Regularization analysis of three-dimensional magnetotelluric inversion

Arun Kumar, Le Wan , and Michael S. Zhdanov , University of Utah , Salt Lake City*

SUMMARY

Inversion of MT data is an inherently nonunique and unstable problem due to the ill-posedness of the electromagnetic inverse problem. A variety of models may fit the data very well. To overcome this ill-posed nature of the inverse problem, we use Tikhonov's regularization in which the ill-posed problem is replaced by a family of well-posed problems. We also analyze the behavior of the Tikhonov regularization parameter to find out its optimal value for a typical model of a hydrocarbon reservoir in a marine environment. We have compared two regularization techniques: rigorous and adaptive regularizations. The results of this numerical study demonstrate that adaptive regularization provides practically the same inverse image as the rigorous regularization, while reducing the computational time dramatically.

INTRODUCTION

Controlled-source electromagnetic (CSEM) and magnetotelluric (MT) techniques have been used as complementary tools in oil and gas exploration in the deep sea environment, to overcome the ambiguity of the seismic reflection method resulting from low impedance contrasts between a hydrocarbon-bearing formation and its host sediments (Hoversten et al., 1998). Seismic methods provide the geometry and layer structure of a formation. Marine CSEM and marine MT help determine whether the fluids imaged in the seismic sections are hydrocarbons or water (Eidesmo et al., 2002).

In this paper we introduce a method of 3-D inversion of MT data, based on the integral equation (IE) method. We use the regularized conjugate gradient method (RRCG) for the nonlinear MT inversion. The main distinguishing feature of our algorithm is application of the effective form of the Frechet derivative calculation based on the integral formulas.

We also carry out a model study using synthetic marine MT data to test the significance of our selection of an optimal Tikhonov regularization parameter (α). We consider a geological model comprising a sea-bottom gas hydrate layer underlain by a petroleum reservoir, both embedded in sea sediments. The synthetic data were generated by the integral equation (IE) modeling code INTEM3D, based on the Contraction Integral Equation (CIE) method (Hursan and Zhdanov, 2002).

TIKHONOV REGULARIZATION IN THE SOLUTION OF MT INVERSE PROBLEMS

The interpretation of magnetotelluric data is based on the calculation of the transfer functions between the horizontal components of the electric and magnetic fields, using the *magnetotelluric impedance matrix*:

$$Z = \begin{bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{bmatrix}, \tag{1}$$

We can describe the forward MT problem by an operator equation:

$$\mathbf{d} = \mathbf{A}(\mathbf{m}), \tag{2}$$

where $\mathbf{d}$ stands for a *data vector* formed by the components of the MT impedance, $\mathbf{A}$ is the nonlinear forward operator symbolizing the governing equations of the MT modeling problem, and $\mathbf{m} = \Delta\sigma$ is the anomalous conductivity within the targeted domain.. We call equation (2) an *impedance equation*. The impedance equation is ill-posed, i.e., the solution can be nonunique and unstable. The conventional way of solving ill-posed inverse problems, according to the regularization theory (Tikhonov and Arsenin, 1977, Zhdanov, 2002), is based on minimization of the Tikhonov parametric functional:

$$P(\mathbf{m}) = \varphi(\mathbf{m}) + \alpha S(\mathbf{m}) = \min, \tag{3}$$

where $\varphi(\mathbf{m}) = ||\mathbf{A}(\mathbf{m}) - \mathbf{d}||^2$ is the misfit functional between the predicted data $A(\mathbf{m})$ and the observed data $\mathbf{d}$, $S(\mathbf{m})$ is a stabilizing functional, and α is a regularization parameter. The optimal value of α_{opt} is determined from the misfit condition,

$$\varphi\left(\mathbf{m}_{\alpha_{opt}}\right) = \delta^2, \tag{4}$$

where δ is the noise level of the data.

A REGULARIZED CONJUGATE GRADIENT METHOD BASED ON THE INTEGRAL EQUATION FORMULATION OF THE FRECHET DERIVATIVE COMPUTATION

The minimization problem (3) can be solved using any gradient type technique. We use the regularized conjugate gradient (RCG) method. Implementation details of this algorithm are specified in Zhdanov and Hursán (2000) and Zhdanov (2002). The key element of the RCG algorithm is the computation of the gradient direction, $\mathbf{l}_n$ for every iteration n:

$$\mathbf{l}_n = \mathbf{F}_n^* \mathbf{R}_n,$$

where $\mathbf{R}_n = A(\mathbf{m}_n) - \mathbf{d}$ is a residual vector at the current iteration, and $\mathbf{F}_n^*$ is the adjoint matrix for the Frechet derivative.

The corresponding expression for the adjoint Fréchet derivative operator of the residual fields is given by the following integral formulas (Zhdanov, 2002):

$$\mathbf{F}_n^* \mathbf{R}_n = \mathrm{Re}\left[\mathbf{E}_n^*(\mathbf{r}) \cdot \int\int_S \widehat{\mathbf{G}}_{E_n}^{T*}(\mathbf{r}'|\mathbf{r}) \cdot \mathbf{R}_n(\mathbf{r}')\, ds'\right], \tag{5}$$

where $\widehat{\mathbf{G}}_{E_n}(\mathbf{r}'|\mathbf{r})$ is the electric Green's tensor computed at the nth iteration for the conductivity distribution, $\sigma_n(\mathbf{r}) = \sigma_b(\mathbf{r}) + \Delta\sigma_n(\mathbf{r})$.

The last expression can be written using the EM migration theory as (Zhdanov, 2002;):

$$\mathbf{F}_n^* \mathbf{r}_n = \mathrm{Re}\left[\mathbf{E}_n^*(\mathbf{r}) \cdot \mathbf{E}_n^{m*}(\mathbf{r})\right] = \mathrm{Re}\left[\mathbf{E}_n(\mathbf{r}) \cdot \mathbf{E}_n^{m}(\mathbf{r})\right], \tag{6}$$

where $\mathbf{E}_n^m$ is the migration electric field:

$$\mathbf{E}_n^m(\mathbf{r}) = \int\int_S \widehat{\mathbf{G}}_{E_n}(\mathbf{r}|\mathbf{r}') \cdot \mathbf{R}_n^*(\mathbf{r}')\, ds'. \tag{7}$$

Once the forward operator with the corresponding Fréchet matrices and the minimization scheme are implemented, an inversion can be successfully performed.

Regularization analysis of 3-D MT Inversion

SELECTION OF THE OPTIMAL REGULARIZATION PARAMETER

The regularization parameter α describes the trade-off between the best-fitting and most reasonable stabilization.The critical question in the regularized solution of the inverse problem is the selection of the optimal regularization parameter α. The solution of this problem can be based on the following consideration.

Let us assume that data $\mathbf{d}_\delta$ are observed with some noise, $\mathbf{d}_\delta = \mathbf{d}_t + \delta\mathbf{d}$, where $\mathbf{d}_t$ is the true solution of the problem and the level of the errors in the observed data is equal to δ :

$$\|\mathbf{d}_\delta - \mathbf{d}_t\| \leq \delta. \tag{8}$$

For any number α_k we can find an element $\mathbf{m}_{\alpha k}$,minimizing $P^{\alpha k}(\mathbf{m})$, calculate the misfit $\|\mathbf{A}(\mathbf{m}_{\alpha k0}) - \mathbf{d}\|^2$. According to the regularization theory, the optimal value of the parameter α is the number α_{opt}, for which we have (Zhdanov, 2002):

$$\varphi\left(\mathbf{m}_{\alpha_{opt}}\right) = \left\|\mathbf{A}(\mathbf{m}_{\alpha_{opt}}) - \mathbf{d}\right\|^2 = \delta^2. \tag{9}$$

A numerical method for determining the parameter α_{opt} can be described as follows. Let us consider the progression of numbers:

$$\alpha_k = \alpha_1 q^{k-1},\ k = 1, 2, ..., n;\ \ 0 < q < 1. \tag{10}$$

It is proven in the regularization theory that the misfit functional $\varphi\left(\mathbf{m}_{\alpha_k}\right)$, is a monotonically decreasing function of α_k. We solve the problem of the parametric functional minimization (3) several times for different values of the regularization parameter α_k, selected according to the rule (10), until the misfit condition (9) is met. This approach is based on rigorous application of the principles of the regularization theory. However, it requires multiple regularized solutions of the inverse problem for the different values of the regularization parameter α.

In practice, another approach is widely used. It is based on application of the iterative algorithms of the parametric functional minimization. For example, in this paper we use the re-weighted regularized conjugate gradient (RRCG) method, referenced above. In the framework of the iterative approach, for any subsequent iteration we update the value of the regularization parameter α_k according to the progression (10). The iterative inversion is terminated when the misfit condition is reached:

$$\varphi\left(\mathbf{m}_{\alpha_{k0}}\right) = \left\|\mathbf{A}(\mathbf{m}_{\alpha_{k0}}) - \mathbf{d}\right\|^2 = \delta^2. \tag{11}$$

This approach is called the a*daptive regularization*. It requires just one inversion run but with variable values of α.

In the model study presented in the next section, we will compare the method of rigorous regularization with the adaptive regularization as applied to the MT inverse problem solution.

SYNTHETIC MARINE MAGNETOTELLURIC DATA

We consider the scenario of a sea-bottom gas hydrate-bearing layer underlain by a petroleum reservoir. As part of the numerical experiment, we conduct a synthetic MT survey in deep seawater. The set-up consists of receivers located five meters above the sea floor with a receiver separation of 500 m. The background geoelectrical model consists of a seawater layer with a thickness of 2000 m and a resistivity of 0.3 Ohm-m, underlain by a homogeneous sediment layer with a thickness of 5000 m and a resistivity of 1 Ohm-m (Figure 1). The gas hydrate layer is underlain by a petroleum reservoir with 100 Ohm-m resistivity and dimensions of 5,000 m x 10,000 m x 100 m, embedded at a depth 1000 m below the sea floor. Figures 1 and 2 show a vertical cross-section of Model 1 and a 3-D image of the true model respectively.

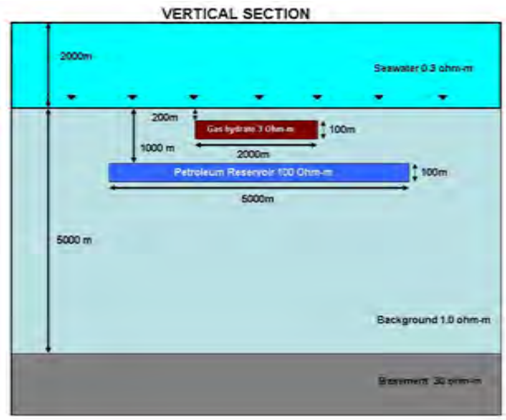

Figure 1: Vertical section of a model showing a rectangular gas hydrate reservoir underlain by a petroleum reservoir located in sea bottom sediments. The gas hydrate layer is located at a depth of 200 m below the sea bottom, has a resistivity of 3 Ohm-m, a thickness 100 m, and a horizontal extent of 2000 m x 2000 m. The petroleum reservoir is located at a depth of 1000 m below the sea bottom and has a resistivity of 100 Ohm-m, a thickness of 100 m, and a horizontal extent of 5,000 m x 10,000 m. The resistivities of the seawater, the sea sediments, and the homogeneous basement are 0.3 Ohm-m, 1.0 Ohm-m, and 30 Ohm-m respectively.

The synthetic MT data were generated using the INTEM3D integral forward modeling code (Hursan and Zhdanov, 2002) for seven different frequencies : 0.01, 0.03, 0.1, 0.3, 1, 3 and 10 Hz.The synthetic data were contaminated by 1% random noise.

Following the traditional approach used in a practical MT survey, we have calculated the synthetic observed apparent resistivities and phases based on two off-diagonal elements of the magnetotelluric tensor, Z_{yx} and Z_{xy}, at each observation point.

INVERSION OF SYNTHETIC MARINE MAGNETOTELLURIC DATA

We have conducted a numerical study of two regularized inversion techniques for this model. In order to speed up the computations, the MT data were inverted using the fast version of IE inversion code based on the quasi-analytical (QA) approximation (Zhdanov et al., 2000b; Zhdanov and Hursán, G., 2000)

Rigorous regularization

We begin with the rigorous regularization. The inversion was carried out at several values of the regularization parameter α varying from 10^{-3} to 25**.** At a particular value of α,we computed the misfit, parametric, and stabilizer functionals. The optimal value of α is obtained when the misfit reaches the level of error (misfit condition) in the data. Figure 3 shows the behavior of the misfit, parametric, and stabilizing functionals corresponding to a particular value of α, and 0.02 is the optimal value of α selected as per the condition of misfit. The total computational time required for each inversion at a particular value of α was about 17 to 20 minutes on a 3.40 GHz PC.

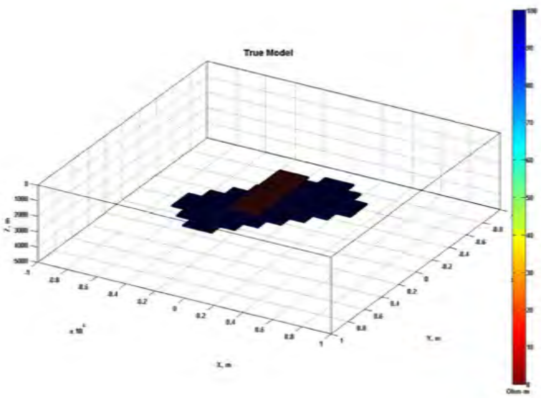

Figure 2: 3-D image of the true model of a gas hydrate reservoir underlain by a petroleum reservoir located in sea-bottom sediments.

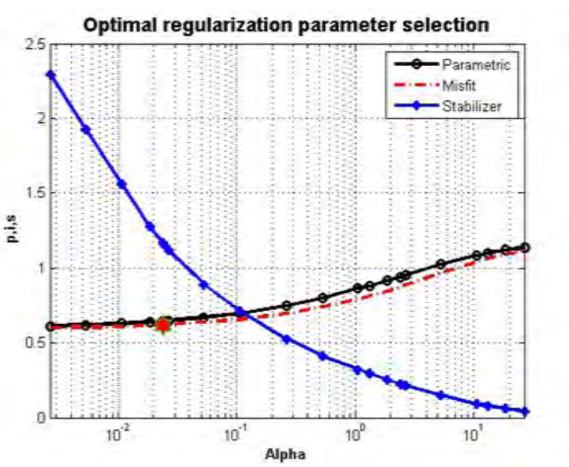

Figure 3: The plot illustrating the principles of optimal regularization parameter selection.

In order to better understand the behavior of the Tikhonov regularization parameter α, we conducted an L-curve analysis (Hansen, 1998). It is based on plotting for all possible α the curve of the misfit functional, $\varphi(\mathbf{m}_\alpha)$, versus the stabilizing functional, $S(\mathbf{m}_\alpha)$. The L-curve shown in Figure 4 illustrates the trade-off between the best fit (minimizing a misfit) and most reasonable stabilization (minimizing a stabilizer).

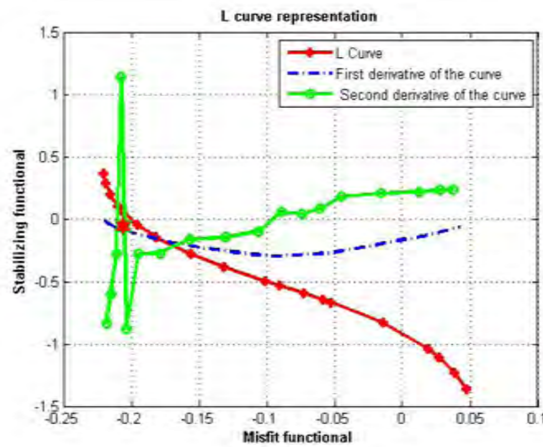

Figure 4: Model 1. L-curve plot for selection of the quasi-optimal regularization parameter α. The red star dot shows the point where the curvature of the curve is greatest, corresponding to the quasi-optimal value of the regularization parameter α.

Figures 5 and 6 show a horizontal cross-section at a depth of 2275 m and 3075 m for a 3-D smooth inversion result at four different values of α. One can see in these figures that the image of the gas hydrate layer and petroleum reservoir is more diffused at α values of 0.001, 0.05, and 1. In the horizontal cross-section, at an optimal value of $\alpha = 0.02$, the image is very clear and recoverable. Therefore, the Tikhonov regularization parameter balances a trade-off between the misfit and the stabilization and provides a stable solution in any situation. The corresponding final volume image of Model 1 obtained by inversion at α value of 0.02 is shown in 7. One can see from the figures that the complete image of the model is not recovered at α values that are too small and too large. The inverse resistivity image which is closest to the true model was obtained for optimal $\alpha = 0.02$. This synthetic model study, therefore, shows the importance of the selection of regularization parameter in the inversion.

Adaptive regularization

In the second numerical experiment, we have carried out the inversion of the same model by a adaptive regularization scheme in which α values are updated in the process of the iterative inversion. We run only 70 iterations to produce a smooth image of the inverse model. The iterative process is terminated when the relative norm of the difference between the observed and predicted data reaches the noise level in the observed data. Figure 8 represents the 3-D volume inverse image of Model 1. The inverse image provides a 3-D model which is very close to the true conductivity model. The computational time for this inversion was about 18 to 20 minutes on a single PC with 3.40 GHz CPU.

CONCLUSIONS

In this paper, we have analyzed the importance of the optimal selection of the Tikhonov regularization parameter to get an accurate and stable solution of the inverse problem in any situation. The synthetic study provides a stable inversion result for a gas hydrate and petroleum reservoir at an optimal value of the regularization parameter. We have com-

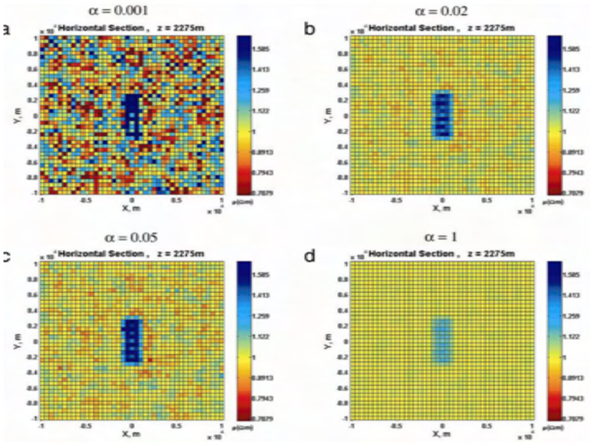

Figure 5: Model 1. A horizontal cross-section of the 3-D smooth inversion result at a depth of 2275 m for different values of the α.

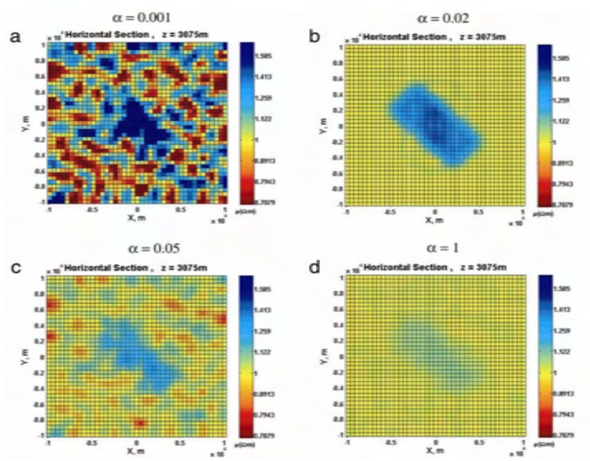

Figure 6: Model 1. A horizontal cross-section of the 3-D smooth inversion result at a depth of 3075 m for different values of the α.

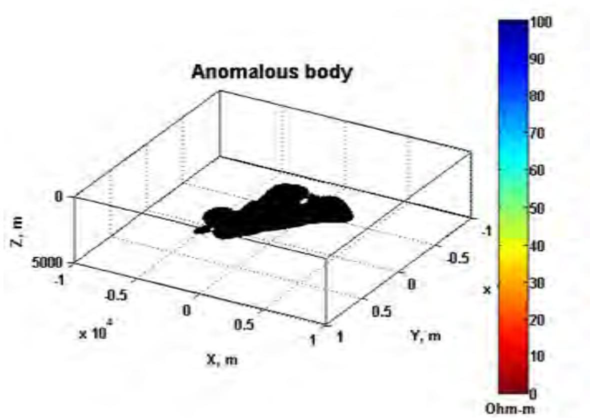

Figure 7: Model 1. A 3-D volume image of the QA inversion result at $\alpha. = 0.02$

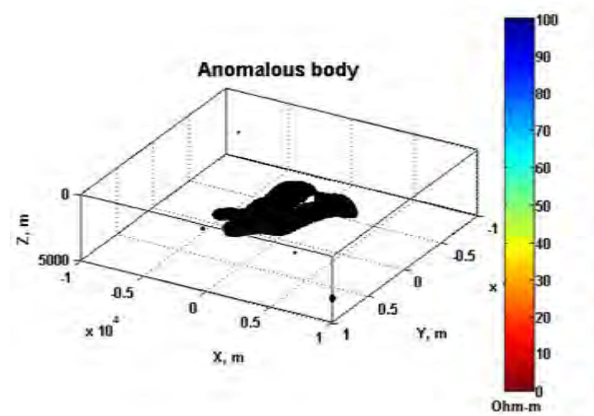

Figure 8: Model 1. 3-D volume image obtained by a QA adaptive regularization inversion method

pared two regularization techniques: 1) rigorous regularization based on multiple inversions of the observed data with the different regularization parameter α, and

2) adaptive regularization with updating of the regularization parameter in the process of the iterative inversion.

The results of this numerical study demonstrate that adaptive regularization provides practically the same inverse image as the rigorous regularization, while reducing the computational time dramatically.

In summary, the results of our research demonstrate that the sea-bottom MT survey can be a powerful tool for offshore petroleum exploration.

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