Joint Inversion of Gravity and Magnetotelluric Data for the Depth-to-Basement Estimation

Hongzhu Cai and Michael S. Zhdanov

***Abstract*— It is well known that both gravity and magnetolluric (MT) methods can be used for the depth-to-basement estimation due to the density and conductivity contrast between the sedimentary basin and the underlaid basement rocks. In this case, the primary targets for both methods are the interface between the basement and sedimenary rocks as well as the physical properties of the rocks (density and conductivity). The solution of this inverse problem is typically nonunique and unstable, especially for gravity inversion. In order to overcome this difficulty and provide a more robust solution, we have developed a method of joint inversion to recover both the depth to the basement and the physical properties of the sediments and basement using gravity and MT data simultaneously. The joint inversion algorithm is based on the regularized conjugate gradient method. To speed up the inversion, we use an effective forward modeling method based on the surface Cauchy-type integrals for the gravity field and the surface integral equation representations for the MT field, respectively. We demonstrate the effectiveness of the developed method using several realistic model studies.**

***Index Terms*— Cauchy-type integrals, gravity field, joint inversion, magnetotelluric method.**

I. Introduction

IN RECONNAISSANCE geophysical exploration for oil and gas and other natural resources, it is of great importance to estimate the depth to the basement and physical properties of the sedimentary rocks. Seismic methods are widely used in the detection of sediment-basement interface. However, the large scale active source seismic surveys for reconnaissance exploration are expensive. Alternatively, the gravity data can be used to detect the geometry of the sedimentary basin, and the survey expense is much cheaper (see [1]–[4]). At the same time, it is well known that the gravity inversion is a nonunique problem. Combining gravity data with another cost-effective technique, e.g., magnetotelluric method (see [5], [6]), helps to reduce the nonuniqueness significantly. In this letter, we propose the workflow of joint gravity and magnetolluric (MT) inversion, which is expected to produce a high-resolution image of the sediment-basement interface with acceptable cost comparing to the seismic survey. One can recover 3-D volume distributions of density and conductivity from the inversion of the gravity and MT data. However, the images produced by the conventional voxel based 3-D inversion of either gravity or MT field data are diffusive by nature of the corresponding physical fields.

In estimation of the depth to the basement, it is desirable to recover a sharp boundary between the sediments and crystalline basement rocks [7]. In order to solve this problem, one can adopt a discretization of the interface between the sediments and basement rocks instead of a 3-D discretization of the subsurface. By assuming that the physical properties, such as density and conductivity, of each geological units take a simple uniform value, the depth of the discretized sediment-basement at each horizontal cell becomes the primary inversion parameter. Note that, the physical parameters, densities, and conductivities, can also be inverted simultaneously with the depth to the basement.

The methods for estimating the depth to the basement using potential field data have been widely used (see [1], [2]). Most of the published papers are based on a column discretization of the sedimentary basin; the thickness of the columns is determined based on the observed gravity data [2]. Cai and Zhdanov [8] proposed a new method based on the Cauchy-type integrals for solving this type of inverse problems. Within the framework of this approach, the sediment-basement interface is discretized, and the vertical location of each cell in the interface is recovered during the inversion (see [3], [8], [9]). The Fréchet derivatives with respect to the depth to basement and the density values can be derived analytically using the Cauchy-type integral method. This method has been successfully applied to realistic synthetic models and field gravity data collected in the sedimentary basin area.

A similar approach can be applied to the inversion of electromagnetic (e.g., MT) data, where the sediment-basement interface shows a significant conductivity contrast (see [3], [7]). Chen *et al.* [10] implemented a 2-D stochastic inversion method to invert for the location and shape of the conductivity contrast surface and to recover sharp boundaries between the layers. However, the stochastic inversion requires a large number of forward modeling computations, which is not feasible in a general 3-D case. Cai [3] and Cai and Zhdanov [7] proposed a gradient-type inversion approach for 3-D case based on the discretization of the sediment-basement interface. The forward modeling was based on efficient contraction integral equation (IE) method, and the Fréchet derivatives with respect to the conductivity and the depth to the basement were calculated effectively using a Quasi-Born approximation.

However, separate inversions for both gravity and electromagnetic (MT) data suffer from the nonuniqueness problem.

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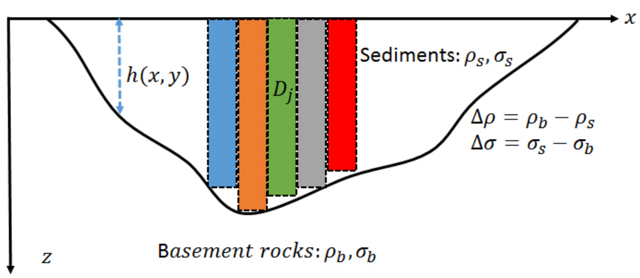

Fig. 1. Sketch of the model of sedimentary basin with uniform density and conductivity for both sediments and basement rocks. For modeling of MT data, the sediments are discretized by vertical prismatic columns.

In the case of the depth-to-basement inversion, this nonuniqueness can be significant, especially for potential field data to recover both the depth to the basement and the density contrast simultaneously. At the same time, the inversion of MT data also suffers from the data noise and nearsurface distortions. The uncertainty is increased significantly, due to the diffusive property of electromagnetic field in the earth medium, when both the conductivity contrast and depth to the basement are unknown. As a result, it is of great importance to improve the resolution of both the MT and gravity methods by considering the joint inversion approach in order to reduce the model ambiguity (see [11], [12]). Conventional joint inversion of different geophysical data sets usually assumes that there is some correlation between different physical parameters or their spatial gradients. Gallardo and Meju [13] implemented a joint inversion algorithm based on the structural similarity between different model parameters. Zhdanov *et al.* [14] proposed a new joint inversion approach using the Gramian constraints between different model parameters.

For the depth-to-basement analysis using gravity and MT data, the primary model parameters are the depth to the basement (which is the same for gravity and MT data), and density and conductivity of the sediments (see [3], [7], [8]). As the result, the joint inversion in this case can be simplified, since both gravity and MT data share the same primary model parameters of the depth to the basement. The recovered depth to the basement could be well constrained by these two different types of geophysical data. Furthermore, we expect that the inverted density and conductivity of the sediment will be closer to the true values as well. In the joint inversion approach, the well-known Bouguer slab formula for gravity field can provide a reasonable initial model for the joint inversion to speed up the convergence. In this letter, we test the effectiveness and robustness of the proposed joint inversion approach using several realistic model studies.

II. Forward Modeling of Gravity and MT Data for a Sedimentary Basin Model

We consider first the forward modeling techniques for gravity and MT data for the sedimentary basin model shown in Fig. 1. We assume that both the sediment and basement have a uniform density and conductivity values. The gravity and MT anomalies are caused by the density and conductivity contrast between these two layers.

It has been demonstrated by Cai and Zhdanov [8] that the gravity anomaly caused by such sediment-basement interface model can be represented using the Cauchy type integral over the sediment-basement interface, denoted by S

$$g_\alpha = -G\Delta\rho \iint_S \frac{\Delta_{\alpha z\gamma\eta} h(x,y)\left(r_\eta - r'_\eta\right)}{|\mathbf{r}-\mathbf{r}'|^3} b_\gamma \, dx\, dy \qquad \alpha,\gamma,\eta = x,y,z \quad (1)$$

where G is the gravitational constant, $\Delta\rho$ is the density contrast value, $h(x, y)$ represents the depth to the basement, each symbol α, γ, and η can be x, y, or z. The four-index Δ symbol is represented in terms of symmetric Kronecker delta symbol $\delta_{\alpha\eta}$, as follows [3], [8], [9]:

$$\Delta_{\alpha\beta\gamma\eta} = \delta_{\alpha\eta}\delta_{\beta\gamma} + \delta_{\alpha\beta}\delta_{\gamma\eta} - \delta_{\alpha\gamma}\delta_{\beta\eta}. \quad (2)$$

We also have

$$b_x = -\frac{\partial h}{\partial x}, \quad b_y = -\frac{\partial h}{\partial y}, \quad b_z = 1. \quad (3)$$

The discretized form of (1) for numerical calculations can be found in [8].

We use the 3-D contraction IE method for modeling of MT data. We consider the basement conductivity as the background conductivity and discretize the sedimentary basin into a grid of vertical columns (Fig. 1). Within the framework of the IE approach, the anomalous fields can be expressed as an integral over the excess currents within domain *D,* as follows:

$$\mathbf{E}^a(\mathbf{r}_i) = \iiint_D \hat{G}_E(\mathbf{r}_i|\mathbf{r})\Delta\sigma(\mathbf{r}) \cdot [\mathbf{E}^b(\mathbf{r}) + \mathbf{E}^a(\mathbf{r})]dv \quad (4)$$

$$\mathbf{H}^a(\mathbf{r}_i) = \iiint_D \hat{G}_H(\mathbf{r}_i|\mathbf{r})\Delta\sigma(\mathbf{r}) \cdot [\mathbf{E}^b(\mathbf{r}) + \mathbf{E}^a(\mathbf{r})]dv. \quad (5)$$

Formula (4) becomes an IE with respect to anomalous electric field, when $\mathbf{r}_i \in \mathbf{r}$. After solving this IE, the electric and magnetic fields in the receivers can be obtained by directly applying (4) and (5) [15]. The corresponding MT impedance data can be calculated based on the determined electric and magnetic fields for two different polarizations [15]–[17].

III. Regularized Joint Inversion of Gravity and MT Data

For the joint inversion of gravity and MT data, we consider a data vector $\mathbf{d}$, as follows:

$$\mathbf{d} = [\mathbf{d_1}; \mathbf{d_2}] \quad (6)$$

where we use indices 1 and 2 to represent gravity and MT data $\mathbf{d_1}$ and $\mathbf{d_2}$, respectively.

The model parameter m includes the depth to the basement, density contrast and sediment conductivity

$$\mathbf{m} = [\mathbf{h}; \Delta\rho; \sigma_s] \quad (7)$$

where $\mathbf{h}$ is the depth to the basement for both gravity and MT problems, $\Delta\rho$ is the density contrast, and σ_s is the conductivity of the sediments. The basement conductivity is already known from the previous 1-D inversion.

It is well known that, the inversions of both gravity and MT data are ill-posed problems. In order to get a stable solution for the joint inversion, we consider the minimization of the Tikhonov parametric functional [11], [12], [18], [19]

$$\begin{aligned} P^\alpha(\mathbf{m},\mathbf{d}) = {} & (\mathbf{W}_{d1}\mathbf{A}^1(\mathbf{m}) - \mathbf{W}_{d1}\mathbf{d_1})^T(\mathbf{W}_{d1}\mathbf{A}^1(\mathbf{m}) - \mathbf{W}_{d1}\mathbf{d_1}) \\ & + (\mathbf{W}_{d2}\mathbf{A}^2(\mathbf{m}) - \mathbf{W}_{d2}\mathbf{d_2})^T(\mathbf{W}_{d2}\mathbf{A}^2(\mathbf{m}) - \mathbf{W}_{d2}\mathbf{d_2}) \\ & + \alpha\ (\mathbf{W}_m\mathbf{m} - \mathbf{W}_m\mathbf{m}_{\mathrm{apr}})^T(\mathbf{W}_m\mathbf{m} - \mathbf{W}_m\mathbf{m}_{\mathrm{apr}}) \end{aligned} \quad (8)$$

where $\mathbf{A}^1$ and $\mathbf{A}^2$ are the forward modeling operators for gravity and MT problems, respectively; $\mathbf{W}_m$ is a diagonal model weighting matrix based on integrated sensitivity; $\mathbf{m}_{\mathrm{apr}}$ is some *a priori* model, $\mathbf{W}_{d1}$ and $\mathbf{W}_{d2}$ are the data weighting matrices for gravity and MT data, respectively. In this letter, we chose the ratio between the average value of the gravity field data and MT impedance components as the data weighting in order to preserve a balance between these two different data sets.

We use the regularized conjugate gradient method to solve the minimization problem (8) [19]. One critical problem of the gradient type inversion is the calculation of Fréchet derivative. Note that, both the gravity and MT inversions in our application are nonlinear problems, since the primary model parameter is the depth to the basement. For gravity data, we can directly take the derivative of both sides of (1) with respect to $\Delta\rho$ and h to find the Fréchet derivative of gravity data with respect to the density contrast and the depth to basement.

In order to calculate the Fréchet derivative for MT data, we consider the conductivity model shown in Fig. 1 with a column discretization. The anomalous electric field for this model can be written as follows [7]:

$$\mathbf{E}^a(\mathbf{r}_i) = \iiint_D \hat{\mathbf{G}}_E(\mathbf{r}_i|\mathbf{r}) \cdot [\Delta\sigma(\mathbf{r})\mathbf{E}(\mathbf{r})]dv = \sum_{j=1}^{N} \mathbf{E}_j^a(\mathbf{r}_i) \quad (9)$$

where $\mathbf{E}_j^a(\mathbf{r}_i)$ represents the anomalous field contributed from the jth column, D_j

$$\mathbf{E}_j^a(\mathbf{r}_i) = \iiint_{D_j} \hat{\mathbf{G}}_E(\mathbf{r}_i|\mathbf{r}) \cdot [\Delta\sigma\ (\mathbf{r})\mathbf{E}(\mathbf{r})]dv. \quad (10)$$

Equation (10) can be written as a combination of the surface integral over the horizontal section of the jth column S_j, and a linear integral from the surface, $z = 0$, down to the bottom of the jth column, $z = z_j$

$$\mathbf{E}_j^a(\mathbf{r}_i) = \int_0^{z_j} \left\{ \iint_{S_j} \hat{G}_E(\mathbf{r}_i|(x,y,z))[\Delta\sigma\ (\mathbf{r})\mathbf{E}(x,y,z)]dxdy \right\} \times dz. \quad (11)$$

By taking the variation of both sides of (10) with respect to the depth of the jth column z_j, and taking into account formula (11), we can find the Fréchet derivative of electric field with respect to the depth to the basement, as follows:

$$F_{ij} = \iint_{S_j} \hat{\mathbf{G}}_E(\mathbf{r}_i|(x,y,z)) \cdot [\Delta\sigma \mathbf{E}(x,y,z)]dxdy. \quad (12)$$

The Fréchet derivative of electric field with respect to the conductivity can be obtained by taking the variation of (9) with respect to $\Delta\sigma$. The Fréchet derivative of magnetic field can be calculated in a similar way. Finally, the Fréchet derivative of the MT impedance can be obtained by considering two different plane wave polarizations [3], [7], [19]. For the joint inversion, we write the Fréchet derivative as follows:

$$\mathbf{F} = [\mathbf{F}_1; \mathbf{F}_2]. \quad (13)$$

The model weighting matrix in (8) for the joint inversion can be written as follows:

$$\mathbf{W}_m = \mathrm{diag}(\mathbf{F}^T\mathbf{F})^{1/4}. \quad (14)$$

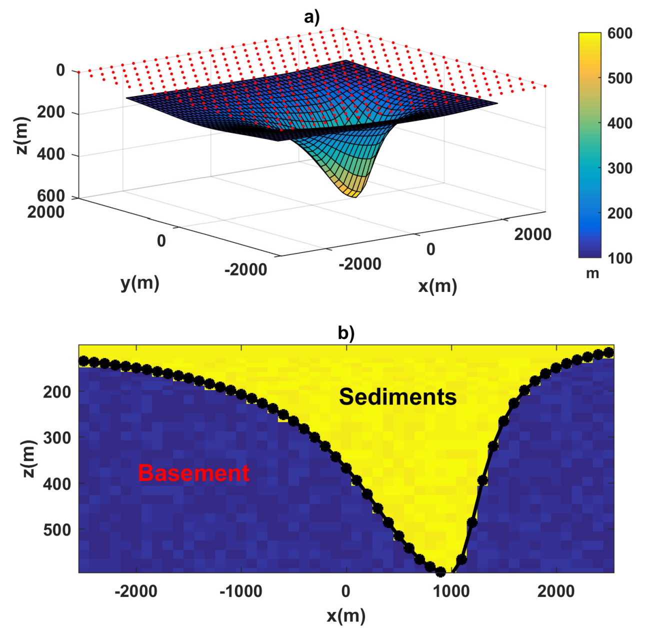

Fig. 2. Synthetic model of the sedimentary basin. (a) Three-dimensional view of the model with the dots indicating the receivers' positions. (b) Vertical section of the model at $y = 0$. The black circles in (b) indicate the sediment-basement interface.

We select the initial model based on the well-known Bouguer slab formula in order to speed up the convergence [20]

$$h = \frac{g_B}{41.89\Delta\rho_0} \quad (15)$$

where g_B is the Bouguer gravity anomaly, and $\Delta\rho_0$ is some initial guess of the density contrast value. However, we should emphasize that our final inversion result is not affected by the selection of this type of initial model. For example, we will demonstrate later that an initial model with a flat sediment-basement interface also works well in a general case.

IV. Model Studies

We have applied our method to synthetic sediment-basement interface model shown in Fig. 2. Fig. 2(a) shows a 3-D view of the interface, while Fig. 2(b) represents the vertical section of the model at $y = 0$. The maximum depth of the sedimentary basin is of 600 m, as shown in Fig. 2. The density contrast between sediment and basement is of 0.4 g/cm^3. The conductivities of the sediment and basement rocks are 0.05 and 0.001 S/m, respectively. The synthetic observed gravity and MT data were contaminated by approximately 5% random noise. We consider both separate and joint inversions of gravity and MT data to recover the depth to the basement. In our inversion, we also assume that the density contrast and the sediment conductivity are unknown. We also use the known basement conductivity, which can be accurately recovered from 1-D inversion using low frequencies.

We first consider a separate inversion of the vertical gravity field component g_z. Our initial model is a horizontal flat sediment-basement interface at $z = 300$ m, and the initial density contrast is of 0.7 g/cm^3. The circle in the top of Fig. 3 indicates the separate gravity inversion result for the depth to basement. From Fig. 3, we can see that the shape of the sedimentary basin is well determined by the inversion, but the recovered maximum depth in this case is less

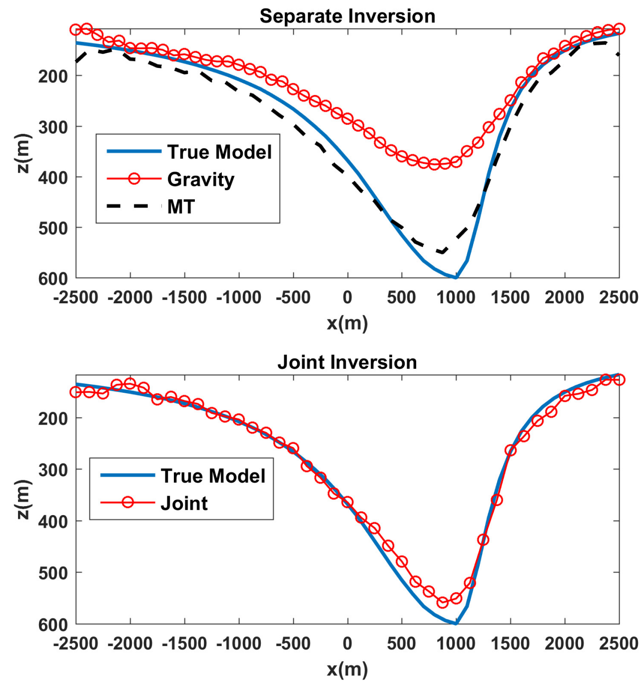

Fig. 3. Comparison between the inversion results and the true model at $y = 0$. (Top) Comparison between the separate inversions and the true model. (Bottom) Result of joint inversion compared to the true model.

than 400 m, while the actual maximum depth is of 600 m. The recovered density contrast is 0.43 g/cm^3, while the true value is 0.4 g/cm^3. As we can see, the recovered depth to the basement depends significantly on the selection of the accurate density contrast value. A small change in the density contrast can result in large change in the depth to the basement. In the case of inverting both depth to the basement and the density contrast, we also have observed that the recovered model depends significantly on the selection of initial model.

Now, we consider the inversion of MT data only to recover the depth to the basement and the conductivity of the sediments. For this inversion, we use 11 frequencies uniformly distributed from 0.01 to 100 Hz in the logarithmic space. Our initial model is a horizontal sediment-basement interface at a depth of 300 m with the sediment conductivity of 0.1 S/m. We use the 1-D inversion to recover the basement conductivity, which provides a value very close to the true conductivity. The dashed line in the top of Fig. 3 represents the MT inversion result. We can see that, the recovered model is much closer to the true model comparing to the gravity inversion. The recovered maximum depth is 546 m and the recovered sediment conductivity is of 0.0435 S/m. However, we can still observe some systematic mismatch from the true model due to the unknown sediment conductivity. We also want to emphasize that, comparing to gravity inversion, the MT inversion is less dependent on the initial model selection. We have tried several different initial models within a reasonable range and all converges to the similar result.

Finally, we consider the joint inversion of gravity and MT data to recover the depth to the basement, the density contrast, and the sediment conductivity simultaneously. We select a flat horizontal surface at a depth of 300 m as the initial model; the initial density contrast and sediment conductivity are 0.7 g/cm^3 and 0.1 S/m, respectively. The bottom of Fig. 3 shows a comparison between the recovered sediment-basement interface from the joint inversion and the true model at $y = 0$. In comparison with the separate inversions, shown in the top of Fig. 3, the recovered shape and location of the sediment-basement interface is much closer to the true model. The recovered sediment conductivity is of 0.0549 S/m, which is also closer to the true value. The maximum depth obtained from the joint inversion is equal to 559 m. The recovered density contrast is of 0.403 g/cm^3, which is practically the same as the true value. We also should emphasize that, the joint inversion is more stable and less dependent on the selection of the initial model than the separate inversions, especially for gravity data.

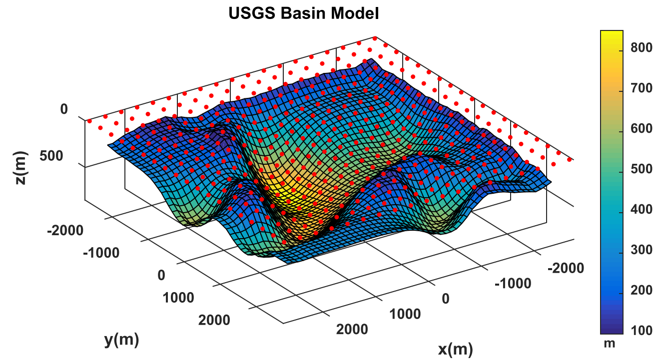

Fig. 4. Scaled model of the USGS basin with the dots representing receivers' locations for gravity and MT data.

V. Joint Inversion of Synthetic Gravity and MT Data for USGS Basin Model

In this section, we apply our method for the joint inversion of gravity and MT data computer simulated for a realistic U.S. Geological Survey (USGS) basin model in Big Bear Lake area (California). This area has been well studied with drilling and gravity data analysis [21]. Previously, we have inverted the Bouguer gravity anomaly for this area to reconstruct a sedimentary basin model [8]. In our previous publications, we also scaled this model to make the size feasible for the MT inversion algorithm.

In this letter, we have numerically simulated both gravity and MT data for the scaled USGS basin model shown in Fig. 4, where the dots represent the locations of the gravity and MT receivers. The density contrast and the sediment/basement conductivities are exactly the same as in the previous model. We have also contaminated the data with approximately 5% random noise. The initial density contrast and sediment conductivity were selected as of 0.6 g/cm^3 and 0.15 S/m, respectively. We use (15) with the initial density contrast and observed gravity anomaly to estimate a realistic initial depth to the basement. The frequencies of MT field data used for this inversion are the same as in the previous MT model. The background conductivity (basement) is determined from 1-D inversion, which is very close to its true value.

It took only 18 iterations of joint inversion for the combined misfit to converge to the noise level of the data (around 5%). Fig. 5 shows the recovered sediment-basement interface and its comparison with the true model along one profile. We can see that both the shape and depth of the sedimentary basin were recovered well. The maximum depth from this joint inversion is about 837 m, which is very close to its true

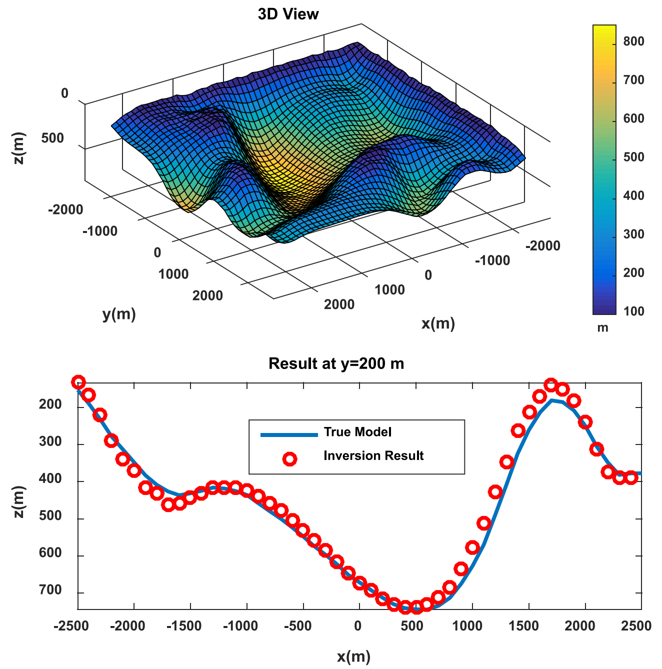

Fig. 5. Result of joint inversion for the scaled USGS model. (Top) Three-dimensional view of the recovered sediment-basement interface. (Bottom) Comparison between the inversion result and the true model at $y = 200$ m.

value of 850 m. The recovered density contrast and sediment conductivity are 0.405 g/cm^3 and 0.0551 S/m, respectively. We should note that, using the initial model based on the Bouguer slab approximation speeds up the convergence of the joint inversion significantly. At the same time, the inversion is robust with respect to the choice of initial model.

VI. Conclusion

We have developed a novel approach to the joint inversion of the gravity and MT data for the depth to the basement. Within the framework of this approach, we discretize the sediment-basement interface only and select the depth of the interface as the primary model parameter for the inversion of both gravity and MT data. As a result, the gravity and MT data are correlated automatically during the inversion for this shared model parameters. The forward modeling of the gravity data is based on the surface Cauchy-type integral over the sediment-basement interface, and the predicted MT data are computed using a surface IE representation as well. The corresponding Fréchet derivatives are calculated using direct differentiation of the Cauchy-type integral for the gravity field and a Quasi-Born approximation for the MT field.

In our inversion, we also consider that the density contrast and the sediment conductivity are unknown. These two parameters are inverted simultaneously with the inversion for the depth to the basement. We have demonstrated the effectiveness of this joint inversion approach in application to the study of sedimentary basins using two realistic synthetic models of the sediment-basin interface.

Note that, the inversion results presented above were based on an assumption of constant densities and conductivities within each geological unit. However, the method can be extended to the cases of variable conductivities and densities. We can also take into consideration the realistic geological model with multiple interfaces between the layers with different physical properties. These more complex cases will be a subject of another publication.

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