NEAR SURFACE GEOSCIENCE'20 CONFERENCE & EXHIBITION

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Increasing the effectiveness of 3D modeling visco-acoustic wave propagation with a solver based on contraction operator

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Summary

In this paper, we develop and study a novel approach to numerical modeling the visco-acoustic wave phenomena and complex 3D rock formations. We examine performance of the solver based on the contraction operator (CO), introduced by the authors, as applied to 3D time-harmonic visco-acoustic wave propagation problem. We demonstrate that, the performance of the CO solver significantly depends on the efficiency of the corresponding FFT operation used in the code. The latter has rough dependence on the array size. The results of numerical experiments indicate, however, that the optimal grid dimensions could be selected at every 8-10 points, reducing modeling time dramatically. These optimal dimensions have a minor or no dependence of particular machine architecture and, thus, could be hardcoded or calculated on-the-fly. We have implemented this approach in the developed CO based solver and illustrated its effectiveness by numerical modeling examples.

Introduction

Acoustic modeling plays significant role in geophysical exploration, especially due to today's interest in full waveform inversion. The main computational challenge in 3D frequency-domain modelling is related to the need of solving a large ill-conditioned system of linear equations. A novel preconditioned iterative solver was earlier designed and tested by the authors [Yavich et al, 2018]. The solver combines the shifted-Laplacian preconditioner [Erlangga et al, 2004] with a special contraction operator (CO) transformation. We have demonstrated that our preconditioning scheme provides a fast convergence of the iterative solver when modeling complex high-contrast velocity models at different frequencies.

The arithmetical complexity of a single iteration of the CO solver behaves asymptotically as $O(n\, log\, n)$, where n is the number of computational cells. In practice, however, CPU time has a very rough dependence of n. This is related to the performance of the 2D fast Fourier transform (FFT) operation, which is involved in the preconditioner. The 2D FFT is known having complex dependency of the speed versus the array size [Baksheev et al, 2010]. In this paper, we investigate how the parameters of the FFT impact modeling time and develop the recommendations to minimize this time, which is critical in the solution of the full-waveform inverse problems

Modeling Method Based on the Contraction Operator

Propagation of a time-harmonic visco-acoustic wave of angular frequency ω in a medium of velocity $c = c(x, y, z)$ can be described by the following partial differential equation:

$$-\varDelta P - \frac{\omega^2}{c^2}(1 - iq)P = F, \tag{1}$$

where $P = P(x, y, z)$ is acoustic pressure, $F = F(x, y, z)$ is the pressure source, and q is attenuation. Numerical solution of equation (1) is typically defined on a rectangular modeling domain, D. Equation (1) is completed with zero Dirichlet boundary condition on the top face of the modeling domain, while some absorbing boundary conditions are commonly applied on the other five faces. We apply the PML boundary condition introduced by [Berenger, 1994].

The second-order cell-centered FD discretization of equation (1) completed with the introduced boundary conditions results in a system of the following linear equations:

$$A\, p = f, \quad A = L_{PML} - \omega^2(1 - iq)\Sigma, \tag{2}$$

where A is a large sparse complex-valued symmetric matrix; $f \in \mathbb{C}^n$ is a discrete representation of the source, F; L_{PML} is the matrix of the discrete second derivatives, with seven nonzero diagonals; and Σ is a diagonal matrix of averaged squared slowness, $1/c(x, y, z)^2$, over each cell. The main algorithmic challenge in numerical modeling of visco-acoustic wave propagation is to efficiently solve the system of equations (2).

In [Yavich et al, 2018], we have presented a preconditioned iterative solver for system (2) which is based on the solutions of an auxiliary system of equations,

$$(I - C)p' = f', \tag{3}$$

where C is a contraction operator, which norm tends to unity in the limiting cases corresponding to high horizontal contrast of the velocity. We have demonstrated that this preconditioning scheme provides a fast convergence of an iterative solver when modeling the wavefield propagation complex high-contrast velocity models at different frequencies. The arithmetical complexity of solving equations (3) is dominated by the complexity of 2D fast sine transforms, making the overall complexity estimate as $O(n\, log\, n)$.

A rigorous numerical study of the performance of the developed CO solver versus the grid size indicated a very rough dependence, inherited from the FFT routine [Baksheev et al, 2010]. More interestingly, a minor increase in the size of the grid easily resulted in the decrease of CPU time. We thus looked for a strategy that would result in the shortest CPU time while increasing the resolution of the modeling grid. We tested the following strategy: dividing the sizes of the grid axis into ranges of 8-10 consecutive points and selecting the optimal size within every range. Fig. 1 illustrates this strategy for 2D FFT performance. In the next section, we present numerical examples of using this strategy in modeling the visco-acoustic waves in 3D structures.

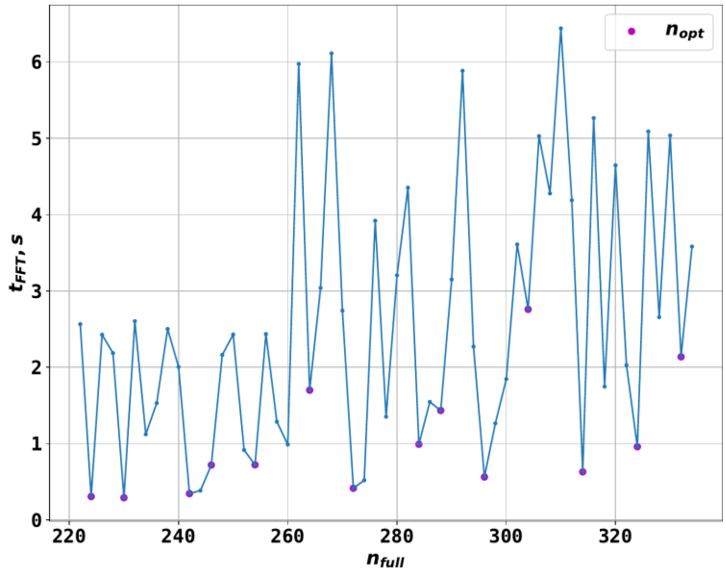

***Figure 1** 2D FFT CPU time of a 3D array $n_{full} \times n_{full} \times n_{full}$ of random numbers. Optimal dimensions are marked by magenta dots.*

Numerical Modeling Examples

In this section, we present the results of studying a sequential version of the developed CO preconditioned iterative solver which is implemented in C/C++ with FFTW3 library [Frigo and Johnson, 2005]. In particular, we trace the impact of the grid size on the modeling speed. The velocity model (Figure 2) occupies the volume [-1; 1]x[0; 1]x[0; 1] km^3 and is formed by three curved layers with velocities of 1500, 3000, and 6000 m/s, respectively, and attenuation q is 0.05. An acoustic source is located at a point with coordinates (0, 0, 255) m and operates with a frequency of 10 Hz.

The performance of the solver at a sequence of grids was recorded (Figure 3). Each grid was further completed with 20 PML layers in the five directions. We see that the performance dependence of the grid size is very rough though the optimal grid dimensions can be picked in this plot. In particular, switching from the grid size of 292×292×146 to the size of 296×296×148 resulted in a dramatic decrease of the CPU time (Table 1) – from 26.8 s per iteration to 5.7 s, i.e. the decrease of 4.7 times. Note that, modeling with both grids resulted in identical responses (Figure 4). We conclude that selecting an optimal size in every range of 8-10 points results in dramatic optimization of the performance of the solver. It is commonly assumed that rounding array size to the nearest power of 2 minimizes run-time. However, from the example with the 256×256×128 grid (Table 1) we see that this not the case.

***Table** 1 Performance of the solver on 256×256×128, 292×292×146 and 296×296×148 grids; horizontal grid dimension n_{full}, grid step h, number of steps per minimum wavelength n_{λ}, iteration count N_{it}, solver CPU time t_{cpu}, and CPU time per iteration t_{it}.*

n_{full}	h, m	n_{λ}	N_{it}	t_{cpu}, s	t_{it}, s
256	9.26	16.2	76	847	11.2
292	7.93	18.9	77	2065	26.8
296	7.81	19.2	78	446	5.7

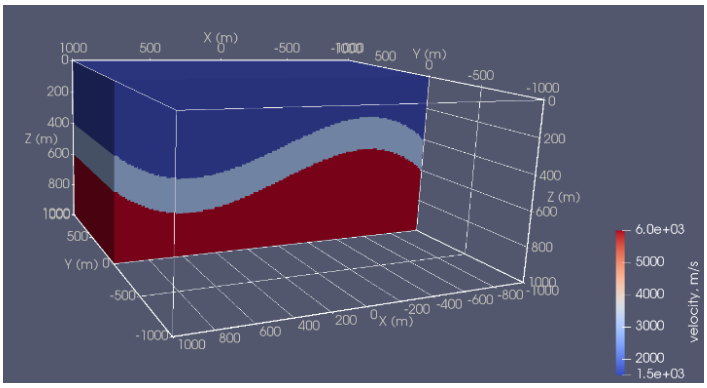

***Figure 2**. Velocity model formed by three curved layers with velocities of 1500, 3000 and 6000 m/s, respectively.*

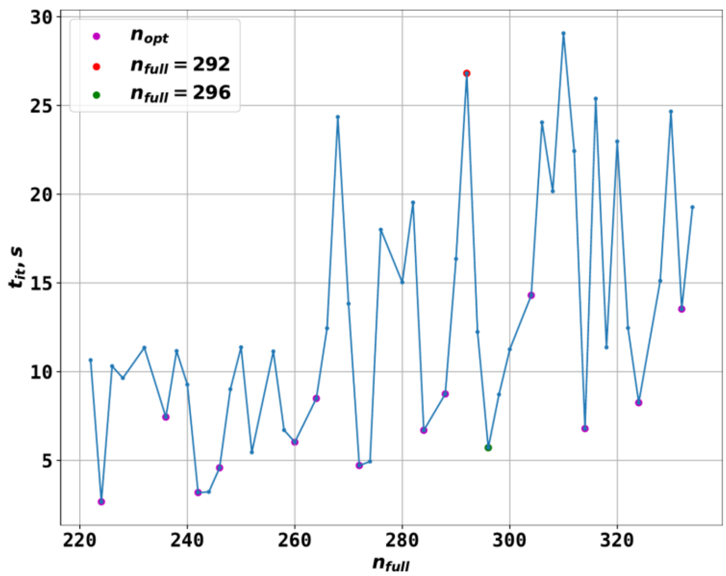

***Figure 3** CPU time per iteration of a CO solver versus modeling grid size $n_{full} \times n_{full} \times \frac{1}{2} n_{full}$ in the horizontal direction.*

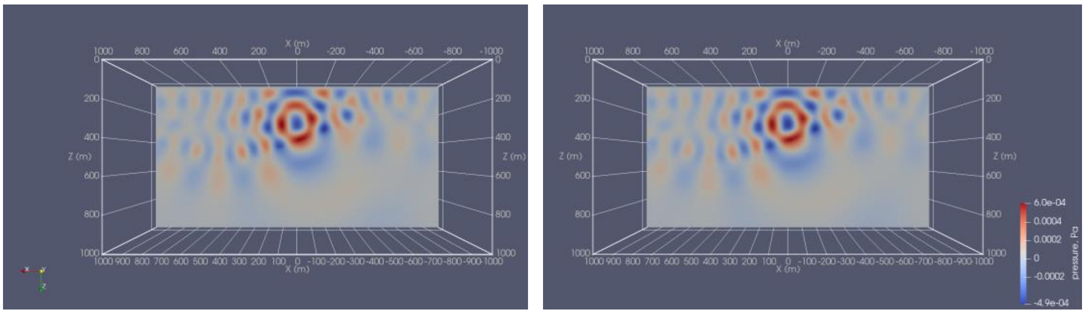

Figure 4 *Acoustic pressure modelled within the plane Y=100 m with different sizes of the grid 292× 292×146 (left panel) and 296×296×148 (right panel).*

Conclusions

This paper demonstrates that the preconditioned iterative solver based on the contraction operator transformation [Yavich et al., 2018] depends heavily on the performance of the FFT transform used in the code. The latter has a rough dependence of the size of the modeling grid. Our numerical experiments indicate that by selecting the optimal grid dimensions one could reduce modeling time dramatically. These optimal dimensions are of minor or no dependence of the particular machine architecture and thus could be hardcoded or calculated on-the-fly. The developed algorithm, if incorporated in the solution of the inverse problem, can dramatically reduce the computational time of full-waveform inversion as well. Future research will be aimed at application of the developed method in seismic full waveform inversion.

Acknowledgements

The authors acknowledge the MIPT and the University of Utah Consortium for Electromagnetic Modeling and Inversion (CEMI) for support of this research and permission to publish. We would like to acknowledge the Skoltech CDISE's high performance computing cluster, Zhores [Zacharov et. al., 2019], for providing the computing resources that have contributed to the results reported herein. This research was supported by the Russian Science Foundation, project No. 18-71-10071.

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