Regularized focusing inversion of marine CSEM data using minimum vertical support stabilizer

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SUMMARY

Marine controlled-source electromagnetic (MCSEM) surveys have become an important part of off-shore petroleum exploration. In this paper we discuss new advances in the development of 3D inversion methods for the interpretation of MCSEM data. Our method is based on rigorous integral equation (IE) forward modeling and a new IE representation of the sensitivity (Fréchet derivative matrix) of observed data to variations in sea-bottom conductivity. We use quasi-analytical approximation for models with variable background conductivity (QAVB) for more efficient Fréchet derivative calculations. In our regularized focusing inversion algorithm we introduce a new stabilizing functional, a minimum vertical support stabilizer. This stabilizer helps generate a focused image of the relatively thin and flat resistive structure of a hydrocarbon (HC) reservoir. The methodology is tested on a 3D inversion of the synthetic EM data and the interpretation of an MCSEM survey conducted in the Troll West Gas Province (TWGP).

INTRODUCTION

In this paper we discuss new advances in the development of 3D inversion methods for the interpretation of MCSEM data using the integral equation (IE) method. We have also developed a new form of efficient Fréchet derivative calculations based on the IE representation (Gribenko and Zhdanov, 2007). As a result, the IE inversion method requires just one forward modeling on every iteration step, which speeds up the computations and results in a relatively fast but rigorous inversion method.

Another distinguished feature of our inversion method is the use of focusing regularization (Portniaguine and Zhdanov, 1999), which provides a sharp boundary image of the petroleum reservoir. In the current paper we extend this approach by introducing a new stabilizing functional, a minimum vertical support stabilizer. This stabilizer helps generate a focused image of the relatively thin and flat resistive structure of a hydrocarbon (HC) reservoir. This new type of focusing inversion is tested on synthetic models of an HC reservoir. We also apply this new technique to the interpretation of an MCSEM survey conducted in the Troll West Gas Province (TWGP), offshore Norway.

PRINCIPLES OF THE 3D REGULARIZED FOCUSING INVERSION OF MCSEM DATA

The Tikhonov parametric functional with focusing stabilizers

A typical MCSEM survey consists of a set of sea-bottom electrical and magnetic receivers and a horizontal electric dipole transmitter towing at some elevation above the sea bottom. The transmitter generates a frequency domain EM field. The main goal of MCSEM data interpretation is to determine the anomalous conductivity distribution, $\Delta\sigma$, within the sea-bottom geological formations, where $\Delta\sigma$ is the difference between the total conductivity, σ, and some known background conductivity, σ_b:

$$\Delta\sigma = \sigma - \sigma_b.$$

Mathematically, we can represent the corresponding EM inverse problem in the form of the operator equation:

$$\mathbf{d} = A(\Delta\sigma), \quad (1)$$

where A is a forward modeling operator, $\mathbf{d}$ stands for the observed EM data in the sea-bottom receivers, and $\Delta\sigma$ is the anomalous conductivity within the targeted domain.

Equation (1) describes an ill-posed inverse problem. The regularized solution of this problem can be obtained by minimization of the corresponding Tikhonov parametric functional, $P^{\alpha}(\Delta\sigma)$ (Tikhonov and Arsenin, 1977):

$$P^{\alpha}(\Delta\sigma) = \|\mathbf{W}_d (A(\Delta\sigma) - \mathbf{d})\|^2_{L_2} + \alpha s(\Delta\sigma) = \min, \quad (2)$$

where $\mathbf{W}_d$ is the data weighting matrix, α is a regularization parameter, and $s(\Delta\sigma)$ is a stabilizing functional.

The major role of stabilizing functionals is selecting the appropriate solution of the inverse problem from the class of models with assigned properties. There are several possible choices for the stabilizer (Zhdanov, 2002). In the paper by Gribenko and Zhdanov (2007) the following stabilizers have been used:

1) the minimum norm stabilizer (s_{MN}), which is equal to the square L_2 norm of the difference between the current model $\Delta\sigma$ and an appropriate a priori model $\Delta\sigma_{apr}$:

$$s_{MN}(\Delta\sigma) = \left\|\mathbf{W}_m(\Delta\sigma - \Delta\sigma_{apr})\right\|^2_{L_2},$$

where $\mathbf{W}_m$ is the weighting matrix of the model parameters ;

2) the minimum support stabilizer (s_{MS}), which is proportional to the volume (support) of the nonzero values of the difference between the current model $\Delta\sigma$ and the a priori model $\Delta\sigma_{apr}$:

$$s_{MS}(\Delta\sigma) = \int\int\int_D \frac{(\Delta\sigma - \Delta\sigma_{apr})^2}{(\Delta\sigma - \Delta\sigma_{apr})^2 + e^2} dv, \quad (3)$$

where e is the focusing parameter.

In the current paper we introduce a new stabilizer, which is specially designed to invert for a thin, subhorizontal structure typical for HC reservoirs. Let us assume for simplicity that we have no a priori model: $\Delta\sigma_{apr} = 0$.

We define a vertical minimum support functional by the formula:

$$s_{VMS}(\Delta\sigma) = \int\int\int_V \left[\frac{(\Delta\sigma)^2}{\int\int_S (\Delta\sigma)^2\, dxdy + e^2}\right] dv, \quad (4)$$

where S is a horizontal section of the rectangular domain V. We also introduce a vertical support of $\Delta\sigma$ (denote vspt($\Delta\sigma$)) as the combined closed subdomains of S where $\Delta\sigma \neq 0$. Then functional $s_{VMS}(\Delta\sigma)$ can be written as:

$$s_{VMS}(m) = vspt(\Delta\sigma) - e^2 \int_{vspt(\Delta\sigma)} \left[\frac{1}{\int\int_S (\Delta\sigma)^2\, dxdy + e^2}\right] dz. \quad (5)$$

From the last equation we can see that:

$$s_{VMS}(\Delta\sigma) \rightarrow vspt(\Delta\sigma)\, if\, e \rightarrow 0,$$

where vspt($\Delta\sigma$) is a vertical support of $\Delta\sigma$. Thus, our new functional is equal to the vertical support of $\Delta\sigma$ for small values of the focusing parameter e.

The minimum norm stabilizer selects the inverse model from the class of models with the least square norm. The minimum support stabilizer insures that the solution belongs to the class of models with the smallest domain of anomalous conductivity. The minimum vertical support stabilizer provides solutions with smallest vertical dimensions of the domain of anomalous conductivity.

Re-weighting minimization

The most common approach to minimization of the parametric functional $P(\Delta\sigma)$ is based on using gradient-type methods. For example, the regularized conjugate gradient (RCG) algorithm of the parametric functional minimization in the case of the minimum norm stabilizer can be summarized as follows (Zhdanov, 2002):

$$\begin{gathered}
\mathbf{r}_n = A(\Delta\sigma_n) - d, \\
\mathbf{l}_n = \mathbf{l}(\Delta\sigma_n) = Re\mathbf{F}_n^{\star}\mathbf{W}_d^{\star}\mathbf{W}_d\mathbf{r}_n + \alpha\mathbf{W}_m^{\star}\mathbf{W}_m(\Delta\sigma_n - \Delta\sigma_{apr}), \\
\beta_n = \|\mathbf{l}_n\|^2 / \|\mathbf{l}_{n-1}\|^2, \quad \tilde{\mathbf{l}}_n = l_n + \beta_n\tilde{\mathbf{l}}_{n-1}, \; \tilde{\mathbf{l}}_0 = \mathbf{l}_0, \\
k_n = \left(\tilde{\mathbf{l}}_n, \mathbf{l}_n\right) / \left\{ \left\|\mathbf{W}_d\mathbf{F}_n\tilde{\mathbf{l}}_n\right\|^2 + \alpha \left\|\mathbf{W}_m\tilde{\mathbf{l}}_n\right\|^2 \right\}, \\
\Delta\sigma_{n+1} = \Delta\sigma_n - k_n\,\tilde{\mathbf{l}}_n,
\end{gathered} \tag{6}$$

where k_n is a length of the iteration step, and $\mathbf{l}_n$ is $\mathbf{l}_n$ is the gradient direction, computed using the derivative matrix, $\mathbf{F}_n^{\star}$.

The appropriate selection of the data and model parameters weighting matrices is very important for the success of the inversion. We determine the data weights as a diagonal matrix formed by the inverse absolute values of the normal field. Computation of the model weighting matrix is based on sensitivity analysis (Zhdanov, 2002). As a result, we obtain a uniform sensitivity of the data to different model parameters.

In order to restrict the variation of the anomalous conductivity within some reasonable bounds, we use the logarithmic model parameters, vector $\widetilde{\mathbf{m}}$, with the scalar components $\widetilde{m}_i$ given by the formula:

$$\widetilde{m}_i = \ln\left(\frac{\Delta\sigma_i - \Delta\sigma_i^-}{\Delta\sigma_i^+ - \Delta\sigma_i}\right). \tag{7}$$

This log parameterization has a property that the scalar components of the original conductivity vector $\Delta\sigma$ always remain within the given lower and upper bounds $\Delta\sigma_i^-$ and $\Delta\sigma_i^+$:

$$\Delta\sigma_i^- \leq \Delta\sigma_i \leq \Delta\sigma_i^+, \; i = 1, 2, ...L. \tag{8}$$

In the case of focusing inversion with minimum support or minimum vertical support stabilizers, we use the re-weighted regularized conjugate gradient (RRCG) method introduced in Zhdanov (2002, pp. 161-166). This algorithm is similar to RCG algorithm (6). However, the inversion is conducted in the space of the weighted model parameters $\mathbf{m}_n^w$, which are related to the original parameters by the formula:

$$\mathbf{m}_n^w = \mathbf{W}_m\tilde{\mathbf{W}}_{en}\widetilde{\mathbf{m}}_n. \tag{9}$$

In the case of minimum vertical support, we should use the following expression for $\tilde{\mathbf{W}}_{en}$:

$$\tilde{\mathbf{W}}_{en} = \tilde{\mathbf{W}}_{en}^{VMS} = \left\{ diag\left[\left(\sum_{ij} m_{ijk}^2 + e^2\right)^{1/2}\right]\right\}^{-1},$$

where

$$\Delta\sigma_{ijk} = \Delta\sigma\left(x_i, y_j, z_k\right).$$

The set of formulas (6) demonstrates that every iteration step requires at least one forward modeling solution to find the predicted data, $A(\Delta\sigma_n)$. Additional computations are needed to find the Fréchet derivative, $\mathbf{F}_n$, and the optimal length of the iteration step k_n (Gribenko and Zhdanov, 2007).

SYNTHETIC MCSEM DATA INVERSION

In this numerical experiment we consider a CSEM survey over a 3D target: a petroleum reservoir in the presence of a salt dome structure. Figures 1 and 2 show a plan view and a vertical cross-section of the model. The sea-bottom reservoir is approximated by a thin resistive body located at a depth of 900 m below sea level, with a thickness of 50 m, and a horizontal size of 800 m $\times$ 800 m. The resistivity of the reservoir is 100 Ohm-m. There is located, also, an irregular-shaped salt dome structure close to the reservoir at a depth of 700 m below the sea bottom. The resistivity of the salt dome is 30 Ohm-m. The depth of the sea bottom is 500 m from the surface, and the sea water resistivity is assumed to be equal to 0.25 Ohm-m. The salt dome and the reservoir are submerged into the conductive thickness of the sea-bottom sediments with a resistivity of 1 Ohm-m.

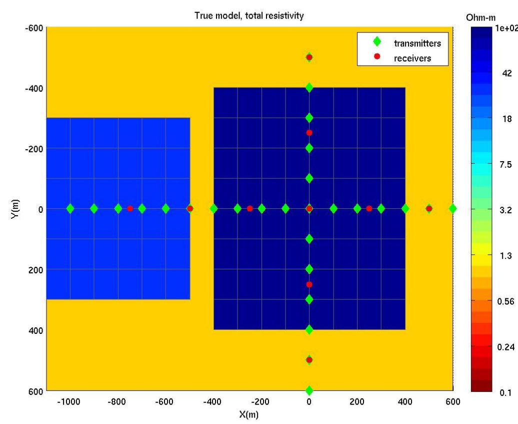

Figure 1: A petroleum reservoir in the presence of a salt dome structure (plan view). The positions of the receivers are shown by red dots, while the green diamonds show the locations of the transmitters.

A synthetic CSEM survey consists of fourteen sea-bottom receivers and an electric dipole transmitter moving along two mutually orthogonal lines at an elevation of 50 m above the sea bottom. The positions of the receivers are shown by red dots in Figure 1. The separation between the receivers is 250 m. The locations of the transmitters are shown by green diamonds in the same figure. The transmitter sends a frequency domain EM signal with two frequencies of 0.25 Hz and 0.75 Hz from points located every 100 meters along the transmitter's line. The receivers measure the in-line components of the electric field only. The observed data are computed with the rigorous IE forward modeling code and are contaminated by random Gaussian noise, with the noise level increasing linearly from 2% at zero offset up to 7% at 3,000 m offset to simulate the typical noise behavior in the field data. The area of inversion is extended from -600 m to 600 m in the x direction, from -600 m to 600 m in the y direction, and from 700 m to $1,200$ m at the depth. We discretize the inversion domain into 2,880 prismatic cells with the cell sizes equal to 100 m, 100 m, and 25 m in the x, y, and z directions, respectively.

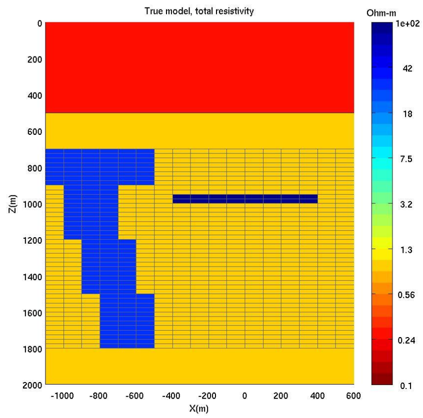

Figure 2: A petroleum reservoir in the presence of a salt dome structure (vertical section). The area shown with grids defines the extent of the anomalous domain in the inversions.

In our numerical experiments, we have used an approach based on inhomogeneous background conductivity (Zhdanov and Wilson, 2004). We have assumed that the position of the salt dome is known, and we have included a salt dome in the inhomogeneous background. This approach seems to be quite realistic. There are practical cases of offshore geophysical exploration where the salt dome structure is known from seismic data, but the location of the petroleum reservoir is unknown. Our inversion method makes it possible to include this known information in the background geoelectrical model.

We have conducted two numerical experiments. First, we ran the rigorous IE-based inversion with minimum support stabilizing functional and after 40 iterations we obtained a normalized weighted residual between the observed noisy data and predicted data equal to 5%. The corresponding inverse model is shown in Figure 3. One can see that the depth and the horizontal extent of the reservoir and its resistivity are recovered well in the inverse image. However, the thickness of the recovered anomaly reaches approximately 200 meters in the middle of the anomaly, while the true thickness is 50 meters.

In the second experiment, we have applied newly developed minimum vertical support inversion in an attempt to reduce the thickness of the recovered resistive anomaly associated with the HC reservoir. We run the rigorous IE-based inversion with the same inversion parameters as in the previous experiment, except that we use the minimum vertical support stabilizer in this experiment. After 40 iterations of the rigorous inversion, the normalized residual reaches almost 5%. Figure 4 shows the vertical section of the result of the inversion. We can clearly see the resistive reservoir in these pictures. The thickness of the reservoir is reduced compared to the minimum support inversion result.

INVERSION OF THE TROLL FIELD SEA BED LOGGING (SBL) DATA

We have applied the rigorous inversion method developed in this paper to the interpretation of sea bed logging data collected at the Troll West Gas Province (TWGP), offshore Norway. The Troll field is located in the northeastern part of the North Sea. The SBL data set was acquired by EMGS and Statoil together in June 2003 across the Troll West Gas Province (TWGP) (Johansen et al., 2005). The survey consists of 24 receivers, deployed along a line crossing the Oil Province, the Western Gas Province and the Eastern Gas Province of the Troll Field. The transmitting dipole generated a square wave signal with fundamental frequency of 0.25 Hz. In this paper we will present the results of interpretation of the in-line electric field component only. Note that the inversion work on this data set have also been done by Hoversten et. al. (2004, 2005, 2006) and Hou et. al. (2006).

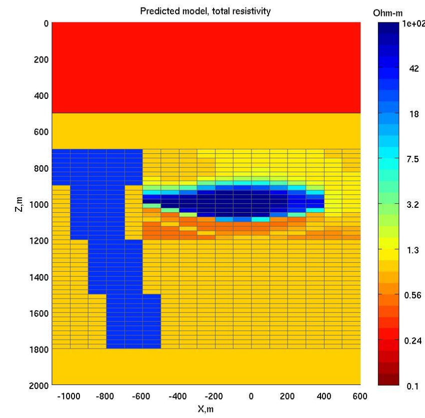

Figure 3: Focusing inversion with the minimum support stabilizer. The vertical section of the result of the inversion of the data contaminated by random noise.

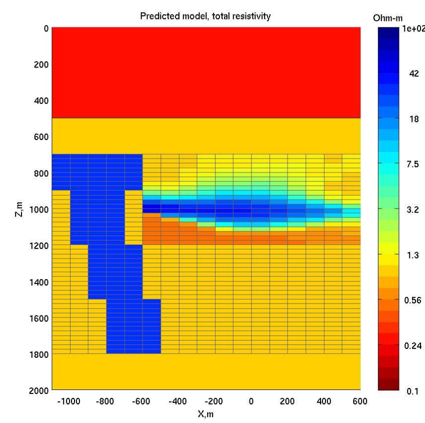

Figure 4: Focusing inversion with the minimum vertical support stabilizer. The vertical section of the result of the inversion of the data contaminated by random noise.

In order to understand better the characteristics and behavior of the observed EM signal in this survey, we have conducted an extensive computer simulation for a geoelectrical model of the Troll Field area. The sea-bottom depth in the area varies between 300 and 360 m. According to the drilling results and well-logging data, the reservoir interval is Jurassic (Sognefjord Fm) sandstones located at a depth between 1500 and 1600 m from the sea level. The gas-filled layer has resistivity around 70 Ohm-m, while the water-bearing sands and overburden are generally very conductive with resistivities in the 0.5 - 2 Ohm-m range (Amundsen et al., 2004).

In the first stage of the inversion we applied 1D parametric inversion to the data recorded in all 24 receivers, assuming that the water depth was equal to 338 m, which corresponds to the water depth record in the data set provided by EMGS. As a result of 1D inversion we found the following parameters of the horizontally layered geoelectrical model:

water conductivity at 3 S/m and sea-bottom sediments conductivity at 0.42 S/m. We have used a two-layered background model with these parameters in our regularized 3D inversion.

The area of inversion was discretized in $84 \times 9 \times 60 = 45,360$ cells, whose cell sizes were 250 m, 1000 m, and 25 m in the x, y, and z directions, respectively. In order to reduce the computational time, we used the data from twelve receivers only (#2, #4, #6, #8, #10, #12, #14, #16, #18, #20, #22, and #24). We chose transmitter points located every 500 meters along the transmitter's line with a maximum offset equal to 10 km. Altogether, we used 41 transmitter positions for every receiver, which amounts to 492 observation pairs. We have also used only the fundamental frequency of 0.25 Hz to speed up the computations.

For this practical data set we ran 10 iterations of the smooth inversion and an additional 60 iterations of the focusing inversion. To increase the convergence rate, we selected an a priori model at the initial smooth inversion stage formed by a 150 m-thick horizontal layer with a resistivity of 3 Ohm-m located at a depth of 1550 m. After the tenth iteration this a priori model was excluded from the inversion scheme to allow for a free (without any a priori constraints) convergence of the inversion. The normalized residual reaches 11% after 70 iterations. We suggest that the relatively high level of the misfit between the observed and predicted data can be explained by the fact that we used a limited area of inversion with a relatively coarse grid, and a small number of the inversion cells, which limits the flexibility of the inversion algorithm. As a result, we do not take into consideration the possible variations in the conductivity of the near-bottom layers of the sediments ("geological noise"). In addition, there is significant noise in the observed data themselves.

The inversion is able to recover a strong resistivity anomaly in the area of the Jurassic sandstone reservoir. Figure 5 shows a simplified geological model along the survey line, overlaying the resistive structure obtained by the 3D inversion. We can observe a clear correlation between the location of the resistivity anomaly obtained from the SBL data and the TWGP reservoir.

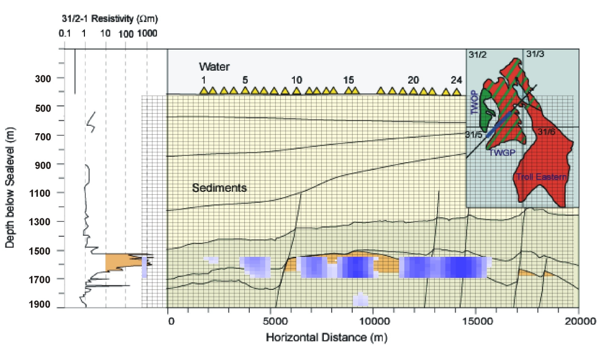

Figure 5:

CONCLUSIONS

In this paper we have presented the results of the new development of a rigorous method for 3D inversion of MCSEM data based on the IE formulation. We have introduced a new type of focusing regularized inversion based on the minimum vertical support stabilizing functional. This kind of regularization is very well suited in inversion for the thin, quasi-horizontal resistive structures of HC reservoirs. We have tested this method by inversion of synthetic MCSEM data computer-simulated over typical models of a sea-bottom petroleum reservoir. The results of these tests demonstrate that the inverse images generated by the focusing regularized inversion with the minimum vertical support stabilizer provides better reconstruction of the true thickness of the reservoir than does the original minimum support regularization.

We have applied the developed method for the inversion of the practical SBL data collected by EMGS and Statoil in the Troll West Gas Province (TWGP), offshore Norway. The inversion results on a large grid with the minimum vertical support stabilizer produce a clear image of the gas reservoir. The geoelectrical image corresponds well to the available seismic depth sections.

The method is implemented in a working draft of a serial version of the code, which can be run on a single PC. The typical inversion on a grid of up to a few thousands inversion cells requires just less than half an hour of computational time on an AMD 4400+ (2.2 GHz) Windows PC with 3,25 GB of RAM.

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