# Modeling large-scale geoelectrical structures with inhomogeneous backgrounds using the integral equation method: applications to the bathymetry effect study in marine CSEM data

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## Summary

We apply a new formulation of the integral equation (IE) method with inhomogeneous background conductivity (IBC) for modeling marine controlled source electromagnetic (MCSEM) data in areas with significant bathymetric inhomogeneities. We also introduce an approach for accuracy control of the IBC IE method. This new approach provides us with the ability to improve the accuracy of computations by applying the IBC technique iteratively. This approach seems to be extremely useful in computing EM data for multiple geological models with some common geoelectrical features, like terrain, bathymetry, or other known structures. It may find wide application in inverse problem solution, where we have to keep some known geological structures unchanged during the iterative inversion. The effectiveness of this approach is illustrated by modeling marine controlled source electromagnetic (MCSEM) data in model with significant bathymetric inhomogeneities.

## Introduction

The problem of repeated calculation of the EM fields for large-scale geoelectrical models with variable background conductivity arises in the solution of many practical problems of EM geophysics. Until recently this problem could be solved only by using the finite-difference (FD) or finite element (FE) methods. The integral equation (IE) approach with its requirements for a simple horizontally layered reference (background) model was not considered to be an appropriate tool for studying the EM field in large-scale inhomogeneous models. However, it has been demonstrated in the paper by Zhdanov and Lee (2005) that the traditional formulation of the IE method for models with horizontally layered background conductivity can be extended to the more general case of models with inhomogeneous background conductivity (IBC). One possible application of this extension is computing EM data for multiple geological models with some common geoelectrical features, like a known inhomogeneous overburden, bathymetry, or salt dome structures. This technique is also useful in inverse problem solution, where we have to keep some known geological structures unchanged during the process of iterative inversion. In the current paper we will investigate the effectiveness of this approach for modeling marine controlled source electromagnetic (MCSEM) data in areas with significant bathymetric inhomogeneities.

Variations in the sea floor depth could have a profound effect on observed sea-bottom EM data, because the receivers of the EM field are located in close proximity to the sea bottom. That is why the bathymetry effects in marine CSEM data should be carefully taken into account in interpretation of the observed data. The accurate simulation of the EM field caused by bathymetric inhomogeneities is a challenging numerical problem, because it requires a huge number of discretization cells in order to represent the bathymetric structures properly. In the framework of the IBC approach to the construction of the IE method, one can precompute the bathymetry effect only once and keep it unchanged during the entire modeling and/or inversion process. Taking into account that precomputing the bathymetry effect constitutes the most time-consuming part of the forward EM modeling, this approach would allow us to increase the effectiveness of the computer simulation in the interpretation of the MCSEM data significantly.

We should note, however, that the true complexity of the sea floor structure may require using hundreds of thousands and even millions of discretization cells for accurate numerical representation on a discrete grid. The corresponding large-scale forward EM problem can be solved only by application of a parallel version of the IE forward modeling code designed for a PC cluster (Yoshioka and Zhdanov 2005). That is why we have incorporated the IBC approach into the CEMI parallel IE code PIE3D. In the current paper we demonstrate the effectiveness of this technique and the corresponding new CEMI software for modeling large-scale geoelectrical structures with inhomogeneous backgrounds, which is a typical problem in studying the bathymetry effects in marine CSEM data.

## Principles of the IBC IE method

In the framework of the IBC method, the total conductivity in the model is represented as a sum of the horizontally layered (normal) conductivity, $\sigma_n$; the inhomogeneous background conductivity, $\sigma_b = \sigma_n + \Delta\sigma_b$ ($\Delta\sigma_b \neq 0$ within some domain $D_b$); and the anomalous conductivity, $\Delta\sigma_a$, ($\Delta\sigma_a \neq 0$ within some domain $D_a$).

We represent the frequency domain EM field in this model as a sum of: 1) the normal fields, $\mathbf{E}^n$, $\mathbf{H}^n$, generated by the given source(s) in the model with normal distribution of conductivity, $\sigma_n$; 2) a variable background effect, $\mathbf{E}^{\Delta\sigma_b}$, $\mathbf{H}^{\Delta\sigma_b}$, produced by the inhomogeneous background conductivity, $\Delta\sigma_b$; 3) and the anomalous fields, $\mathbf{E}^{\Delta\sigma_a}$, $\mathbf{H}^{\Delta\sigma_a}$, due to the anomalous conductivity distribution, $\Delta\sigma_a$. The background EM field, $\mathbf{E}^b$, $\mathbf{H}^b$ in this case is a sum of the normal fields and those caused by the inhomogeneous background conductivity:

$$\mathbf{E}^b = \mathbf{E}^n + \mathbf{E}^{\Delta\sigma_b},\ \mathbf{H}^b = \mathbf{H}^n + \mathbf{H}^{\Delta\sigma_b}. \tag{1}$$

Using these notations, the integral equation for the anomalous electric field inside domain $D_a$ can be written in the form:

$$\mathbf{E}^{\Delta\sigma_a}(\mathbf{r}_j) = \mathbf{G}_E^{D_a}\left(\Delta\sigma_a\left(\mathbf{E}^b + \mathbf{E}^{\Delta\sigma_a}\right)\right),\ \mathbf{r}_j \in D_a, \tag{2}$$

where the symbol $\mathbf{G}_E^{D_a}$ denotes the electric Green's operator with a volume integration of $D_a$.

We can find the background electric field from a rigorous integral equation similar to equation (2) but written for domain $D_b$:

$$\mathbf{E}^{\Delta\sigma_b} = \mathbf{E}^b(\mathbf{r}_j) - \mathbf{E}^n(\mathbf{r}_j) = \mathbf{G}_E^{D_b}\left(\Delta\sigma_b\left(\mathbf{E}^b + \mathbf{E}^{\Delta\sigma_a}\right)\right). \tag{3}$$

The basic idea of the IBC formulation of the IE method is that we can use some approximation for this background field to simplify the forward modeling solution (Zhdanov and Lee, 2005). For example, we can neglect the return induction effects from the scatterer (anomalous domain) to the inhomogeneous background. In this case, equation (3) is reduced to the following integral equation for the background electric field inside the domain of integration, $D_b$ :

$$\mathbf{E}^b(\mathbf{r}_j) = \mathbf{G}_E^{D_b}\left(\Delta\sigma_b\mathbf{E}^b\right) + \mathbf{E}^n(\mathbf{r}_j),\ \mathbf{r}_j \in D_b. \tag{4}$$

Thus, the forward EM modeling problem is now reduced to a separate solution of two systems of integral equations: one, equation (4), for the background electric field $\mathbf{E}^b$ due to inhomogeneous background conductivity, and another, equation (2), for the anomalous electric field $\mathbf{E}^{\Delta\sigma_a}$. We use the Contraction Integral Equation (CIE) method of Hursán and Zhdanov (2002) as a main algorithm for the solution of the corresponding EM field integral equations (4) and (2). This algorithm ensures the fast convergence of the corresponding system iterations.

## Accuracy control of the IBC IE method

The main assumption of the IBC IE method is that this returned induction field is very small compared with the normal field and an anomalous part of the background field induced in the background conductivity. The obvious condition where this approximation can be employed is that the effect of the induced field $\mathbf{G}_E^{D_b}\left(\Delta\sigma_b\left(\mathbf{E}^{\Delta\sigma_a}\right)\right)$ in inhomogeneous background from the anomalous body is much smaller than the effect of the background field itself inside the domain of integration, $D_b$ :

$$\left\|\mathbf{E}^b - \mathbf{G}_E^{D_b}\left(\Delta\sigma_b\left(\mathbf{E}^b + \mathbf{E}^{\Delta\sigma_a}\right)\right) - \mathbf{E}^n\right\|_{D_b} / \left\|\mathbf{E}^b\right\|_{D_b}, \tag{5}$$

where $\|...\|_{D_b}$ denotes the $L_2$ norm calculated over domain $D_b$ :

$$\left\|\mathbf{E}^b\right\|_{D_b}^2 = \int_{D_b} |\mathbf{E}(\mathbf{r})|^2\, dv.$$

We can also evaluate the possible errors of ignoring the return response of the currents induced in the inhomogeneous background on the field in the anomalous domain $D_a$ :

$$\left\|\mathbf{E}^a - \mathbf{G}_E^{D_a}\left(\Delta\sigma_a\left(\mathbf{E}^a + \mathbf{E}^{\Delta\sigma_b(1)}\right)\right) - \mathbf{E}^n\right\|_{D_a} / \left\|\mathbf{E}^a\right\|_{D_a}, \tag{6}$$

where:

$$\mathbf{E}^{\Delta\sigma_b(1)}(\mathbf{r}_j) = \mathbf{G}_E^{D_b}\left(\Delta\sigma_b\left(\mathbf{E}^b + \mathbf{E}^{\Delta\sigma_a}\right)\right),\ \mathbf{r}_j \in D_a,$$

and:

$$\mathbf{E}^a = \mathbf{E}^n + \mathbf{E}^{\Delta\sigma_a}$$

Condition (5) makes it possible to evaluate the accuracy of the IBC IE method.

The remarkable fact is that the above condition not only provides us with the ability to control the accuracy of our computations, but it also shows how to improve the accuracy by applying the IBC technique iteratively. Indeed, if we find the error $\varepsilon_1^b$ is too large, we can solve the rigorous integral equation (3) for the background electric field, taking into account the anomalous field $\mathbf{E}^{\Delta\sigma_a}$ computed in the previous step. We denote by $\mathbf{E}^{b(2)}$ a new solution of equation (3). Now we can use this updated background field $\mathbf{E}^{b(2)}$ in integral equation (2) for the anomalous field.

A solution of that equation gives us a second iteration of the anomalous electric field, $\mathbf{E}^{\Delta\sigma_a(2)}$.

The iterative process described above is continued until we reach the required accuracy of the background field calculations in both domains, $D_a$ and $D_b$. This iterative process always converges because we use the contraction form of the corresponding integral equations in our computations (Hursán and Zhdanov, 2002).

## Application of the IBC IE method to study of the bathymetry effects in marine CSEM data

The IBC IE method was implemented in a new version of parallel integral equation PIE3D code (Yoshioka and Zhdanov, 2005).

We present the application of this new version of PIE3D code for computer simulation of the bathymetry effects in the marine CSEM data. We consider a theoretical model of a complex hydrocarbon reservoir in the presence of the bathymetry.

A vertical section of the geoelectrical model is shown in Figure 1. One can see in this figure a resistive structure of a hydrocarbon reservoir located within the conductive sea-bottom sediments. The reservoir has a complex 3-D geometry and contains three layers: a water-filled layer with a resistivity of 0.5 Ohm-m, a gas-filled layer with a resistivity of 1,000 Ohm-m, and an oil-filled layer with a resistivity of 100 Ohm-m, as shown in Figure 2. The resistivity of the seawater layer is 0.25 Ohm-m, and the depth

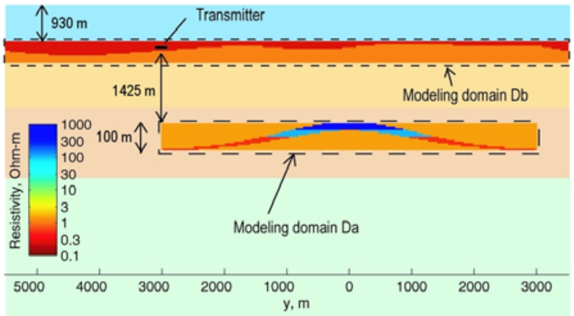


Fig. 1: A vertical section of a geoelectrical model of a hydrocarbon reservoir in the presence of rough seafloor bathymetry.

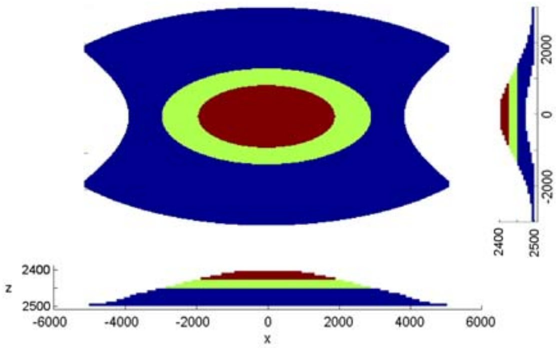

Fig. 2: Detailed plan and side views of the hydrocarbon reservoir located within the conductive sea-bottom sediments.

of the sea floor varies from 900 m to 1,200 m below the sea level. Figure 3 shows a 3-D relief of the bathymetry

The EM field in the model is excited by an electric horizontal bipole oriented in the $y$ direction and located at the point with horizontal coordinates $x = -2000\ m$ and $y = -3,000\ m$. The elevation of the transmitter bipole is 50 m above the sea bottom. The transmitter generates the frequency domain EM field at a frequency of 0.25 Hz.

In our numerical study, following the general principles of the IBC IE method, we have included geoelectrical inhomogeneities associated with the sea-bottom bathymetry in the inhomogeneous background conductivity $\Delta\sigma_b$, while the hydrocarbon reservoir structure is described by the anomalous conductivity distribution $\Delta\sigma_a$. We have introduced two modeling domains, $D_a$ and $D_b$, accordingly. These domains are outlined by the black dashed lines in Figure 1. Modeling domain $D_b$ covers the area with conductivity variations associated with the seafloor structure, while modeling domain $D_a$ corresponds to the location of the reservoir. We use 2.2 million cubic cells (441 x 361 x 14) with each cell size 25 x 25 x 25 $m^3$ to represent accurately the bathymetry structure of the model. The domain $D_a$ of the hydrocarbon reservoir is discretized in 1.5 million (401 x 241 x 16) cells of the same size. Obviously in this case we have to deal with a really massive numerical problem. The solution of this problem by any conventional IE method would require

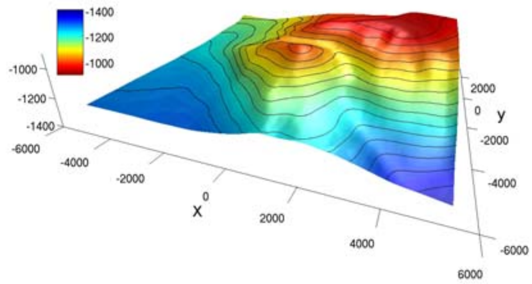

Fig. 3: A 3-D relief of the bathymetry.

the simultaneous solution of the corresponding system of integral equations on a grid formed by at least a combination of two domains, $D_a$ and $D_b$, which have together 3.7 million cells. At the same time, the application of the IBC IE method allows us to separate the modeling domain into two subdomains, $D_a$ and $D_b$, with relatively smaller numbers of cells. We solve the corresponding IE of the IBC method in these two domains separately, which saves a lot of computer memory and computational time. However, what is even more important, by including the bathymetry structure in the inhomogeneous background conductivity, we can save the precomputed background field and keep it unchanged for future computations with modified parameters of the reservoir. This results in an enormous reduction of the computing cost in interpretation of the practical EM field data and in the inverse problem solution.

We have applied the IBC IE method to modeling the electric field in a system of sea-bottom receivers located on the rectangular grid with a separation between the receivers of 100 m in both the x and y directions. For comparison, we have also used an iterative version of the IBC method to calculate the electric fields for the same model. The convergence plot for iterative IBC modeling is shown in Figure 4. One can observe a very good convergence rate in this figure. After just two iterations the relative errors reach about $9.8 \times 10^{-9}$ and $8.9 \times 10^{-9}$ within the anomalous (reservoir) and inhomogeneous background (bathymetry) domains, respectively.

Figures 5 and 6 present the maps of the absolute values of the $x$ component of the electric field computed using the IBC and iterative IBC, respectively. Direct comparison between these two sets of maps shows an excellent correlation in field values.

## Discussion and conclusions

In this paper we have developed a parallel implementation of the new integral equation (IE) method with in-

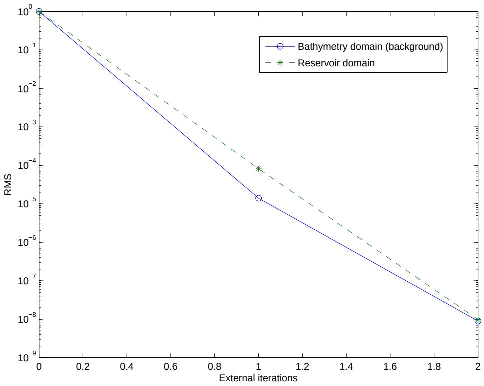


Fig. 4: Model 1. The convergence plot for iterative IBC modeling. The solid (blue) line shows the relative errors vs. iteration number for the inhomogeneous background (bathymetry) domain, while the dashed (green) line presents the same curve for the anomalous (reservoir) modeling domain.

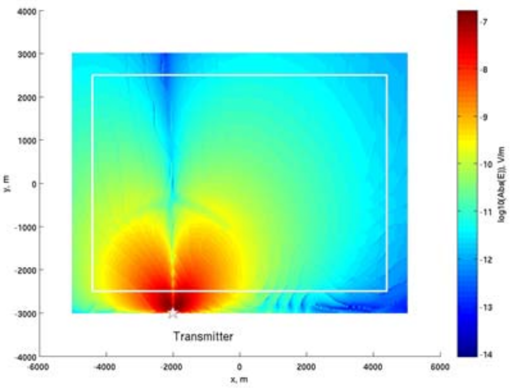


Fig. 5: The map of the absolute values of the $x$-component of the electric field computed using the IBC method. The yellow contour shows the location of the reservoir domain.

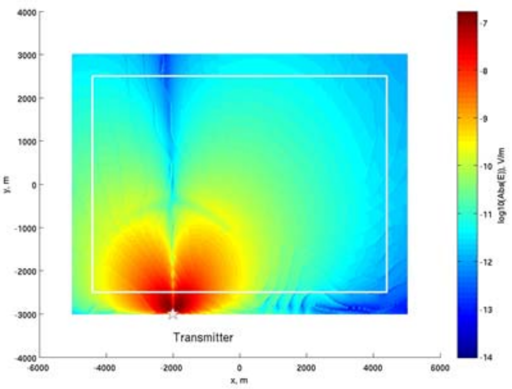


Fig. 6: The map of the absolute values of the $x$-component of the electric field computed using the iterative IBC method. The yellow contour shows the location of the reservoir domain.

homogeneous background conductivity (IBC). We have also introduced an approach to the accuracy control of the IBC IE method. This new approach provides us with the ability to improve the accuracy of computations by applying the IBC technique iteratively.

We have applied a new parallel code based on the IBC IE method for modeling the marine controlled-source electromagnetic (MCSEM) data in the areas with significant bathymetric inhomogeneities. Application of the IBC EM method of numerical modeling allows us to separate this massive computational problem into at least two problems, with relatively smaller sizes.

Another advantage of the IBC IE method, which is even more important in practical applications, is related to the fact that interpretation of the field data usually requires multiple solutions of the forward problem with different parameters of the target (in our examples, a salt dome structure or a sea-bottom hydrocarbon reservoir). Using the IBC approach, we can precompute the bathymetry effect only once, and then repeat the computations on a smaller grid covering the anomalous domain only. A similar advantage will take place for a rigorous inversion algorithms based on IE methods.

## Acknowledgments

The authors acknowledge the support of the University of Utah Consortium for Electromagnetic Modeling and Inversion (CEMI), which includes BAE Systems, Baker Atlas Logging Services, BGP China National Petroleum Corporation, BHP Billiton World Exploration Inc., Centre for Integrated Petroleum Research, EMGS, ENI S.p.A., ExxonMobil Upstream Research Company, INCO Exploration, Information Systems Laboratories, MTEM, Newmont Mining Co., Norsk Hydro, OHM, Petrobras, Rio Tinto - Kennecott, Rocksource, Schlumberger, Shell International Exploration and Production Inc., Statoil, Sumitomo Metal Mining Co., and Zonge Engineering and Research Organization.

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