Multi-grid IE method for large-scale models with inhomogeneous background

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SUMMARY

We present a multigrid integral equation (IE) method for three-dimensional (3D) electromagnetic (EM) field computations in large-scale models with inhomogeneous background conductivity (IBC). This method combines the advantages of the iterative IBC IE method and the multigrid quasi-linear (MGQL) approximation. The new EM modeling method solves the corresponding systems of linear equations within the domains of anomalous conductivity, D_a and inhomogeneous background conductivity, D_b, separately on coarse grids. The observed EM fields in the receivers are computed using grids with fine discretization. The developed MGQL IBC IE method can be also applied iteratively by taking into account the return effect of the anomalous field inside the domain of the background inhomogeneity D_b, and vice versa. The iterative process described above is continued until we reach the required accuracy of the EM field calculations in both domains, D_a and D_b. The method was tested for modeling the marine CSEM field for complex geoelectrical structures with hydrocarbon petroleum reservoirs and a rough sea bottom bathymetry.

INTRODUCTION

In the framework of the IE method, the conductivity distribution is divided into two parts: 1) the background conductivity, σ_b, which is used for the Green's functions calculation, and 2) the anomalous conductivity, $\Delta\sigma_a$, within the domain of integration, D. One principal advantage of the IE method over another numerical techniques, e.g., over a finite-different (FD) or finite-element (FE) methods is that the IE method requires the discretization of the anomalous domain D only, while the FD and FE method need a huge grid covering the entire modeling domain.

It is very well known, however, that the main limitation of the IE method is that the background conductivity model must have a simple structure to allow for an efficient Green's function calculation. The most widely used background models in EM exploration are those formed by horizontally homogeneous layers. The theory of the Green's functions for layered one-dimensional (1D) models is very well developed and lays the foundation for efficient numerical algorithms. Any deviation from this 1D background model must be treated as an anomalous conductivity.

In some practical applications, however, it is difficult to describe a model using a horizontally layered background conductivity. As a result, the domain of integration may become too large, which increases significantly the size of the modeling domain and of the required computer memory and computational time for IE modeling. It was demonstrated in the paper by Zhdanov et al. (2006) that we can overcome these computational difficulties by developing the IE method with an inhomogeneous background conductivity (IBC). This method is based on the separation of the effects due to excess electric current, $\mathbf{j}^{\Delta\sigma_b}$, induced in the inhomogeneous background domain, and those due to the anomalous electric current, $\mathbf{j}^{\Delta\sigma_a}$, in the location of the anomalous conductivity, respectively. As a result, we arrive at a system of integral equations which uses the same simple Green's functions for the layered model, as in the original IE formulation. However, the new equations take into account the effect of the variable background conductivity distribution. The accuracy control of this method is based on application of the IBC technique iteratively.

In the current paper, we have expanded this technique by incorporating the principles of multigrid quasi-linear (MGQL) modeling (Ueda and Zhdanov, 2006) in the framework of the iterative IBC IE modeling. The new version of the parallel software PIE3DMG allows us to use the multigrid approach together with the iterative IBC method.

We apply this new technique to study the bathymetry effects in marine CSEM data.

INTEGRAL EQUATION FORMULATION

We consider a 3-D geoelectrical model with horizontally layered (normal) conductivity σ_n, inhomogeneous background conductivity $\sigma_b = \sigma_n + \Delta\sigma_b$ within a domain D_b, and anomalous conductivity $\Delta\sigma_a$ within a domain D_a.

One can represent the EM field in this model as a sum of the normal fields $\mathbf{E}^n$ and $\mathbf{H}^n$ generated by the given source(s) in the model with normal distribution of conductivity σ_n, a variable background effect $\mathbf{E}^{\Delta\sigma_b}$ and $\mathbf{H}^{\Delta\sigma_b}$ produced by the inhomogeneous background conductivity $\Delta\sigma_b$, and the anomalous fields $\mathbf{E}^{\Delta\sigma_a}$ and $\mathbf{H}^{\Delta\sigma_a}$ related to the anomalous conductivity distribution $\Delta\sigma_a$:

$$\begin{aligned} \mathbf{E} &= \mathbf{E}^n + \mathbf{E}^{\Delta\sigma_b} + \mathbf{E}^{\Delta\sigma_a}, \\ \mathbf{H} &= \mathbf{H}^n + \mathbf{H}^{\Delta\sigma_b} + \mathbf{H}^{\Delta\sigma_a}. \end{aligned} \tag{1}$$

The total EM fields in this model can be written as:

$$\mathbf{E} = \mathbf{E}^b + \mathbf{E}^{\Delta\sigma_a}, \ \ \mathbf{H} = \mathbf{H}^b + \mathbf{H}^{\Delta\sigma_a}, \tag{2}$$

where the background EM fields $\mathbf{E}^b$ and $\mathbf{H}^b$ are sums of the normal fields and those caused by the inhomogeneous background conductivity:

$$\mathbf{E}^b = \mathbf{E}^n + \mathbf{E}^{\Delta\sigma_b}, \ \ \mathbf{H}^b = \mathbf{H}^n + \mathbf{H}^{\Delta\sigma_b}. \tag{3}$$

The basic idea of this IE formulation is that the EM field induced in the anomalous domain by the excess currents in the background inhomogeneity $\mathbf{j}^{\Delta\sigma_b}$ can be taken into account, while the return induction effects by the anomalous currents $\mathbf{j}^{\Delta\sigma_a}$ would be ignored. In other words, the anomalous electric fields $\mathbf{E}^{\Delta\sigma_a}$ are assumed to be much smaller than the background fields $\mathbf{E}^b$ inside the domain of integration D_b.

It was demonstrated by Zhdanov et al. (2006) that the accuracy of the IBC IE method can be improved by applying the IBC method iteratively. This means that we can take into account the return effect of the anomalous field inside the domain of the background inhomogeneity D_b and evaluate the accuracy of this solution. After that we can use this updated background field for the anomalous field. The iterative process described above is continued until we reach the required accuracy of the EM fields calculations in both domains, D_a and D_b.

MULTIGRID QL APPROXIMATION

The multigrid QL approximation, introduced by Ueda and Zhdanov (2006), is based on the following principles. A general forward EM problem is formulated in such a way that the anomalous conductivity can be treated as a perturbation from a known background (or "normal") conductivity distribution. The solution of the EM problem in this case contains two parts: 1) the linear part, which can be interpreted as a direct scattering of the source field by the inhomogeneity without taking into account coupling between scattering (excess) currents, and 2) the nonlinear part, which is composed of the combined effects of the anomalous conductivity and the unknown scattered field in the inhomogeneous structure. The QL approximation is based on the assumption that this last part is linear proportional to the background field $\mathbf{E}^b$ through some electrical reflectivity vector λ (Zhdanov and Fang, 1996; Gao et al., 2004):

$$\mathbf{E}^a(\mathbf{r}) \approx \lambda(\mathbf{r}) \left|\mathbf{E}^b(\mathbf{r})\right|. \tag{4}$$

In the framework of the multigrid approach, we discretize the conductivity distribution in the model and the electric fields using two grids, $\sum_c$ and $\sum_f$, where $\sum_c$ is a coarse discretization grid and $\sum_f$ is a fine discretization grid, where each block of the original grid $\sum_c$ is divided into additional smaller cells. First, we solve the integral equation for the electric field on a coarse grid to determine the total electric field $\mathbf{E}$. After that we can find the anomalous field $\mathbf{E}^a$ on the coarse grid $\sum_c$:

$$\mathbf{E}^a(\mathbf{r}_c) = \mathbf{E}(\mathbf{r}_c) - \mathbf{E}^b(\mathbf{r}_c), \tag{5}$$

where $\mathbf{r}_c$ denotes the centers of the cells of the grid $\sum_c$ with coarse discretization.

The components of the electrical reflectivity vector on a coarse grid are found by direct calculations as:

$$\lambda_x(\mathbf{r}_c) = E_x^a(\mathbf{r}_c) / \left|\mathbf{E}^b(\mathbf{r}_c)\right|, \tag{6}$$

$$\lambda_y(\mathbf{r}_c) = E_y^a(\mathbf{r}_c) / \left|\mathbf{E}^b(\mathbf{r}_c)\right|, \tag{7}$$

$$\lambda_z(\mathbf{r}_c) = E_z^a(\mathbf{r}_c) / \left|\mathbf{E}^b(\mathbf{r}_c)\right|, \tag{8}$$

assuming that $\left|\mathbf{E}^b(\mathbf{r}_c)\right| \neq 0$.

After we have found $\lambda(\mathbf{r}_c)$, we determine the $\lambda(\mathbf{r}_f)$ values on the fine discretization grid $\sum_f$ by linear interpolation (where $\mathbf{r}_f$ denotes the centers of the cells of the grid $\sum_f$ with fine discretization). We compute the anomalous electric field $\mathbf{E}^a(\mathbf{r}_f)$ in the centers of the cells of the new grid $\sum_f$ with fine discretization using expression (4):

$$\mathbf{E}^a(\mathbf{r}_f) \approx \lambda(\mathbf{r}_f) \left|\mathbf{E}^b(\mathbf{r}_f)\right|.$$

We can find now the total electric field $\mathbf{E}(\mathbf{r}_f)$ on a new grid, as:

$$\mathbf{E}(\mathbf{r}_f) = \mathbf{E}^a(\mathbf{r}_f) + \mathbf{E}^b(\mathbf{r}_f). \tag{9}$$

Finally, we compute the observed fields in the receivers using the discrete analog of the IE form of Maxwell's equations for the grid with fine discretization.

The application of the multigrid QL technique in the framework of the IBC IE method is straightforward. We introduce two pairs of coarse and fine grids, $\sum_c^b, \sum_f^b$ and $\sum_c^b, \sum_f^b$, in the inhomogeneous background, D_b, and anomalous, D_a, domains, respectively. We solve on the coarse grids and interpolate these solution in the corresponding fine grids using the QL method described above. After that we compute the observed fields in the receivers using grids with fine discretization.

APPLICATION OF THE MULTIGRID QL IBC IE METHOD TO STUDY OF THE BATHYMETRY EFFECTS IN MARINE CSEM DATA

In this section we will present the application of the new PIE3DMG CEMI code for computer simulation of the bathymetry effects in the marine CSEM data. This is a very important problem in marine EM geophysics, because the effect of the sea bottom bathymetry can significantly distort the useful EM response from a hydrocarbon (HC) reservoir, which is the main target of offshore geophysical exploration.

In order to investigate better the response of the HC reservoir, we consider first a synthetic model of a reservoir with a horizontal flat sea floor. In the second example we present a practical case of modeling the MCSEM data in the Sabah area, Malaysia, which is characterized by extremely strong bathymetry inhomogeneities.

Model 1: a synthetic hydrocarbon reservoir

A vertical section of the geoelectrical model is shown in Figure 1. One can see in this figure that a resistive structure of a hydrocarbon reservoir is located within the conductive sea bottom sediment. The reservoir has a complex three-dimensional geometry and contains three layers: a water-filled layer with a resistivity of 0.5 Ohm-m, a gas-filled layer with a resistivity of 1,000 Ohm-m, and an oil-filled layer with resistivity of 100 Ohm-m, as shown in Figure 1. The parameters of the sea bottom sediment are also shown in Figure 1. Figure 2 presents a more detailed plan view and cross-section of the reservoir. The resistivity of the seawater layer is 0.3 Ohm-m, and the depth of the sea floor is 1,350 m below sea level.

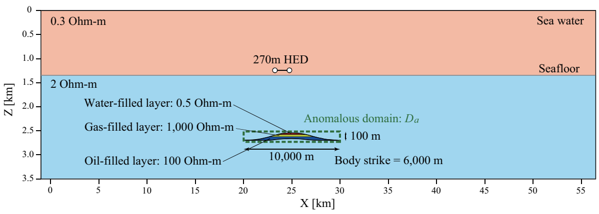

Figure 1: A model of a hydrocarbon reservoir located within a conductive sea-bottom sediment. The reservoir has a complex 3-D geometry and contains three layers: a water-filled layer with a resistivity of 0.5 Ohm-m, a gas-filled layer with a resistivity of 1,000 Ohm-m, and an oil-filled layer with a resistivity of 100 Ohm-m.

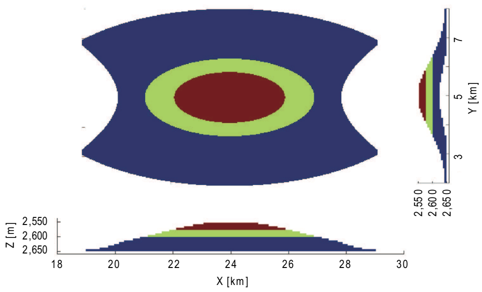

Figure 2: Detailed plan and side views of the hydrocarbon reservoir located within the conductive sea-bottom sediment.

The EM field in the model is excited by an x-directed electric horizontal bipole with a length of 270 m and located at the point with horizontal coordinates $x = 24,000$ m and $y = 5,000$ m, as shown in Figure 1. The elevation of the transmitter bipole is 50 m above the sea bottom. The transmitter is assumed to generates the frequency domain EM fields at a frequency of 0.25 Hz.

In our numerical study, following the general principles of the IE method, the hydrocarbon reservoir structure is described by the anomalous conductivity distribution $\Delta\sigma_a$. The modeling domain D_a corresponds to the location of the reservoir, and this domain is discretized in 1.5 million cells (400 × 240 × 16) with each cell sized 25 × 25 × 6 m^3 to represent accurately the reservoir structure of the model. We have applied the multigrid QL approach, which can decrease the computation cost without losing accuracy. This method computes the EM fields on the coarse grid first, and then interpolates the results to the fine grid using the QL technique described above. In this case the coarse grid has 200 × 120 × 8 = 192,000 cells. We have checked the accuracy of the multigrid approach for large-scale modeling using this model of the HC reservoir.

Figure 3 shows amplitude plot of the in-line electric field, E_x, as well as data for the model with a hydrocarbon reservoir obtained using the rigorous IE solution on the fine grid (black line) and the MGQL approach (red circles) at sea bottom receivers located along, $y = 5,000$ m line. Figure 4 shows the corresponding phase plot for the same model. One can see from these figures that the MGQL results fit the rigorous solution very accurately. At the same time, the rigorous IE solution on the fine grid using the PC cluster requires 34 minutes on 32 CPUs, while it takes only six minutes for MGQL modeling on four CPUs.

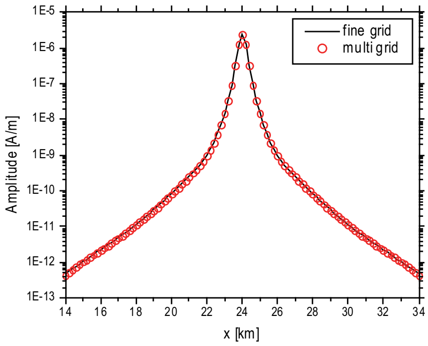

Figure 3: Amplitude plots of in-line electric field data for the model with a hydrocarbon reservoir obtained using fine-grid (black line) and multiple grids (red circles).

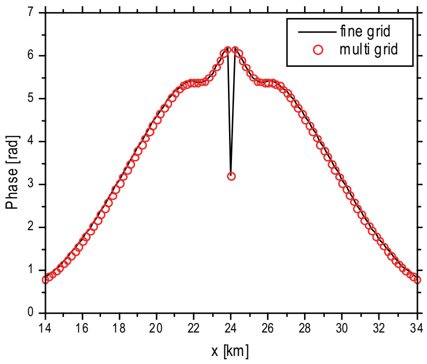

Figure 4: Phase plots of in-line electric field data for the model with a hydrocarbon reservoir obtained using fine-grid (black line) and multiple grids (red circles).

Model 2: Sabah area model

In this section we apply the developed IBC IE forward modeling method to a computer simulation of a synthetic MCSEM survey in the area of Sabah, Malaysia. Sarawak Shell Berhad, Shell International Exploration and Production, and PETRONAS Managing Unit planned a SeaBed Logging^{TM} (SBL) acquisition program to test the viability of the technology by acquiring data over geologically favorable target reservoirs in the Sabah area in 2004. They also carried out a survey for the bathymetry. We have included the detailed bathymetry data provided by Shell in this geoelectrical model. The location of the hydrocarbon reservoir was estimated from the seismic survey. We have used approximately the same location as in the real Sabah area, but we have assumed that the HC reservoir can be described by the same geoelectrical structure as in our Model 1.

The EM fields in this model are generated by an horizontal electric dipole (HED) transmitter with a length of 270 m, located at the point $(x, y) = (24 \text{ km}, 5 \text{ km})$ km at a depth of 50 m above the sea bottom. The transmitter generates the EM fields with a transmitting current of 1 A at a frequency of 0.25 Hz. An array of seafloor electric receivers is located 5 m above the sea bottom along a line with the coordinates $(x = \{14 \text{ km}, 34 \text{ km}\}, y = 5 \text{ km})$ with a spacing of 0.2 km (Figure 5).

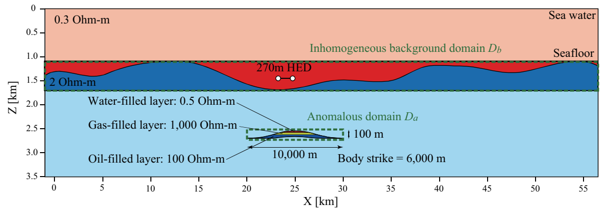

Figure 5: Sabah area model. A vertical section of a geoelectrical model of a hydrocarbon reservoir in the presence of rough seafloor bathymetry.

Following the main principles of the IBC IE method, the modeling area was represented by two modeling domains, D_a and D_b, outlined by the green dashed lines in Figure 5. Modeling domain D_b covers the area with conductivity variations associated with the bathymetry of the sea bottom, while modeling domain D_a corresponds to the location of the hydrocarbon reservoir. We used 7,193,600 (1124 × 200 × 32) cells with a cell size of 50 × 50 × 20 m^3 for a discretization of the bathymetry structure. The domain D_a of the hydrocarbon reservoir area was discretized in 1,536,000 (400 × 240 × 16) cells with a cell size of 25 × 25 × 6 m^3, as in model 1. A 3-D relief of the bathymetry is plotted in Figure 6.

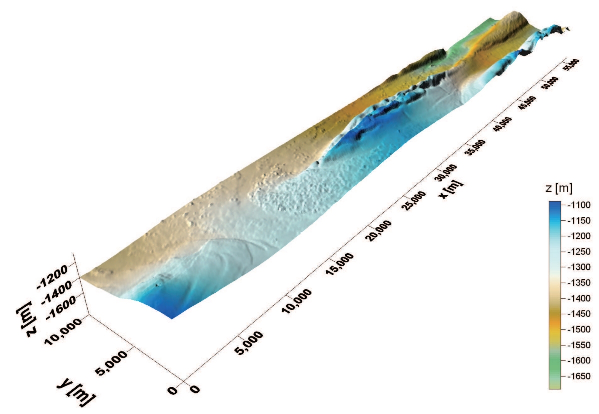

Figure 6: A 3D relief of the bathymetry for the Sabah model.

The solution of this model by any conventional IE method would require the simultaneous solution of the corresponding system equations on a grid formed by at least a combination of two domains, D_a and D_b, which have together 8.7 million cells. At the same time, the application of the IBC IE method allows us to separate the modeling domain into two subdomains, D_a and D_b. We solve the corresponding IE of the IBC method in these domains separately, which can save a lot of computer memory and computational time. Moreover, we can save the precomputed background field, which includes an inhomogeneous background field, and keep it unchanged for future computations with modified parameters of the reservoir. This fact results in an enormous reduction of the computing cost in interpretation of practical EM field data and also in the inverse problem solution. In addition, we have used the MGQL inversion of the iterative IBC method, which resulted in significant additional reduction of the required computer resources and computational time.

We have applied the iterative version of the MQGQL IBC IE method to modeling electric fields in a system of sea bottom receivers located on a rectangular grid with a separation between the receivers of 100 m in both x and y directions. The convergence plot for iterative IBC modeling is shown in Figure 7. One can see an excellent convergence rate in this figure. After just two iterations the relative errors reach about 2.3×10^{-13} within the inhomogeneous background (bathymetry) domain and about 2.7×10^{-13} within the anomalous (reservoir) domain.

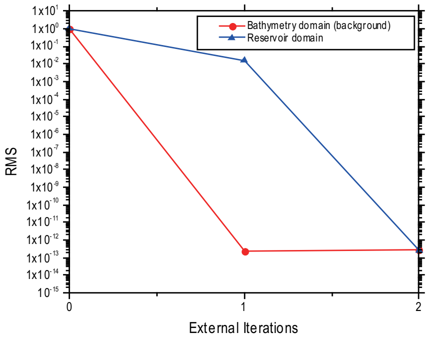

Figure 7: The convergence plot for iterative IBC modeling. The red line shows the relative error vs. iteration number for the inhomogeneous background (bathymetry) domain, while the blue line presents the same for the anomalous (reservoir) domain.

Figures 8 and 9 present maps of the absolute values of the x and y components of the total electric field computed using the iterative IBC in each iteration. Usually, the background fields are much larger than both of the anomalous and the inhomogeneous background fields. Therefore, the total fields seem not to change significantly from iteration to iteration, while each of the anomalous and the inhomogeneous field changes in each iteration as shown in Figure 7.

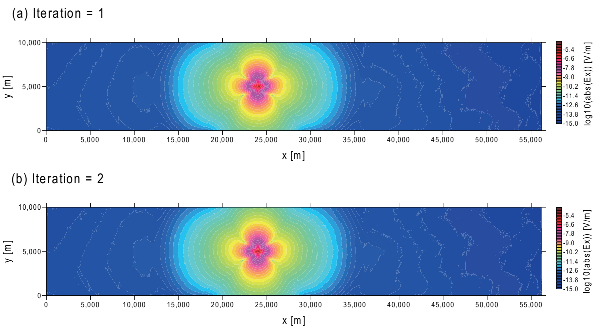

Figure 8: The map of absolute values of the x component of the electric field computed using the iterative IBC method (multigrid) in each iteration; (a) iteration =1, (b) iteration = 2.

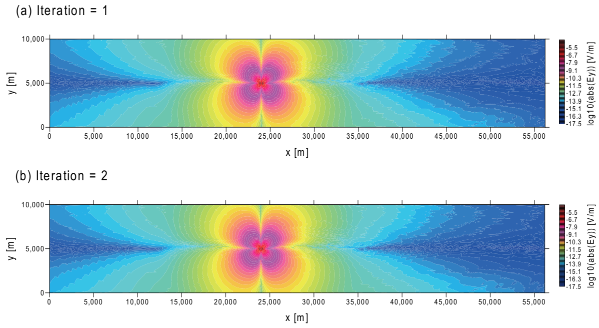

Figure 9: The map of absolute values of the y component of the electric field computed using the iterative IBC method (multigrid) in each iteration; (a) iteration =1, (b) iteration = 2.

CONCLUSIONS

In this paper, we have developed a parallel implementation of the new integral equation (IE) method. This new method can improve the accuracy of the solution by iterative calculation, and can reduce the computational cost using the multigrid QL approach.

We have applied a new parallel code based on the IBC IE method for modeling the marine controlled-source electromagnetic (MCSEM) data in the area with significant bathymetric inhomogeneities. Generally, it requires a huge number of discretization cells to describe three-dimensional targets in the presence of the complex seafloor bathymetry adequately. The multigrid QL version of IBC EM method allows us to separate this massive computational problem into at least two problems, which require relatively smaller number of discretizations.

At the same time, using the MQGQL IBC IE method,, we can precompute the bathymetry effect only once, and then repeat the computations on a smaller grid covering the anomalous domain only. Also, we have demonstrated that, the multigrid QL approach allows us to compute the EM fields with the less computational cost without loosing the accuracy.

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