



## Electromagnetic geophysics: Notes from the past and the road ahead

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### ABSTRACT

During the last century, electrical geophysics has been transformed from a simple resistivity method to a modern technology that uses complex data-acquisition systems and high-performance computers for enhanced data modeling and interpretation. Not only the methods and equipment have changed but also our ideas about the geoelectrical models used for interpretation have been modified tremendously. This paper describes the evolution of the conceptual and technical foundations of EM methods. It outlines a framework for further development, which should focus on multi-transmitter and multireceiver surveys, analogous to seismic data-acquisition systems. Important potential topics of future research efforts are in the areas of multidimensional modeling and inversion, including a new approach to the formulation and understanding of EM fields based on flux and voltage representation, which corresponds well to geophysical experiments involving the measurement of voltage and flux of electric and magnetic fields.

### INTRODUCTION

The electrical method was one of the first geophysical exploration techniques to become widely used at the end of the 1920s and at the beginning of 1930s in oil, gas, and mineral-deposit exploration. The development of a practical use of electrical fields to explore the earth's interior began with the pioneering work of the Schlumberger brothers, Conrad and Marcel, who went on to build one of the world's most successful geophysical service companies.

Since the time when the Schlumberger brothers began their work, electrical prospecting methods have transformed dramatically, starting from a simple resistivity method and developing into a complicated technology based on complex electromagnetic (EM) surveys with natural and controlled sources. Not only have the methods and equipment changed, but our ideas about EM fields, their generation and measurement, and geoelectrical models used for interpretation

have also advanced tremendously. For many years, the basic model for interpretation was a 1D model of a layered earth or a 2D model. However, during the last 20 years, geophysicists have begun using 3D models for interpretation as well. This advancement has required developing the corresponding mathematical methods of interpretation, based on modern achievements of EM theory and computer science in numerical modeling and inversion.

It is important to emphasize that the areas of practical application of electrical and EM methods are very diversified. EM measurements are conducted on land, in the air, in the sea, and within boreholes. These methods are applied in the mining industry; in oil and gas exploration; in geotechnical, engineering, groundwater, and environmental geophysics; and in regional and crustal tectonic studies. The two main applications of electrical and EM methods are sounding, mostly for petroleum and other stratigraphic studies, and searching for discrete conductors in base-metal exploration.

The first successful application of electrical and EM methods was in exploring for highly conductive metal ores. The methods were applied in the exploration for massive sulfide orebodies and disseminated metal ores. Even with the development of other applications in more recent years, the use of EM methods in the search for metallic ores remains one of the most important commercial applications. The fact is that in the big picture of expenditures in exploration geophysics, EM methods are still small compared to seismic methods; and within EM methods, the search for minerals has held the dominant position.

There is a large area of application of electrical methods in groundwater studies as well because the electrical resistivity of a rock is closely related to its water content. Another area of application is the search for geothermal resources because the resistivity of high-temperature geothermal zones is very low. In exploring for oil and gas, joint applications of the seismic, electrical, and gravity methods have been very successful, especially in recent years.

The diversity of problems defines the variety of electrical and EM methods that can be applied. The number of different modifications of EM methods is extremely large because we have the possibility of using different types of transmitters and measuring the different components of EM fields in a variety of receivers.

The behavior of an EM field is controlled by three primary proper-

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ties of the medium — the electrical conductivity, the dielectric permittivity, and the magnetic susceptibility — as well as a number of other physical phenomena. Electrical conductivity is the most useful of all physical properties that characterize rocks and minerals in geophysical exploration with relatively low frequency. For high-frequency methods such as ground-penetrating radar (GPR), dielectric permittivity is the most important property. Electrical conductivity in the earth varies by many orders of magnitude; in contrast, the other physical properties used in geophysical exploration (acoustic wave speed, density, magnetic susceptibility) vary over quite limited ranges. Many different parameters affect electrical conductivity, the most important being the presence of water or hydrocarbons in a rock, groundwater salinity, amounts of electronically conducting minerals in a rock, temperature, and pressure. All of these factors make electrical conductivity an obvious tracer for economic mineral deposits of various types.

It is impossible to include in one relatively short paper a long and rich history of EM geophysics. However, I would like to write here about some critical turning points in its development and some major discoveries. Often we do not think about the people who made these discoveries, but in fact, the stories of these people and of the circumstances that enabled them to make those contributions are as important as the science itself. So in this paper, I present a few stories, describing some of these individuals and their discoveries.

I also discuss recent developments in data-acquisition, modeling, and interpretation methods. During the last decade, geophysicists have begun using more realistic 3D models for interpretation. This conceptual change has required the development of corresponding mathematical methods of interpretation based on the modern achievements of EM theory and numerical modeling and inversion. I briefly review the most important results in this field as well.

Finally, I discuss a new approach to EM-field characterization and modeling based on the flux and voltage representation of a field instead of using conventional vectorial representations (Zhdanov, 2009a). Indeed, it is very well known that the flux of a field through a given surface and the work (or voltage) of the field along a given path represent the most important physical entities studied and measured in geophysical experiments. That is why the analytical and numerical representations of the EM field for flux and voltage appear to be extremely well suited for describing EM phenomena. A new paradigm for representing EM fields using flux and voltage opens new possibilities for accurate simulation in geophysical applications. This new approach to formulating and understanding the basic properties of the laws of electromagnetism has strong potential to stimulate the future development of EM geophysics.

## IN THE BEGINNING

Many of the theories and methods of modern-day EM geophysics are based on a few fundamental laws developed in the nineteenth century, such as Ohm's law, Ampere's law, and Faraday's law. The early study of electrical and EM phenomena in relation to the problems of finding ore and other mineral deposits was stimulated by a growing demand from the mining industry.

Robert Fox (1830) was probably the first scientist who considered a possible application of spontaneous polarization phenomena in locating orebodies. The first experiments measuring electrical conductivity to locate copper-bearing loads were conducted by James Fisher in Michigan, U.S.A., in 1893 (Jakosky, 1940). From 1912 to 1914, the Schlumberger brothers conducted the first DC geophysical sur-

veys in France for tectonic study. In 1916, Wenner advanced a method for measuring earth resistivity (Wenner, 1928) that was taken up for ore prospecting by Lundberg (1922), Gish and Rooney (1925), and Heiland (1926). Wenner made significant efforts in developing the principles of interpreting DC data. However, the DC method did not gain much practical ground until the concept of apparent resistivity was introduced by the Schlumberger brothers in 1922, which made data analysis and interpretation much more geologically meaningful and easier to conduct.

The intriguing saga of the Schlumberger family is told in probably the most comprehensive way in the recent book by Michael Oristaglio and Alexander Dorozynski (2009). The exciting idea of using an electric field for "seeing" through the ground was conceived of by Conrad Schlumberger as early as 1912. However, the advent of World War I in 1914 delayed the practical implementation of the idea. Only in 1919 did Paul Schlumberger offer financial support to his sons, Conrad and Marcel, to develop the revolutionary new technology of electrical prospecting. On 12 November 1919, the father and sons formalized their agreement, marking the beginning of a new era in exploration geophysics.

Initially, the Schlumberger brothers' field experiments to use electrical measurements to map subsurface formations did not result in significant discoveries. The first major commercial success came in the 1930s when the firm founded by the Schlumberger brothers was invited to work in the former Soviet Union. It was in the Azerbaijan oil fields and the Caucasus Mountains that resistivity well logging started its triumphant march over the planet.

The pioneering work of the Schlumberger brothers stimulated the creation and development of the Russian school of EM geophysicists, which produced a significant number of distinguished geophysicists and led to major geologic discoveries in the Ural Mountains and eastern Siberia in the 1940s and 1950s.

## BIRTH OF GEOPHYSICAL INVERSION AND REGULARIZATION THEORY

With the practical application of electrical methods in exploring the subsurface, the need for mathematical solutions to interpret the behavior of EM fields interacting with reasonable models of the earth became apparent. One of the most influential figures in the field of mathematical geophysics was Andrey Nikolaevich Tikhonov.

Tikhonov was born in 1906 in Gzhatsk (now Gagarin), about 120 km east of Moscow, Russia. In 1919, at 13 years of age, he started working as a railway clerk. In 1922, not quite 16 years of age, he was accepted as a student in the Department of Physics and Mathematics at Moscow State University, where he later spent most of his career. After graduation, he continued on the faculty in the Department of Physics and Geophysics and collaborated with the Geophysical Institute of the USSR Academy of Sciences. Although his early work dealt with essentially mathematical topics such as topology, he gradually became more involved with applying mathematics to geological and geophysical problems. During World War II, his professional career took a crucial turn — he was charged by the Geophysical Institute of the USSR Academy of Sciences to carry out a mathematically based evaluation of the effectiveness of DC electrical prospecting methods. Tikhonov was working closely with the Russian geophysicists, who conducted extensive searches for oil and gas in areas close to the Ural Mountains.

As a mathematician, Tikhonov knew that reconstructing the geoelectrical properties of subsurface formations from surface electrical

data was a typical ill-posed problem of mathematical physics. A classical definition of the ill-posed problem was given by French mathematician [Hadamard \(1902\)](#). According to Hadamard, a problem is ill posed if the solution is not unique or if it is not a continuous function of the data (i.e., if to a small perturbation of data there corresponds an arbitrarily large perturbation of the solution). Unfortunately, from the point of view of classical theory, all geophysical inverse problems are ill posed because their solutions are nonunique or unstable. This fact made Tikhonov believe that any attempt to recover the electrical properties of rocks from observed DC field data that was limited and noisy was doomed to failure. However, to his great surprise, the results of the field work with the DC method led to the discovery of a significant oil field in the Ural region. Tikhonov realized that practical geophysicists were able to solve this ill-posed problem and obtain geologically reasonable results by using an intuitive estimation of the possible solutions and selecting a geologically adequate model.

The successful results of applying electrical geophysical methods to oil exploration had an enormous effect on Tikhonov. He realized that classical methods of mathematical physics, as they were known in the first half of the twentieth century, with their restrictions regarding which numerical problems could be solved and which could not, had nothing to do with the practical problems of geophysics. The key idea developed by Tikhonov at that time was the introduction of a mathematical equivalent to human expertise and intuition, which played a key role in the oil discoveries in the Ural region. This led him to a formulation of the theory of ill-posed inverse problems, which up until then had been considered by mathematicians to be unsolvable.

In 1943, Tikhonov's first paper on inversion showed that such ill-posed problems could be solved ([Tikhonov, 1943](#)). This paper laid the basis for a new topic in applied mathematics, the Tikhonov regularization theory of the solution of ill-posed set problems ([Tikhonov and Arsenin, 1977](#)).

Tikhonov's fundamental discoveries played a major role in developing EM methods in geophysics. His early theoretical suggestions about the design of natural-field and controlled-source EM methods led his colleagues to the highly successful application of these methods in the search for oil and gas in the USSR immediately following World War II. However, his interests in applied mathematics covered many fields of the natural sciences, including atmospheric physics, ecology, medical tomography, and nuclear physics. His academic career culminated when, in 1969, he founded the College of Computational Mathematics and Cybernetics at Moscow State University, where he served as dean for 20 years.

## MAGNETOTELLURIC METHOD

A group of methods for determining the electrical structure of the earth using naturally existing EM fields rather than fields generated by a controlled source came into use around 1960, based on theoretical concepts originally proposed by [Andrey N. Tikhonov \(1950\)](#) in the USSR, [Louix Cagniard \(1953\)](#) in France, and [Tsuneji Rikitake \(1950\)](#) in Japan. Knowledge of the existence of telluric currents is far from recent. As early as in 1868, Sir George Biddell Airy, English mathematician and astronomer, made the first coordinated study of earth currents and their relationship to magnetic variations ([Airy, 1868](#)). In 1862, one of the first field experiments to measure telluric currents was carried out by [Lamont \(1862\)](#) in the Alps. [Terada \(1917\)](#) appears to have been the first to measure the dependence of

the magnetic-field relationships on the conductivity of the ground.

The Schlumberger brothers observed telluric currents during their experiments with DC measurements on the ground as well. They were also the first to suggest that telluric currents could be used for oil and gas exploration. However, practical measurements showed significant variations and instability in telluric-current behavior, which made it difficult to develop any reasonable technique for interpreting telluric-current data. The main sources of this instability were associated with complex processes in the ionosphere and magnetosphere, which were unknown at that time.

The core of the discovery made independently by Tikhonov and by Cagniard was that the effect of processes in the ionosphere and magnetosphere could be cancelled if the electric-field components of the telluric field were normalized by the magnetic-field components. Tikhonov and Cagniard introduced a concept of magnetotelluric impedances defined as follows:

$$Z_{xy} = \frac{E_x}{H_y}, \quad (1)$$

$$Z_{yx} = -\frac{E_y}{H_x}, \quad (2)$$

which has the dimensions of volts per ampere, or ohms.

At the time, this was a revolutionary idea because it let geophysicists transform observed field data in the predictions of the resistivity of rock formations. This opened the way for the development of a new exploration technique, the magnetotelluric (MT) method. Interestingly, as early as 1934, Hirayama found the explicit form for the ratio of  $E$  to  $H$  at the surface of the earth for an incident plane wave ([Hirayama, 1934](#)), and [Hatakayama \(1938\)](#) even used tensor conductivities to explain differences between  $E_x/H_y$  and  $E_y/H_x$ . However, Tikhonov and Cagniard should be credited for creating a solid physical and mathematical foundation of the MT sounding method.

The power of the Tikhonov and Cagniard approach is that, on the one hand, it is based on a simple geoelectrical model; on the other hand, it provides a geophysically and geologically meaningful result using a simple interpretation technique. Further development of the MT method was accomplished in the works of M. Berdichevsky, J. Booker, T. Cantwell, V. Dmitriev, J. Gough, G. Jiracek, T. Madden, F. Morrison, U. Schmucker, L. Vanyan, K. Vozoff, P. Wannamaker, S. Ward, J. Weaver, P. Weidelt, and many others, whose collective efforts transformed the method into a practical geophysical tool (e.g., [Berdichevsky, 1965](#); [Morrison et al., 1968](#); [Schmucker, 1970](#); [Vozoff, 1972](#); [Weidelt, 1975](#); [Berdichevsky and Dmitriev, 1976, 2002, 2008](#); [Vanyan and Butkovskaya, 1980](#); [Berdichevsky and Zhdanov, 1984](#); [Egbert and Booker, 1986](#); [Jiracek et al., 1987](#); [Booker and Chave, 1989](#); [Gough et al., 1989](#); [Madden and Mackie, 1989](#); [Wannamaker et al., 1989](#); [Wannamaker, 1991](#); [Weaver, 1994](#)).

The MT method has had a long and rich history full of great discoveries and setbacks. In the 1960s and early 1970s, the MT method became widely used for oil and gas exploration. Originally, the interpretation of MT data was based on simple layered-earth models, which made it easy to provide MT sounding curves in the form of the corresponding plots of the apparent resistivities versus the period of the observed data or square root of the period (which is proportional to the depth of investigation). The MT sounding curves were then transformed into 1D geoelectrical sections. The entire geoelectrical model was recovered by stitching together multiple 1D sections.

In real earth, however, there is always some departure from ideal

one-dimensionality, or horizontal inhomogeneity. Consequently, there is always some departure of an observed MT sounding curve from an ideal 1D curve computed for a specific layered-earth model. Such departures are called distortions to an MT sounding curve, and the observed curves in such cases are called distorted curves. Formal interpretation of these MT sounding curves in terms of 1D structure, ignoring such distortions, unfortunately often resulted in creating false geoelectrical structures of the depth sections, sometimes misinterpreted as potential hydrocarbon reservoirs.

Mark Naumovich Berdichevsky was the first to realize the importance of accounting for the effects of horizontal geoelectrical inhomogeneities on MT data. He introduced the tensor measurements in the MT method, which soon became widely used all over the world. The transition to tensor-based data processing resulted in a major increase in the amount of information extracted from MT observations. Another major contribution of Berdichevsky in geoelectrics was development of a distortion theory of MT sounding curves. His work on the distortion theory resulted in the method of deep geomagnetic sounding of the earth, created in collaboration with M. S. Zhdanov (Berdichevsky and Zhdanov, 1984).

The development of effective numerical modeling methods during the 1980s and 1990s made it possible to move the interpretation of MT data from simplistic stitched 1D sections to more realistic 2D and even 3D geoelectrical models, which has provided new opportunities for the practical application of the MT method in geophysical exploration.

## FROM FREQUENCY-DOMAIN TO TIME-DOMAIN METHODS

In the 1950s and 1960s, controlled-source frequency-domain electromagnetic (FDEM) sounding became widely used in exploration. The concept of using a monochromatic transmitted field to construct a sounding curve with frequency or transmitter-receiver separation being varied was developed early in the history of electrical geophysics. The method is detailed in various works, including Frischknecht (1967), Vanyan (1967, 1997), Keller (1968), Wait (1982), and Kaufman and Keller (1983).

Beginning in the 1960s, however, interest developed in the use of EM sounding methods to explore to depths of importance for oil and gas reservoirs and for geothermal systems. This required penetration of the EM field through several or even many kilometers of rock with a relatively low resistivity. Considering that the depth of investigation scales by the inverse wavenumber,  $1/k = 1/(i\omega\mu_0\sigma)^{1/2}$ , the frequency content transmitted by the controlled source decreases by the square of the depth to be investigated. Moreover, the frequency must be lowered in proportion to the square of the conductivity, inasmuch as typical oil and gas or geothermal exploration problems involve earth resistivities that are lower by factors of 10 or more than the average resistivity of the earth. Instead of using frequencies ranging from a few hertz to a few kilohertz, in these new problems it is often necessary to consider frequencies ranging from a few megahertz to a fraction of hertz.

Moving to a frequency window several decades wider, a number of practical problems arise. For example, generating a large-amplitude alternating current is not an easy task at megahertz frequencies. Moreover, the sequential observation of field components at various frequencies in the megahertz range can require an extraordinary amount of time. An obvious solution to both practical problems is to use a time-domain signal. For example, a signal generated by a step-

on/step-off current in the transmitter contains a wide spectrum of frequencies within a short transmitting time.

In other words, the key to achieving a greater depth of investigation was in transitioning from frequency-domain (FD) to time-domain (TD) EM methods. During the 1960s, an additional advantage of TDEM over FDEM was discovered: an ability to sound to a great depth while the source and receiver were close together. This fact seems to be quite obvious today; however, it took some time and effort for geophysicists to understand the physics and mathematics of the phenomenon.

It was very well known at that time that one could not use FDEM in the near zone of the transmitter because the near-zone field was insensitive to the electrical conductivity of the medium. The transition from the frequency domain to the time domain was treated as a simple superposition of the frequency-domain signals within a wide frequency range. Based on this concept, it seemed obvious that one could not use the TDEM method in the near zone of the transmitter as well.

This misconception was overturned by Russian geophysicist Grigory Grigorievich Obukhov in the late 1960s, who demonstrated that the TDEM field in the near zone is linearly proportional to the resistivity of the medium (Obukhov, 1968). Unfortunately, Obukhov's original result was not well received by his peers. At that time, many prominent Russian geophysicists could not believe that the time-domain pulse, which was treated as a simple superposition of the frequency-domain signals, would possess such radically different properties. However, the mathematics behind Obukhov's discovery was solid, and his result was later confirmed theoretically and practically in several field experiments.

Obukhov's result can be understood in this way. Let us consider a field generated by a horizontally directed current dipole located in a homogeneous unbounded conductive medium with a conductivity  $\sigma$  and a magnetic permeability equal to that of free space  $\mu_0$ , energized with a current impulse that has the form of a delta function. It is very well known that the vertical component of the magnetic field of a harmonic dipole source with the unit horizontal moment  $p_x = 1$  is

$$H_z(\omega) = -\frac{1}{4\pi} \frac{\partial}{\partial y} \left( \frac{e^{ikr}}{r} \right), \quad (3)$$

where  $r$  is the distance from the transmitter to the receiver,  $\omega$  is frequency, and  $k$  is a wavenumber. Applying to this expression a Fourier transform from the frequency domain to the time domain, we find for a magnetic field excited by a delta current pulse the following expression:

$$H_z^\delta(t) = -\frac{1}{4\pi} \frac{\partial}{\partial y} \left( \frac{1}{r} \left( \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ikr} e^{-i\omega t} d\omega \right) \right), \quad (4)$$

where  $t$  is time. After some algebra, equation 4 can be rewritten as (Zhdanov and Keller, 1994)

$$\begin{aligned} H_z^\delta(t) &= -\frac{1}{4\pi} \frac{\partial}{\partial y} \left( \frac{1}{r} \frac{8\pi^2 r (2\pi)^{1/2}}{\mu_0 \tau^3 \sigma} e^{-2(\pi r/\tau)^2} \right) \\ &= \frac{8\pi^3 (2\pi)^{1/2}}{\mu_0 \sigma} \frac{y}{\tau^5} e^{-2(\pi r/\tau)^2}, \end{aligned} \quad (5)$$

where the time-domain parameter  $\tau$  is defined as a combination of various constants that group together naturally:

$$\tau = \frac{2\pi}{\left(\frac{\mu_0\sigma}{2t}\right)^{1/2}}. \quad (6)$$

This parameter has the dimension of length and is usually calculated to be

$$\tau = (2\pi\rho t \times 10^7)^{1/2}. \quad (7)$$

Let us analyze expression 5. First of all, we see that the characteristic decline of the field from an impulsive source as a function of distance from the source is determined by the exponential multiplier  $e^{-2(\pi r/\tau)^2}$ , in which  $r$  appears as a ratio to the length of  $\tau$ . This ratio, as with a harmonically driven source, is called electrical distance:

$$r_\tau = \frac{r}{\tau}. \quad (8)$$

Note that  $\tau$  plays the same role as does the wavelength  $\lambda$  in the frequency domain. As is the case in the frequency domain, we can recognize three zones of behavior for the fields: (1) the near zone, for which  $r \ll \tau$ ; (2) the intermediate zone, in which  $r \approx \tau$ ; and (3) the far zone, for which  $r \gg \tau$ .

In the definition of  $\tau$ , there is a trade-off between time and distance in defining the various zones; that is, the far zone can be associated with the early part of the transient response and the near zone can be associated with the late part of the transient waveform. Because of this reciprocity in the roles played by time and distance, sometimes the far zone is called the early time zone and the near zone is called the late time zone. The near field corresponds to late times because the late times have a lower frequency content, which extends the near field farther from the transmitter. At very early times, almost everything is in the far field because this corresponds to high frequencies and very short electrical distances.

Let us turn now to the characteristic features of the transient fields in the near zone,  $r \ll \tau$ . In equation 5, taking a limit  $r/\tau \rightarrow 0$ , we have

$$H_z^\delta(t) \approx \frac{8\pi^3(2\pi)^{1/2}\rho y}{\mu_0 \tau^5}, \quad (9)$$

where  $\rho = 1/\sigma$  is electric resistivity. For comparison, let us write the same expression for the magnetic field in the near zone caused by a harmonic source:

$$H_z(\omega) \approx \frac{1}{4\pi} \frac{y}{r^3} e^{-i\omega t}, \quad (10)$$

where  $r = \sqrt{x^2 + y^2 + z^2}$  and where  $x, y, z$  are the coordinates of the receiver in the Cartesian system with the origin in the location of the electric dipole source. Comparing formulas 9 and 10, we see that although  $H_z(\omega)$  is independent of the resistivity of the medium, the time-domain signal  $H_z^\delta(t)$  is linearly proportional to  $\rho$ . Moreover, if we analyze  $H_z^\delta(t)$  and  $|H_z(\omega)|$  as functions of depth  $z$  for fixed values of  $x_0, y_0, t_0$ , and  $\omega_0$ , we see that in the near zone the field of a harmonic dipole decreases with depth but the transient field is uniform with depth. This provides the possibility, in theory at least, of investigating the geoelectric section to great depths with observations made close to the source. In practice, it is only required that a sufficiently strong pulse be applied to the source to explore to very great depths.

<sup>2</sup>The first work on transient EM for base-metal exploration was done by Wait in the early 1950s (see references in Wait [1982]) and was made commercial by Newmont at about the same time.

The discovery made by Obukhov created a paradigm change in exploration EM geophysics. The receiver now could be placed in close proximity to the transmitter, which made the results of EM soundings local. The depth of investigation was controlled by the time of the recorded signal in the transmitter. With the improved accuracy of the late-time measurements and increased power of the source, the depth of investigation was increased as well. All of these facts opened the way for a practical application of TDEM in oil and gas exploration. Key roles in the development of the method of TDEM soundings in the near zone were played by [Sidorov and Tickshaev \(1969\)](#), [Kaufman \(1989\)](#), and [Kaufman and Morozova \(1970\)](#).<sup>2</sup>

Over decades, many different transient EM surveys have been introduced for geophysical exploration. As an example of the most promising techniques, which were developed in the past and recently, I can mention the SIROTEM, UTEM, and LOTEM systems, and the MTEM method. SiroteM and UTEM were developed to find base metal orebodies; LOTEM and MTEM were introduced for EM sounding and oil and gas exploration.

The SIROTEM system is based on the Russian MPP0-1 unit. From 1972 to 1976, the Australian Commonwealth Scientific and Industrial Research Organisation's (CSIRO) Division of Mineral Physics, under the leadership of Ken McCracken and with the support of major Australian mining companies, redesigned the Russian MPP0-1 using the latest technology of the time, including a complementary metal oxide semiconductor (CMOS) processor. This was the first time a microprocessor had been used in an EM geophysical instrument. The CSIRO system was commercialized as SIROTEM and subsequently used extensively in mineral exploration around the world ([Buselli and O'Neill, 1977](#); [McCracken et al., 1986](#)).

The University of Toronto's EM (UTEM) system uses a large, fixed, horizontal transmitter loop as a source. An array of receivers located inside and outside of the transmitter loop measures all three components of the magnetic field ( $H_x, H_y$ , and  $H_z$ ) and the horizontal components of the electric field ( $E_x$  and  $E_y$ ). The UTEM transmitter sends a low-frequency current of precise triangular waveform through the transmitter loop ([Lamontagne et al., 1978](#)). The system has a relatively limited depth range, and it is applied primarily in mineral exploration.

The long-offset transient EM (LOTEM) technique was designed for deep penetration in the ground for EM sounding in oil and gas exploration. The LOTEM system consists of a grounded wire transmitter, an induction loop, and electric-field receivers. A typical distance between transmitter and receiver is approximately equal to or greater than the exploration depth. A detailed description of LOTEM can be found in [Strack \(1999\)](#).

After several years of research and improvement of EM instruments, a modification to TDEM methods has been introduced by [Wright et al. \(2002\)](#), [Hobbs et al. \(2005\)](#), and [Ziolkowski et al. \(2007\)](#); the technique is called multitransient EM (MTEM). The survey configurations of MTEM are very similar to those of seismic surveys with multiple receivers and multiple transmitters. The transmitter is a current bipole source, and the receivers are a line of bipoles with two electrodes. The key characteristic of the MTEM survey is the use of multiple transmitter-receiver separations, which results in a combination of parametric sounding and profiling. Transient current injection at the source may take the form of a step

change in current, such as the reversal in polarity of a DC current, or a coded finite-length sequence, such as a pseudorandom binary sequence (PRBS), which simulates a delta-pulse function in the transmitter.

These new time-domain-based geophysical methods provide additional capabilities for successful exploration for mineral resources.

### EM SOUNDINGS USING HIGH-POWER EM PULSES: THE KhibINI EXPERIMENT

In the development of modern EM methods capable of exploring the earth to depths of many kilometers, the limit to the depths that can be reached is always imposed by the signal strength. For example, to double the depth that can be reached, one must increase the strength of the source by about one order of magnitude. The source strength for an EM field is measured in terms of the source moment.

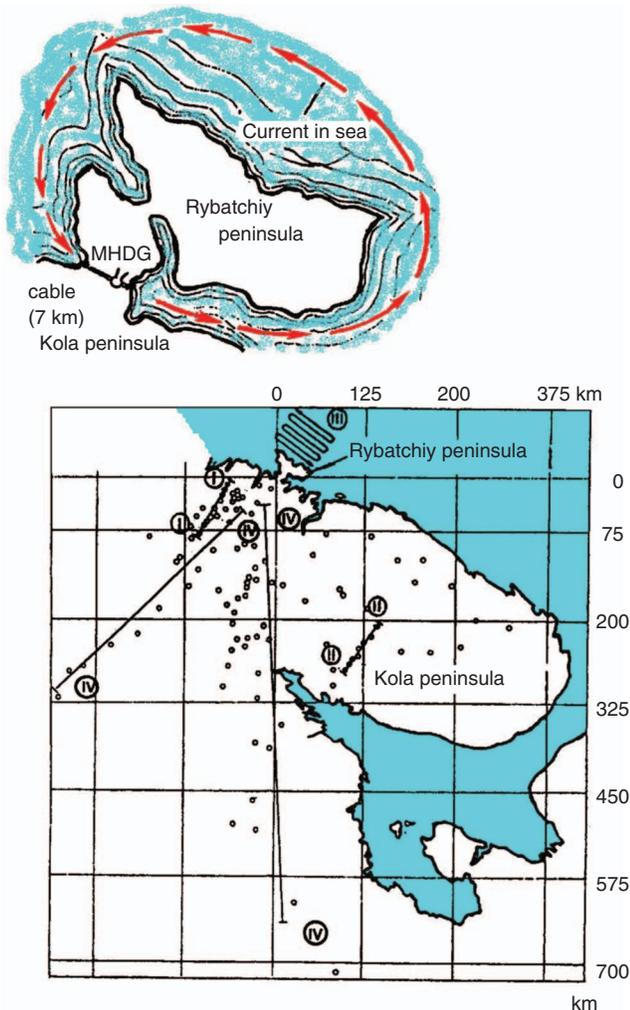


Figure 1. Survey configuration of the Khibini experiment. (a) The red line shows the current flow off the shore of the Kola peninsula over the Barents Sea. (b) Locations of the observation stations in the Kola peninsula (circles). The black lines show the profiles of detailed EM soundings: (I) Pechenga ore-bearing structure, (II) Imandra-Varzug ore-bearing area, (III) Barents Sea shelf, (IV) profile of the deep sounding of the Baltic Shield.

Therefore, an increase in source moment by an order of magnitude requires an increase in source power by two orders of magnitude. This can be achieved in several ways. The most direct approach is through using a transmitter of greater power. Another approach to increase signal strength is by repetitious transmissions of low power with synchronous stacking of the signals at a single receiver site. With long-term robust stacking, the ratio of signal level to noise (S/N) grows roughly as the square root of the number of transmissions. The energy expended in stacking increases in direct proportion to the number of transmissions.

Both of these approaches to increasing effective source moment have very real practical limits. Increasing the power available to energize a source requires a primary energy source whose mass grows more or less directly with power capability if only conventional generators are considered. So the maximum practical generator size and the longest stacking time that can be afforded should be taken into account when designing the EM surveys. To achieve significantly greater depth of sounding requires a departure from the use of conventional EM sources.

An idea for the application of pulsed magneto-hydrodynamic (MHD) generators in EM surveying was introduced in the 1970s by Russian physicist Evgeny Pavlovich Velikhov (Velikhov et al., 1975). One of the first practical applications of an MHD generator for EM soundings was performed during the Khibini experiment, conducted in the 1980s in the USSR. The Khibini experiment was designed to test the concept of using single, immensely strong impulses of an MHD generator (Velikhov, 1989). This powerful generator, which extracts energy from a stream of heated, ionized gas as it flows through a magnetic field, energized a single 11-km-long conductor connecting grounding structures in the Barents Sea on either side of the Sredniy and Rybachiy peninsulas.

The site for the Khibini source was selected so that the current from the two grounding structures would travel around the Sredniy and Rybachiy peninsulas, forming an equivalent loop in the horizontal plane. The maximum current provided to the cable was 22,000 A in 5–10-s pulses. Observations were comprised of the stations to the south on the Precambrian shield of the Kola Peninsula (Figure 1b) or the stations to the north, off the Kola Peninsula and over the Barents basin (Figure 1a). The geologic formations beneath the Barents Sea are formed by young sedimentary rocks with significant potential for oil or gas production, e.g., the massive Shtokman gas field. Evaluation of the deep potential of the Pechenga deposit was another important problem that was solved using EM surveying with MHD-generated pulses (Velikhov et al., 1987).

The Khibini experiment was one of the first large-scale experiments of deep EM sounding using high-power pulses. It provided geophysicists with a unique opportunity to test new technologies for studying a deep-earth interior using a controlled-source EM field. However, further development of an EM surveying system with a very powerful MHD generator was limited by the high cost of this exploration tool. With progress in MHD technology and with more simple and cost-effective operations, this method could become a power tool in regional and crustal tectonic studies associated with exploring the earth's resources.

### MINERAL EXPLORATION

Many difficulties are involved in the exploration of metallic ores. Metal orebodies are relatively small but valuable, and many non-economic minerals can masquerade as conductive orebodies. Metallic

ores respond to EM fields in different ways, depending on their host environment. That is why it is very important to select a specific set of EM methods to solve a specified exploration problem. Here, I outline several most promising electrical and EM techniques for mineral exploration.

The first group of methods is formed by time-domain sounding and profiling. Time-domain sounding was developed in Russia in the 1960s and since then has spawned a multitude of TDEM methods routinely used by exploration companies. The time-domain sounding system has proven to be one of the most effective in exploration. However, it is more efficient to measure the spatial-time structure of the entire EM field excited by a fixed or moving transmitter, an approach called time-domain array sounding (TDAS). In this case, one can consider the observed field as a huge surface EM hologram from which it is possible to extract a detailed image of the subsurface structure. Combinations of the different positions of the transmitters and receivers allow us to “illuminate” the orebodies from the different directions and thereby limit the ambiguity in solving the inverse problem.

TDAS is the method of choice in mineral exploration with systems such as Mount Isa Mines’ (now Geophysical Resource Service) MIMDAS system and BHP Billiton’s Geoferrret. Horizontal loops and grounded dipoles can be used as transmitters. The receivers can include horizontal loops and/or induction coils for measuring the different components of the magnetic field as well as grounded dipoles for electric-field observations. This enables the same survey configuration to be used for TDEM, induced polarization, and MT methods. The fact is that the greater the number of different components of the EM field we can observe, the greater the amount of detailed information about the subsurface structures that can be revealed.

The next group of methods used in mineral exploration is related to the controlled-source magnetotelluric (CSMT) method. CSMT is usually based on the simultaneous observations of electric and magnetic fields and on calculating the impedance similar to the conventional MT method. The main difference between MT sounding and CSMT methods is that in MT the observed EM field is the natural MT field, whereas in CSMT we register the controlled-source EM field. The main advantages of CSMT are connected with the possibility of taking into account the near-surface inhomogeneities and inverting the observed data for 3D geoelectrical structures.

There are some examples of successful application of the MT method in mineral exploration (e.g., Morrison and Nichols, 1997). However, the possibilities of conventional MT study have not yet been fully utilized. Recent improvements in MT equipment and processing tools have significantly increased the accuracy of MT field observations. A combination of the natural MT method for low frequencies and the CSMT method for high frequencies can result in soundings from the depth of the first meters down to several kilometers. The future success of MT methods in mineral exploration depends on the correct design of the receiver array. The best results can be obtained by a dense profile or by array observations similar to the MIMDAS system. Thus, we can expect that the practical applications of MT and CSMT will expand significantly during the next 10–20 years.

Another important application of EM methods are cross-borehole (crosshole) and surface-to-borehole studies. Putting the transmitters and/or the receivers into the hole significantly increases the resolution of EM methods because we can illuminate the target from different directions. Several different systems are used for borehole inves-

tigations. For example, a crosshole survey can be conducted by the frequency-domain vertical magnetic dipole system. The transmitter is located in one of the boreholes, and the receivers are in another borehole. This system can be used for locating and characterizing massive sulfide deposits or continuity of coal seams.

There are many problems in the surface-to-borehole and crosshole methods related to the borehole acquisition system itself, e.g., limited investigation distance, metal mandrel disturbance, and casing effects. Another problem in using this system is related to interpretation. The ultimate goal is to obtain a tomographic image of the target using EM data. This problem is not simple because we must clearly solve a 3D problem. The future success in the practical application of surface-to-borehole and crosshole methods depends significantly on the ability to solve this problem.

Several very successful and important techniques for mineral exploration are based on airborne electromagnetic (AEM) methods. The first commercial application of the AEM method was made as early as in 1951 (Fountain, 1998). Since then, AEM can be credited with directly aiding in the discovery of more than 80 mineral deposits (Witherly, 2000). Many different frequency- and time-domain AEM systems have been developed over the last 60 years. In the excellent papers cited above, the interested reader can find an overview of the history of different systems developed and flown.

In spite of all of the technological achievements and advances in AEM systems, however, the interpretation of AEM data is still based on simple conductivity-depth transforms (e.g., Macnae et al., 1998; Fullagar and Reid, 2001), layered-earth inversions (e.g., Chen and Raiche, 1998; Farquharson et al., 2003), or laterally constrained layered earth inversions (e.g., Auken et al., 2005). Even with widespread use, it has been repeatedly demonstrated that these inherently 1D methods are often invalid for recovering simple 2D and 3D targets (e.g., Ellis, 1998) let alone anything resembling geologic complexity (e.g., Raiche et al., 2001). Though 3D parameterized inversion methods such as thin sheets embedded in conductive hosts have been successful for certain types of targets (Wolfgram and Golden, 2001), the routine use of more generalized 3D modeling and inversion methods is yet to be realized.

In principle, AEM surveys measure the spatial-time or spatial-frequency structure of the entire EM field excited by a moving transmitter over the area of investigation. That is why the observed data can be treated as yet another huge AEM hologram, from which it should be possible to extract a detailed image of the subsurface structure. The question is how it can be done in practice.

The problem with 3D modeling of AEM data is that it is nontrivial given the necessity to solve as many large linear systems of equations as there are transmitter positions in the survey. For 3D inversion, this problem is exacerbated because sensitivities also need to be computed using adjoint operators, and the whole process must be repeated for multiple iterations. Computing time aside, a common limitation with 3D inversion is limited memory for storing the large but sparse sensitivity matrix. Various 3D modeling approximations have been introduced to simplify the nonlinear physics of AEM to a series of linear problems (e.g., Zhdanov and Tartaras, 2002; Zhdanov and Chernyavskiy, 2004). Avoiding such approximations, Ellis (2002), Wilson et al. (2006), and Raiche et al. (2007) introduce various 2.5D and 3D inversion software programs. Although more complete in their modeling of the physics of AEM, they too are limited to inverting “several” hundred stations of AEM data to “only” a few thousand elements, given their implementation and the previously discussed limitations.

To overcome these difficulties, Cox and Zhdanov (2007, 2008) and Wilson et al. (2010) apply the footprint approach to 3D inversion of AEM data. The use of a footprint allows for the inversion only of those parts of the model within the footprint of a particular transmitter-receiver pair. The footprints of all transmitter-receiver pairs superimpose themselves on the same 3D model, and so the sensitivity matrix is constructed. This approach makes 3D inversion of AEM data a practical consideration because entire surveys of frequency-domain AEM data with tens of thousands of stations can be inverted to geoelectric models with hundreds of thousands of cells within a day on a desktop workstation.

An example is a 3D inversion of data acquired for salinity mapping over the Bookpurnong irrigation district in South Australia (Wilson et al., 2010). This area has been the focus of various trials to manage a decline in vegetation, largely in response to floodplain salinization from groundwater discharge in combination with decreased flooding frequency, permanent weir pool levels, and recent drought. The RESOLVE frequency-domain helicopter system with six operating frequencies was flown in the area in August 2008. The

transmitter-receiver separation was 7.91 m for the five horizontal coplanar coil sets and 8.99 m for the single vertical coaxial coil set. This survey consisted of approximately 45,000 stations, distributed over 26 lines oriented in a northwest-southeast direction with 100-m line spacing and seven tie lines.

Let me extend this example with a brief comparison between the results of 1D and 3D inversions of AEM data. Wilson et al. (2010) inverted each station for a layered earth model using the AirBeo code of Raiche et al. (2007). They also applied a rigorous 3D inversion method based on a moving footprint approach. Figure 2a shows a slice of the model at 4 m depth derived from interpolation of the layered-earth inversion results. Figure 2b shows the same image with the same color scale but obtained from the 3D inversion results. The Murray River, which has a lower conductivity than the floodplains, is clearly visible in the 3D inversion results. The layered-earth inversion somewhat smears this result and in some areas completely misses the fact that the Murray River is present. This case study shows that 3D inversion results are in far better agreement with the known geology of the area than results obtained from layered-earth inversion.

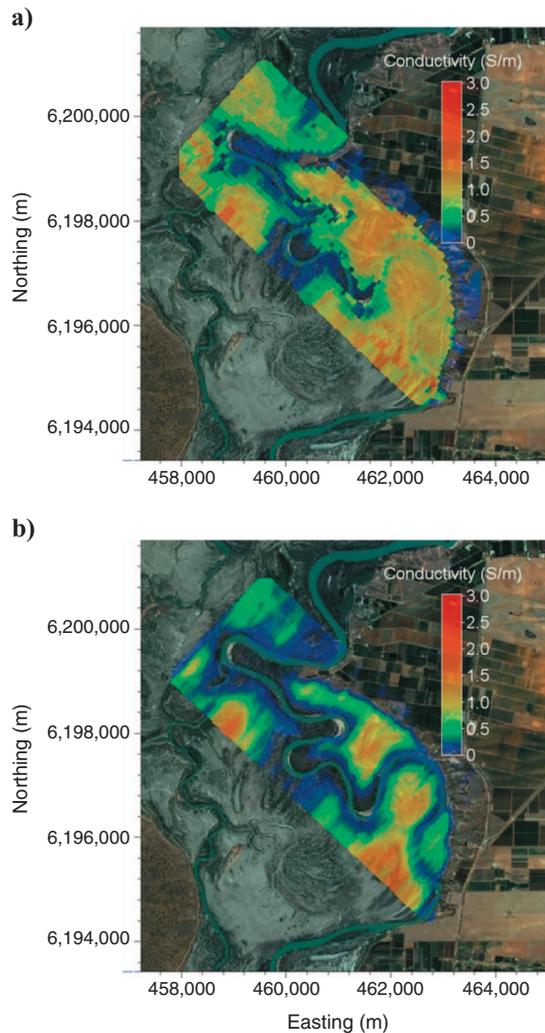


Figure 2. A horizontal cross section at 4 m depth of the conductivity obtained from (a) interpolation of layered-earth inversions of the RESOLVE frequency-domain AEM data using the AirBeo code and from (b) the 3D inversion of the RESOLVE data.

## HYDROCARBON EXPLORATION

Traditionally, the seismic method has been a method of choice for hydrocarbon exploration, but electrical methods have found only limited application in such studies. However, even long ago in the former Soviet Union, electrical methods played an important role in the discovery of oil and gas fields (Spies, 1980). Electrical methods have proven to be especially effective for locating hydrocarbon reservoirs in western Siberia. These reservoirs are difficult to locate using conventional reflection seismic methods. The reservoirs are of lithologic type with little structural reflection. The seismic methods help determine the plane structure of the potentially oil-containing layers, but the electrical methods identify the oil-containing reservoirs themselves. That is why the integrated approach based on a joint application of seismic exploration and electrical exploration was the most successful tool in the discovery of major Siberian oil and gas deposits (Zhdanov and Keller, 1994).

A similar situation is observed in the oil and gas provinces of the Precaspian depression in Russia. The oil-containing layers are located there within a depth interval 3–5 km below the salt-dome structures, which makes the interpretation of seismic data a very complicated and ambiguous problem. At the same time, the salt layers have a very high resistivity and are transparent for the transient EM fields. Therefore, a joint interpretation of the seismic and EM data produces the best result for exploration. The EM methods applied in this case are TDEM sounding and profiling. As with mineral exploration, time domain array sounding (TDAS) seems to be the most effective tool in hydrocarbon exploration as well. In a general case, TDAS can be designed as a complete EM analog of the seismic data acquisition system with multiple transmitters and receivers. The network of receivers should be dense enough to achieve a necessary resolution of the survey. Creating such a system is a major goal for developing EM exploration methods in the very near future. In part, this has been done with MIMDAS but needs to be extended to larger arrays.

Another application of electrical methods in the petroleum industry lies in monitoring the flow of oil and gas from reservoirs as they deplete. In oil-field development, to increase oil output, assistance must be provided to extract oil from a reservoir. This assistance can be in the form of flooding — using water, gas, carbon dioxide, im-

miscible fluids, or a flame front. Reservoir characterization is needed to monitor and regulate this process for uniform extraction of the resource. Some of these processes markedly change the resistivity of the rock within which the process occurs. That is why there is a considerable potential for the use of EM methods in such monitoring.

An important group of EM methods has emerged recently for offshore hydrocarbon exploration. Geophysical EM methods originally were introduced for land observations only. It was thought that seawater was so conductive that useful amounts of an EM field would not penetrate, so electrical exploration methods were not feasible in the oceans. On the contrary, it was discovered in the mid-20th century that various EM methods could be used quite effectively in marine environment.

The first experiments with marine EM field measurements were conducted by Russian geophysicists in the Arctic Ocean (e.g., Novysh and Fonarev, 1966; Trofimov and Fonarev, 1972). Marine geoelectrical investigations were extensively developed in the former Soviet Union (Berdichevsky et al., 1989). In the late 1970s, the Scripps Institution of Oceanography conducted several deepwater marine EM experiments in the Pacific Ocean (e.g., Filloux, 1979; Cox, 1981). There have been a number of EM experiments used for mapping and imaging subsea-bottom geoelectrical structures (e.g., Sinha et al., 1990; Chave et al., 1991), especially for application to the study of the oceanic lithosphere and actively spreading mid-ocean ridges (Shneyer et al., 1991; Evans et al., 1994; Constable and Cox, 1996; MacGregor et al., 2000, 2001).

Marine EM exploration for sea-bottom hydrocarbon reservoirs has been carried out routinely in Russia since the 1970s. In the West, this method was not widely used by industry until the late 1990s, when several major oil corporations, including ExxonMobil, Statoil, and Shell, began using marine controlled-source electromagnetic (CSEM) surveys for offshore hydrocarbon exploration (Srka et al., 2006). The modern-day success of the application of EM methods to offshore hydrocarbon exploration is based on the fundamental fact that oil- and gas-bearing structures are characterized by very high resistivity and the surrounding sea-bottom formations filled with salt water are very conductive. Marine EM methods enable geologists to distinguish between a hydrocarbon-filled reservoir, which has high resistivity, and one filled with water or shale, which has lower resistivity.

Most existing EM technologies for marine geophysical exploration are based on magnetotelluric (MT) methods or a marine version of CSEM. Marine CSEM, the version most widely used at present, involves transmitting a low-frequency EM signal from a subsea source towed behind a ship. The signal returned from sediments below the seafloor is then recorded by a number of receivers dropped on the sea bottom at differing distances from the source. The measured EM response depends on the resistivity of the material through which the EM fields have propagated. Thus, the marine CSEM method utilizes a dipole transmitter to create vertical loops of current that can be distorted by the presence of thin, resistive layers of the hydrocarbon-filled reservoir, making marine CSEM highly sensitive to hydrocarbon formations (e.g., Ellingsrud et al., 2002).

Ground-breaking developments in EM technology over recent years and their subsequent use in offshore hydrocarbon exploration have shown encouraging results in detecting hydrocarbon reserves in potential reservoirs prior to drilling wells. As such, marine EM

heralds a new epoch for the oil and gas industry, with tremendous potential as the technology is exploited (Stefatos et al., 2009).

### 3D NUMERICAL MODELING AND INVERSION

One of the most challenging problems in EM methods is the development of effective interpretation schemes for 3D inhomogeneous geologic formations. Seminal papers on EM modeling for 3D geoelectrical structures were published 35 years ago by Art Raiche (1974), Gerald W. Hohmann (Hohmann, 1975), and Peter Weidelt (Weidelt, 1975). These publications inspired several generations of EM geophysicists to develop new methods of EM modeling and to study the complex behavior of EM fields in 3D geoelectrical structures. In the last decade of the 20th century and in the beginning of the 21st century, methods for numerical and analytical modeling of the interaction of EM fields with earth structures developed rapidly. This development was driven by the availability of high-performance computers, including PC clusters. These modeling capabilities made possible the extraction of much more information from the field data than was possible previously when only heuristic interpretation was feasible.

Several techniques are available for EM forward modeling. They are based on numerical implementation of the differential-equation (DE) approach [finite-difference (FD) or finite-element (FE) methods] or the integral equation (IE) approach. During the last decades, considerable advances have been made in all of these areas. Overviews of the effective modeling methods can be found, for example, in Hohmann (1983), Avdeev (2005), and Zhdanov (2002, 2009a).

Also, we can observe remarkable progress in the development of multidimensional interpretation techniques. Many papers have been published during the last 20 years on 3D modeling and inversion of EM geophysical data (e.g., Eaton, 1989; Lee et al., 1989; Madden and Mackie, 1989; Oldenburg et al., 1993; Zhdanov and Fang, 1996, 1999; Alumbaugh and Newman, 1997; Newman and Alumbaugh, 1997, 2000; Zhdanov and Hursán, 2000; Zhdanov et al., 2000; Sasaki, 2001; Zhdanov and Tartaras, 2002; Zhdanov and Golubev, 2003; Abubakar and van der Berg, 2004; Mackie and Watts, 2004; Siripunvaraporn et al., 2004, 2005; Gribenko and Zhdanov, 2007; Zhdanov, 2009a, 2009b).

The methods for solving 3D EM inverse problems usually are based on optimizing the model parameters by applying different inversion techniques. The simplest are the Monte Carlo methods, which determine the wanted parameters by random or regular trial-and-error procedures. Because of the limitations of computing time, Monte Carlo methods have found limited applications in EM geophysics. The more widely used approaches are based on applying gradient methods for optimization.

We should note, as seen from Maxwell's equations, that EM fields are related nonlinearly to the conductivity of the earth. Therefore, EM inverse problems are characterized first of all by their nonlinearity. The key problem in the optimization technique is calculating the Fréchet derivative (sensitivity matrix). The final goal of the inversion is usually the reconstruction of the conductivity distribution in the model. That is why the Fréchet derivative (or sensitivity matrix) is determined, as a rule, with respect to the conductivity parameters of the model. In a general case, the calculation of the Fréchet derivative matrix is a very challenging problem. The good news, however, is that, in the framework of the gradient methods, one must calculate not the Fréchet derivative itself but the result of application of the adjoint Fréchet derivative operator to the corresponding electric or

magnetic fields, which is a less expensive numerical operation. This approach is now realized in most gradient-inversion algorithms.

Another major difficulty in inversion is related to its ill-posedness, which means that the existence, uniqueness, and/or stability of solutions is in question. The inherent nonlinearity of EM problems makes the ill-posedness more severe. To overcome this difficulty and obtain a stable solution of ill-posed inverse problems, we must apply the regularization theory, which was developed in pioneering work of Tikhonov (e.g., Tikhonov and Arsenin, 1977). This approach gives a solid basis upon which to construct effective inversion algorithms for 3D EM problems.

Traditional inversion methods based on the Tikhonov regularization theory usually provide a stable solution of the inverse problem using maximum smoothness stabilizing functionals. As a result, the obtained solution is a smooth image, which in many practical situations does not describe the examined geologic target properly. Portniaguine and Zhdanov (1999) and Mehanee and Zhdanov (2002) demonstrate that the images with sharp boundaries can be recovered by regularized inversion algorithms based on a new family of focusing stabilizing functionals. In particular, minimum-support (MS) and minimum-gradient-support (MGS) functionals are extremely effective in solving geophysical inverse problems (Zhdanov, 2002, 2009b).

As an example, let's take an application of a 3D rigorous inversion to the interpretation of a marine MT survey conducted in the Gemini Prospect (Zhdanov et al., 2009), which lies about 200 km southeast of New Orleans in about 1-km-deep water in the northern Gulf of Mexico (Figure 3). This work was done in close cooperation with Kerry Key and Steve Constable of the Scripps Institution of Oceanography. The main target of this sea-bottom MT survey was the salt bodies located in the area of the Gemini Prospect. Salt bodies are usually characterized by a higher seismic wave velocity and higher resistivity than the surrounding sea-bottom sediments. They represent a very difficult target for marine seismic exploration but can be identified clearly by marine EM methods.

It is generally well known that rocks with high seismic velocity and impedance contrast are also higher in electrical resistance than surrounding sediments. The Gemini salt body lies 1.5 km beneath the seafloor in 1-km-deep water and has a high electrical resistivity compared with the surrounding sediments. The subsalt gas deposit at Gemini is located at a depth of about 4 km on the southeastern edge of the Gemini structure. The high contrast in electrical conductivity between the salt and the surrounding sediments makes the Gemini

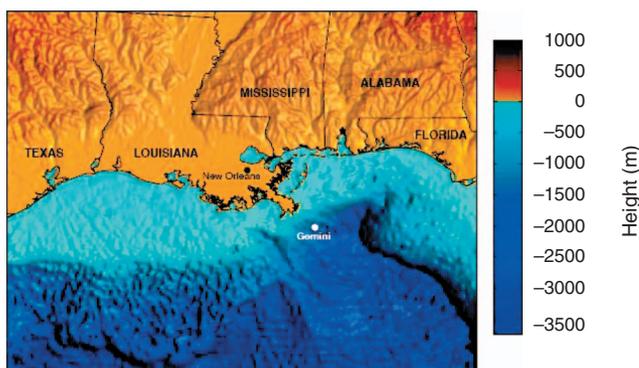


Figure 3. Location of Gemini Prospect in the northern Gulf of Mexico.

Prospect an attractive target for the MMT method. Sea-bottom MT surveys were conducted by the Scripps Institution of Oceanography in the late 1990s and early 2000s, at a total of 42 MT sites. Figure 4 shows the location of the MT sites.

Constable and Key conducted a detailed analysis of the Gemini MT data, using a 2D Occam's inversion code (Constable et al., 1987; de Groot Hedlin and Constable, 1990; Key, 2003; Key et al., 2006). The results obtained by 2D inversion of the MT data are in general agreement with the seismic data. Hoversten et al. (2000), who successfully interpreted a single line over the Gemini Prospect in 2000, also show the power of sharp boundary inversion, which is in excellent agreement with seismic images.

A rigorous 3D MT inversion of the Gemini data using a fine discretization grid has been performed for the transect lines in Figure 4 (Zhdanov et al., 2009). The inversion domain was discretized into 1.6 million cells. The inversion was run using an MT code based on the integral equation method, which is capable of running on massively parallel supercomputers. It took 9 hours to complete 51 iterations on the 832-processor cluster, with a final misfit of 6.2% between the observed and predicted data.

Figure 5 presents a 3D view of the geoelectrical inverse model and the bathymetry in the area of the survey. The inversion results reveal a resistive salt structure, which is confirmed by a comparison with the seismic data. These inversion results demonstrate that we can map resistive geoelectrical structures such as salt domes with reasonable accuracy using 3D inversion of marine MT data.

The main goal of future EM research and development will be 3D modeling and inversion directed toward the use of multitransmitter and multireceiver EM data. In this connection, I reemphasize the importance of having TDAS surveys with a dense profile or array observations. The fact is that success in imaging and inversion depends on good sampling in space.

## ELECTROMAGNETIC IMAGING AND MIGRATION

Along with the development of full 3D forward modeling and inversion capabilities, much attention has been paid to simpler yet rea-

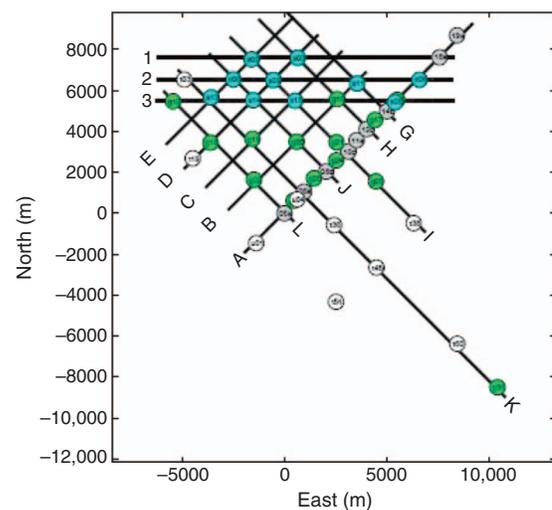


Figure 4. Location of MT profiles with observation sites in Gemini Prospect (after Key, 2003). Letters signify line numbers; circles are the magnetotelluric observation sites.

reasonable imaging methods. The main difference between inversion and imaging can be explained as follows. In inversion, we try to fit the observed data by the theoretical predicted data, which requires repeated forward-modeling calculations. In imaging, we directly transform the EM data observed on the surface of the earth or in the sea bottom into some image of a geologic structure. Usually, imaging is numerically equivalent to one forward-modeling solution. At the same time, it can produce a reasonable picture of the underground structures, which can be used for a geologic interpretation. One can compare EM imaging with seismic imaging, based, for example, on seismic wave migration, which is the basic tool for seismic data interpretation. That is why a new approach to EM imaging, called electromagnetic migration (Zhdanov, 1988, 2009a), seems to be important in practical applications.

EM migration is a type of transformation of the EM field observed at the earth's surface downward into the lower half-space. Using such a transformation, we can view the geoelectric structure from a more advantageous perspective and in some cases actually construct an image of the earth's interior. The principles of EM migration have been developed in several works (e.g., Zhdanov and Frenkel, 1983a, 1983b; Zhdanov, 1988, 1999; Zhdanov et al., 1988; Zhdanov and Keller, 1994; Zhdanov et al., 1996; Tompkins, 2004; Mittet et al., 2005). EM migration is based on a special form of downward continuation of the observed EM field or one of its components. This downward continuation is obtained as a solution of the boundary value problem in the lower half-space for the adjoint Maxwell's equations, in which the boundary values of the migration field on the earth's surface are determined by the observed EM data. However, it is important to stress that EM migration is not the same as analytic continuation because it does not reconstruct the true EM field within the earth but merely transforms the field.

The physical principles of EM migration are similar to those of optical holography, which reconstructs a volume image of an object by using a hologram displaying the amplitude and the phase structure of the wavefront of light. To make it possible to record by photographic emulsion amplitude as well as phase, the reference wave of light is added. This additional wave is coherent with the object's light and interferes with it, producing diffraction patterns that form an optical hologram on the photographic emulsion. To generate a volume image, it is sufficient to illuminate a hologram with a reference light wave. The wave scattered by photographic diffraction patterns is identical to the original wavefront of light scattered by an object and reproduces the volume image of the object.

A similar interpretation can be provided for EM migration. In a CSEM experiment, we place the EM transmitting/receiving system on the surface of the ground or at the sea bottom. The transmitters generate a harmonic (frequency-domain) or pulse (time-domain) primary EM field that propagates through the medium, containing the target, and is recorded by the receivers. Just as in the case of an optical holography, it is necessary to provide a reference signal to measure relative phases in the frequency domain. The recorded amplitudes and phases of an EM field scattered by an object form a broadband EM hologram. As in optical holography, we can reconstruct the volume image of the object by "illuminating" the broadband EM hologram by the reference signal. In the optical case, this can be performed optically, yielding a visible image; in the case of a broadband EM field, the reconstruction is done numerically using computer transformation.

We should note, however, that in the relatively low-frequency range used in exploration geophysics, the EM field propagates in

geologic formations according to the diffusion equation, which results in a relatively low resolution of the geoelectrical image obtained by migration. To improve the resolution of the EM migration imaging, we should apply the migration iteratively.

One can introduce a residual EM field as the difference between the predicted EM field for the geoelectrical model produced by the migration image and the actual EM field. EM migration imaging is considered as the initial step in a general EM inversion procedure, based on the minimization of the misfit functional between the observed and predicted data. By applying migration iteratively, we arrive at iterative EM migration, which provides a rigorous solution of EM inverse problems.

EM holography/migration was introduced for interpretation of land EM data. However, this technique is most effective in the case of relatively dense EM surveys, which are difficult to implement on land. At the same time, marine CSEM surveys with their dense system of transmitters and receivers are extremely well suited to the EM migration technique.

I present here an example application of the iterative migration to interpreting synthetic marine CSEM survey data that has been computer simulated for the famous Shtokman gas field located in the center of the Russian sector of the Barents Sea, about 500 km (370 miles) north of the Kola Peninsula (Figure 6). The Shtokman gas field is one of the world's largest known natural gas fields, with reserves of 3.8 trillion m<sup>3</sup> of gas and 37 million tons of gas condensate (Gazprom, 2009). Discovered in 1988, it was named after Russian geophysicist Vladimir Shtokman, a descendant of German emigrants. It is now operated by a consortium of three companies: Russia's Gazprom, the French energy company TOTAL, and Norway's Statoil. The field so far has not been developed owing to extreme Arctic conditions, distance from infrastructure, and depth of the sea, varying from 320 to 340 m.

The Shtokman gas deposit is formed by an anticline structure containing gas condensate in its crest zone. The productive horizons are located within Middle Jurassic sandstones. A 3D geoelectric model of the field was constructed based on available geological and geophysical information (Zhdanov et al., 2010). Figure 7 presents a ver-

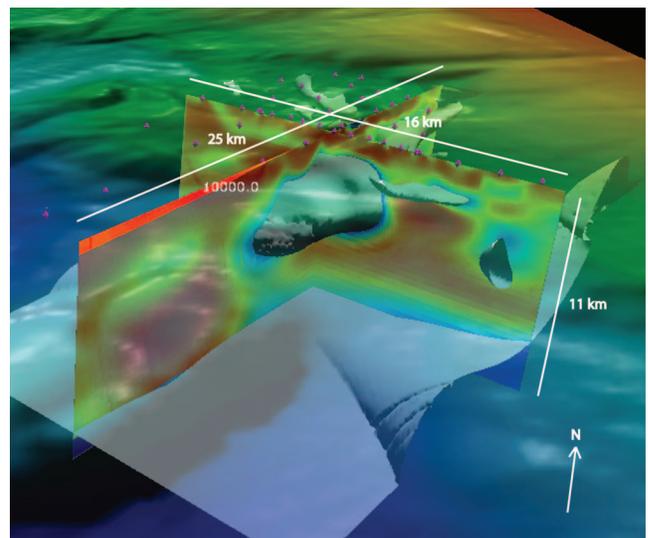


Figure 5. A 3D image of the inversion result for the Gemini prospect MT data in the presence of the sea-bottom bathymetry. Note the isosurface of the salt bodies.

tical section of the constructed geoelectrical model. The host sediments have a resistivity of 1 ohm-m, and the reservoir units have a resistivity of 100 ohm-m. This model was used for simulating 3D marine CSEM surveys, shown in the top part of Figure 8. The survey is formed by nine observational lines with sea-bottom receivers distributed on a  $2 \times 2$ -km grid, and an electric dipole transmitter towing above the corresponding receiver lines. The observed  $E_x$ ,  $E_z$ , and  $H_y$  components of the EM field in the receivers were calculated at 0.25, 0.5, and 0.75 Hz. Random noise was added as a function of transmitter-receiver offset and data threshold above a noise floor.

The observed data were migrated downward using a fully parallelized computer code. The initial migration image was used to construct the initial inverse geoelectrical model, and the corresponding predicted fields measured at the receiver positions resulting from this 3D resistivity model were calculated. These residual fields were then migrated, and an updated resistivity model was obtained. The process was reiterated until (1) the misfit reached a preset threshold, (2) the decrease in error between multiple iterations was less than a preset threshold, or (3) the maximum number of iterations was



Figure 6. The Shtokman gas field is located in the center of the Russian sector of the Barents Sea, about 370 miles (500 km) north of the Kola peninsula.

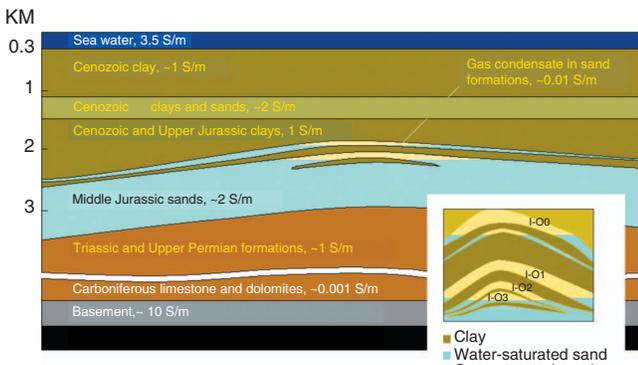


Figure 7. Vertical section of a geoelectrical model of the Shtokman gas field. A panel inserted in the lower right corner presents a detailed view of the four productive horizons of the deposit located within the Middle Jurassic sandstones.

reached. Figures 8 and 9 present a 3D view and a vertical section of the final migration resistivity image. As shown, the anticline structure defining the Shtokman field was recovered quite well from the iterative migration.

Thus, when applied iteratively, EM migration is analogous to iterative inversion methods in that it provides a rigorous solution to the corresponding inverse problem (Zhdanov, 2002, 2009a). The main difference between iterative migration and inversion is in the physical interpretation of the gradient directions. The approach based on the ideas of EM migration makes it possible to use physical properties of the migration field to construct effective numerical methods of underground imaging and inversion.

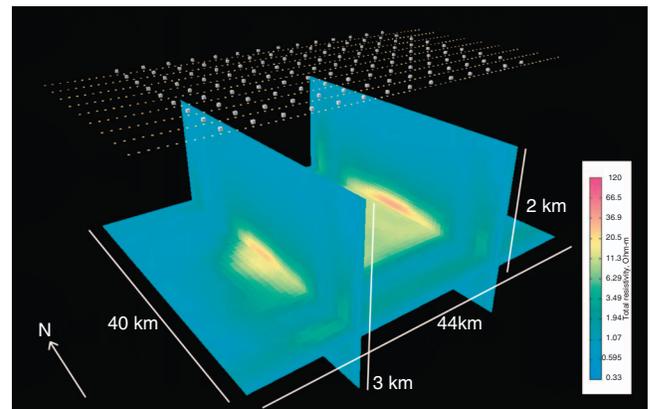


Figure 8. Results of iterative migration of synthetic marine CSEM data computer simulated for the Shtokman gas field: a 3D view of the migration image and a sketch of the marine CSEM survey (above the image). White squares are receiver positions distributed on a  $2 \times 2$ -km grid. Note the anticline structure defining the Shtokman gas field, which is manifested by a yellow zone with a resistivity of about 20 ohm-m.

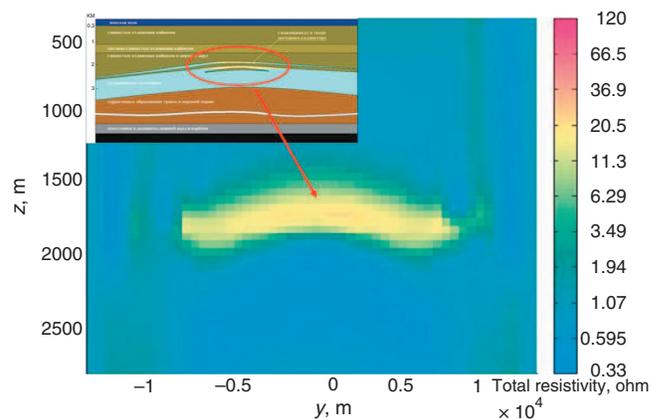


Figure 9. Results of iterative migration of the computer-simulated synthetic marine CSEM data for the Shtokman gas field. Note a vertical section of the 3D resistivity model obtained by iterative migration. The anticline structure defining the Shtokman gas field is manifested by a yellow zone with a resistivity of about 20 ohm-m. The insert in the upper-left corner presents a schematic view of the productive horizons of the deposit located within Middle Jurassic sandstones.

## NEW PARADIGM IN EM MODELING: FLUX AND VOLTAGE REPRESENTATION OF EM FIELDS

The fundamental principles of EM geophysics have been developed in the framework of classical EM theory, where the EM field is described by the electric and magnetic vector fields and Maxwell's equations represent a system of differential equations with respect to these vector fields. However, it is more natural from the physical and geophysical points of view to describe the EM field by the corresponding flux and work (or voltage) of the field instead of using traditional vector representations (Zhdanov, 2009a, 2010). Indeed, it is very well known that the flux of a field through a given surface and the work (or voltage) of the field along a given path represent the most important physical entities studied and measured in geophysical experiments. That is why a representation of an EM field in the form of corresponding flux and voltage appears to be extremely well suited for a description of EM phenomena.

A new paradigm for EM field representation using flux and voltage opens a new possibility for an accurate simulation of EM phenomena in geophysics. Practically all numerical modeling methods produce vector electric and magnetic fields at a given point, whereas in EM geophysics we measure the potential difference (voltage) between two electrodes and/or the magnetic flux in the sensor. Zhdanov (2010) shows that although any discretization of the vector fields can be done only approximately, one can always derive exact discretizations for the flux and voltage of a field. To provide an adequate description of practical EM geophysical data, EM modeling should be based on calculations of flux and voltage.

Maxwell's equations were introduced by generalizing the basic laws of electromagnetism established in the first half of the 19th century. During the last decades, a novel approach has been developed regarding the formulation of Maxwell's equations, based on the algebraic theory of differential forms (Deschamps, 1981; Lindell, 2004; Fecko, 2006; Zhdanov, 2009a, 2010). The key idea of this new paradigm in EM theory is that, in fact, Maxwell's equations can be derived directly from the basic differential equations of the field theory, formulated for flux and voltage (see Appendix A). Moreover, the whole system of Maxwell's equations naturally appears from the general theory of nonstationary fields if we consider flux and work (voltage) as the fundamental characteristics of EM fields. This remarkable fact demonstrates that the basic laws of electromagnetism are actually hidden in the fundamental differential relationships between the flux and work of a nonstationary field. No other equations exist for a pair consisting of flux and work but equations of the Maxwell type. Thus, the new mathematical form of Maxwell's equations emphasizes the importance of the fluxes and work of an EM field, which corresponds well to geophysical experiments that involve, as a rule, measuring the flux and voltage of magnetic and electric fields, respectively.

The flux of a field through a given surface and the work of a field along a given path indeed represent the most fundamental physical entities that are studied and measured in geophysical experiments. At the same time, a discrete character of flux and work, as the integrals over the corresponding surfaces and lines, provides a clear indication about the possible discrete nature of EM phenomena. In the framework of this approach, it is unnecessary to describe the EM phenomena by the fields continuously distributed in space and time, as done in classical EM theory. Instead, one can think of a discrete distribution of the corresponding flux and work (or voltage) of mag-

netic and electric fields, respectively, which opens a natural way for field quantization.

As a result, we can arrive at a system of algebraic equations for the fluxes and voltages (work) of an EM field that provide an exact representation of the original system of Maxwell's equations for the differential forms. Any discretization of the classical system of Maxwell's equations for the vector fields based on finite-difference or finite-element methods results in some approximate representation of the vector fields. This property of the numerical methods based on differential-form equations opens the possibility for developing a very accurate technique for EM modeling, especially in the case of high conductivity contrast, which is important in geophysical applications.

## CONCLUSION

Using natural and artificial sources, EM methods in geophysics have proven to be one of the major tools for surveying underground structures. From EM measurements conducted on the earth's surface, in the air, in the sea, and in wells, we are able to map the earth from surface to mantle. EM methods have found and continue to find a wide use in geologic mapping, mineral and oil prospecting, tectonic studies, earthquake studies, and environmental assessment and monitoring.

The road ahead will be based on exciting new discoveries in the ways we transmit, measure, process, and interpret data. The future success of EM exploration will be founded on the development of multitransmitter and multireceiver methods with array-observation systems analogous to seismic data-acquisition systems. The main efforts in the development of EM data interpretation in the future will be concentrated in three directions: (1) fast and accurate 3D modeling, (2) rapid imaging, and (3) large-scale 3D inversion. The regularization theory will play a key role in solving all of these problems.

The road ahead will also include the development of a new approach to formulating and understanding EM fields based on flux and voltage representation. This approach has a strong potential to stimulate the future development of EM geophysics.

Although very powerful, EM data represents only one component of an integrated geophysical exploration strategy. Whereas EM data responds directly to the resistivity of hydrocarbons and other mineral deposits, seismic data are well suited to identify structures that may contain hydrocarbons and mineralization zones. Furthermore, geophysical well-logging data, together with a sound geologic understanding, can provide important information on expected properties of subsurface lithologies. Thus, only an integrated approach, based on a shared earth model, will establish an advanced interpretation technology that will ensure maximum benefit from geophysical data, including EM data.

The further development of EM geophysics will require significant efforts of dedicated researchers in all the areas mentioned above.

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## APPENDIX A

### MAXWELL'S EQUATIONS IN THE CONTEXT OF THE THEORY OF DIFFERENTIAL FORMS

During the last decades, a novel approach has been developed to formulating Maxwell's equations, based on the algebraic theory of differential forms (Deschamps, 1981; Lindell, 2004; Fecko, 2006; Zhdanov, 2009a, 2010). The differential forms are introduced as expressions on which integration operates. In particular, a differential expression  $\mathbf{B} \cdot d\mathbf{r}$ , which is integrated over a curve, represents the elementary work  $dW$  of the vector field  $\mathbf{B}$  along an infinitesimally small vector element of curve  $d\mathbf{r}$ . It is called a differential 1-form  $\varphi_{(1)}$ :

$$\varphi_{(1)} = \mathbf{B} \cdot d\mathbf{r} = dW. \quad (\text{A-1})$$

A differential expression  $\mathbf{B} \cdot d\mathbf{s}$ , which is integrated over a surface, describes an elementary scalar flux  $dF_{\mathbf{B}}^{ds}$  of the vector field  $\mathbf{B}$  through an infinitesimally small vector element of surface  $d\mathbf{s}$ . It is called a differential 2-form  $\psi_{(2)}$ :

$$\psi_{(2)} = \mathbf{B} \cdot d\mathbf{s} = dF_{\mathbf{B}}^{ds}. \quad (\text{A-2})$$

A differential expression  $\text{div}\mathbf{B}dv$ , which is integrated over a volume, is equal to an elementary source  $dQ$  of the vector field  $\mathbf{B}$  within an infinitesimally small element of volume  $dv$ . We call this expression a differential-3 form  $\theta_{(3)}$ :

$$\theta_{(3)} = qdv = dQ, \quad (\text{A-3})$$

where  $q = \text{div}\mathbf{B}$ . Thus, all three forms represent the scalar values of the work, flux, and source of the vector field, respectively (Figure A-1).

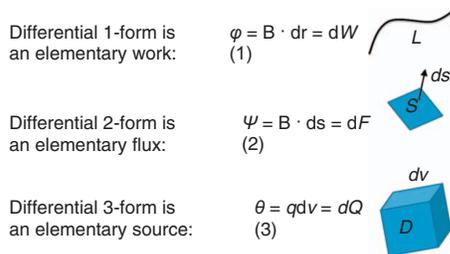


Figure A-1. Definition of differential forms. The differential forms are the expressions on which integration operates.

We also introduce a 0-form as a scalar function  $f$ , which is integrated over a region of zero dimension:  $\varphi_{(0)} = f$ .

The calculus of differential forms is based on a special differential operation called the exterior derivative (Fecko, 2006). The beauty of this operator is that all three classical vector differential operations (gradient, divergence, and curl) can be represented as a single exterior differential operator  $d$ :

$$0\text{-forms: } d\varphi_{(0)} = df = \text{grad } f(\mathbf{r}) \cdot d\mathbf{r} = \nabla f(\mathbf{r}) \cdot d\mathbf{r}, \quad (\text{A-4})$$

$$1\text{-forms: } d\varphi_{(1)} = d \wedge (\varphi \cdot d\mathbf{r}) = \text{curl}\varphi \cdot d\mathbf{s} = [\nabla \times \varphi] \cdot d\mathbf{s}, \quad (\text{A-5})$$

$$2\text{-forms: } d\psi_{(2)} = d \wedge (\psi \cdot d\mathbf{s}) = (\text{div}\psi)dv = (\nabla \cdot \psi)dv, \quad (\text{A-6})$$

$$3\text{-forms: } d\theta_{(3)} = d \wedge (\theta dv) = 0, \quad (\text{A-7})$$

where the wedge ( $\wedge$ ) denotes an exterior product.

In a general case, we can consider an arbitrary nonstationary vector field in 4D Euclidean space-time  $E_4$  that has three conventional spatial coordinates,  $x_1 = x$ ,  $x_2 = y$ , and  $x_3 = z$ . The fourth coordinate  $x_4$  is equal to time:  $x_4 = t$ . Note that any scalar  $U$  or vector  $\mathbf{A}$  functions of the spatial coordinates  $\mathbf{r} = (x_1, x_2, x_3)$  and the time coordinate  $t$  can be treated as the functions defined in the 4D space  $E_4$ . We can also introduce arbitrary vector functions  $\mathbf{H}(\mathbf{r}, t)$ ,  $\mathbf{D}(\mathbf{r}, t)$ , and  $\mathbf{j}(\mathbf{r}, t)$  and a scalar function  $q(\mathbf{r}, t)$ . The remarkable fact is that any pair of nonstationary fields  $\mathbf{H}(\mathbf{r}, t)$  and  $\mathbf{D}(\mathbf{r}, t)$  satisfies a set of differential equations that have exactly the same structure as Maxwell's equation of EM theory.

One can define differential forms of five different orders in the 4D space  $E_4$ :

$$0\text{-forms: } \Omega_{(0)} = U, \quad (\text{A-8})$$

$$1\text{-forms: } \Omega_{(1)} = \alpha = \mathbf{A} \cdot d\mathbf{r} - Udt, \quad (\text{A-9})$$

$$2\text{-forms: } \Omega_{(2)} = \psi = \mathbf{D} \cdot d\mathbf{s} - (\mathbf{H} \cdot d\mathbf{r}) \wedge dt, \quad (\text{A-10})$$

$$3\text{-forms: } \Omega_{(3)} = \gamma = qdv - (\mathbf{j} \cdot d\mathbf{s}) \wedge dt, \quad (\text{A-11})$$

$$4\text{-forms: } \Omega_{(4)} = \theta = qdv \wedge dt. \quad (\text{A-12})$$

These forms satisfy the following basic differential equations:

$$0\text{-forms: } dU = \text{grad } U \cdot d\mathbf{r} + \frac{\partial U}{\partial t} Udt, \quad (\text{A-13})$$

$$1\text{-forms: } d \wedge \alpha = \text{curl}\mathbf{A} \cdot d\mathbf{s} - \left( \text{grad } U + \frac{\partial \mathbf{A}}{\partial t} \right) \cdot d\mathbf{r} \wedge dt, \quad (\text{A-14})$$

$$2\text{-forms: } d \wedge \psi = (\operatorname{div} \mathbf{D})dv + \left( \frac{\partial}{\partial t} \mathbf{D} - \operatorname{curl} \mathbf{H} \right) \cdot ds \wedge dt, \quad (\text{A-15})$$

$$3\text{-forms: } d \wedge \gamma = - \left[ \operatorname{div} \mathbf{j} + \frac{\partial}{\partial \tau} q \right] dv \wedge dt, \quad (\text{A-16})$$

$$4\text{-forms: } d \wedge \theta = 0. \quad (\text{A-17})$$

A reader familiar with EM theory will immediately recognize the pieces of Maxwell's equations hidden in formulas A-13–A-17. According to equation A-15, we have the following differential equation for any 2-form  $\psi = \mathbf{D} \cdot ds - (\mathbf{H} \cdot d\mathbf{r}) \wedge dt$ :

$$d \wedge \psi = \gamma^\psi, \quad (\text{A-18})$$

where the corresponding four-current 3-form  $\gamma^\psi$  is equal to

$$\gamma^\psi = q^\psi dv - (\mathbf{j}^\psi \cdot ds) \wedge dt \quad (\text{A-19})$$

and

$$\mathbf{j}^\psi = \operatorname{curl} \mathbf{H} - \frac{\partial}{\partial t} \mathbf{D} \text{ and } q^\psi = \operatorname{div} \mathbf{D}. \quad (\text{A-20})$$

Maxwell's equations were introduced by generalizing the basic laws of electromagnetism established in the first half of the 19th century. It is remarkable that all of these equations can be derived directly from the basic differential equations of the field theory. Indeed, we can introduce two EM differential 2-forms — Maxwell's field  $M$  and a force field  $F$  — according to

$$M = \mathbf{D} \cdot ds - (\mathbf{H} \cdot d\mathbf{r}) \wedge dt = D - H \wedge dt, \quad (\text{A-21})$$

$$F = \mathbf{B} \cdot ds + (\mathbf{E} \cdot d\mathbf{r}) \wedge dt = B + E \wedge dt, \quad (\text{A-22})$$

where  $\mathbf{H}$  and  $\mathbf{B}$  are the vector magnetic and induction fields, respectively;  $\mathbf{E}$  and  $\mathbf{D}$  are the vector electric and displacement fields, respectively;  $D = \mathbf{D} \cdot ds$ ;  $H = \mathbf{H} \cdot d\mathbf{r}$ ;  $B = \mathbf{B} \cdot ds$ ; and  $E = \mathbf{E} \cdot d\mathbf{r}$ .

Thus, the force field is a linear combination of the magnetic induction flux  $B$  and a time differential multiplied by the electric voltage (work)  $E \wedge dt$ , and Maxwell's field is a linear combination of electric displacement flux  $D$  and a time differential multiplied by the work of the magnetic field  $H \wedge dt$ . These physical entities — magnet-

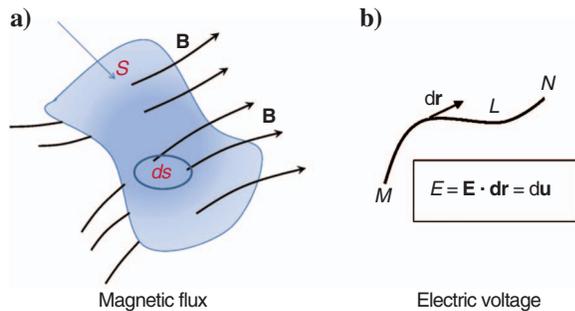


Figure A-2. Illustration of (a) magnetic flux  $\mathbf{B}$  and (b) electric voltage  $E$ . The flux of a field through a given surface  $S$  and the work of the field along a given path  $L$  represent the most important physical entities studied and measured in geophysical experiments.

ic induction flux and electric displacement flux, electric voltage, and magnetic work — represent the fundamental properties of what we call an EM field (Figure A-2).

Using the basic properties of differential 2-forms results in the following differential equations:

$$d \wedge M = \gamma^e, \quad (\text{A-23})$$

$$d \wedge F = 0, \quad (\text{A-24})$$

where the corresponding electric four-current  $\gamma^e$  is equal to

$$\gamma^e = qdv - (\mathbf{j} \cdot ds) \wedge dt \quad (\text{A-25})$$

and we take into account the absence of magnetic charges and current in equation A-24.

Equations A-23 and A-24 show that Maxwell's equations can be derived directly based on the mathematical theory of differential forms. Indeed, according to formulas A-18 and A-20, from equation A-23 for Maxwell's field  $M$ , we immediately obtain Maxwell's first and fourth equations:

$$\operatorname{curl} \mathbf{H} = \mathbf{j} + \frac{\partial}{\partial t} \mathbf{D} \text{ and } \operatorname{div} \mathbf{D} = q. \quad (\text{A-26})$$

Equation A-24 written in vectorial notation brings us to Maxwell's original second and third equations:

$$\operatorname{curl} \mathbf{E} = - \frac{\partial}{\partial t} \mathbf{B} \text{ and } \operatorname{div} \mathbf{B} = 0,$$

which completes the proof of our statement.

An important feature of the differential form of Maxwell's equations, A-23 and A-24, is that they describe the relationships between the elementary fluxes and work of the different EM field components, whereas the classical Maxwell's equations deal with the vectors of EM fields themselves.

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