Generalized effective-medium theory of the complex resistivity of multi-phase heterogeneous rocks

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SUMMARY

In this paper, I introduce a new mathematical model of the IP effect based on the effective-medium theory, which provides a unified mathematical method to study heterogeneity, multi-phase structure, and polarizability of rocks. The geoelectrical parameters of a new composite conductivity model are determined by the intrinsic petrophysical and geometrical characteristics of composite media: mineralization and/or fluid content of rocks, matrix composition, porosity, anisotropy, and polarizability of formations. The new model of multi-phase conductive media can provide a quantitative tool for evaluation of the type of mineralization, the volume content of different minerals, and/or hydrocarbon saturation, using electromagnetic data.

INTRODUCTION

The electromagnetic data observed in geophysical experiments, in general, reflect two phenomena: 1) electromagnetic induction (EMI) in the earth, and 2) induced polarization (IP) effect related to the relaxation of polarized charges in rock formations. The induced polarization (IP) effect is caused by the complex electrochemical reactions that accompany current flow in the earth. These reactions take place in a heterogeneous medium representing the rock formations in the areas of mineralization.

It is well known that the effective conductivity of rocks is not necessarily a constant and real number but may vary with frequency and be complex. There are several explanations for these properties of effective conductivity. Most often they are explained by the physical-chemical polarization effects of mineralized particles of the rock material, and/or by the electrokinetic effects in the poroses of reservoirs (Wait, 1959; Marshall and Madden, 1959; Luo and Zhang, 1998). This phenomenon is usually explained as a surface polarization of the mineralized particles and the surface of the moisture-porous space, which occurs under the influence of the external electromagnetic field. It is manifested by accumulating electric charges on the surface of different grains forming the rock.

In this paper, I introduce a new composite geoelectrical model of rock formations based on the effective-medium approach, which generates a conductivity model with parameters directly related by analytical expressions to the physical characteristics of the microstructure of rocks and minerals (micro geometry and conductivity parameters). A new composite geoelectrical model provides more realistic representation of the complex rock formations than conventional unimodal conductivity models. It allows us to model the relationships between the physical characteristics of different types of rocks and minerals (e.g. conductivities, grain sizes, porosity, anisotropy, and polarizability) and the parameters of the relaxation model.

Effective medium approximation for composite media has been discussed in many publications (e. g. Norris et al., 1985; Kolundzija and Djordjevic, 2002). However, the existing form of EMT does not allow including the IP effect in the general model of the heterogeneous rocks, In this paper I demonstrate that the EMT formalism can be used in the theory of induced polarization (IP) effect as well. This new theory allows us to develop a unified physical - mathematical model which can be used for examining the EM effects in the complex rock formations with different mineral structures and electrical properties. It takes into account the mineralization and/or fluid content of the rocks, the matrix composition, porosity, anisotropy, and polarizability of the formations.

PRINCIPLES OF THE EFFECTIVE-MEDIUM APPROACH

We represent a complex heterogeneous rock formation as a composite model formed by a homogeneous host medium of a volume V with a (complex) conductivity tensor $\widehat{\sigma}_0(\mathbf{r})$ (where $\mathbf{r}$ is an observation point) filled with grains of arbitrary shape and conductivity. A typical example of a multi-phase model of the rock is shown in Figure 1.

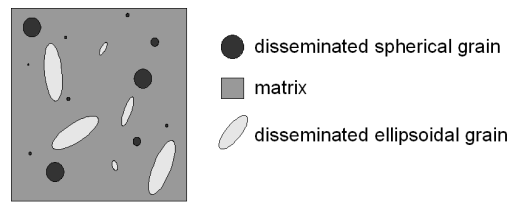

Figure 1: A typical example of a multi-phase model of the rock composed of a set of different types of randomly oriented grains.

In the present problem, the rock is composed of a set of N different types of grains, the lth grain type having (complex) tensor conductivity $\widehat{\sigma}_l$. The grains of the lth type have a volume fraction f_l in the medium and a particular shape and orientation. Therefore, the total conductivity tensor of the model, $\widehat{\sigma}(\mathbf{r})$, has the following distribution for volume fraction f_l and volume fraction $f_0 = \left(1-\sum_{l=1}^{N} f_l\right)$, respectively:

$$\widehat{\sigma}(\mathbf{r}) = \begin{cases} \widehat{\sigma}_0 \text{ for volume fraction } f_0 = \left(1-\sum_{l=1}^{N} f_l\right) \\ \widehat{\sigma}_l \text{ for volume fraction } f_l\,. \end{cases} \tag{1}$$

The polarizability effect is usually associated with surface polarization of the coatings of the grains. The surface polarization is manifested by accumulating electric charges on the surface of the grain. A double layer of charges is created on the grain's surface, which results in the voltage drop at this surface (Wait, 1982). It has been shown experimentally that for relatively small external electric fields used in electrical exploration, the voltage drop, Δu, is linear proportional to the normal current flow at the surface of the particle, $j_n = (\mathbf{n}\cdot\mathbf{j})$. That is, at the surface of the grain we have

$$\Delta u = k\,(\mathbf{n}\cdot\mathbf{j})\,, \tag{2}$$

where $\mathbf{n}$ is a unit vector of the outer normal to the grain's surface, and k is a surface polarizability factor, which, in general, is a complex frequency dependent function. This function is usually treated as the interface impedance which characterizes the boundary between the corresponding grain and surrounding host medium and describes the interfacial or membrane polarization.

Following the standard logic of the EMT, we substitute a homogeneous effective medium with the conductivity tensor $\widehat{\sigma}_e$ for the original heterogeneous composite model and subject it to a constant electric field, $\mathbf{E}^b$, equal to the average electric field in the original model:

$$\mathbf{E}^b = \langle\mathbf{E}\rangle = V^{-1}\iiint_V \mathbf{E}(\mathbf{r})\,dv. \tag{3}$$

The effective conductivity is defined from the condition that the current density distribution $\mathbf{j}_e$ in an effective medium is equal to the average current density distribution in the original model:

$$\mathbf{j}_e = \widehat{\sigma}_e\cdot\mathbf{E}^b = \widehat{\sigma}_e\cdot\langle\mathbf{E}\rangle = \langle\widehat{\sigma}\cdot\mathbf{E}\rangle\,. \tag{4}$$

In order to find the effective conductivity tensor $\widehat{\sigma}_e$, we represent the given inhomogeneous composite model as a superposition of a homogeneous infinite background medium with the conductivity tensor $\widehat{\sigma}_b$ and the anomalous conductivity $\Delta\widehat{\sigma}(\mathbf{r})$:

$$\widehat{\sigma}(\mathbf{r}) = \widehat{\sigma}_b + \Delta\widehat{\sigma}(\mathbf{r}). \tag{5}$$

From (5) and (4), we have:

$$\widehat{\sigma}_e \cdot \mathbf{E}^b = \widehat{\sigma}_b \cdot \mathbf{E}^b + \langle \Delta\widehat{\sigma} \cdot \mathbf{E} \rangle. \tag{6}$$

Thus, we can see that the effective conductivity tensor, $\widehat{\sigma}_e$, can be found from equation (6), if one determines the average excess electric current $\langle \Delta\widehat{\sigma} \cdot \mathbf{E} \rangle$. The last problem can be solved using the integral form of Maxwell's equations.

Following the ideas of the QL approximation (Zhdanov, 2002), we can represent the electric field as follows:

$$\widehat{\mathbf{m}}(\mathbf{r}') = \Delta\widehat{\sigma}(\mathbf{r}') \cdot \left(\widehat{\mathbf{I}} + \widehat{\lambda}(\mathbf{r}')\right). \tag{7}$$

where $\widehat{\mathbf{m}}(\mathbf{r}')$ is material property tensor.

Note that exact representation (**??**) always exists because the corresponding material property tensor can always be found for any given fields $\mathbf{E}(\mathbf{r}')$ and $\mathbf{E}^b$ (Zhdanov, 2002).

Let us substitute (**??**) into (4), taking into account (5):

$$\mathbf{j}_e = \widehat{\sigma}_e \cdot \mathbf{E}^b = \widehat{\sigma}_b \cdot \mathbf{E}^b + \langle \widehat{\mathbf{m}} \rangle \cdot \mathbf{E}^b.$$

From the last formula we see that:

$$\widehat{\sigma}_e = \widehat{\sigma}_b + \langle \widehat{\mathbf{m}} \rangle. \tag{8}$$

Thus, in order to determine the effective conductivity of the composite polarized medium, we have to find the average value of the material property tensor, $\langle \widehat{\mathbf{m}} \rangle$.

INTEGRAL REPRESENTATIONS FOR THE EM FIELD IN HETEROGENOUS POLARIZABLE MEDIA

One can represent the electric field $\mathbf{E}(\mathbf{r})$ generated in a homogeneous anisotropic background medium by the currents induced within the anomalous conductivity $\Delta\widehat{\sigma}(\mathbf{r})$ using integral form of the Maxwell's equations:

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}^b + \iiint_V \widehat{\mathbf{G}}_b(\mathbf{r} \mid \mathbf{r}') \cdot \left[\Delta\widehat{\sigma}(\mathbf{r}') \cdot \mathbf{E}(\mathbf{r}')\right] dv', \tag{9}$$

where V is the volume occupied by all inhomogeneities, and $\widehat{\mathbf{G}}_b(\mathbf{r} \mid \mathbf{r}')$ is a Green's tensor for the homogeneous anisotropic full space.

We assume, however, that in addition to electrical heterogeneity, the medium is characterized by polarizability effects which are manifested by the surface polarization of the grains. Mathematically, the surface polarization effect can be included in the general system of Maxwell's equations by adding the following boundary conditions on the surfaces S_l of the grains (Luo and Zhang, 1998):

$$\left[\mathbf{n} \times \left(\mathbf{E}^+(\mathbf{r}') - \mathbf{E}^-(\mathbf{r}')\right)\right]_{S_l} = -\left[\mathbf{n} \times \nabla' \Delta u(\mathbf{r}')\right]_{S_l}, \tag{10}$$

where $\mathbf{E}^+$ designates the boundary value of electric field $\mathbf{E}(\mathbf{r})$ when the observation point tends to the boundary S_l of the lth grain from the inside of the grains, and $\mathbf{E}^-$ if this point tends to the boundary from the outside of the grains.

According to (2), we assume that the voltage drop at the surface of the grain is proportional to the normal current:

$$\Delta u = k\left(\mathbf{n}(\mathbf{r}') \cdot \mathbf{j}(\mathbf{r}')\right) = k\left(\mathbf{n}(\mathbf{r}') \cdot \widehat{\sigma}(\mathbf{r}') \cdot \mathbf{E}(\mathbf{r}')\right), \tag{11}$$

where current $\mathbf{j}(\mathbf{r}')$ is taken for the internal side of the grain's surface.

Therefore, electric field due to the surface polarization effect $\mathbf{E}^p(\mathbf{r})$ can be represented as an electric field of a specified discontinuity (10) (Zhdanov, 1988):

$$\mathbf{E}^p(\mathbf{r}) = \iint_S \widehat{\mathbf{G}}_b(\mathbf{r} \mid \mathbf{r}') \cdot \mathbf{n}(\mathbf{r}') k\sigma_b \left(\mathbf{n}(\mathbf{r}') \cdot \widehat{\sigma}(\mathbf{r}') \cdot \mathbf{E}(\mathbf{r}')\right) ds'. \tag{12}$$

where S stands for the superposition of all surfaces S_l of the entire ensemble of grains, $S = l = 1\overset{N}{\cup} S_l$, and vector $\mathbf{n}(\mathbf{r}')$ is directed outside the grains.

Substituting expression (**??**) into formula (**??**), we can find the total electric field caused by the effects of both the electromagnetic induction and induced polarization:

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}^b + \iiint_V \widehat{\mathbf{G}}_b(\mathbf{r} \mid \mathbf{r}') \cdot \left[\widehat{\mathbf{m}}(\mathbf{r}') \cdot \mathbf{E}^b\right] dv' +$$
$$\iint_S \widehat{\mathbf{G}}_b(\mathbf{r} \mid \mathbf{r}') \cdot \mathbf{n}(\mathbf{r}') \left(\mathbf{n}(\mathbf{r}') \cdot \widehat{\xi}(\mathbf{r}') \cdot \left[\widehat{\mathbf{m}}(\mathbf{r}') \cdot \mathbf{E}^b\right]\right) ds', \tag{13}$$

where $\widehat{\xi}(\mathbf{r}')$ is equal to:

$$\widehat{\xi}(\mathbf{r}') = k\sigma_b \widehat{\sigma}(\mathbf{r}') \cdot \left(\Delta\widehat{\sigma}(\mathbf{r}')\right)^{-1}. \tag{14}$$

As usual, we restrict our discussion to the low frequency approximation (quasi-static model of the field), where $a_l/w_l << 1$; a_l is a characteristic size of a grain of the lth type, and w_l is a wavelength in that grain. In this case, we can use a QL approximation for the integrals over V_l and S_l and assume that the material property tensor is constant in the grain with the volume V_l :

$$\widehat{\mathbf{m}}(\mathbf{r}') = \widehat{\mathbf{m}}_l, \ , \ \mathbf{r}' \in V_l. \tag{15}$$

Note, however, that in the case of spherical or elliptical grains the material property tensor is always constant within the spherical and/or elliptical inclusions.

After some algebra, expression (13) can be written in the form:

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}^b + \iiint_V \widehat{\mathbf{G}}_b(\mathbf{r} \mid \mathbf{r}') \cdot \widehat{\mathbf{q}}(\mathbf{r}') dv' \cdot \mathbf{E}^b. \tag{16}$$

In the last formula we use the following notations:

$$\widehat{\mathbf{q}}(\mathbf{r}') = \left[\widehat{\mathbf{I}} + \widehat{\mathbf{p}}(\mathbf{r}')\right] \cdot \widehat{\mathbf{m}}(\mathbf{r}'), \ \widehat{\mathbf{q}}_l = \widehat{\mathbf{q}}(\mathbf{r}'), \ \mathbf{r}' \in V_l, \tag{17}$$

$$\widehat{\mathbf{p}}(\mathbf{r}') = \widehat{\Gamma}_l^{-1} \cdot \widehat{\Lambda}_l \cdot \widehat{\xi}(\mathbf{r}'), \ \widehat{\mathbf{p}}_l = \widehat{\mathbf{p}}(\mathbf{r}'), \ \mathbf{r}' \in V_l, \tag{18}$$

where $\widehat{\mathbf{q}}$ and $\widehat{\mathbf{p}}$ are volume and surface polarizability tensors, respectively, and $\widehat{\Gamma}_l$ and $\widehat{\Lambda}_l$ are volume and surface depolarization tensors:

$$\widehat{\Gamma}_l = \iiint_{V_l} \widehat{\mathbf{G}}_b(\mathbf{r} \mid \mathbf{r}') dv', \tag{19}$$

$$\widehat{\Lambda}_l = \iint_{S_l} \widehat{\mathbf{G}}_b(\mathbf{r} \mid \mathbf{r}') \cdot \mathbf{n}(\mathbf{r}') \mathbf{n}(\mathbf{r}') ds'. \tag{20}$$

Equation (16) shows that the surface polarization effect introduced by formula (12) can be represented by the equivalent volume polarization effect and combined with the electromagnetic induction phenomenon in one integral expression.

EFFECTIVE CONDUCTIVITY OF THE HETEROGENEOUS POLARIZABLE MEDIUM

In this section we will derive a constructive approach for determining the effective conductivity of the heterogeneous polarizable medium. We have established above that, in order to solve this problem, we have to find the average value of the material property tensor, $\langle \widehat{\mathbf{m}} \rangle$. According to equation (17) this tensor is related to the volume polarizability tensor $\widehat{\mathbf{q}}$ by the following formula:

$$\widehat{\mathbf{m}} = \left[\widehat{\mathbf{I}} + \widehat{\mathbf{p}}\right]^{-1} \widehat{\mathbf{q}}. \tag{21}$$

Therefore, in order to find $\widehat{\mathbf{m}}$, we need to determine tensor $\widehat{\mathbf{q}}$ first. We introduce a "polarized" anomalous conductivity $\Delta\widehat{\sigma}^p(\mathbf{r})$ as:

$$\Delta\widehat{\sigma}^p(\mathbf{r}) = \left[\widehat{\mathbf{I}} + \widehat{\mathbf{p}}(\mathbf{r})\right] \cdot \Delta\widehat{\sigma}(\mathbf{r}). \tag{22}$$

Multiplying both sides of (16) by $\Delta\widehat{\sigma}^p(\mathbf{r})$, we arrive at equation for $\widehat{\mathbf{q}}$:

$$\widehat{\mathbf{q}}(\mathbf{r}) = \Delta\widehat{\sigma}^p(\mathbf{r}) + \Delta\widehat{\sigma}^p(\mathbf{r}) \cdot \iiint_V \widehat{\mathbf{G}}_b(\mathbf{r} \mid \mathbf{r}') \cdot \widehat{\mathbf{q}}(\mathbf{r}')\, dv'. \tag{23}$$

Solving equation (23), we determine the volume polarizability tensor, $\widehat{\mathbf{q}}_l$, for every grain:

$$\widehat{\mathbf{q}}_l = \left[\widehat{\mathbf{I}} - \Delta\widehat{\sigma}_l^p \cdot \widehat{\Gamma}_l\right]^{-1} \cdot \Delta\widehat{\sigma}_l^p \cdot \left[\widehat{\mathbf{I}} - \widehat{\Gamma}_l \cdot \langle \widehat{\mathbf{q}} \rangle\right]. \tag{24}$$

Taking an average value of both sides of (24), and solving the resulting equation for $\langle \widehat{\mathbf{q}} \rangle$, we finally find:

$$\langle \widehat{\mathbf{q}} \rangle = \left\langle \left[\widehat{\mathbf{I}} - \Delta\widehat{\sigma}^p \cdot \widehat{\Gamma}\right]^{-1} \right\rangle^{-1} \left\langle \left[\widehat{\mathbf{I}} - \Delta\widehat{\sigma}^p \cdot \widehat{\Gamma}\right]^{-1} \cdot \Delta\widehat{\sigma}^p \right\rangle. \tag{25}$$

According to equation (17), the average value of the material property tensor is:

$$\langle \widehat{\mathbf{m}} \rangle = \left\langle \left[\widehat{\mathbf{I}} + \widehat{\mathbf{p}}\right]^{-1} \widehat{\mathbf{q}} \right\rangle. \tag{26}$$

Substituting (26) into (8), we finally have:

$$\widehat{\sigma}_e = \widehat{\sigma}_b + \left\langle \left[\widehat{\mathbf{I}} + \widehat{\mathbf{p}}\right]^{-1} \widehat{\mathbf{q}} \right\rangle =$$

$$\widehat{\sigma}_b + \left[\widehat{\mathbf{I}} + \widehat{\mathbf{p}}_0\right]^{-1} \widehat{\mathbf{q}}_0 f_0 + \sum_{l=1}^{N} \left[\widehat{\mathbf{I}} + \widehat{\mathbf{p}}_l\right]^{-1} \widehat{\mathbf{q}}_l f_l. \tag{27}$$

Formula (8) allows us to calculate the effective conductivity for any multi-phase polarized composite medium. This formula can be treated as an IP analog of the "average-t-matrix approximation" (ATA) of the theory of electronic propagation in disordered binary alloys (Soven, 1967).

EFFECTIVE RESISTIVITY OF THE ISOTROPIC MEDIUM FILLED WITH ISOTROPIC GRAINS OF ARBITRARY SHAPE: ANISOTROPY EFFECT

We consider first a composite model with isotropic grains of arbitrary shape. In this case all conductivities become scalar functions:

$$\widehat{\sigma}_0 = \widehat{\mathbf{I}}\sigma_0,\ \Delta\widehat{\sigma}_l = \widehat{\mathbf{I}}\Delta\sigma_l,\ \Delta\widehat{\sigma}_l^p = \left(\widehat{\mathbf{I}} + \widehat{\mathbf{p}}_l\right)\Delta\sigma_l$$

and, according to formula (18):

$$\widehat{\mathbf{p}}_l = \xi_l \widehat{\Gamma}_l^{-1} \cdot \widehat{\Lambda}_l, \tag{28}$$

where ξ_l is equal to:

$$\xi_l = k_l \sigma_0 \sigma_l (\Delta\sigma_l)^{-1}. \tag{29}$$

Therefore, we can write:

$$\widehat{\sigma}_e = \widehat{\sigma}_b + \left\langle \left[\widehat{\mathbf{I}} + \widehat{\mathbf{p}}\right]^{-1} \widehat{\mathbf{q}} \right\rangle =$$

$$\widehat{\sigma}_e = \widehat{\mathbf{I}}\sigma_0 + \sum_{l=1}^{N} \left[\widehat{\mathbf{I}} + \widehat{\mathbf{p}}_l\right]^{-1} \left[\widehat{\mathbf{I}} - \left(\widehat{\mathbf{I}} + \widehat{\mathbf{p}}_l\right)\Delta\sigma_l \widehat{\Gamma}_l\right]^{-1} \left[\widehat{\mathbf{I}} + \widehat{\mathbf{p}}_l\right]\Delta\sigma_l f_l. \tag{30}$$

It can be demonstrated that if the grains have nonisometric shape (e.g., ellipsoidal shape) but random orientation (see Figure 1, averaging of the tensor terms in expression (30) will result in scalarization.. Therefore, the effective medium conductivity will become a scalar function. However, if all the grains are oriented in one specific direction as shown in Figure 2, the effective conductivity of this medium will become anisotropic. Thus, the effective conductivity may be a tensor in spite of the fact that the background medium and all the grains are electrically isotropic.

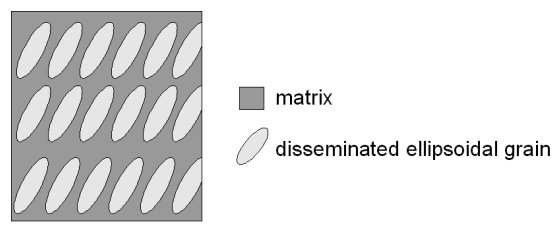

Figure 2: An example of electrically anisotropic media: a multi-phase model of the rock is composed of a set of ellipsoidal grains oriented in one direction.

FUNDAMENTAL GEMTIP MODEL: EFFECTIVE RESISTIVITY OF THE ISOTROPIC MULTI-PHASE HETEROGENEOUS MEDIUM FILLED WITH SPHERICAL INCLUSIONS

It was demonstrated in the pioneer work of Pelton (1977), that the Cole-Cole relaxation model (Cole and Cole, 1941) can represent well the typical complex conductivity of polarized rock formations. In the framework of this model, the complex resistivity, $\rho(\omega)$, is described by the following well known expression:

$$\rho(\omega) = \rho\left(1 - \eta\left(1 - \frac{1}{1 + (i\omega\tau)^{C}}\right)\right), \tag{31}$$

where ρ is the DC resistivity [Ohm-m]; ω is the angular frequency [rad/sec]; τ is the time parameter; η is the intrinsic chargeability; and

C is the relaxation parameter. The dimensionless intrinsic chargeability, η, characterizes the intensity of the IP effect.

One of the reasons for electrical conductivity relaxation in rocks is the heterogeneity of formations containing microscopic inclusions of different minerals. In the pioneered work by Wait (1982, p.77), a simplified model of the composite medium as a loading of spherical conducting particles in a resistive background was introduced. The effective conductivity for this model was determined based on the equations of the static electric field. This model provided a foundation for the phenomenological theory of induced electrical polarization.

In this section we will show that Wait's model appears as a special case of a GEMT model, developed in this paper. We consider, as an example, an isotropic multi-phase composite model, with all model parameters described by the scalar functions. A composite model is formed by a homogeneous host medium of a volume V with a conductivity σ_0 filled with grains of spherical shape. We assume also that we have a set of N different types of grains, the lth grain type having radius a_l, conductivity σ_l, and surface polarizability k_l. In this model, both the volume and the surface depolarization tensors are constant scalar tensors equal to:

$$\widehat{\Gamma}_l = \Gamma_l \widehat{\mathbf{I}} = -\widehat{\mathbf{I}} \frac{1}{3\sigma_b} \widehat{\mathbf{I}}, \ \widehat{\Lambda}_l = \Lambda_l \widehat{\mathbf{I}} = -\frac{2}{3\sigma_b a_l} \widehat{\mathbf{I}}. \tag{32}$$

The corresponding tensor formulas for conductivities, tensors $\widehat{\mathbf{m}}$ and $\widehat{\mathbf{q}}$, can be substituted by the scalar equations. For example, assuming $\sigma_b = \sigma_0$ and, therefore, $\Delta\sigma_0 = 0$, we obtain the following scalar formula for the effective conductivity of the polarized inhomogeneous medium:

$$\sigma_e = \sigma_0 + \sum_{l=1}^{N} \left[1 - (1+p_l)\Delta\sigma_l \Gamma_l\right]^{-1} \Delta\sigma_l f_l. \tag{33}$$

Substituting expression (32) for volume depolarization tensor, we finally find an expression for the effective resistivity of the composite polarized medium:

$$\rho_e = \rho_0 \left\{ 1 + 3 \sum_{l=1}^{N} \left[f_l \frac{\rho_0 - \rho_l}{2\rho_l + \rho_0 + 2k_l a_l^{-1}} \right] \right\}^{-1}, \tag{34}$$

where $\rho_0 = 1/\sigma_0$, $\rho_l = 1/\sigma_l$.

It is well-known from the experimental data that the surface polarizability factor is a complex function of frequency. Let us represent the surface polarizability of the lth grain as:

$$k_l = b_l \left(i\omega\tau_l\right)^{-C_l}, \tag{35}$$

where:

$$b_l = a_l \left(2\rho_l + \rho_0\right)/2.$$

Thus, after some algebra, we have:

$$\rho_e = \rho_0 \left\{ 1 + \sum_{l=1}^{N} \left[f_l M_l \left[1 - \frac{1}{1 + \left(i\omega\tau_l\right)^{C_l}} \right] \right] \right\}^{-1}, \tag{36}$$

where

$$M_l = 3 \frac{\rho_0 - \rho_l}{2\rho_l + \rho_0}. \tag{37}$$

In the case of a two-phase composite model, we have a homogeneous host medium of a volume V with a (complex) resistivity ρ_0 and spherical inclusions with the resistivity ρ_1. Formula (34) is simplified:

$$\rho_e = \rho_0 \left\{ 1 + f_1 M_1 \left[1 - \frac{1}{1 + \left(i\omega\tau_1\right)^{C_1}} \right] \right\}^{-1}. \tag{38}$$

The last formula is equivalent to the conventional Cole-Cole formula (31) with the following parameters:

$$\eta = \frac{3 f_1 \left(\rho_0 - \rho_1\right)}{2\rho_1 + \rho_0 + 3 f_1 \left(\rho_0 - \rho_1\right)}, \tag{39}$$

and

$$\tau = \left[\frac{a_1}{2\alpha_1} \left(2\rho_1 + \rho_0 + 3 f_1 \left(\rho_0 - \rho_1\right)\right) \right]^{1/C}. \tag{40}$$

CONCLUSIONS

We have developed a rigorous mathematical model of heterogeneous conductive media based on the effective-medium approach. The new generalized effective-medium theory provides a unified mathematical model of heterogeneity, multi-phase structure, and polarizability of rocks. The geoelectrical parameters of this model are determined by the intrinsic petrophysical and geometrical characteristics of the composite medium: the mineralization and/or fluid content of the rocks, the matrix composition, porosity, anisotropy, and polarizability of the formations. Therefore, in principle, the effective complex conductivity of this new model may serve as a basis for determining the intrinsic characteristic of the polarizable rock formation from the observed electrical data, such as the volume content of the different minerals and/or hydrocarbon saturation. As a result, the parameters of the new conductivity model can be used for mineral discrimination and/or hydrocarbon saturation evaluation and monitoring using EM methods, which is an important goal in mineral exploration (Zhdanov, 2005).

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