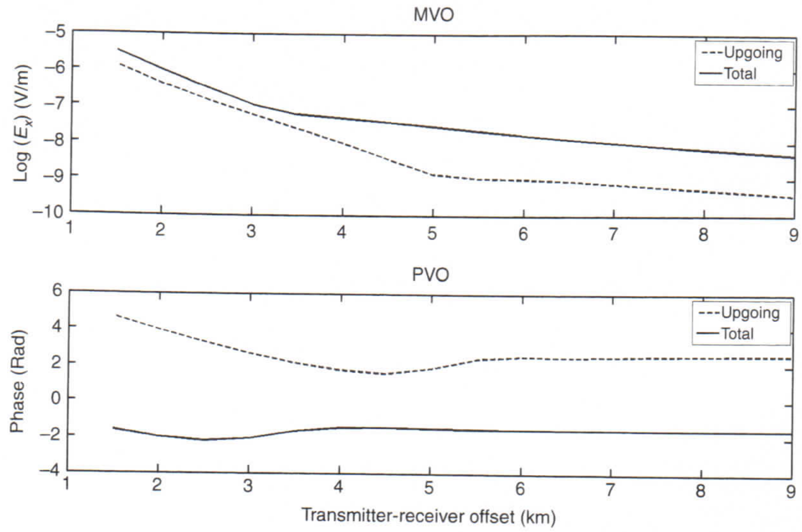

Figure 4 Model 1: Example of the separation of the in-line electric field E_x at a frequency of 0.5 Hz into the upgoing and downgoing parts. The top panel shows the plots of the amplitude vs. offset curves for the total and upgoing fields, while the bottom panel presents similar plots of the phase vs. offset curves.

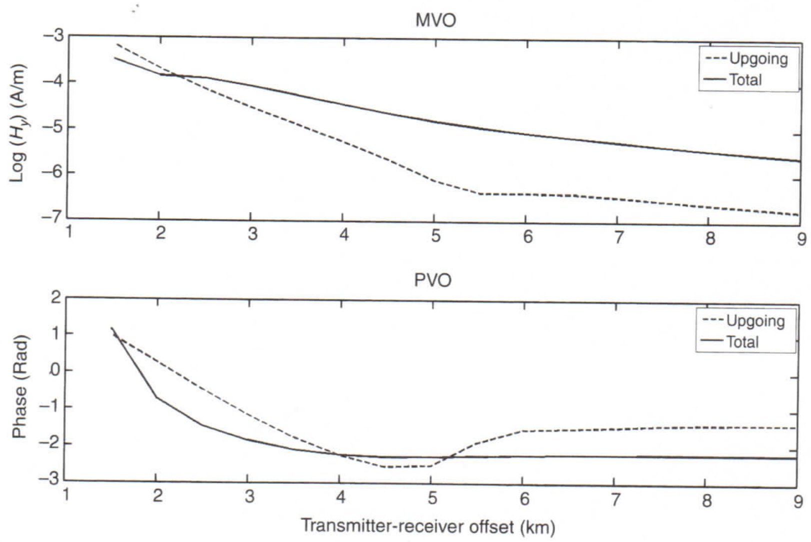

Figure 5 Model 1: Example of the separation of the in-line magnetic field H_y at a frequency of 0.5 Hz into the upgoing and downgoing parts. The top panel shows the plots of the amplitude vs. offset curves for the total and upgoing fields, while the bottom panel presents similar plots of the phase vs. offset curves.

that the separation of the upgoing part of the total electric field results in effective imaging of the horizontal location of the reservoir. A similar effect is observed in the maps of the horizontal component of the magnetic field H_y after a decomposition transformation.

Note that in Figures 6 and 7 a dashed white rectangle outlines the location of a resistive reservoir. From these results, it is clear, the upgoing field (real and/or imaginary parts, amplitude and/or phase) separated using the technique, reflects the location of the target much better than the total EM field data. Numerical data, shown above, were generated for a frequency of 0.5 Hz. We also used other frequencies from 0.1 Hz to 10 Hz and other locations of the transmitting bipole source to test our technique. We have obtained similar results for all these frequencies and different transmitter locations.

5.2. Model 2

We have constructed Model 2 to test the separation technique for a more complicated model representing a geological structure with two reservoirs embedded within the conductive sediments at a depth of 500 m and 600 m below the seafloor, respectively. Figure 8 shows a 3D view of the model, while Figure 9 presents a plan view of the reservoir locations. As an example, we assume that a horizontal electric bipole transmitter is located in the central survey line at a distance of 7 km to the left side of the reservoir 1 shown in Figure 9. The receiver array is the same as in Model 1 with spatial steps, both along the survey line and perpendicular to the survey line, equal to 500 m, as shown in Figure 3.

Figures 10 and 11 indicate the effectiveness of separation for Model 2, in which a dashed white rectangle outlines the location of 3D resistive reservoirs. All these results demonstrate that the developed technique of the total field decomposition into upgoing and downgoing parts can be effectively used for fast analysis of observed sea-bottom EM data. The upgoing field produced using the separation technique reflects well the locations of the sea-bottom resistive structures.

6. Conclusions

The modern technique of EM field decomposition into upgoing and downgoing parts has its roots in the classical methods of geomagnetic field separation into external and internal parts. The most general approach to the solution of this problem is based on the theory of the Stratton-Chu type integrals. This approach makes it possible to separate MCSEM data observed on the seafloor in areas with rough bathymetry. In the case of a flat seafloor, the separation of MCSEM data can be done using the Fourier transform in the (k, ω) domain. However, spatial spectrum transformation, in a general case, requires measurement of the data in a relatively large area

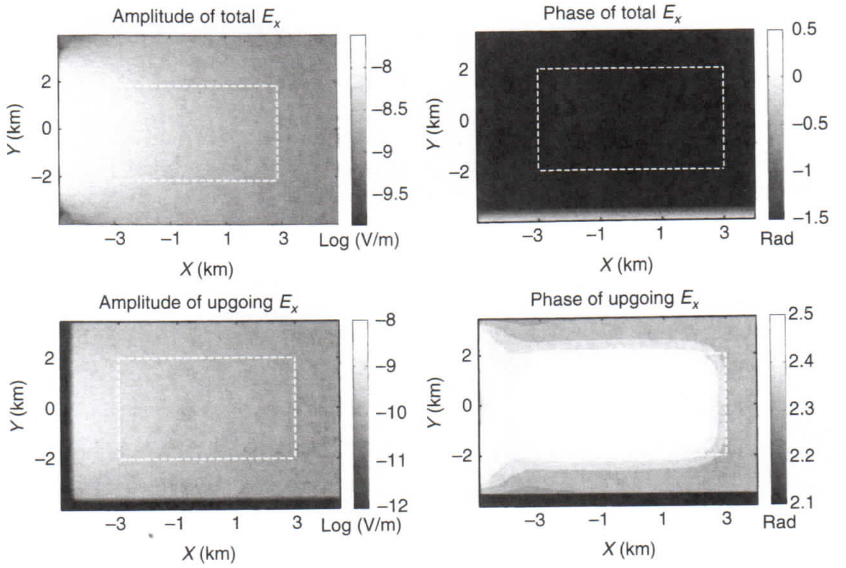

Figure 6 Model 1: The top panels show maps of the amplitude (top left panel) and phase (top right panel) of the total in-line electric field E_x at a frequency of 0.5 Hz. The bottom panels present maps of the amplitude (bottom left panel) and phase (bottom right panel) of the upgoing in-line electric field E_x^u separated using the method of horizontal gradients.

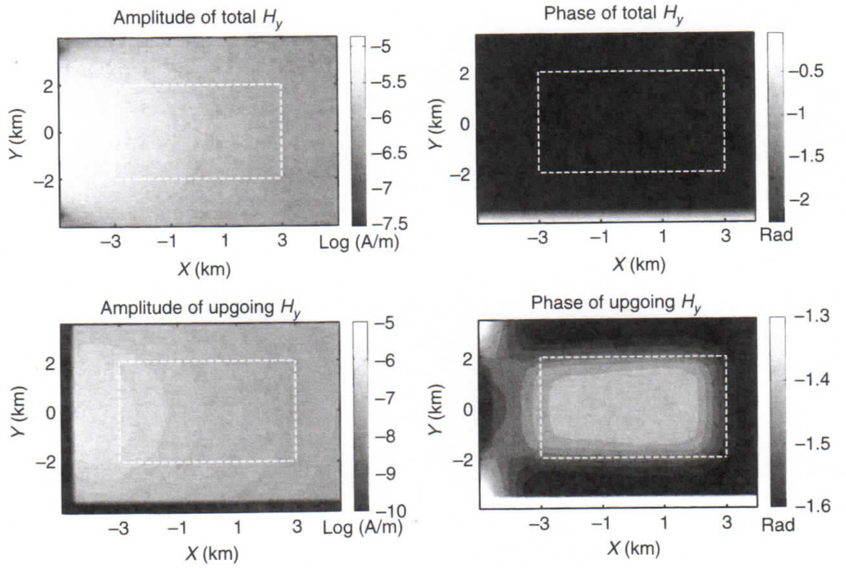

Figure 7 Model 1: The top panels show maps of the amplitude (top left panel) and phase (top right panel) of the total magnetic field H_y at a frequency of 0.5 Hz. The bottom panels present maps of the amplitude (bottom left panel) and phase (bottom right panel) of the upgoing magnetic field H_y^u separated using the method of horizontal gradients.

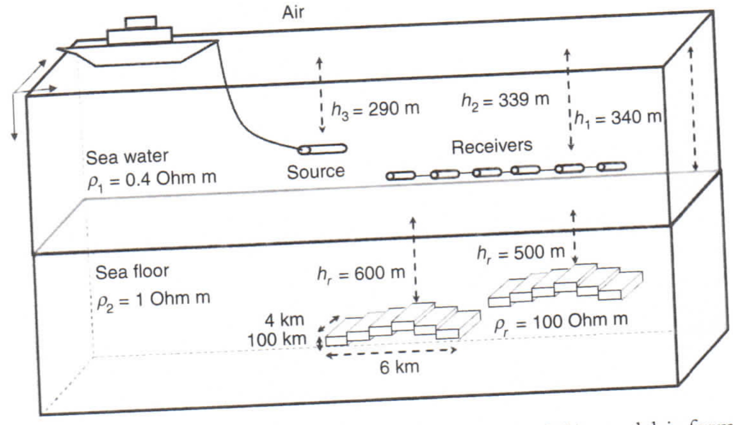

Figure 8 3D view of Model 2. The geoelectrical section of this model is formed by a seawater layer with a resistivity of 0.4 Ohm m and a thickness of 340 m, and conductive sea-bottom sediments with a resistivity of 1 Ohm m, respectively. Two reservoirs with 100 Ohm m resistivity are embedded in the sea bottom at depths of 500 m and 600 m below the seafloor.

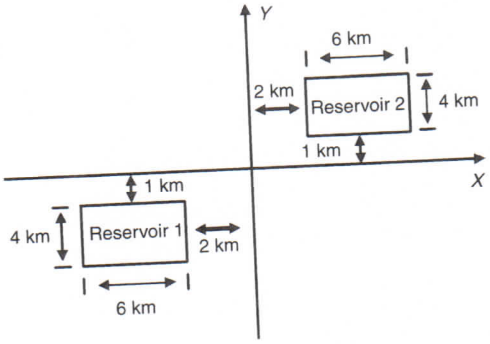

Figure 9 Model 2: Plan view of the resistive reservoir locations.

of observation. This method can be used for a single point separation under the assumption that the field can be approximated by a plane wave, which is valid in the far field of the source only.

We have developed a method of solving this problem which is free of these limitations of the spectral decomposition method. It is based on the calculations of the horizontal gradients of the field, and therefore it can be used for a decomposition of MCSEM data measured at two points only. The numerical study shows that this novel method can be used as a rapid transformation of MCSEM data for a qualitative evaluation of the location of the typical exploration targets, e.g., HC reservoirs.

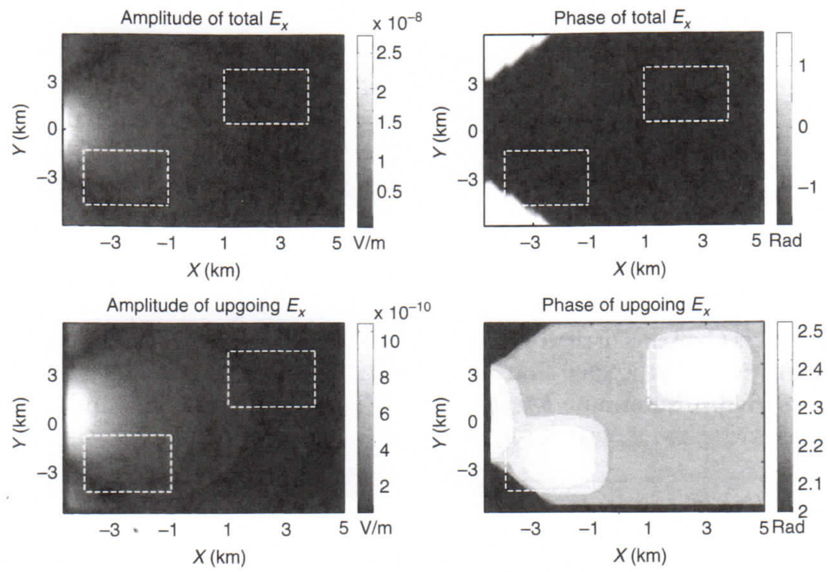

Figure 10 Model 2: The top panels show maps of the amplitude (top left panel) and phase (top right panel) of the total in-line electric field E_x at a frequency of 0.5 Hz. The bottom panels present maps of the amplitude (bottom left panel) and phase (bottom right panel) of the upgoing in-line electric field E_x^u separated using the method of horizontal gradients.

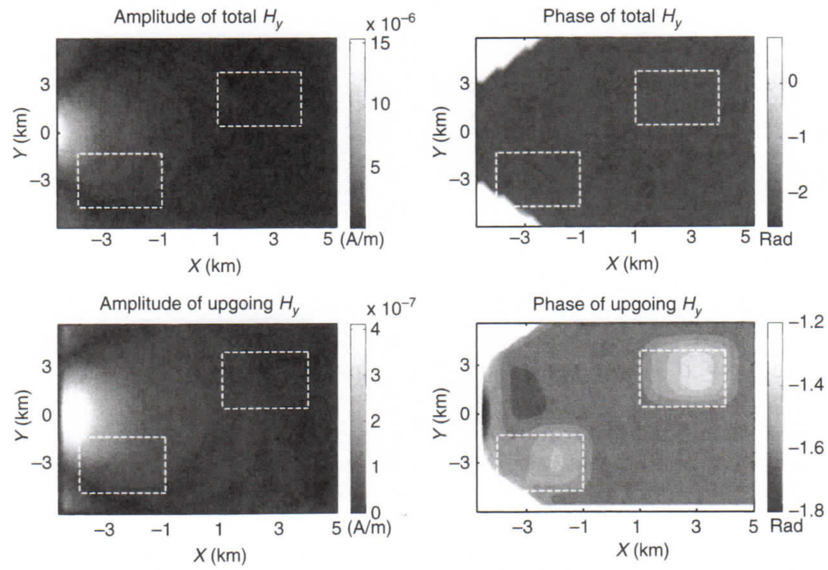

Figure 11 Model 2: The top panels show maps of the amplitude (top left panel) and phase (top right panel) of total magnetic field H_y at a frequency of 0.5 Hz. The bottom panels present maps of the amplitude (bottom left panel) and phase (bottom right panel) of the upgoing magnetic field H_y^u separated using the method of horizontal gradients.

Acknowledgments

The authors acknowledge the support of the University of Utah Consortium for Electromagnetic Modeling and Inversion (CEMI), which includes BAE Systems, Baker Atlas Logging Services, BGP China National Petroleum Corporation, BHP Billiton World Exploration Inc., BP, EMGS, ENI S.p.A., ExxonMobil Upstream Research Company, Fugro, Halliburton Energy Services, Information Systems Laboratories, NewmontMining Co., OHM, Petrobras, PGS, Rio Tinto - Kennecott, Rocksource, Russian Research Center Kurchatov Institute, Schlumberger, Science Applications International Co., Shell International Exploration and Production Inc., StatoilHydro, Sumitomo Metal Mining Co., Woodside Energy, and Zonge Engineering and Research Organization.

APPENDIX A. Stratton-Chu Integral Formulas

In these appendices we present a short review of the theory of the Stratton-Chu type integrals, developed by Zhdanov (1988).

Let us consider a 3D domain D bounded by a smooth surface S. We assume that domain D is filled by a homogeneous medium with the conductivity σ, magnetic permeability μ, and dielectric constant ε. The frequency-domain electromagnetic field, $\mathbf{E}$, $\mathbf{H}$, in the model is generated by some sources located outside domain D. The time dependence is expressed by an exponential function $\exp(-i\omega t)$. This field satisfies inside D (up to its boundary S) the following Maxwell's equations:

$$\begin{aligned} \nabla \times \mathbf{H} &= \tilde{\sigma}\mathbf{E}, \\ \nabla \times \mathbf{E} &= i\omega\mu\mathbf{H}, \\ \nabla \cdot \mathbf{H} &= 0, \\ \nabla \cdot \mathbf{E} &= 0, \end{aligned} \tag{78}$$

where $\tilde{\sigma} = \sigma - i\omega\varepsilon$ stands for the complex electric conductivity of the medium.

One can formulate a boundary-value problem for the EM field as follows. We assume that the EM field components are known everywhere on the surface S. The goal is to find the values of the electric, $\mathbf{E}(\mathbf{r}')$, and magnetic, $\mathbf{H}(\mathbf{r}')$, fields inside the domain, $\mathbf{r}' \in \mathrm{D}$. A solution of this problem is provided by the classical Stratton-Chu integral formulas (Stratton, 1941):

$$\iint_S [(\mathbf{n} \cdot \mathbf{E})\nabla G + (\mathbf{n} \times \mathbf{E}) \times \nabla G + i\omega\mu(\mathbf{n} \times \mathbf{H})G]ds = \mathbf{E}(\mathbf{r}'), \quad \mathbf{r}' \in \mathrm{D}, \tag{79}$$

$$\iint_S [(\mathbf{n}\cdot\mathbf{H})\nabla G + (\mathbf{n}\times\mathbf{H})\times\nabla G + \tilde{\sigma}(\mathbf{n}\times\mathbf{E})G]ds = H(\mathbf{r}'), \quad \mathbf{r}' \in \mathrm{D}, \tag{80}$$

where $\mathbf{n}$ is the unit vector of an inward pointing normal to S; $G = G(\mathbf{r}'|\mathbf{r})$ is the fundamental Green's function for the Helmholtz equation:

$$G(\mathbf{r}'|\mathbf{r}) = -\frac{1}{4\pi|\mathbf{r}-\mathbf{r}'|}\exp(\mathrm{i}\tilde{k}|\mathbf{r}-\mathbf{r}'|),$$

and $\tilde{k}$ is the corresponding wave number:

$$\tilde{k} = \sqrt{\mathrm{i}\omega\mu\tilde{\sigma}} = \sqrt{\mathrm{i}\omega\mu\sigma + \omega^2\mu\varepsilon}\ \mathrm{Re}\,\tilde{k} > 0. \tag{81}$$

The Green's function satisfies the equation:

$$\nabla^2 G + \tilde{k}^2 G = \delta(\mathbf{r}-\mathbf{r}'), \tag{82}$$

where $\delta(\mathbf{r}-\mathbf{r}')$ is a delta-function.

APPENDIX B. Stratton-Chu Type Integrals and Their Properties

The concept of the Stratton-Chu type integrals was introduced by Zhdanov (1980, 1988).

Let some vector fields $\mathcal{E}$ and $\mathcal{H}$ be specified on the inner side of a closed smooth boundary S of a domain D_i and have continuously differentiable tangential components, $\mathcal{E}_\tau$ and $\mathcal{H}_\tau$. We assume that the normal components of these fields, $\mathcal{E}_n$ and $\mathcal{H}_n$, are related to the tangential components by the following expressions:

$$\mathcal{E}_n = -\frac{1}{\tilde{\sigma}}\nabla_s\cdot(\mathrm{n}\times\mathcal{H}_\tau), \quad \mathcal{H}_n = -\frac{1}{\mathrm{i}\omega\mu}\nabla_s\cdot(\mathrm{n}\times\mathcal{E}_\tau). \tag{83}$$

Equations (83) represent a two-dimensional form of Maxwell's equations (Zhdanov, 1988).

We consider the expressions:

$$C_S^E(\mathbf{r}') = \iint_S [(\mathrm{n}\cdot\mathcal{E})\nabla G + (\mathrm{n}\times\mathcal{E})\times\nabla G + \mathrm{i}\omega\mu(\mathrm{n}\times\mathcal{H})G]ds, \tag{84}$$

and

$$C_S^H(\mathbf{r}') = \iint_S [(\mathrm{n} \cdot \mathcal{H})\nabla G + (\mathrm{n} \times \mathcal{H}) \times \nabla G + \tilde{\sigma}(\mathrm{n} \times \mathcal{E})G] ds. \quad (85)$$

These expressions are equal to the integrals from formulas (79) and (80), if $\mathcal{E} = \mathbf{E}$ and $\mathcal{H} = \mathbf{H}$. They are called Stratton-Chu type integrals (Zhdanov, 1980, 1988), and functions $\mathcal{E}$ and $\mathcal{H}$ are their densities.

The Stratton-Chu type integrals have several important properties which make them very useful in EM applications.

(1) Everywhere in the whole space outside the surface S, the integrals C_S^E and C_S^H satisfy Maxwell's equations:

$$\begin{aligned} \nabla \times \mathbf{C}_S^H &= \tilde{\sigma}\mathbf{C}_S^E \\ \nabla \times \mathbf{C}_S^E &= i\omega\mu\mathbf{C}_S^H, \\ \nabla \cdot \mathbf{C}_S^H &= 0, \\ \nabla \cdot \mathbf{C}_S^E &= 0. \end{aligned} \quad (86)$$

(2) If the densities of the Stratton-Chu type integrals are equal to the boundary values of the electromagnetic fields $\mathbf{E}$ and $\mathbf{H}$, satisfying the Maxwell's equations (78) in the domain D right up to the surface S and vanishing at infinity, then we have:

$$\mathbf{C}_S^E(\mathbf{r}') = \begin{cases} \mathbf{E}(\mathbf{r}'), & \mathbf{r}' \in D_i, \\ 0, & \mathbf{r}' \in D_e, \end{cases} \quad \mathbf{C}_S^H(\mathbf{r}') = \begin{cases} \mathbf{H}(\mathbf{r}'), & \mathbf{r}' \in D_i, \\ 0, & \mathbf{r}' \in D_e, \end{cases} \quad (87)$$

where D_i and D_e are the inner and outer domains bounded by S.

(3) At the points on the inner $(\mathbf{r}_0^+)$ and outer (r_0^-) sides of the surface S there exist limit values of the Stratton-Chu type integrals equal to:

$$\begin{aligned} \mathbf{C}_i^E(\mathbf{r}_0) &= \lim_{\mathbf{r}' \to \mathbf{r}_0^+} \mathbf{C}_S^E(\mathbf{r}') = \mathbf{C}_S^E(\mathbf{r}_0) + \frac{1}{2}\mathcal{E}(\mathbf{r}_0), \\ \mathbf{C}_i^H(\mathbf{r}_0) &= \lim_{\mathbf{r}' \to \mathbf{r}_0^+} \mathbf{C}_S^H(\mathbf{r}') = \mathbf{C}_S^H(\mathbf{r}_0) + \frac{1}{2}\mathcal{H}(\mathbf{r}_0), \quad \mathbf{r}' \in D_i, \mathbf{r}_0 \in S \end{aligned} \quad (88)$$

and

$$\begin{aligned} \mathbf{C}_e^E(\mathbf{r}_0) &= \lim_{\mathbf{r}' \to \mathbf{r}_0^-} \mathbf{C}_S^E(\mathbf{r}') = \mathbf{C}_S^E(\mathbf{r}_0) - \frac{1}{2}\mathcal{E}(\mathbf{r}_0), \\ \mathbf{C}_e^H(\mathbf{r}_0) &= \lim_{\mathbf{r}' \to \mathbf{r}_0^-} \mathbf{C}_S^H(\mathbf{r}') = \mathbf{C}_S^H(\mathbf{r}_0) - \frac{1}{2}\mathcal{H}(\mathbf{r}_0), \quad \mathbf{r}' \in D_e, \mathbf{r}_0 \in S, \end{aligned} \quad (89)$$

where the singular integrals $\mathbf{C}_S^E(\mathbf{r}_0)$ and $\mathbf{C}_S^H(\mathbf{r}_0)$ are determined in the sense of the Cauchy principal value.

Thus, the Stratton-Chu type integrals have discontinuities on S, which are equal to their densities:

$$\mathbf{C}_i^E(\mathbf{r}_0) - \mathbf{C}_e^E(r_0) = \mathcal{E}(\mathbf{r}_0), \quad \mathbf{C}_i^H(\mathbf{r}_0) - \mathbf{C}_e^H(\mathbf{r}_0) = \mathcal{H}(\mathbf{r}_0). \quad (90)$$

At the same time, the sums of the limit values of the Stratton-Chu type integrals are equal to the double singular integrals:

$$\mathbf{C}_i^E(\mathbf{r}_0) + \mathbf{C}_e^E(\mathbf{r}_0) = 2\mathbf{C}_0^E(\mathbf{r}_0), \quad \mathbf{C}_i^H(\mathbf{r}_0) + \mathbf{C}_e^H(\mathbf{r}_0) = 2\mathbf{C}_0^H(\mathbf{r}_0). \quad (91)$$

The interested reader can find the proof of properties (1) through (3) in the monograph by Zhdanov (1988).

REFERENCES

Arfken, G., Weber, H.J., 1995. Mathematical Methods for Physicists, Fourth Edition. Academic Press.
Amundsen, L., Løseth, L., Mittet, R., Ellingsrud, S., Ursin, B., 2006. Decomposition of electromagnetic fields into upgoing and downgoing components. Geophysics 71, G211–G223.
Berdichevsky, M.N., Zhdanov, M.S., 1984. Advanced Theory of Deep Geomagnetic Sounding. Elsevier.
Chapman, S., Bartels, J., 1940. Geomagnetism, vol. 2. Clarendon Press.
Kertz, W., 1954. Modelle für erdmagnetischen induzierte elektrische Ströme in Untergrund. Nachr. Akad. Wiss. Göttingen Math.-Phys. Klasse, Abt. 11 A, 101–110.
Siebert, M., 1962. Die Zerlegung eines zweidimensionalen Magnetfeldes in äusseren und inneren Antei mit Hilfe der Cauchyschen Integralformed. Z. Geophys. 5, 231–236.
Stratton, J.A., 1941. Electromagnetic Theory. McGraw-Hill Book Company.
Zhdanov, M.S., 1980. Cauchy integral analogs for the separation and continuation of electromagnetic fields within conducting matter. Geophysical Surveys 4, 115–136.
Zhdanov, M.S., 1988. Integral Transforms in Geophysics. Springer-Verlag.
Zhdanov, M.S., 2002. Geophysical Inverse Theory and Regularization Problems. Elsevier.
Zhdanov, M.S., 2009. Geophysical Electromagnetic Theory and Methods. Elsevier.
Zhdanov, M.S., Keller, G., 1994. The Geoelectrical Methods in Geophysical Exploration. Elsevier.
Zhdanov, M.S., Traynin, P., Booker, J., 1996. Underground imaging by frequency domain electromagnetic migration. Geophysics 61, 666–682.
Zhdanova, O.N., Zhdanov, M.S., 1999. Methods for the analysis and interpretation of the sea floor electromagnetic fields. In: Deep Electromagnetic Exploration. Springer-Verlag.