Rigorous 3-D inversion of marine CSEM data based on the integral equation method

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SUMMARY

Marine controlled source electromagnetic (MCSEM) surveys have become an important part of offshore petroleum exploration. In this paper we present a new approach to 3-D inversion of MCSEM data. It is based on rigorous integral equation (IE) forward modeling and a new IE representation of the sensitivity (Frechet derivative matrix) of observed data to variations in sea-bottom conductivity. This approach requires just one forward modeling on every iteration of the regularized gradient type inversion algorithm, which speeds up the computations significantly. We also use a regularized focusing inversion method, which provides a sharp boundary image of the petroleum reservoir. The methodology is tested on a 3-D inversion of the synthetic EM data representing typical MCSEM surveys conducted for offshore petroleum exploration.

INTRODUCTION

During recent years marine controlled source electromagnetic (MCSEM) surveys have become intensively used for off-shore petroleum exploration (Eidesmo et al., 2002; Ellingsrud et al., 2002; Carazzone et al., 2005; Hesthammer and Boulaenko, 2005). The success of the EM method's application for the search of oil and gas reservoirs is based on the fundamental fact that oil- and gas- containing structures are characterized by very high resistivity, while the surrounding sea-bottom formations filled with salt water are very conductive. Therefore, a petroleum reservoir represents a clear target for EM methods. However, the interpretation of MCSEM data is still a very challenging problem, especially if one would like to take into account a real three-dimensional (3-D) structure of a sea-bottom geological formation.

In this paper we present a new method of 3-D inversion of MCSEM data which uses a rigorous integral equation (IE) based forward modeling and regularized focusing inversion algorithm. To obtain a stable solution of a 3-D inverse problem we apply a regularization method based on a focusing stabilizing functional (Zhdanov, 2002). This stabilizer helps generate a sharp and focused image of anomalous conductivity distribution, which is important in petroleum exploration with the goal of delineating the boundaries of the prospective reservoir.

We present in this paper the results of the application of the rigorous inversion method to the inversion of the synthetic MCSEM data.

IE METHOD IN 3-D INVERSION OF MCSEM DATA

We consider, first, the typical MCSEM survey consisting of a set of sea-bottom electrical and magnetic receivers and a horizontal electric dipole transmitter towing at some elevation above the sea bottom. The transmitter generates a frequency domain EM field. The operating frequencies are usually selected to be low enough (in a range of 0.1 - 5 Hz) to propagate through the conductive sea water and sea-bottom layers of the sediments and to illuminate the sea-bottom geological structures. The field recorded by the receivers can be represented as a sum of the normal EM field, $\{\mathbf{E}^{norm}, \mathbf{H}^{norm}\}$, generated in the horizontally layered background model formed by the sea water and the sedimental layers, and an anomalous part, $\{\mathbf{E}^{a}, \mathbf{H}^{a}\}$, related to the horizontal conductivity inhomogeneities, $\Delta\sigma$, present in the sea bottom. The anomalous electric field is related to the electric current induced in the inhomogeneity, $\mathbf{j} = \Delta\sigma\mathbf{E}$, according to the following integral formula:

$$\mathbf{E}^{a}\left(\mathbf{r}_j\right) = \iiint_D \widehat{\mathbf{G}}_E\left(\mathbf{r}_j \mid \mathbf{r}\right) \cdot \left[\Delta\sigma\left(\mathbf{r}\right)\mathbf{E}\left(\mathbf{r}\right)\right] dv, \tag{1}$$

where $\widehat{\mathbf{G}}_E\left(\mathbf{r}_j \mid \mathbf{r}\right)$ is the electric Green's tensor defined for an unbounded conductive medium with the normal (horizontally layered) conductivity σ_{norm}; and domain D represents a volume with the anomalous conductivity distribution $\sigma\left(\mathbf{r}\right) = \sigma_{norm} + \Delta\sigma\left(\mathbf{r}\right)$, $\mathbf{r} \in D$. Simailar equation can be written for magnetic field as well.

We use integral equation (1) to formulate both the forward and inverse problems of the MCSEM method. Indeed, in short form this equation can be written as:

$$\mathbf{d} = A\left(\Delta\sigma\right), \tag{2}$$

where A is a forward modeling operator, $\mathbf{d}$ stands for the observed EM data in the sea-bottom receivers, and $\Delta\sigma$ is a vector formed by the anomalous conductivities within the targeted domain. The inversion is based on minimization of the Tikhonov parametric functional, $P^{\alpha}\left(\Delta\sigma\right)$, with the focusing stabilizer $s\left(\Delta\sigma\right)$ (Zhdanov, 2002):

$$P^{\alpha}\left(\Delta\sigma\right) = \left\|A\left(\Delta\sigma\right) - \mathbf{d}\right\|^2 + \alpha s\left(\Delta\sigma\right). \tag{3}$$

The most common approach to minimization of the parametric functional $P(\Delta\sigma)$ is based on using gradient type methods. For example, the conjugate gradient (CG) algorithm of the parametric functional minimization can be summarized as follows (Zhdanov, 2002):

$$\begin{gathered}
\mathbf{r}_n = A(\Delta\sigma_n) - \mathbf{d}, \quad \mathbf{l}_n = \mathbf{l}\left(\Delta\sigma_n\right) = F^{\star}_{m_n}\mathbf{r}_n + \alpha W^{\star}_m W_m\left(\Delta\sigma_n - \Delta\sigma_{apr}\right), \\
\beta_n = \left\|\mathbf{l}_n\right\|^2 / \left\|\mathbf{l}_{n-1}\right\|^2, \quad \tilde{\mathbf{l}}_n = \mathbf{l}_n + \beta_n\tilde{\mathbf{l}}_{n-1}, \quad \tilde{\mathbf{l}}_0 = \mathbf{l}_0, \\
k_n = \left(\tilde{\mathbf{l}}_n, \ \mathbf{l}_n\right) / \left\{\left\|F_{m_n}\tilde{\mathbf{l}}_n\right\|^2 + \alpha\left\|W_m\tilde{\mathbf{l}}_n\right\|^2\right\}, \\
\Delta\sigma_{n+1} = \Delta\sigma_n - k_n\tilde{\mathbf{l}}_n.
\end{gathered} \tag{4}$$

where W_m is the model parameters weighting matrix, k_n is a length of the iteration step, and $\mathbf{l}_n$ is the gradient direction, computed using the adjoint Fréchet derivative operator, $F^{\star}_{m_n}$.

The last formula demonstrates that every iteration step requires at least one forward modeling solution to find the predicted data, $A(\Delta\sigma_n)$. Additional computations are needed to find the Fréchet derivative, F_{m_n}, and the optimal length of the iteration step k_n.

Thus, the critical element of the inversion is computing the Fréchet derivative of the forward modeling operator. Direct computation of the Fréchet derivative is very time consuming even when the reciprocity principle is utilized. In the current paper we develop a new form of the QA approximation for the models with the variable background conductivity and apply this form for more efficient Fréchet derivative calculations. We use this approach for developing a fast and rigorous method of the MCSEM data inversion.

An advantage of using the QA approximation for forward modeling, as it is mentioned in Zhdanov and Hursán, 2000, is the ability to generate a simple formula for the Fréchet derivative operator which can be used in inversion algorithms. For example, we have derived the following integral representation for the Fréchet derivative of the electric field:

$$\left.\frac{\partial \mathbf{E}(\mathbf{r}_j)}{\partial \Delta\sigma_a(\mathbf{r})}\right|_{\Delta\sigma_n} = \mathbf{F}_E(\mathbf{r}_j \mid \mathbf{r}), \tag{5}$$

where the vector function $\mathbf{F}_E$ is the kernel of the integral Fréchet derivative operators:

$$\mathbf{F}_E(\mathbf{r}_j \mid \mathbf{r}) = \left[\frac{1}{1-g^Q(\mathbf{r})}\widehat{\mathbf{G}}_E(\mathbf{r}_j \mid \mathbf{r}) + \widehat{\mathbf{K}}(\mathbf{r}_j \mid \mathbf{r})\right]\mathbf{E}^{(n)}(\mathbf{r}), \tag{6}$$

and

$$\widehat{\mathbf{K}}(\mathbf{r}_j \mid \mathbf{r}) = \iiint_{D_a} \frac{\Delta\sigma_a(\mathbf{r}')}{(1-g^Q(\mathbf{r}'))^2}\widehat{\mathbf{G}}_E(\mathbf{r}_j \mid \mathbf{r}') \cdot \mathbf{E}^b(\mathbf{r}') \times$$
$$\left[\frac{\mathbf{E}^{b*}(\mathbf{r}')}{\mathbf{E}^b(\mathbf{r}') \cdot \mathbf{E}^{b*}(\mathbf{r}')} \cdot \widehat{\mathbf{G}}_E(\mathbf{r}' \mid \mathbf{r})\right] dv'. \tag{7}$$

Function g^Q is determined by expression:

$$g^Q(\mathbf{r}_j) = \frac{\mathbf{E}^B(\mathbf{r}_j) \cdot \mathbf{E}^{b*}(\mathbf{r}_j)}{\mathbf{E}^b(\mathbf{r}_j) \cdot \mathbf{E}^{b*}(\mathbf{r}_j)}, \tag{8}$$

where $\mathbf{E}^B$ is Born approximation. These formulas show that the Fréchet derivative can be found by direct integration of the expression involving the electric field $\mathbf{E}^{(n)}$ computed on the previous iteration for the anomalous conductivity distribution.

SYNTHETIC MCSEM DATA INVERSION

We have investigated several models of marine CSEM surveys. In the majority of these model studies we have considered a 2-D MCSEM survey, which is currently the most widely used in offshore exploration. The typical 2-D survey consists of a set of receivers located along a line at the sea bottom and of an electric bipole transmitter towed parallel (above) the receivers.

Model 1

We consider a model formed by two resistive reservoirs located at the different depths (Figure 1). The synthetic MCSEM survey consists of seventeen sea-bottom receivers and an electric dipole transmitter moving along a line passing directly above the receivers at an elevation of 50 m above the sea bottom. The separation between the receivers is 1000 m. We assume that transmitter sends a frequency domain EM signal with two frequencies of 0.25 and 0.75 Hz from points located every 200 meters along the transmitter's line. The maximum transmitter - receiver offset is 10 km. The background geoelectrical model consists of a sea-water layer with a thickness of 300 m, a resistivity of 0.25 Ohm-m, and homogeneous seabottom sediments with the resistivity of 1 Ohm-m. We assume that we have two petroleum reservoirs with the same thickness of 100 m and a resistivity of 100 Ohm-m, but located at the different depths of 1350 m and 1150 m respectively below the sea level (Figure 1). The horizontal dimensions of the reservoirs are 2 km and 1 km in the x direction and 2 km in the y direction, respectively.

We have applied the rigorous inversion algorithm described above to the inversion of the synthetic CSEM data computed for this model. The area of inversion was discretized in $16 \times 2 \times 10$ cells in the x, y, and z directions with the cells' sizes equal to 500 m, 1000 m, and 50 m, respectively. Figure 2 shows a vertical section of the inversion result. Figures 3 and 4 present the volume images of the true and inverse models respectively. As one can see, the depth and the horizontal extent of both reservoirs are recovered well in the inverse image.

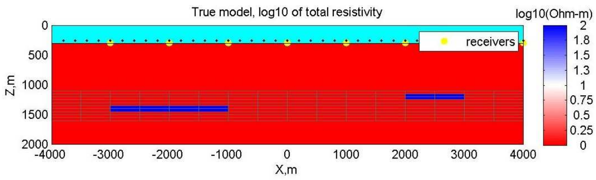

Figure 1: Model 1 formed by two resistive reservoirs located at different depths.

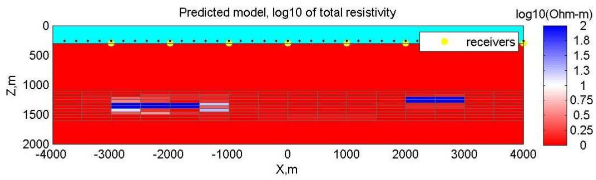

Figure 2: A vertical section of the inversion result for Model 1.

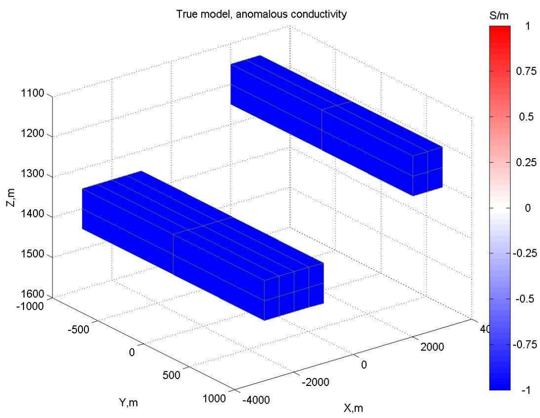

Figure 3: A volume image of true Model 2.

The typical in-line electric field data recorded in receiver #3 for two frequencies are shown in Figures 5 and 6. We plot here the real and imaginary parts of the total electric field E_x normalized by the absolute values of the normal electric field, E_x^{norm}, generated in the horizontally layered background model formed by the sea water and the conductive sea-bottom sediments. One can see rather complex anomalous behavior of the observed field. Nevertheless, the predicted data computed for the inverse model shown in Figures 2 and 4 fit the synthetic observed data extremely well.

Model 2

In the next numerical experiment we consider a 3-D MCSEM survey over a truly 3-D target: a petroleum reservoir in a presence of a salt dome structure. We have used a similar model in our research on marine MT data inversion (Wan and Zhdanov, 2004). Figures 7 and 8 show a plan view and a vertical cross-section of the model. The sea-bottom reservoir is approximated by a thin resistive body located at a depth of 900 m below the sea level, with a thickness of 100 m, and a horizontal size of 800 m $\times$ 800 m. The resistivity of the reservoir

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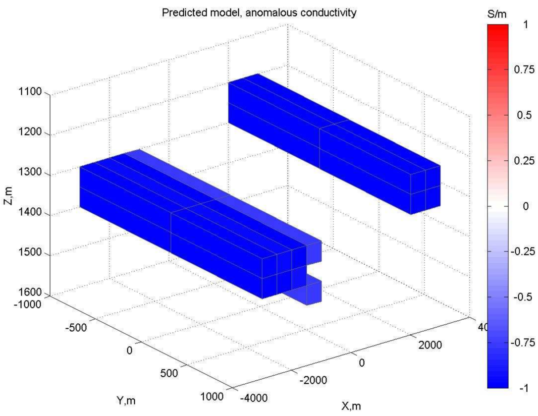

Figure 4: A volume image of the inversion result for Model 2.

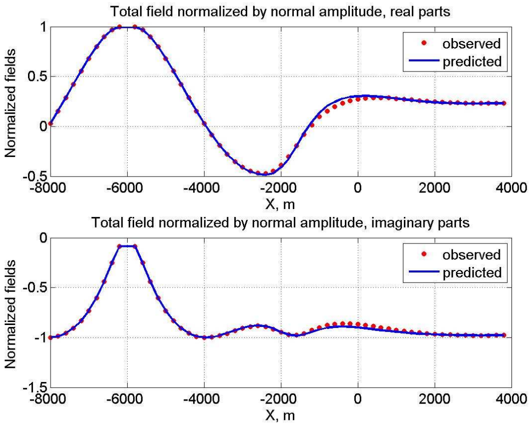

Figure 5: Model 2. The plots of the real (top panel) and imaginary (bottom panel) parts of the normalized observed and predicted in-line electric fields E_x at a frequency of 0.25 Hz for receiver #3.

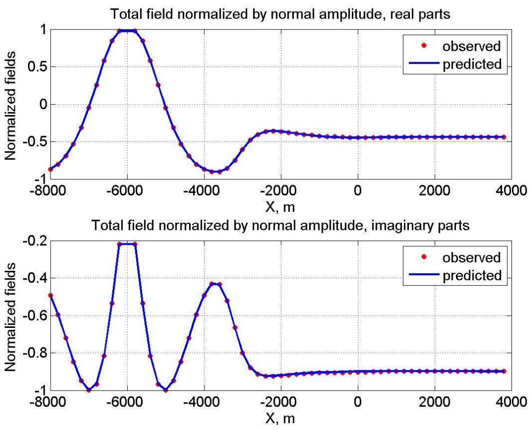

Figure 6: Model 2. The plots of the real (top panel) and imaginary (bottom panel) parts of the normalized observed and predicted in-line electric fields E_x at a frequency of 0.75 Hz for receiver #3.

is 100 Ohm-m. There is located, also, an irregular shaped salt dome structure close to the reservoir, at a depth of 700 m below the sea level. The resistivity of a salt dome is 30 Ohm-m. The depth of the sea bottom is 500 m from the surface, and the sea water resistivity is assumed to be equal to 0.25 Ohm-m. A 3-D image of the true model is shown in Figure 9.

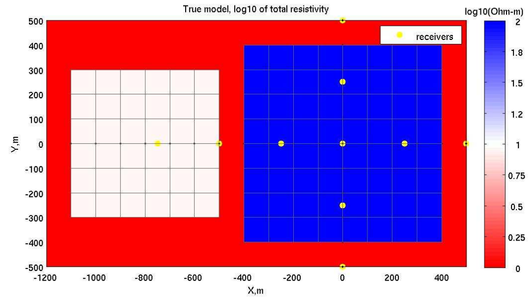

Figure 7: Model 2: a petroleum reservoir in a presence of a salt dome structure (plan view).

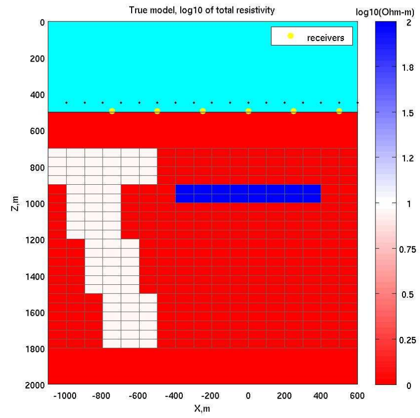

Figure 8: Model 2: a petroleum reservoir in a presence of a salt dome structure (vertical section).

A synthetic MCSEM survey consists of fourteen sea-bottom receivers and an electric dipole transmitter moving along two mutually orthogonal lines at an elevation of 50 m above the sea bottom. The positions of the receivers are shown by yellow dots in Figure 8. The separation between the receivers is 250 m. The transmitter's locations are shown by diamonds in the same figure. The transmitter sends a frequency domain EM signal with three frequencies of 0.1, 1, and 10 Hz from points located every 100 meters along the transmitter's line. The receivers measure the in-line components of the electric field only. The observed data is contaminated by 5% random noise.

We discretize the inversion domain into 1320 prizmatic cells. The cell dimensions are 100 m horizontally and 50 m vertically. After 45 iterations we have arrived at the inverse model shown in Figures 10 and 11. Note that in this inversion we have used an approach based on the inhomogeneous background conductivity (Zhdanov and Wilson, 2004). We have assumed that the position of the salt dome is known, and we have included a salt dome in the inhomogeneous background. One can see that, the depth and the horizontal extent of the reservoir and its resistivity are recovered well in the inverse image.

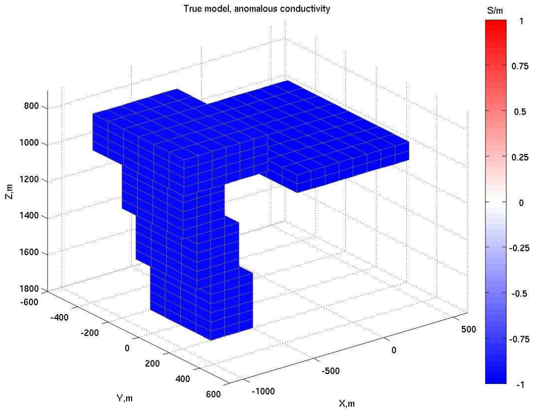

Figure 9: A 3-D image of true Model 2.

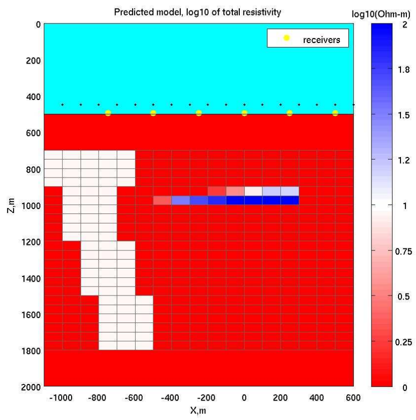

Figure 10: A vertical section of the inversion result for Model 2.

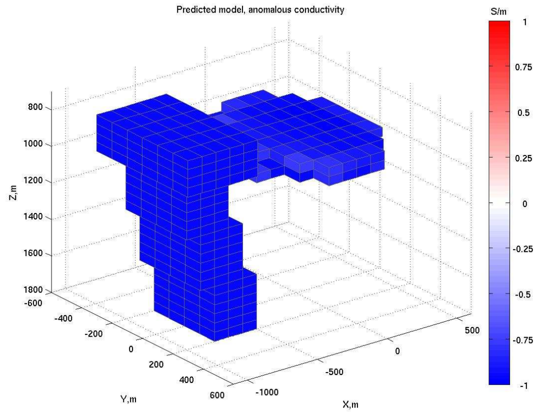

Figure 11: A 3-D image of the inversion result for Model 2.

CONCLUSIONS

We have developed a rigorous method for 3-D inversion of MCSEM data based on the integral equation formulation. This method can be used for interpretation of real MCSEM data collected for offshore petroleum exploration. We have tested this method on simple synthetic MCSEM surveys, simulating the typical transmitter-receiver layouts which are currently used by the EM acquisition companies. The results of these tests demonstrate that the inverse images generated by this method provide a reasonable reconstruction of the true location and the resistivity of the targets.

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