Integral electric current method in 3-D electromagnetic modeling for large conductivity contrast

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SUMMARY

We introduce a new approach to 3-D electromagnetic (EM) modeling for models with large conductivity contrast. It is based on the equations for integral current within the cells of the discretization grid, instead of the electric field or electric current themselves, which are used in the conventional integral equation (IE) method. We obtain these integral currents by integrating the current density over each cell. The integral currents can be found accurately for the bodies with any conductivity. As a result, the method can be applied, in principle, for the models with high conductivity contrast. At the same time, knowing the integral currents inside the anomalous domain allows us to compute the EM field components in the receivers using the standard integral representations of the Maxwell's equations. We call this technique an *integral electric current (IEC) method*. The method is carefully tested by comparison with an analytical solution for a model of a sphere with large conductivity embedded in the homogenous whole space.

INTRODUCTION

One of the difficult problems in electromagnetic (EM) modeling is accurate numerical solution for models with large conductivity contrast. This problem appears, for example, in modeling EM data for mineral exploration when we have a conductive target embedded in relatively resistive host rocks. The study of the topography effect on EM data requires the solution of a similar problem, because the contrast in conductivity between the conductive earth and nonconductive air can be as large as $10^8 - 10^{10}$ times. Well-logging is another area where one should take into account a strong contrast between the cased borehole, for example, and surrounding rock formations.

In this paper, we introduce a new approach to the solution of this problem based on the integral equation (IE) method. The conventional IE algorithms are usually written for the electric field or electric current components within the domain with anomalous conductivity. This domain is divided in the number of cells, which are selected to be so small that the field components vary slowly within the cell. If the conductivity of the body and/or frequency are high, it is difficult to satisfy this condition. The EM field varies extremely fast within a good conductor, which may result in errors of numerical modeling. In order to overcome this difficulty, Newman and Hohmann (1988) used a special grouping of the boxcar basis functions to form current loops within the conductor. Farquharson and Oldenburg (2002) implemented the more sophisticated edge element basis functions to avoid the inaccuracy of the conventional boxcar basis function approach.

In this paper we consider a novel approach for solving this problem. We develop a new form of the IE method, which is based on the equations for integral current within the cells, instead of the electric field or electric current themselves. We obtain these integral currents by integrating the current density over each cell. The integral currents can be found accurately for a body with any conductivity. We do not use anymore the requirements that the field varies slowly inside the cell, because we deal with the integral of this field. As a result, the method can be applied, in principle, for models with arbitrary conductivity contrast. At the same time, knowing the integral currents inside the anomalous domain allows us to compute the EM field components in the receivers using the standard integral representations of Maxwell's equations. We call this technique an *integral electric current (IEC) method*.

FORMULATION OF THE IEC METHOD

The conventional IE method is based on the following equation for the total electric **E** fields:

$$\mathbf{E}(\mathbf{r}') = \iiint_D \widehat{\mathbf{G}}_E(\mathbf{r}' \mid \mathbf{r}) \cdot [\Delta\widetilde{\sigma}(\mathbf{r})\mathbf{E}(\mathbf{r})]\, dv + \mathbf{E}^b(\mathbf{r}'). \quad (1)$$

where $\widehat{\mathbf{G}}_E(\mathbf{r}_j \mid \mathbf{r})$ is the electric Green's tensor defined for an unbounded conductive medium with the complex background conductivity $\widetilde{\sigma}_b = \sigma - i\omega\varepsilon$; $\mathbf{E}^b$ is the background electric field; domain D corresponds to the volume with the anomalous conductivity distribution $\widetilde{\sigma}(\mathbf{r}) = \widetilde{\sigma}_b + \Delta\widetilde{\sigma}(\mathbf{r})$, $\mathbf{r} \in D$.

The conventional approach to discretization of the integral equation (1) is based on dividing domain D into N elementary cells, D_n, formed by some rectangular grid in the domain $D = \bigcup_{n=1}^{N} D_n$, and assuming that $\Delta\widetilde{\sigma}(\mathbf{r})$ has the constant value $\Delta\widetilde{\sigma}_n$ within the cell.

We also assume that each cell D_n is so small that the electric field is approximately constant within the cell, $\mathbf{E}(\mathbf{r}) \approx \mathbf{E}(\mathbf{r}_n)$, where $\mathbf{r}_n$ is a center point of rectangular cell D_n. Under this condition the discrete analog of equation (1) can be written as (Zhdanov, 2002):

$$\mathbf{e}_D = \widehat{\mathbf{G}}_D \widehat{\sigma} \mathbf{e}_D + \mathbf{e}_D^b, \quad (2)$$

where $\widehat{\sigma}$ is a $(3N \times 3N)$ diagonal matrix of anomalous conductivities, $\mathbf{e}_D$ and $\mathbf{e}_D^b$ are the vectors of the total and background electric fields formed by the x, y and z components of these fields at the centers of the cells D_n of the anomalous domain D. The $3N \times 3N$ matrix $\widehat{\mathbf{G}}_D$ is formed by the volume integrals over the elementary cells D_n of the components of the corresponding electric Green's tensor $\widehat{\mathbf{G}}_E$, acting inside domain D.

Note that equation (1), or equivalent matrix equation (2) provides an adequate approximation of the original integral equation, if the following conditions hold: 1) the linear size h of elementary cell D_n is much smaller than the wave length λ_b of the EM field in the background medium,

$$h << \lambda_b, \quad (3)$$

and 2) h is much smaller than the wave length λ_a of the EM field in a medium with anomalous conductivity:

$$h << \lambda_a. \quad (4)$$

The first condition (3) usually holds for typical geophysical EM modeling problems. The second condition (4) may fail in the case of high anomalous conductivity, which is the subject of this paper.

Our goal is to construct a discrete analogue of integral equation (1), which would provide an accurate approximation only under condition (3). In order to obtain a system of linear equations with respect to integral currents, let us multiply both sides of equation (1) by $\Delta\widetilde{\sigma}(\mathbf{r}')$. As a result we have:

$$\mathbf{j}(\mathbf{r}') = \Delta\widetilde{\sigma}(\mathbf{r}') \iiint_D \widehat{\mathbf{G}}_E(\mathbf{r}' \mid \mathbf{r}) \cdot \mathbf{j}(\mathbf{r}) dv + \mathbf{j}^b(\mathbf{r}'), \quad (5)$$

where:

$$\mathbf{j}^b(\mathbf{r}') = \Delta\widetilde{\sigma}(\mathbf{r}')\,\mathbf{E}^b(\mathbf{r}'),\ \mathbf{r}' \in D,$$

is the induced current due to background field $\mathbf{E}^b$.

Integrating both sides of equation (5) over elementary cell D_p and assuming that anomalous conductivity is constant within the cell D_p, $\Delta\widetilde{\sigma} = \Delta\widetilde{\sigma}_p$, we find

$$\mathbf{I}_p = \Delta\widetilde{\sigma}_p \sum_{n=1}^{N} \iiint_{D_n} \left[\iiint_{D_p} \widehat{\mathbf{G}}_E(\mathbf{r}' \mid \mathbf{r})\,dv'\right] \cdot \mathbf{j}(\mathbf{r})dv + \mathbf{I}_p^b, \quad (6)$$

where $\mathbf{I}_p^b$ is the integral current in the cell D_p due to background field $\mathbf{E}^b$:

$$\mathbf{I}_p^b = \iiint_{D_p} \mathbf{j}^b(\mathbf{r}')\,dv'.$$

The last equation can be written using matrix notation:

$$\mathbf{I}_D = \widehat{\sigma}\widetilde{\widehat{\mathbf{G}}}_D \mathbf{I}_D + \mathbf{I}_D^b, \quad (7)$$

where $\widehat{\sigma}$ is a $(3N \times 3N)$ diagonal matrix of anomalous conductivities, $\mathbf{I}_D$ and $\mathbf{I}_D^b$ are the vectors of the total and background electric field intensities formed by the x, y and z components of these fields at the centers of the cells D_n of the anomalous domain D. Matrix $\widetilde{\widehat{\mathbf{G}}}_D$ is a transposed matrix of the original linear system (2), $\widehat{\mathbf{G}}_D$, for the vector of electric current $\mathbf{e}_D$ (which justifies the notation we use for $\widetilde{\widehat{\mathbf{G}}}_D$).

Thus, forward electromagnetic modeling based on the IE method is reduced to the solution of the matrix equation (7) for the unknown vector $\mathbf{I}_D$ of integral electric current components inside domain D. We use contraction integral equation (CIE) method of Hursán and Zhdanov (2002) to solve this equation.

NUMERICAL ANALYSIS OF THE ELECTRIC CURRENT DISTRIBUTION INSIDE THE CONDUCTIVE BODY

Consider a model of a prismatic conductive body with a resistivity of 0.1 Ohm-m embedded in a two-layered background (Figure 1). The resistivity contrast between the second layer and the prism is 10^4. The incident field is a vertically propagated plane EM wave at the 25 Hz frequency, containing both the TM and TE modes. We investigated the effect of different vertical discretizations of the prismatic body on the electric current calculations. Three different discretizations were used in the vertical direction: 5, 15, and 25 cells. The discretizations in the x and y directions remained the same: 10 and 20 cells, respectively. The horizontal components of the TE mode electric and magnetic fields obtained using all three discretizations are shown in the left panels of Figures 2 and 3, respectively. The right panels in these figures present the relative errors, ε_{E_y} and ε_{H_x}, in the real (top) and imaginary (bottom) parts of the corresponding components computed as the difference between the field for the finest discretization (25) and the field, obtained for the coarsest discretization (5), normalized by the field at the finest discretization (25):

$$\varepsilon_{E_y} = \frac{E_y^{(25)} - E_y^{(5)}}{E_y^{(25)}},\ \varepsilon_{H_x} = \frac{H_x^{(25)} - H_x^{(5)}}{H_x^{(25)}}.$$

One can see that these errors do not exceed 1.5%.

Figure 1, panel b, presents the vertical distribution of the electric field within the conductive prism computed using different vertical disretizations. To produce these plots, we selected a central vertical column of the cells within the prism for each discretization. The electric field, $\mathbf{E}(\mathbf{r}_n)$, was calculated in the center of each elementary cell from this column according to the following formula:

$$\mathbf{E}(\mathbf{r}_n) = \mathbf{I}_n \,/\, (\Delta\widetilde{\sigma}_n D_n), \quad (8)$$

using the integral electric current, $\mathbf{I}_n$, computed for this cell with the IEC method (where D_n and $\Delta\widetilde{\sigma}_n$ are the volume and the anomalous conductivity of the corresponding elementary cell, respectively). Figure 1, panel b, shows that the electric field computed for the finest discretization (25 cells in the vertical direction) describes well the skin effect within the conductive body, while the field on the coursed discretization of 5 vertical cells is practically insensitive to the skin effect. At the same time, the difference between the observed EM field components at the surface is within just 1.5% (Figures 2 and 3). This remarkable property of the IEC solution is related to the main principle of the IEC method, which is based on computing the integral electric current, $\mathbf{I}_n$, within every cell. In this case the electric field computed according to formula (8) should also describe the averaged electric field within the cell, which corresponds well to the plots shown in Figure 1, panel b. One can see that the plots of the horizontal electric field components for the coarser discretization describe the average values of the same plot for the finer discretization. The plots of the vertical component of the electric field behaves a little bit differently, because the vertical field is 10^3 times smaller than the horizontal fields. Nevertheless, the plots for 15-cell and 25-cell discretizations practically coincide, which is a clear manifestation that we reached the optimal level of discretization at 25 cells in the vertical direction. The solution will not change if we will use the finer discretization. Thus, another important property of the new IEC method is that it does not require a very fine discretization to produce an accurate result, because it does not operate with the discretized electric field but with the integral currents, instead.

COMPARISON BETWEEN THE IEC METHOD AND ANALYTICAL SOLUTION FOR A CONDUCTIVE SPHERE

In order to check the accuracy of the new IEC method, we apply this technique to model a response of the conductive sphere excited by the vertically propagated plane EM wave. This problem represents one of a few EM problems which allow for an analytical solution. The mathematical solution of this problem has been described in several publications (see, for example, March, 1953; Berdichevsky and Zhdanov, 1984; Ward and Hohmann, 1987; Balanis, 1989). This problem is usually solved by means of the Debye potentials. We compare this analytical solution with numerical modeling using the IEC method.

Figure 4, a, shows a model of the conductive sphere with a radius of 50 m embedded in the homogeneous whole space with a background (normal) resistivity of $\rho_n = 1000$ Ohm-m. In the model study we use different resistivities of the sphere: $\rho_d = 100$, 10, 1, 0.1, and 0.01 Ohm-m. The incident field is an E-polarized (TE mode) vertically propagated plane EM wave at a frequency of 25 Hz. The origin of the Cartesian coordinate system is located in the center of the sphere. The receiver profile runs from -410 to 410 m in the x direction at an elevation of 350 m above the center of the sphere. The receivers are located every 20 m. To calculate the sphere response by the IEC method, we approximated the sphere with a model formed by cells with a side of 6.25 m (see Figure 4, b).

Using both the analytical solution and the IEC method, we computed an apparent magnetotelluric (MT) resistivity for a sphere model according to the formula:

$$\rho_a = \frac{1}{\omega\mu_0}\left(\frac{E_y}{H_x}\right)^2.$$

Note that, according to the method of Debye potentials (Berdichevsky and Zhdanov, 1984), the EM field components are represented in the

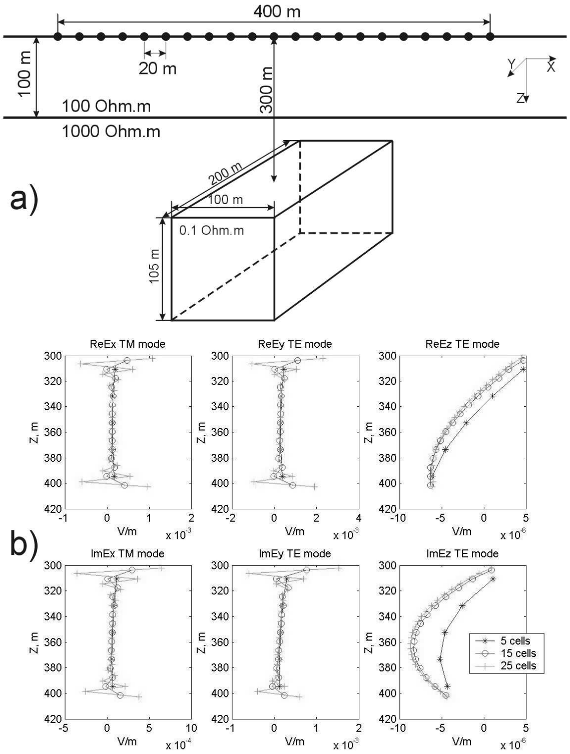

Figure 1: Panel **a** shows a model of a prismatic conductive body with a resistivity of 0.1 Ohm-m embedded in the two-layered background. Panel **b** presents the vertical distribution of the electric field within a conductive prism computed using three different discretizations in the vertical direction: 5 cells (stars), 15 cells (circles), and 25 cells (crosses).

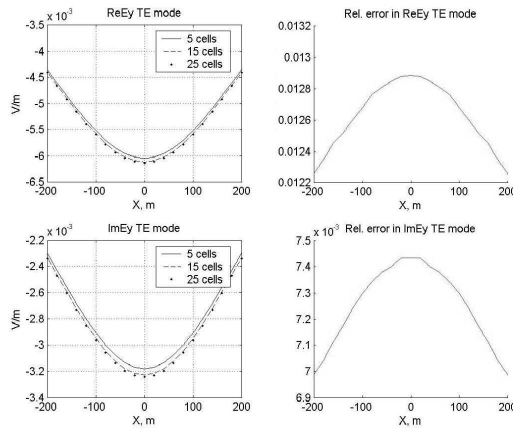

Figure 2: The y component of the electric field, E_y, obtained using three discretizations in the vertical direction, 5, 15, and 25 cells, respectively.

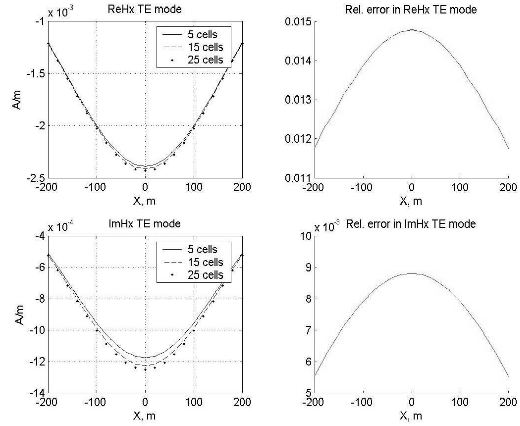

Figure 3: Two left panels show the real part (top) and imaginary part (bottom) of the x component of the magnetic field, H_x, obtained using three discretizations in the vertical direction, 5, 15, and 25 cells, respectively. Two right panels show the relative errors in real (top) and imaginary (bottom) parts of the component.

form of series. Therefore, the result may depend on the number of the terms kept in these series in calculations. However, these series converge extremely fast. Figure 5 represents the plot of the maximum value of the apparent resistivity vs. the number of terms used in the series in analytical calculations for the model with maximum conductivity contrast (1e+5). One can see that the result practically does not change after adding the third term.

Figure 6 shows the plots of the real and imaginary parts of the apparent resistivity, $Re\rho_a$ and $Im\rho_a$, for the different resistivity contrasts between the homogeneous background and the conductive sphere, $c = \rho_n/\rho_d$, equal to 10, 10^2, 10^3 , 10^4, and 10^5, respectively. The solid lines correspond to the analytical solution, while the dashed lines present the numerical IEC results. One can see that the difference between the analytical and the numerical IEC solutions does not exceed 0.15 % at the extremum value of the apparent resistivity for the highest conductivity contrast of 10^5. This result demonstrates that the developed new method of integral current equations produces an accurate result even for the models with high conductivity contrast.

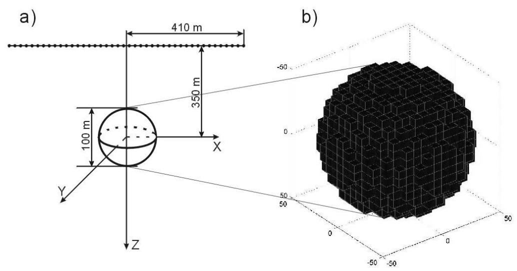

Figure 4: a) Model of a conductive sphere with a radius of 50 m embedded in the homogeneous whole space with a background resistivity of 1000 Ohm-m; b) approximation of the sphere with a model formed by cubic cells with a side of 6.25 m.

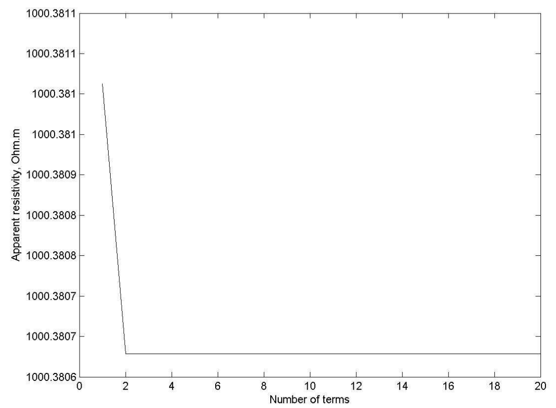

Figure 5: The plot of the maximum value of the apparent resistivity vs. the number of terms used in the series in analytical calculations for the model with maximum conductivity contrast (1e+5).

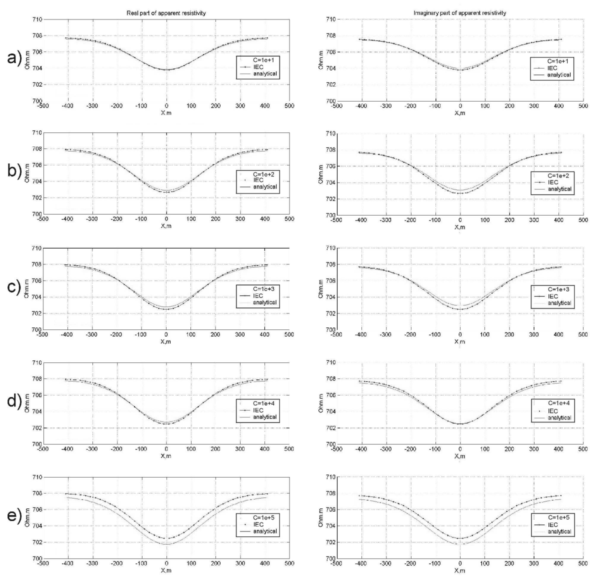

Figure 6: The plots of the real and imaginary parts of the apparent resistivity, $Re\rho_a$ and $Im\rho_a$, computed using the analytical solution and the IEC method. The solid lines show the data obtained by the analytical solution, while the dashed lines present the results of numerical modeling with the IEC method for the following resistivity contrasts, $c = \rho_n/\rho_d$, between the homogeneous background and the conductive sphere : a) $c = 10$, b) $c = 10^2$, c) $c = 10^3$, d) $c = 10^4$, and e) $c = 10^5$.

CONCLUSIONS

For a long time the main limitation of the integral equation method was modeling the EM field for models with high conductivity contrast. In this paper we have developed a new approach to the construction of the IE method. It is based on using integral electric currents, calculated over the elementary cells of the discretization grid, instead of the electric field itself within the cells, as is commonly used in the conventional IE method. As a result the method is capable of modeling the EM response in geoelectrical structures with high contrast of conductivity.

The method was carefully tested. We compared the numerical modeling results with the exact analytical solution for a model of a conductive sphere. Future work will be directed to application of the new method for examining the complex models of geological targets with the large conductivity contrast, typical for mineral exploration.

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