DATA-SPACE IMPLEMENTATION OF REGULARIZED GAUSS-NEWTON METHOD IN 3D INVERSION OF THE LITHOPROBE AND EARTHSCOPE MT DATA

E. Nadasi[1], A.V. Gribenko[2], M.S. Zhdanov[2,3]

[1] University of Miskolc; [2] TechnoImaging LLC; [3] University of Utah

Summary

The Western Superior region is part of the Canadian shield and is located north from the Superior Lake. The area has huge economic significance because several gold and base metal mineral deposits can be found there. Magnetotellurics is a useful tool to investigate the deep structure of the craton and find geological connections with the existing ore deposits. In this paper we present the results of magnetotelluric (MT) inversion of a subset of Lithoprobe and EartScope data collection covering this region. 92 MT stations were selected for 3D inversion, 79 from the Lithoprobe project and 13 from the EarthScope database. The regularized Gauss-Newton method was used applying data-space implementation. The algorithm inverts the full MT impedance and tipper data simultaneously. Based on the results, several conductive anomalies were identified in the Earth's crust and upper mantle. Recovered 3D conductivity model was compared with known tectonic structures, earlier geoelectric studies and seismic measurements. The deepest conductor appeared below the depth of 300 km. Three elongated quasi-vertical conductive anomalies between 100 and 300 km depth may represent the zones of partially melted material rising through the upper mantle.

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Introduction

This paper presents the results of 3D inversion of the magnetotelluric (MT) data collected as the parts of Lithoprobe and EarthScope projects over the Western Superior region of the Canadian shield. The developed and applied algorithm is based on the integral equation (IE) method of numerical modeling and uses the Gauss-Newton optimization in the data space. In addition, it implements simultaneous inversion for 3D conductivity distribution and distortion matrix.

The Lithoprobe project is a large regional geophysical and geological research program supported by Canada's Natural Sciences and Engineering Research Council. It covers several specific regions in Canada, one of them was the Western Superior Region, the target area of this paper. The MT data were collected between 1997 and 2000. These data were supplemented by Earth Scope US Array data from northern Minnesota (Figure 1). The EarthScope US Array program is a large continental-scale geophysical research program supported by the National Science Foundation of the USA. The EarthScope MT data were collected between 2011 and 2013 in northern Minnesota as part of the Mid-Continent Rift subproject. The Incorporated Research Institutions for Seismology (IRIS) has archived the observed MT data, which can be downloaded from IRIS webpage: http://ds.iris.edu/spud/emtf.

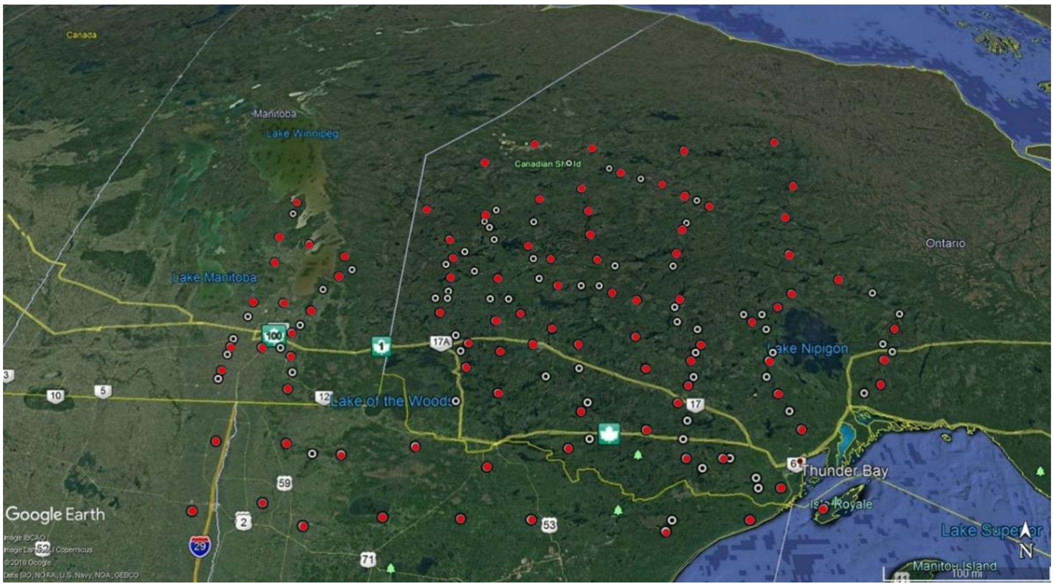

Figure 1 *Map of the area covered by the data set used in the study. White circles and red dots show the locations of all available MT stations. Red dots indicate the locations of the selected MT stations (92: 79 Lithoprobe and 13 EarthScope) used in the 3D inversion.*

The deep geological structure of the Western Superior was studied in several papers. Interpretation of MT data can be found in Ferguson et al. (2005) and in Roots and Craven (2017). Analyses of seismic datasets (e.g. Musacchio et al. 2004) revealed evidence of an anisotropic upper mantle. The findings of these papers were taken into account when we evaluated the 3D MT inversion results.

Theory

The forward MT problem can be written in operator notations as follows (Zhdanov 2018):

$$\boldsymbol{d} = \begin{bmatrix} \boldsymbol{c}\boldsymbol{Z}^{reg} \\ \boldsymbol{W} \end{bmatrix} = \boldsymbol{A}(\sigma), \tag{1}$$

where **d** stands for the observed MT data; **A** is a forward modeling operator based on the contraction form of the integral equation (CIE) formulation (Hursan and Zhdanov 2002) and MT transfer functions used to compute the undisturbed impedance $\mathbf{Z}^{reg}$. **W** is the tipper vector; matrix **c** contains components of the distortion matrix, and $\boldsymbol{\sigma}$ is a vector of the conductivity distribution within the modeling domain.

In order to solve equation (1) and to find the conductivity distribution and the distortion matrix from the given observed MT impedances and tippers, we follow the standard procedure of Tikhonov regularization (Tikhonov and Arsenin 1977, Zhdanov 2015) based on minimization of the parametric (cost) functional,

$$P(\sigma, \boldsymbol{c}) = \|\boldsymbol{r}\|^2 + \alpha\|\boldsymbol{S}\|^2, \quad (2)$$

$$\boldsymbol{r} = \boldsymbol{W}_d(\boldsymbol{cA}(\sigma) - \boldsymbol{d}),$$

$$\boldsymbol{S} = \begin{bmatrix} \boldsymbol{S}_\sigma \\ \boldsymbol{S}_\boldsymbol{c} \end{bmatrix} = \begin{bmatrix} \boldsymbol{D}(\sigma - \sigma_0) \\ (c - c_0) \end{bmatrix},$$

where σ_0 is a vector of a reference conductivity model; $\mathbf{c_0}$ is a 2 X 2 identity matrix, corresponding to no distortion case; and **D** represents a matrix of the finite difference first derivative operator. Matrix $\mathbf{W_d}$ is a data weighting matrix. The data weights are computed based on the noise level (variance). The stabilizing functional, **S,** consists of two parts: regularization for the conductivity distribution $\mathbf{S}_\boldsymbol{\sigma}$, and regularization for the distortion matrix, $\mathbf{S_c}$. The regularization parameter α balances the effect of the misfit and stabilizer in the parametric functional.

We used the regularized Gauss-Newton (RGN) method in data space (Gribenko and Zhdanov 2017) to minimize the parametric functional (2). A conventional model space RGN method requires the inversion of the large square Hessian matrix, which is one of the main obstacles for using the RGN method in geophysical inversion. The data space implementation involves the inversion of a much smaller matrix, which makes it possible to use the RGN method with limited computer resources. The two formulations are equivalent and produce identical solutions. The developed code was carefully tested on several synthetic models and applied to the field MT data (e.g. Gribenko and Zhdanov 2017).

Inversion results

After the data preparation, we have run 1D inversion of the MT sounding curves produced by averaging the observed MT data over all stations. The resulting 1D conductivity model was used as a starting model for 3D inversion of the entire MT dataset.

We applied 3D joint conductivity and distortion matrix inversion to the MT impedance and tipper data simultaneously. The inversion domain was extended at approximately 930 km in the North-South direction and in 1,157 km in the East-West direction. The horizontal cell size was selected at 11.3 x 11.3 km^2. The vertical discretization consisted of 40 layers with the thickness increasing logarithmically from 1 to 40 km. The total number of discretization cells was 334,560.
The error floors were set to 7.5% for the impedance and 0.075 for the tipper. The 3D RGN inversion was run for 33 iterations until nRMS misfit reached 2.24. The nRMS was computed by the following formulas:

$$\boldsymbol{r}_k = \boldsymbol{W}_d(\boldsymbol{A}(\boldsymbol{m}_k) - \boldsymbol{d}),\ \ \boldsymbol{nRMS} = \sqrt{\frac{\boldsymbol{r}_k^*\boldsymbol{r}_k}{N_d}}, \quad (3)$$

where $\mathbf{A(m_k)}$ is the vector of the predicted data at iteration number k; **d** is the vector of the observed data, N_d is the total number of data entries, and superscript "*" indicates a transposed complex conjugate vector.

Figure 2 shows horizontal sections of the resistivity model produced by 3D inversion at depths from 60 km to 380 km. In general, resistivity is relatively high to the depth of approximately 150 km, with more conductive material present at a greater depth. We have identified four prominent conductive features in the inverse resistivity model - C_d, C_1, C_2, and C_3. The deepest conductor, C_d, appears below the depth of 350 km, and extends in the W-E direction. The conductive features C_1, C_2, and C_3 are apparently originated from the deep conductor C_d and extended up to at least 150 km depth. These features are quite narrow and extend in the S-N direction. One interpretation of C_1- C_3 conductors could be the rise of hot conductive material from the deep mantle through the fractured zones in the extended upper mantle.

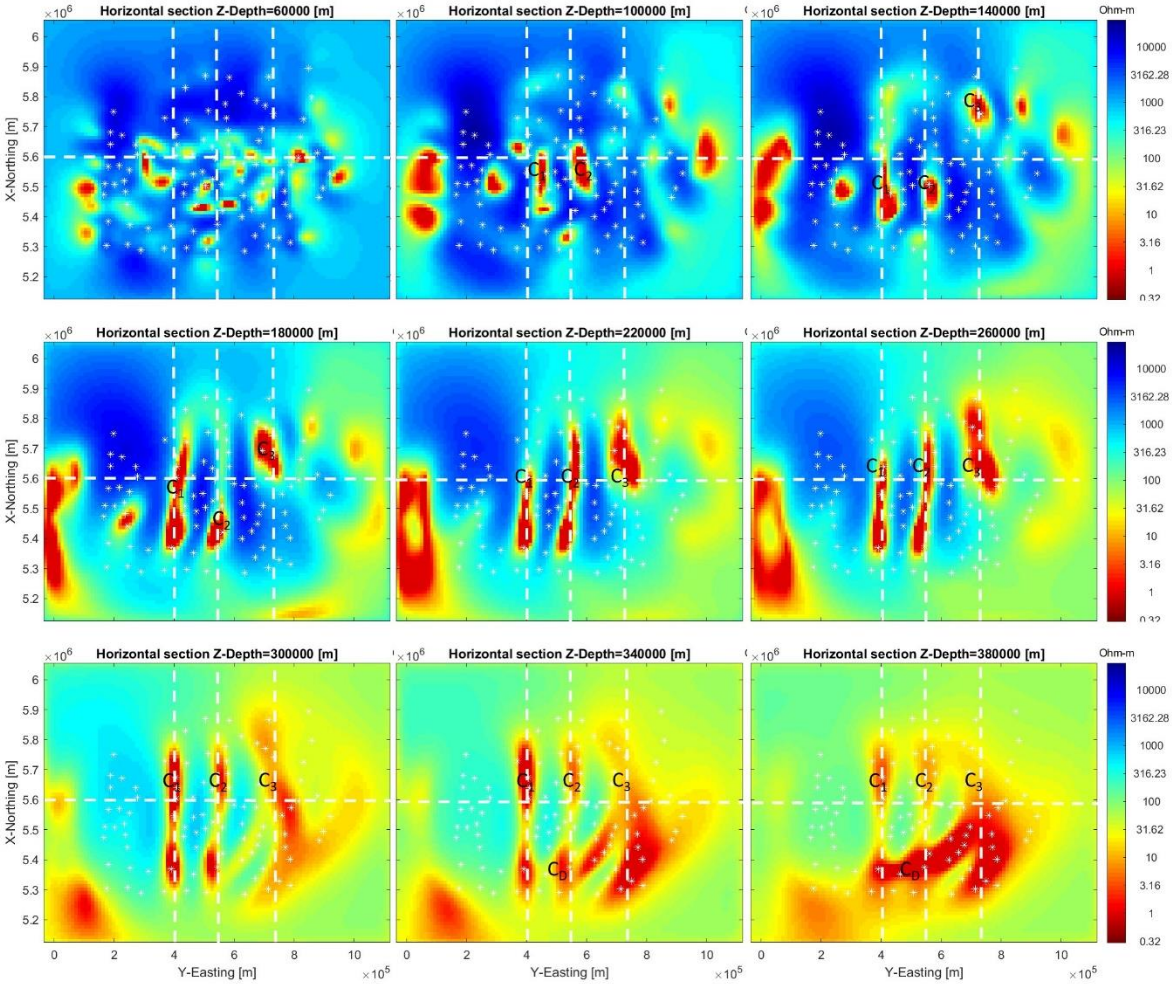

Figure 2 *Horizontal sections of the recovered resistivity distribution from 60 to 380 km. Dashed lines indicate locations of the vertical sections of the inversion result. Four conductive features are labeled by C_d, C_1, C_2, and C_3.*

Another possibility for such features is the apparent anisotropy of the upper mantle conductivity. Interchanging conductive and resistive features elongated in X (N-S) direction could be placed by the inversion in an effort to reproduce the actual anisotropic model with a low resistivity in the X (N-S) direction. Several researchers who studied geological and geophysical properties of the Western Superior province noticed the presence of anisotropy in the upper mantle (e.g. Musacchio et al. 2004).

In order to further examine the aforementioned conductive features, we present the W-E vertical section at 5,600 km Northing in Figure 3 (along the horizontal white dashed line shown in Figure 2). All three conductors are nearly vertical at the bottom, with a slight westerly strike at the top. Series of shallow conductors are also visible above 100 km depth. There is a possible connection between C_1, C_2, and C_3 and these shallow conductors.

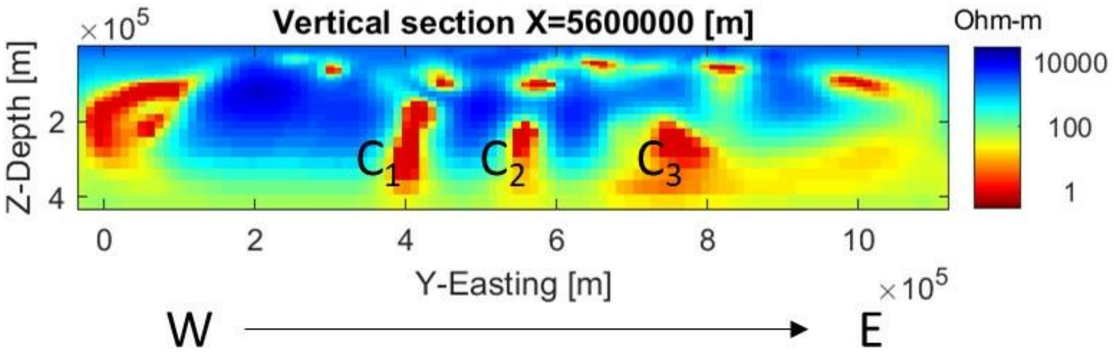

***Figure 3** W-E vertical section of inverse resistivity model along a profile located at 5,600 km Northing.*

Conclusion

We have applied the developed data-based RGN method for the 3D inversion of the MT data from the Western Superior region, collected by Lithoprobe and EarthScope projects. The results of inversion demonstrate that the inclusion of the distortion matrix provides superior data fit and recovers conductivity anomalies with greater details. A deep conductivity anomaly has been identified below 300 km depth. Three elongated conductive features are present in the subsurface between 100 and 300 km depth. These anomalies can be interpreted as ascending conductive material of the mantle through the fractures of the upper mantle or can be connected to conductivity anisotropy.

Acknowledgment

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