Hybrid method for 3D modeling of electromagnetic fields in complex structures with inhomogeneous background conductivity

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SUMMARY

We present a new formulation of the hybrid method for three-dimensional (3D) electromagnetic (EM) modeling in complex structures with inhomogeneous background conductivity (IBC). This method overcomes the standard limitation of the conventional IE method related to the use of a horizontally layered background only. The new method allows us to compute the effect of IBC structures by using any appropriate numerical method which may be able to build a model with a flexible grid. This approach seems to be extremely useful in computing EM data for multiple geologic models with some common geoelectrical features, like terrain, bathymetry, or other known structures. It may find wide application in inverse problem solution, where we have to keep some known geologic structures unchanged during the iterative inversion. The method was carefully tested for modeling the EM field for complex structures with known variable background conductivity. The effectiveness of this approach is illustrated by modeling marine magnetotelluric (MT) data.

INTRODUCTION

Over the last decades, significant progress has been made in the development of three-dimensional (3D) electromagnetic (EM) modeling algorithms. The most widely used approaches include finite-element (FE) techniques, finite-difference (FD) methods and closely related finite-volume methods. There was also a number of publications dedicated to the integral equation (IE) technique. In addition to these frequency domain approaches, new efficient time domain methods are emerging. The FD methods are probably the simplest ones in concept and in practical implementation, while the FE techniques are the most flexible in accounting for the model geometry (Adveev, 2005), however, the FE methods are not as straightforward to implement as the FD methods. The IE methods can be highly efficient as well, but their computational complexity increases as the model complexity increases (Mackie et al., 1993; Zhdanov, 2002, 2009).

Zhdanov et al. (2006) and Endo et al. (2009) have developed 3D EM modeling techniques based on the extended formulation of the IE method to a more general case of models with inhomogeneous background conductivity (IBC). This extended method is based on the separation of the effects related to the excess current induced in the inhomogeneous background domain, from those effects related to the anomalous electric current in the location of the anomalous conductivity, respectively. As a result, one can arrive at a system of integral equations that use the same Green's functions for the layered model as in the original IE formulation. However, these equations take into account the effect of the variable background conductivity distribution.

In the present paper, we extend this IBC IE method to the hybrid technique, i.e., we calculate the EM fields due to the inhomogeneous background domain by other numerical methods than the IE method and compute the anomalous fields by the IE method. This new method allows us to build models with more flexible geoelectrical structures.

The accurate simulation of the EM field caused by inhomogeneities, such as bathymetry, is a challenging numerical problem because it requires a huge number of discretization cells to represent the bathymetric structures properly. We can choose FD or FE methods to solve this kind of problem. However, these methods require the discretization of the entire modeling domain. This computational problem becomes more challenging when we need to investigate the EM response of the anomalous domain (target) by using different parameters, because we have to repeat the massive computation. In the framework of the hybrid method, one can precompute the effect of the IBC structure only once by the FD (or an other) method, which allows us to build a flexible geoelectrical model and keep it unchanged during the entire modeling and/or inversion process. Taking into account that precomputing the effect of the IBC structure constitutes the most time-consuming part of the EM forward modeling, this approach would allow us to increase the effectiveness of the computer simulation in the interpretation of the EM data significantly. We illustrate this approach by modeling marine MT data.

As an example, we have developed an algorithm and a numerical code, which uses the FD method for IBC field calculations. The method was tested on typical geoelectrical models with variable backgrounds.

THE HYBRID METHOD IN A MODEL WITH INHOMOGENEOUS BACKGROUND CONDUCTIVITY

We consider a 3D geoelectrical model with horizontally layered (normal) conductivity σ_n, inhomogeneous background conductivity $\sigma_b = \sigma_n + \Delta\sigma_b$ within a domain D_b, and anomalous conductivity $\Delta\sigma_a$ within a domain D_a (Figure 1). The model is excited by an EM field generated from an arbitrary source which is time-harmonic as $e^{-i\omega t}$. The EM field in this model satisfies Maxwell's equations:

$$\begin{aligned} \nabla \times \mathbf{H} &= \sigma_n \mathbf{E} + \mathbf{j} = \sigma_n \mathbf{E} + \mathbf{j}^{\Delta\sigma_b} + \mathbf{j}^{\Delta\sigma_a} + \mathbf{j}^e, \\ \nabla \times \mathbf{E} &= i\omega\mu_0 \mathbf{H}, \end{aligned} \quad (1)$$

where

$$\mathbf{j}^{\Delta\sigma_a} = \begin{cases} \Delta\sigma_a \mathbf{E}, & r \in D_a \\ 0, & r \notin D_a \end{cases} \tag{2}$$

is the anomalous current within the local inhomogeneity D_a and

$$\mathbf{j}^{\Delta\sigma_b} = \begin{cases} \Delta\sigma_b \mathbf{E}, & r \in D_b \\ 0, & r \notin D_b \end{cases} \tag{3}$$

is the excess current within the inhomogeneous background domain D_b.

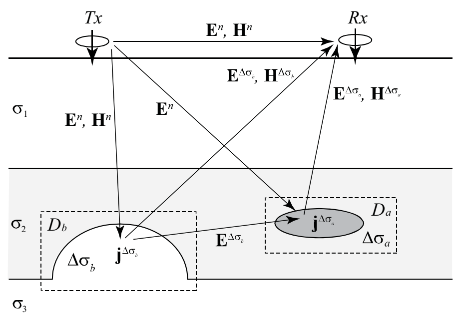

Figure 1: A sketch of a 3D geoelectric model with horizontally layered (normal) conductivity σ_n, inhomogeneous background conductivity $\sigma_b = \sigma_n + \Delta\sigma_b$ within a domain D_b, and anomalous conductivity $\Delta\sigma_a$ within a domain D_a.

Equations (1) - (3) show that one can represent the EM field in this model as a sum of the normal fields $\mathbf{E}^n$ and $\mathbf{H}^n$ generated by the given source(s) in a model with normal distribution of conductivity σ_n, variable background effects $\mathbf{E}^{\Delta\sigma_b}$ and $\mathbf{H}^{\Delta\sigma_b}$ produced by the inhomogeneous background conductivity $\Delta\sigma_b$, and the anomalous fields $\mathbf{E}^{\Delta\sigma_a}$ and $\mathbf{H}^{\Delta\sigma_a}$ related to the anomalous conductivity distribution $\Delta\sigma_a$:

$$\begin{aligned} \mathbf{E} &= \mathbf{E}^n + \mathbf{E}^{\Delta\sigma_b} + \mathbf{E}^{\Delta\sigma_a}, \\ \mathbf{H} &= \mathbf{H}^n + \mathbf{H}^{\Delta\sigma_b} + \mathbf{H}^{\Delta\sigma_a}. \end{aligned} \tag{4}$$

The total EM fields in this model can be written as

$$\begin{aligned} \mathbf{E} &= \mathbf{E}^b + \mathbf{E}^{\Delta\sigma_a}, \\ \mathbf{H} &= \mathbf{H}^b + \mathbf{H}^{\Delta\sigma_a}, \end{aligned} \tag{5}$$

where the background EM fields $\mathbf{E}^b$, $\mathbf{H}^b$ are sums of the normal fields and those caused by the inhomogeneous background conductivity.

Note that all these fields can be calculated by any numerical methods, such as finite-difference, finite-elements, or integral equation methods. Zhdanov et al. (2006) demonstrated the IBC integral equation method where both the background EM fields and the anomalous EM fields were calculated by the integral equation method.

Similar to the logic of the IBC IE method (Zhdanov et al., 2006), we write the EM fields generated by the given current distribution,

$$\mathbf{j}^{\Delta\sigma}(\mathbf{r}) = \mathbf{j}^{\Delta\sigma_b}(\mathbf{r}) + \mathbf{j}^{\Delta\sigma_a}(\mathbf{r}) = \Delta\sigma_b \mathbf{E}(\mathbf{r}) + \Delta\sigma_a \mathbf{E}(\mathbf{r}),$$

within a medium of normal conductivity σ_n:

$$\begin{aligned} \mathbf{E}(\mathbf{r}_j) &= \mathbf{E}^n + \mathbf{E}^{\Delta\sigma_b} + \iiint_{D_a} \widehat{\mathbf{G}}_E(\mathbf{r}_j|\mathbf{r}) \cdot \Delta\sigma_a \mathbf{E}(\mathbf{r})\, dv \\ &= \mathbf{E}^b + \iiint_{D_a} \widehat{\mathbf{G}}_E(\mathbf{r}_j|\mathbf{r}) \cdot \Delta\sigma_a \mathbf{E}(\mathbf{r})\, dv, \\ \mathbf{H}(\mathbf{r}_j) &= \mathbf{H}^n + \mathbf{H}^{\Delta\sigma_b} + \iiint_{D_a} \widehat{\mathbf{G}}_H(\mathbf{r}_j|\mathbf{r}) \cdot \Delta\sigma_a \mathbf{E}(\mathbf{r})\, dv \\ &= \mathbf{H}^b + \iiint_{D_a} \widehat{\mathbf{G}}_H(\mathbf{r}_j|\mathbf{r}) \cdot \Delta\sigma_a \mathbf{E}(\mathbf{r})\, dv. \end{aligned} \tag{6}$$

As we noted above, the background fields $\mathbf{E}^b$, $\mathbf{H}^b$ in equation (6) can be calculated by any numerical method, such as finite-difference, finite-element, or integral equation methods. The last terms of equations (6) describe the anomalous fields generated by the anomalous domain D_a:

$$\begin{aligned} \mathbf{E}^{\Delta\sigma_a}(\mathbf{r}_j) &= \mathbf{E}(\mathbf{r}_j) - \mathbf{E}^n(\mathbf{r}_j) - \mathbf{E}^{\Delta\sigma_b}(\mathbf{r}_j) \\ &= \iiint_{D_a} \widehat{\mathbf{G}}_E(\mathbf{r}_j|\mathbf{r}) \cdot \Delta\sigma_a \mathbf{E}(\mathbf{r})\, dv \\ &= \mathbf{G}_E^{D_a}\left(\Delta\sigma_a\left(\mathbf{E}^b + \mathbf{E}^{\Delta\sigma_a}\right)\right), \end{aligned} \tag{7}$$

$$\begin{aligned} \mathbf{H}^{\Delta\sigma_a}(\mathbf{r}_j) &= \mathbf{H}(\mathbf{r}_j) - \mathbf{H}^n(\mathbf{r}_j) - \mathbf{H}^{\Delta\sigma_b}(\mathbf{r}_j) \\ &= \iiint_{D_a} \widehat{\mathbf{G}}_H(\mathbf{r}_j|\mathbf{r}) \cdot \Delta\sigma_a \mathbf{E}(\mathbf{r})\, dv \\ &= \mathbf{G}_H^{D_a}\left(\Delta\sigma_a\left(\mathbf{E}^b + \mathbf{E}^{\Delta\sigma_a}\right)\right). \end{aligned} \tag{8}$$

In equations (7) and (8), the symbols $\mathbf{G}_E^{D_a}$ and $\mathbf{G}_H^{D_a}$ denote the electric and magnetic Green's operators with a volume integration of D_a. Using equations (7) and (8), one can calculate the EM fields at any point $\mathbf{r}_j$ if the electric field is known within the inhomogeneity.

The basic idea of a new hybrid method is similar to that of the IBC IE method, i.e., we can take into account the EM field induced in the anomalous domain by the excess currents in the background inhomogeneity $\mathbf{j}^{\Delta\sigma_b}$ but would ignore the return induction effects by the anomalous currents $\mathbf{j}^{\Delta\sigma_a}$. In other words, we assume that the anomalous electric fields $\mathbf{E}^{\Delta\sigma_a}$ are much smaller than the background fields $\mathbf{E}^b$ inside the domain D_b.

HYBRID IMPLEMENTATION OF THE IBC IE METHOD WITH THE FD TECHNIQUE

We have stated above that one can calculate the EM fields due to the IBC domain by any numerical method. In the current paper, we use the finite-difference (FD) method of Mackie et al. (1993) to calculate the EM fields produced by the IBC domain for the case of a 3D MT modeling problem.

In order to compute the EM fields at the receivers, we have to solve the field equations (6). We can calculate the background fields $\mathbf{E}^b$, $\mathbf{H}^b$, which include the EM fields due to the IBC structure ($\mathbf{E}^{\Delta\sigma_b}$, $\mathbf{H}^{\Delta\sigma_b}$), using the FD method. To compute the anomalous fields at the receivers, we have to solve the domain equations (equations (7), $\mathbf{r}_j \in D_a$). We solve these equations by the contraction integral equation (CIE) method (Hursán and Zhdanov, 2002).

According to the theory of the CIE method, the total electric fields inside the anomalous domain can be expressed in a discrete form as:

$$\widehat{\mathbf{A}}\mathbf{e} = \mathbf{e}^b, \tag{9}$$

where $\mathbf{e}$ and $\mathbf{e}^b$ are the total and the background (including the effects of the IBC structure) electric fields, respectively, and

$$\widehat{\mathbf{A}} = \widehat{\mathbf{I}} - \widehat{\mathbf{G}}_D\widehat{\mathbf{S}}^a. \tag{10}$$

Matrix $\widehat{\mathbf{A}}$ is a $3N \times 3N$ complex non-Hermitian matrix with a full structure (now we assume that the anomalous body is discretized into N cells). $\widehat{\mathbf{G}}_D$ is a $3N \times 3N$ matrix containing electric Green's tensor integrals, and $\widehat{\mathbf{S}}^a$ is a $3N \times 3N$ diagonal matrix with the anomalous conductivities.

We can calculate $\mathbf{e}^b$ as well as $\mathbf{E}^b$ by using the FD method, and can solve the linear system (9) by iterative solvers, such as the complex generalized minimum residual (CGMRES) method.

SYNTHETIC MODEL EXAMPLE

In order to analyze the overall efficiency of a new approach in comparison with the conventional IE modeling, we have applied both the hybrid IBC method and the original IE IBC method for numerical modeling of the MT data for the same geoelectrical model shown in Figure 2. The results of conventional IE modeling are obtained by the code INTEM3D (Hursán and Zhdanov, 2002).

Figure 2 presents a vertical cross section of the 3D model selected for this modeling experiment. A resistive hydrocarbon reservoir with a conductivity of 0.01 S/m (anomalous domain) is submerged within the complexly stratified sea-bottom sediments (IBC domain). These two domains are located in a horizontally layered half-space. The anomalous domain is discretized into 2640 ($22 \times 20 \times 6$) cells, while the IBC domain is discretized into 56000 ($80 \times 20 \times 35$) cells . The size of an elementary cell in both domains is $200 \times 400 \times 100$ m^3.

We have computed the EM responses for this model in 6400 receivers located at every 200 m in the x direction and every 400 m in the y direction over a 160×40 grid using both the conventional IE algorithm (INTEM3D) and the hybrid code. In our numerical test, we have simulated the EM field generated by a vertically propagated plane EM wave (magnetotelluric (MT) data simulation). The plane waves in two H- and E-polarizations are used as the sources with a frequency of 0.25 Hz. We apply the CGMRES method (Zhdanov, 2002) to solve a system of linear IE equations in both the IE and hybrid methods. The desired misfit level of the matrix solution is 10^{-8} in both modeling experiments.

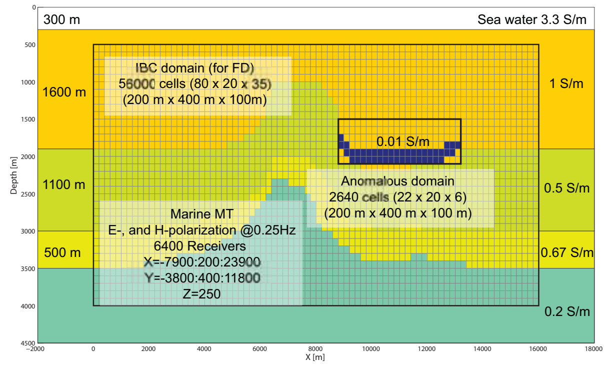

Figure 2: A vertical cross section of the 3D geoelectrical model.

In the case of hybrid algorithm, the EM fields at the receivers (background fields for the field equations) and inside the anomalous domain (background fields for the domain equations) are computed in a model with the IBC domain only by the FD method. After that, we solve the domain equations within the anomalous domain only. In the case of the conventional IE method, we need to solve the domain equations with larger domains, which includes both the IBC and the anomalous domain.

Figures 3 and 4 show the real and imaginary components of the electric fields and magnetic fields for H-polarization, computed using two different codes. The profile is along the x axis at $y = 3800$ m. The solid lines represent the results obtained by the conventional IE method (INTEM3D), whereas the circles represent those computed using a new hybrid scheme. One can see that both results agree reasonably well with each other.

Figures 5 and 6 show the real and imaginary parts of the x component of the electric field, E_x, and of the y component of the magnetic field, H_y, for H-polarization, computed using the IE, hybrid, and FD methods.

As in the case of the profiles (Figures 3 and 4), one can see that the new hybrid method produces practically the same results as the IE method. We should note that the accuracy of the hybrid method can be greatly affected by the accuracy of the FD method, so that the model discretization in the FD method should be investigated to improve the accuracy of the calculation. However, the model study shows that our new hybrid method works adequately for a complex geoelectrical model.

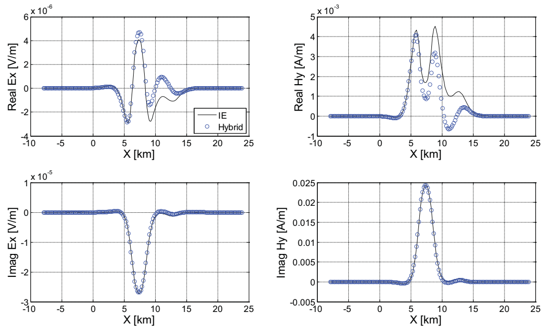

Figure 3: Plots of the real and imaginary parts of the electric and the magnetic fields for H-polarization along the x-directed profile at $y = 3800$. The solid lines represent the results obtained by the conventional IE method (INTEM3D); the circles show the data computed using the hybrid scheme.

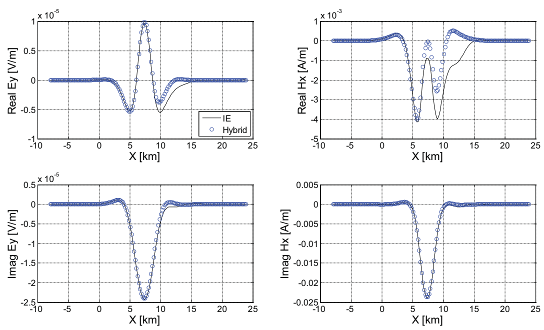

Figure 4: Plots of the real and imaginary parts of the electric and the magnetic fields for E-polarization along the x-directed profile at $y = 3800$. The solid lines represent the results obtained by the conventional IE method (INTEM3D); the circles show the data computed using the hybrid scheme.

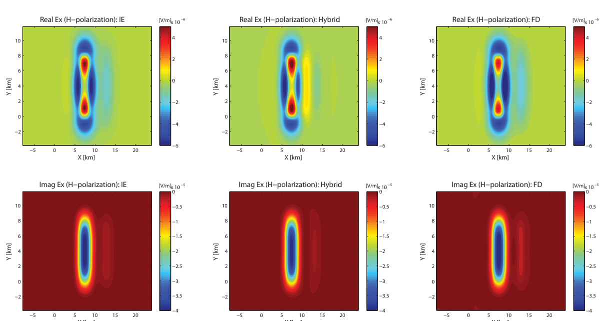

Figure 5: Maps of the real and imaginary parts of the x component of the electric field computed by the conventional IE method (left panels), hybrid method (middle panels) , and FD method (right panels).

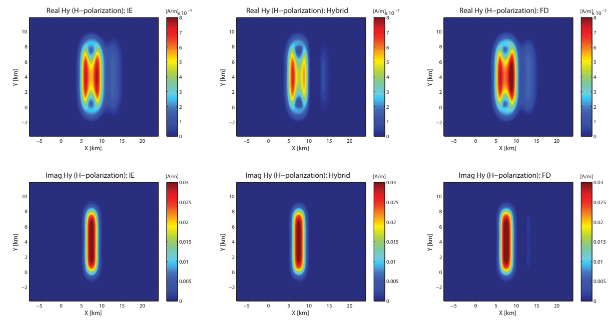

Figure 6: Maps of the real and imaginary parts of the y component of the magnetic field computed by the conventional IE method (left panels), hybrid method (middle panels) , and FD method (right panels).

CONCLUSIONS

The results of this paper clearly demonstrate that a method, based on a combination of different numerical techniques in one computational scheme, can be introduced for the models with a variable background conductivity. This fact opens the possibility of incorporating an inhomogeneous background, such as a known geologic structure or the terrain and bathymetry effects, in an IE-based hybrid forward modeling.

The advantage of the hybrid method is related to the fact that interpretation of the field data usually requires multiple solutions of the forward problem with different parameters of the target (in our example, a sea-bottom hydrocarbon reservoir). Any traditional numerical method would require repeating these massive computations, including hundreds of thousands of cells covering whole inhomogeneous domains, every time we change the model of the target, which is extremely expensive. At the same time, using the hybrid approach, we can precompute the effect of an IBC domain only once using any appropriate modeling technique (e.g., by the FD method) and then repeat the computations on a smaller grid covering the anomalous domain only. These factors may prove to be critical in the effective use of the hybrid method in fast EM inversion over complex geoelectrical structures.

The results of this paper also open a possibility for the development of the hybrid method for 3D modeling of electromagnetic fields in complex structures with inhomogeneous background conductivity using different numerical methods.

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EDITED REFERENCES

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