Joint inversion of gravity gradiometry and magnetic data in the Barents Sea using the probabilistic Gramian

M. Zhdanov[1,2], M. Jorgensen[1,2], M. Tao[2]

[1] TechnoImaging; [2] CEMI, University of Utah

Summary

We have developed a novel probabilistic approach to the joint inversion of multi-modal geophysical data based on the Gramian constraint. The multi-modal geophysical survey is the most effective technique for geophysical exploration because different physical data reflect distinct physical properties of the different components of the geological system. The joint inversion of multi-modal data can produce enhanced subsurface images of the physical property distributions, which enhances our ability to explore natural resources. One effective method of joint inversion is based on the Gramian constraint. This technique enforces the relationships between different model parameters during the inversion process. We demonstrate that the Gramian can be interpreted as a determinant of the covariance matrix between different physical models representing subsurface geology in the framework of the probabilistic approach to inverse theory. This interpretation enables us to use all the power of the modern probability theory and statistics in developing new methods for the joint inversion of multi-modal geophysical data. We apply the developed joint inversion methodology to inversion of gravity gradiometry and magnetic data in the Nordkapp Basin, Barents Sea to image salt diapirs.

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Introduction

Mutually complementary information about rock formations can be provided by different geophysical methods. However, the inversion of the standalone geophysical data sets is subject to considerable uncertainty regarding causative geomorphology and intrinsic physical property contrast. The joint inversion of multiphysics data can reduce this uncertainty. This can be accomplished by using the known petrophysical relationships between different physical properties of the rocks within the framework of the inversion process (e.g., Hoversten et al., 2006; Moorkamp et al., 2016; Zhdanov, 2015).

Gramian constraints present an alternative approach (Zhdanov et al., 2012; Zhdanov, 2015) to joint inversion. They enforce the functional relationships between multiple physical parameters without a priori knowledge of the specific form of these petrophysical relationships. Gramian constraints were introduced in the framework of the deterministic approach to the solution of the inverse problem. However, there is a probabilistic approach to solving inverse problems where the observed data and model parameters are treated as realizations of some random variables (e.g., Tarantola, 1987).

We introduce a novel approach to the joint inversion where the Gramian constraints are represented in the probabilistic form as the determinant of the covariance matrix between the different model parameters. This approach is illustrated by inverting potential field data overlying known salt diapirs in the Nordkapp Basin, the principal salt-producing basin in the western Barents Sea. The results of standalone and probabilistic Gramian inversions are compared.

Theory of probabilistic Gramian approach

The Gramian constraint enforces the correlation of individual model parameters or their transforms (Zhdanov et al., 2012; Zhdanov, 2015). By minimizing the Gramian functional in regularized inversion, we obtain multiphysics inverse models with better cross-model correlation. The deterministic Gramian functional, S_G, is given by the following formula:

$$S_G\left(m^{(1)}, m^{(2)}, \dots, m^{(n)}\right) = \begin{vmatrix} (m^{(1)}, m^{(1)}) & \dots & (m^{(1)}, m^{(n)}) \\ \dots & \dots & \dots \\ (m^{(n)}, m^{(1)}) & \dots & (m^{(n)}, m^{(n)}) \end{vmatrix},$$

where (-,-) represents the inner product of the model parameters in the Hilbert space of the models.

By employing the power of probability and statistical theory, we consider the observed data and the model parameters as realizations of some random variables. The joint inversion requires the correlation between different model parameters, which can be done by adding the term containing the covariance matrix, representing a probabilistic analog of the Gramian functional in the deterministic approach. The probabilistic Gramian functional, S_{G_σ} , can be introduced as the determinant of the covariance matrix between different model parameters:

$$S_{G_\sigma}\left(m^{(1)}, m^{(2)}, \dots, m^{(n)}\right) = \begin{vmatrix} cov(m^{(1)}, m^{(1)}) & \dots & cov(m^{(1)}, m^{(n)}) \\ \dots & \dots & \dots \\ cov(m^{(n)}, m^{(1)}) & \dots & cov(m^{(n)}, m^{(n)}) \end{vmatrix}.$$

The determinant of the covariance matrix is always non-negative, as with the deterministic Gramian. The key difference is how the model parameters are treated as random variables and how the covariance matrix is calculated using the principles of statistical estimation.

We can now introduce a probabilistic parametric functional, P_σ^α, which is a linear combination of the sum of data misfit functionals, $\sum_{i=1}^{n} \varphi(m^{(i)})$, and probabilistic Gramian:

$$P_\sigma^\alpha\left(m^{(1)}, m^{(2)}, \dots, m^{(n)}\right) = \sum_{i=1}^{n} \varphi(m^{(i)}) + \alpha S_{G_\sigma}\left(m^{(1)}, m^{(2)}, \dots, m^{(n)}\right) = min,$$

where $\alpha \in [0, \infty)$ is a regularization parameter. The joint inversion of multiphysics data is now reduced to the minimization of the probabilistic parametric functional, P_{σ}^{α}. One can apply different optimization algorithms to solve this problem (e.g., Tarantola, 1987; Zhdanov, 2015).

Application to joint inversion of gravity gradiometry and TMI data in the Nordkapp Basin

Seismic imaging of salt diapirs is difficult due to weak primaries, strong multiples and diffraction noise. The salt structures are surrounded by a "shadow zone" where continuous seismic reflectors are difficult to understand (Hokstad et al., 2011; Tu and Zhdanov, 2021). Accurate imaging of the diapirs from top to bottom is important, as large salt bodies without an overhang might contain small hydrocarbon volumes, but small salt bodies with an overhang can contain large hydrocarbon volumes.

To overcome this problem, we jointly inverted gravity gradiometry and total magnetic intensity (TMI) data overlying three distinct salt diapirs in the Nordkapp Basin. A GPU-accelerated inversion algorithm employing a moving sensitivity domain was used to carry out the 3D voxel inversions (Cuma and Zhdanov, 2014). The TMI data and all components of the gravity tensor were inverted towards density and induced magnetization models, following Jorgensen and Zhdanov (2021). A map of the G_{zz} component of the gravity tensor data, inversion extent, and profile location is shown in Figure 1. The vertical sections of the density and magnetization models extracted from the standalone inverted models are shown in Figure 2. The salt diapirs are characterized by low density and magnetization opposite the inducing field direction (Paoletti et al., 2020; Tao et al., 2021). Figure 3 presents the same sections extracted from the results of the probabilistic Gramian inversion, which show a stronger correlation across the respective models and give a more complete picture of the diapirs. The Uranus diapir (center) lacks the overhangs required for a structural trap, which was confirmed by drilling. This absence is most apparent in the magnetization model.

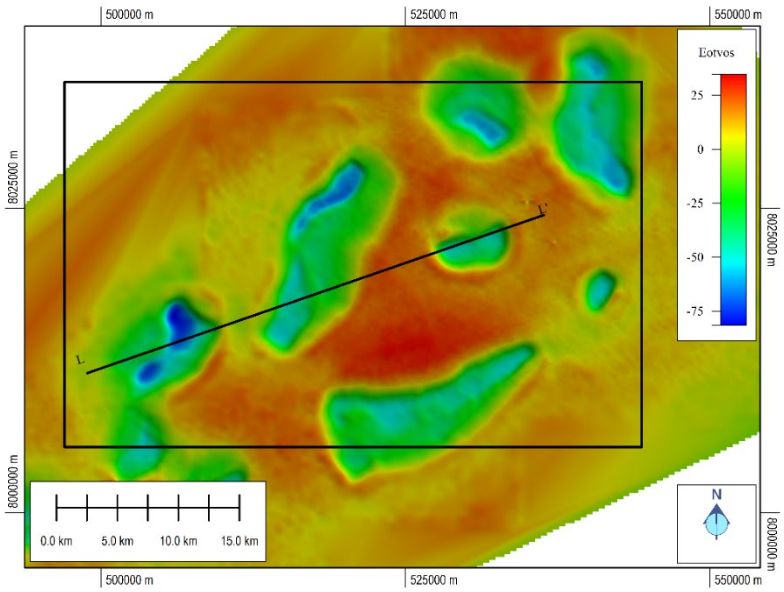

Figure 1 *Map of the observed Gzz component of gravity gradiometry data used in the inversion. The black box outlines the inversion domain. The black line labeled LL' indicates the location of the vertical sections.*

Figure 4 shows the model parameter cross plots of density versus magnetization. The cloud of correlations from the standalone inversions (upper panel) makes it challenging to discern petrophysical relationships. The correlation coefficient for these models is 0.73. We contrast this to the correlations from the probabilistic Gramian inversion (bottom panel), which delineate a clear trend between salt and host rock with a higher correlation coefficient of 0.93.

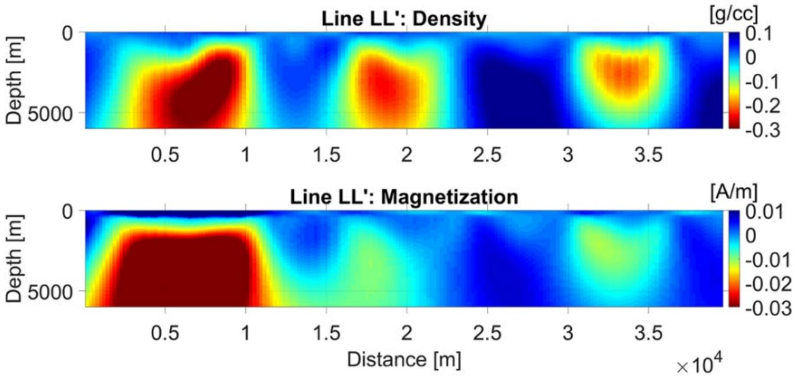

Figure 2 *Vertical sections extracted from the standalone inversions along profile LL'. The top panel shows the anomalous density. The bottom panel shows the induced magnetization projected onto the inducing field.*

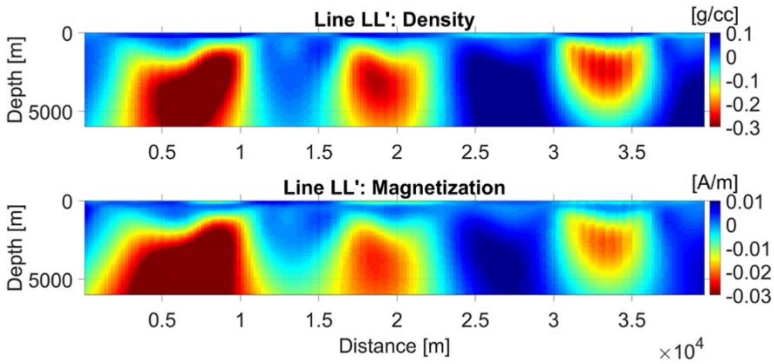

Figure 3 *Vertical sections extracted from the joint probabilistic Gramian inversion along profile LL'.1. The top panel shows the anomalous density. The bottom panel shows the induced magnetization projected onto the inducing field.*

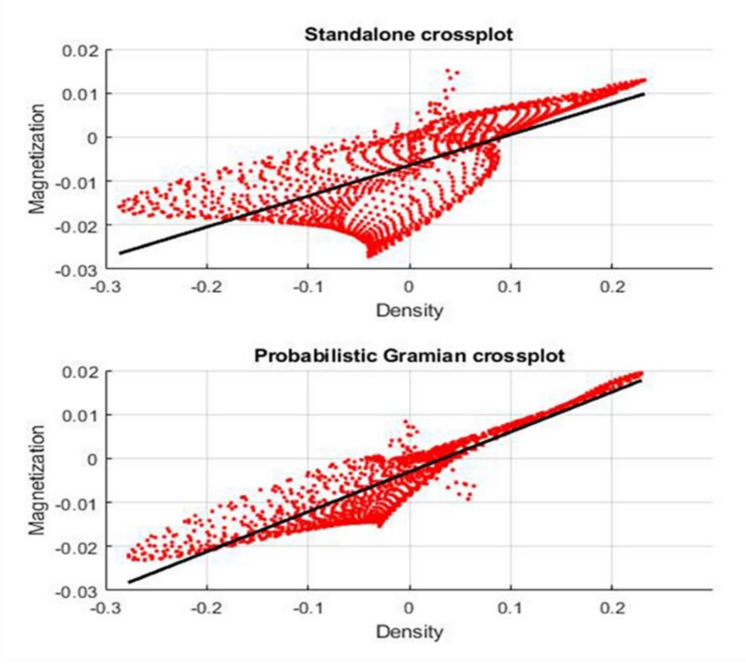

Figure 4 *Model parameter cross plot comparison of the different inversion methodologies. The top panel shows the correlation of the standalone inversions. The bottom panel shows the correlation of the probabilistic Gramian inversion.*

Conclusions

We have demonstrated how the joint Gramian-based inversion could be reformulated using the probabilistic approach. Furthermore, we show that the determinant of the covariance matrix between the different physical properties representing the geologic formations is an analog of Gramian. This helps understand better the role of the Gramian in enforcing the relationships between different physical models. It also presents an alternative numerical implementation of the Gramian-type constraints by using statistical estimates to calculate the components of the covariance matrix.

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References

Cuma, M. and Zhdanov, M. S. [2014] Massively parallel regularized 3D inversion of potential fields on CPUs and GPUs. *Computers and Geosciences*, **62**, 80–87.

Hokstad, K., Fotland, B., Mackenzie, G., Antonsdottir, V., Foss, S., Stadtler, C., et al. [2011] Joint imaging of geophysical data: Case history from the Nordkapp Basin, Barents Sea. *SEG Technical Program Expanded Abstracts*, 1098–1102.

Hoversten, G. M., Cassassuce, F., Gaskeripova, E., Newman, G. A., Chen, J., Rubin, Y., et al. [2006] Direct reservoir parameter estimation using joint inversion of marine seismic AVA and CSEM data. *Geophysics*, **71**, C1–C13.

Jorgensen, M. and Zhdanov, M. S. [2021] Recovering magnetization of rock formations by jointly inverting airborne gravity gradiometry and total magnetic intensity data. *Minerals*, **11**(4), 366.

Moorkamp, M., Lelievre, P., Linde, N. and Khan, A. [2016] *Integrated imaging of the Earth: theory and applications.* Wiley.

Paoletti, V., Milano, M., Baniamerian, J. and Fedi, M. [2020] Magnetic field imaging of salt structures at Nordkapp Basin, Barents Sea. *Geophysical Research Letters*, **47**(18).

Tao, M., Jorgensen, M. and Zhdanov, M. S. [2021] Mapping the salt structures from magnetic and gravity gradiometry data in Nordkapp Basin, Barents Sea. *SEG/AAPG International Meeting for Applied Geoscience & Energy*, Houston, Texas, USA, 874-878.

Tarantola, A. [1987] *Inverse problem theory*. Elsevier.

Tu, X. and Zhdanov, M. S. [2022] Joint focusing inversion of marine controlled-source electromagnetic and full tensor gravity gradiometry data. *Geophysics* **87**, K35–K47.

Zhdanov, M. S., Gribenko, A. V. and Wilson, G. [2012] Generalized joint inversion of multimodal geophysical data using Gramian constraints. *Geophysical Research Letters*, **39**, L09301.

Zhdanov, M. S. [2015] *Inverse theory and applications in geophysics*. Elsevier.