Sensitivity analysis of marine CSEM surveys

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SUMMARY

Marine controlled-source electromagnetic method (MCSEM) is widely used now for offshore hydrocarbon exploration. In this paper we examine the sensitivity MCSEM surveys. We introduce a technique of electromagnetic (EM) data sensitivity analysis based on integrated sensitivity calculations for a survey formed by multiple transmitters and receivers. We use this technique to study the sensitivity of typical EM surveys to local inhomogeneities. We map the spatial distribution of a survey sensitivity with respect to the local anomalies of the conductivity of a geological formation. The calculated sensitivity values provide useful information for designing MCSEM surveys.

INTRODUCTION

Marine controlled-source electromagnetic (MCSEM) surveys become an important part of offshore hydrocarbon (HC) exploration. However, the question still remains open: How sensitive are the MCSEM data to the presence of a resistive reservoir within the conductive sea-bottom sediments? Another important question is related to the optimal survey design for off-shore HC exploration. In this paper we address some aspects of these questions using a novel approach to the sensitivity analysis of MCSEM surveys.

Traditionally, the sensitivity of a geophysical method is determined as the ratio of the variation of the data to the variation of the model parameters. The sensitivity can be found by directly modeling the theoretical response for the given model perturbation. However, this "brute force" approach requires a lot of computations and is extremely time-consuming. A more efficient technique for sensitivity analysis is based on the reciprocity principle (Rodi, 1976; McGillivray and Oldenburg, 1990; McGillivray et al., 1994; Spies and Habashy, 1995; Zhdanov, 2002). In the case of MCSEM surveys, however, even using the reciprocity principle may not completely solve the problem because these surveys are formed by multiple transmitters and receivers.

In this paper we introduce a novel approach to sensitivity analysis based on the integrated sensitivity which is used to evaluate a cumulative response of the observed data to the conductivity perturbations for an entire survey with a multiple-transmitter/receiver observational system. We also apply a new method of Fréchet derivative calculation (Gribenko and Zhdanov, 2007), using quasi-analytical approximation for a variable background (QAVB). The corresponding numerical method of Fréchet derivative computations based on explicit integral expressions simplifies all calculations dramatically.

We have applied this new technique to the sensitivity analysis of typical MCSEM surveys.

SENSITIVITY OF EM FIELD IN A 3-D MEDIUM

Differential sensitivities

We begin our paper with a summary of the basic principles of the sensitivity analysis of the EM field in a 3-D medium. The differential sensitivities $\mathbf{s}_E(\mathbf{r}'|\mathbf{r}'')$ and $\mathbf{s}_H(\mathbf{r}'|\mathbf{r}'')$ of the electric and magnetic fields at the point $\mathbf{r}'$ to the conductivity perturbations at the point $\mathbf{r}''$ are calculated by the formulae:

$$\mathbf{s}_E\left(\mathbf{r}'|\mathbf{r}''\right) = \frac{\delta\mathbf{E}\left(\mathbf{r}'\right)}{\delta\sigma},\ \mathbf{s}_H\left(\mathbf{r}'|\mathbf{r}\right) = \frac{\delta\mathbf{H}\left(\mathbf{r}'\right)}{\delta\sigma}. \tag{1}$$

Consider a 3-D geoelectrical model with an arbitrarily varying conductivity σ. We assume that $\mu = \mu_0 = 4\pi\times10^{-7}H/m$, where μ_0 is the free-space magnetic permeability. The model is excited by an electromagnetic field generated by an arbitrary source with an extraneous current distribution $\mathbf{j}^e$ concentrated within some local domain Q. This field is time harmonic as $e^{-i\omega t}$.

The electromagnetic field in this model satisfies Maxwell's equations:

$$\nabla\times\mathbf{H} = \sigma\mathbf{E} + \mathbf{j}^e, \tag{2}$$

$$\nabla\times\mathbf{E} = i\omega\mu_0\mathbf{H}. \tag{3}$$

We can derive the expressions for sensitivities by differentiating the Maxwell's equations and applying the integral equation method to perturbed solution of these equations. Then the perturbations of the electric and magnetic fields, $\delta\mathbf{E}(\mathbf{r}')$, $\delta\mathbf{H}(\mathbf{r}')$, corresponding to the perturbation of the conductivity tensor at a point $\mathbf{r}''$, are:

$$\begin{aligned}\delta\mathbf{E}\left(\mathbf{r}'\right) &= \widehat{\mathbf{G}}_E\left(\mathbf{r}'\mid\mathbf{r}''\right)\cdot\left[\delta\sigma\left(\mathbf{r}''\right)\mathbf{E}\left(\mathbf{r}''\right)\right],\\ \delta\mathbf{H}\left(\mathbf{r}'\right) &= \widehat{\mathbf{G}}_H\left(\mathbf{r}'\mid\mathbf{r}''\right)\cdot\left[\delta\sigma\left(\mathbf{r}''\right)\mathbf{E}\left(\mathbf{r}''\right)\right],\end{aligned} \tag{4}$$

where $\widehat{\mathbf{G}}_E$ and $\widehat{\mathbf{G}}_H$ are electric and magnetic Green's tensors. We arrive to the following equations:

$$\begin{aligned}\mathbf{s}_E\left(\mathbf{r}'|\mathbf{r}''\right) &= \widehat{\mathbf{G}}_E\left(\mathbf{r}'\mid\mathbf{r}''\right)\cdot\mathbf{E}\left(\mathbf{r}''\right),\\ \mathbf{s}_H\left(\mathbf{r}'|\mathbf{r}''\right) &= \widehat{\mathbf{G}}_H\left(\mathbf{r}'\mid\mathbf{r}''\right)\cdot\mathbf{E}\left(\mathbf{r}''\right).\end{aligned} \tag{5}$$

From the last formulae we see that Green's electromagnetic tensors provide the sensitivity estimation of the electromagnetic field to the model conductivity.

Integrated sensitivity

In many practical applications, it is useful to consider the notion of *integrated sensitivity,* $S_E(\mathbf{r}'')$, which combines differential sensitivities for all frequencies, all transmitters and all receivers. The integrated sensitivity of the data, collected over some surface Σ of observations over a frequency interval Ω, is equal to:

$$S_E\left(\mathbf{r}''\right) = \frac{\|\delta\mathbf{E}\|_{\Omega,\Sigma}}{\delta\sigma} = \sqrt{\int_\Omega\int\int_\Sigma\left|\delta\mathbf{E}\left(\mathbf{r}',\omega\right)\right|^2 ds'd\omega}.$$

Taking into account expressions (4), the integrated sensitivity of the electric field to the local perturbation of the conductivity at the point $\mathbf{r}''$ is:

$$S_E\left(\mathbf{r}''\right) = \sqrt{\int_\Omega\int\int_\Sigma\left|\widehat{\mathbf{G}}_E\left(\mathbf{r}'\mid\mathbf{r}''\right)\cdot\mathbf{E}\left(\mathbf{r}''\right)\right|^2 ds'd\omega}. \tag{6}$$

In a similar way we can find the integrated sensitivities of the magnetic field:

$$S_H\left(\mathbf{r}''\right) = \sqrt{\int_\Omega\int\int_\Sigma\left|\widehat{\mathbf{G}}_H\left(\mathbf{r}'\mid\mathbf{r}''\right)\cdot\mathbf{E}\left(\mathbf{r}''\right)\right|^2 ds'd\omega}. \tag{7}$$

Formulas (6) and (7) can be used for practical computation of the integrated sensitivities if we know the corresponding Green's tensors, $\widehat{\mathbf{G}}_E$ and $\widehat{\mathbf{G}}_H$.

The numerical method of integrated sensitivity computation

We can represent a numerical solution of the system of Maxwell's equations (2) and (3) in the form of a discrete operator equation:

$$\mathbf{d} = \mathbf{A}(\sigma), \tag{8}$$

where $\mathbf{d} = (d_1, d_2, d_3, ...d_N)$ is a vector of the observed EM data, $\mathbf{A}$ is a forward modeling operator for solving the system of Maxwell's equations, and $\sigma = (\sigma_1, \sigma_2, \sigma_3, .., \sigma_L)$ is a vector formed by the conductivity distribution in the model.

Let us analyze the sensitivity of the EM data to the perturbation of one specific parameter, $\delta\sigma_k$. To solve this problem, we apply the variational operator to both sides of equation (8):

$$\delta d_i = F_{ik}\delta\sigma_k. \tag{9}$$

In the last formula, F_{ik} are the elements of the Fréchet derivative matrix $\mathbf{F}$ of the forward modeling operator, and there is no summation over index k. Then the integrated sensitivity of the data to the parameter $\delta\sigma_k$ is determined as the ratio (Zhdanov, 2002):

$$S_k = \frac{\|\delta\mathbf{d}\|}{\delta\sigma_k} = \frac{\sqrt{\sum_i (\delta d_i)^2}}{\delta\sigma_k} = \frac{\sqrt{\sum_i (F_{ik})^2}\delta\sigma_k}{\delta\sigma_k} = \sqrt{\sum_i (F_{ik})^2}. \tag{10}$$

One can see that the integrated sensitivity depends on the parameter number k. In other words, the sensitivity of the data to the different parameters varies, because the contributions of the different parameters to the observation are also variable.

The diagonal matrix with the diagonal elements equal to $S_k = \|\delta\mathbf{d}\| / \delta\sigma_k$ is called *an integrated sensitivity matrix:*

$$\mathbf{S} = \mathbf{diag}\left(\sqrt{\sum_i (F_{ik})^2}\right) = \mathbf{diag}(\mathbf{F}^*\mathbf{F})^{1/2}. \tag{11}$$

In order to compute the integrated sensitivity, one should determine the Fréchet derivative matrix $\mathbf{F}$. We apply the quasi-analytical approximation for a variable background (QAVB) developed by Gribenko and Zhdanov (2007). This method provides explicit integral representations of the Frechet derivative for models with inhomogeneous backgrounds, so the calculations are simplified dramatically.

SENSITIVITY ANALYSIS OF TYPICAL MCSEM SURVEYS

The typical MCSEM survey is formed by a set of sea-bottom electrical and magnetic receivers and a horizontal electric dipole transmitter towed at some elevation above the sea bottom (e.g., Eidesmo et al., 2002; Carazzone et al., 2005). In order to understand better the basic properties of this survey, we begin our sensitivity analysis with a simple basic survey design consisting of just one receiver and an electric dipole transmitter moving above this receiver in the x direction along a 16 km line at an elevation of 50 m above the sea bottom. The receiver is located at a depth of 5 m above the sea floor and measures the E_x component of the electric field and the H_y component of the magnetic field. The transmitter generates a frequency domain EM field with a frequency of 0.25 Hz from points every 100 m along the transmitter line. The maximum and minimum transmitter-receiver offsets are 7 km and 600 m, respectively. The background geoelectrical model consists of a sea-water layer with a thickness of 300 m and a resistivity of 0.25 Ohm-m and a layer of conductive sea-bottom sediments with a resistivity of 1 Ohm-m.

First of all, we should note that, in the case of the MCSEM survey, the observed data are usually normalized by the amplitude of the background field. In other words, we usually work with the weighted data:

$$\mathbf{d}_w = \mathbf{W}_d\mathbf{d}, \tag{12}$$

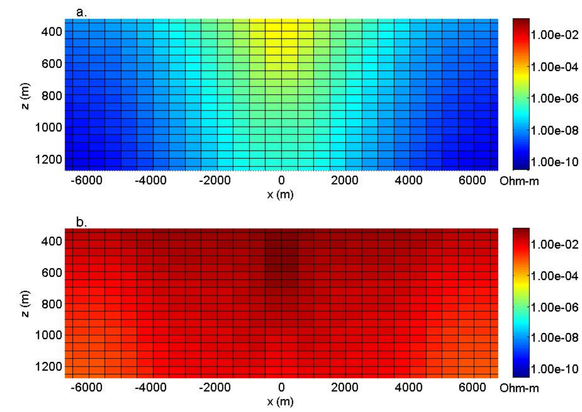

Figure 1: The integrated sensitivity distributions for the basic survey design in the vertical section along the x axis. The top panel presents the normalized sensitivity S_k^{norm} (without data weights), while the bottom panel presents the sensitivity S_k^w with data weighting for the E_x component.

where the data weighting matrix is determined by the inverse absolute value of the background (normal) electric or magnetic fields for electric and magnetic observations, respectively. The integrated sensitivity of the weighted data to the parameter $\delta\sigma_k$ is determined according to the following formula:

$$S_k^w = \frac{\|\delta\mathbf{d}_w\|}{\delta\sigma_k}. \tag{13}$$

Formula (11) for the weighted integrated sensitivity matrix takes the form:

$$\mathbf{S} = \mathbf{diag}(\mathbf{F}^\star\mathbf{W}_d^\star\mathbf{W}_d\mathbf{F})^{1/2}. \tag{14}$$

Taking into account that the weighted data in expression (12) are dimensionless and the inverse conductivity $\frac{1}{\delta\sigma_k}$ is measured in Ohm-m, we immediately conclude that the weighted sensitivities S_k^w are measured in the units of the resistivity, Ohm-m.

In order to demonstrate the importance of data weighting, let us consider Figure 1. This Figure presents the sensitivity distributions for the basic survey design in the vertical section along the x axis with and without data weighting for the E_x component. Note that, in order to be able to compare these two sensitivity distributions, we have plotted the original sensitivities normalized by the norm of the background (normal) data $\|\mathbf{d}_b\|$:

$$S_k^{norm} = S_k / \|\mathbf{d}_b\| = \frac{\|\delta\mathbf{d}\|}{\delta\sigma_k} / \|\mathbf{d}_b\|. \tag{15}$$

One can see that the application of the data weights increases the integrated sensitivity of the survey significantly. That is why data weighting is important in MCSEM data interpretation.

We have conducted several numerical experiments to examine the effects of different survey parameters on the integrated sensitivity distribution. We have considered the effects of the maximum and minimum transmitter-receiver offsets, the separations between the transmitters and receivers, the frequency of the transmitted EM signal, the different components of EM field, and the number and orientation of the lines in the survey. In this paper we presents results of this analysis for last three parameters.

Effect of the frequency of the transmitted EM signal

In this experiment the integrated sensitivity of the E_x component was calculated for different frequencies: 0.25Hz, 0.75Hz, and 1.25Hz (Figure 2). We can observe in this figure that the higher the frequencies are,

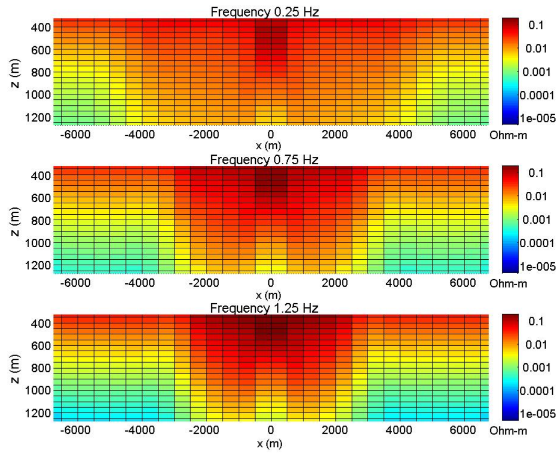

Figure 2: The integrated sensitivity distributions of the E_x component for different frequencies of the transmitted EM signal.

the more concentrated and higher sensitivity is in the vicinity of the receiver. As one would expect, the sensitivity decreases faster with the depth and distance from the transmitter when there is an increase in the frequency, which is a simple manifestation of the skin-effect.

Effect of the different components of the observed EM field

In the next experiment we calculated the sensitivity for the different components of the observed EM field. We have compared the integrated sensitivities for the H_y component of the magnetic field, the E_x component of the electric field, and for the joint electric and magnetic observations (Figure 3). One can see that, the sensitivity is more intense for the magnetic field H_y than for the electric field E_x. The joint electric and magnetic components, E_x and H_y, have stronger sensitivity than the individual components. This is a very important observation, which explains the benefit of the joint inversion of the electric and magnetic data.

Effect of the survey design (number and orientation of lines in the survey)

We have investigated also the effect of the survey design on the integrated sensitivity of the observed data. Figure 4 presents two typical surveys, that we have considered. The first survey has one observational line with three receivers and multiple transmitters (Figure 4, top panel). The second survey is formed by one line of three receivers and two parallel lines of multiple transmitters (Figure 4, bottom panel). The distance between receivers along the corresponding line is 1.5 km. The transmitters generate a frequency domain EM field with a frequency of 0.25 Hz from points located every 100 m along the corresponding transmitter line. The maximum and minimum transmitter-receiver offsets are 7,000 m and 600 m, respectively. For the survey with two parallel transmitter lines, the distance between the lines is 3 km (see Figure 4, bottom panel).

We have computed the integrated sensitivities for these two surveys. The corresponding cross-sections along the x and y sections are presented in Figures 5 and 6, respectively. The most intense sensitivity is the one for survey #2 with two parallel lines, especially around the receivers. In the y direction the domain with the high sensitivity is also twice wider and a little bit deeper than for a one-line survey, but it is bounded between the transmitter lines in the x direction (Figure 5). Overall, we conclude that using two parallel lines of transmitters increases the azimuthal sensitivity of the receivers located along one line.

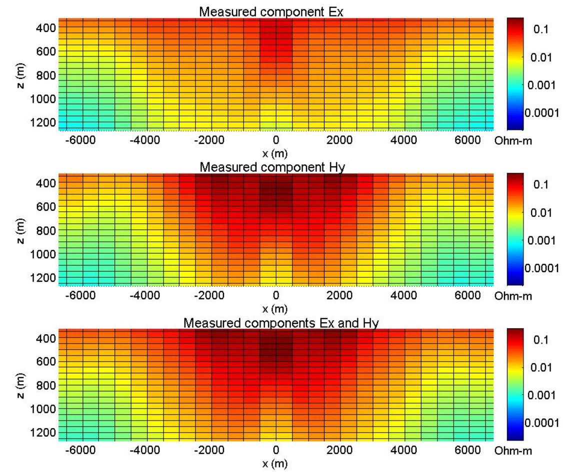

Figure 3: The integrated sensitivity distributions of the E_x component (top panel), the H_y component (midle panel), and the joint electric and magnetic, E_x and H_y, data for the basic survey design.

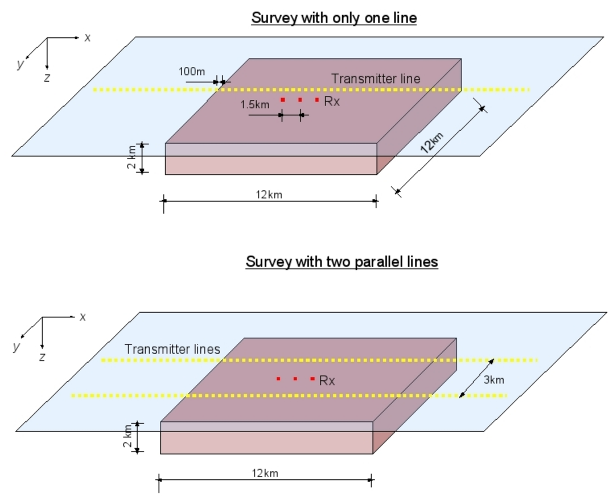

Figure 4: Survey configurations.

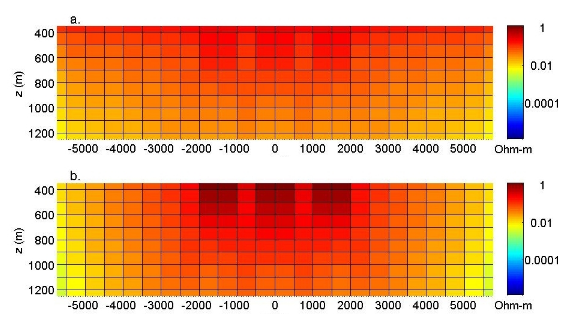

Figure 5: The integrated sensitivity distributions for different survey configurations in the vertical section along the x axis.The top panel corresponds to the one-line survey, the bottom panel corresponds to the two-line survey.

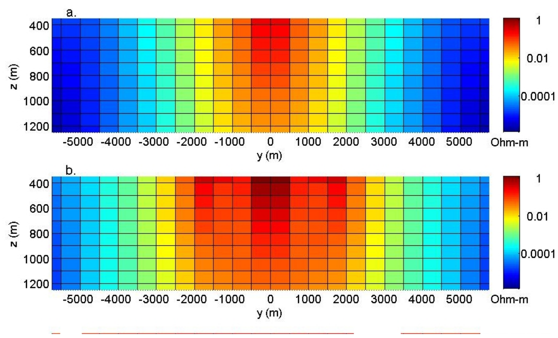

Figure 6: The integrated sensitivity distributions for different survey configurations in the vertical section along the *x* axis. The top panel corresponds to the one-line survey, the bottom panel corresponds to the two-line survey.

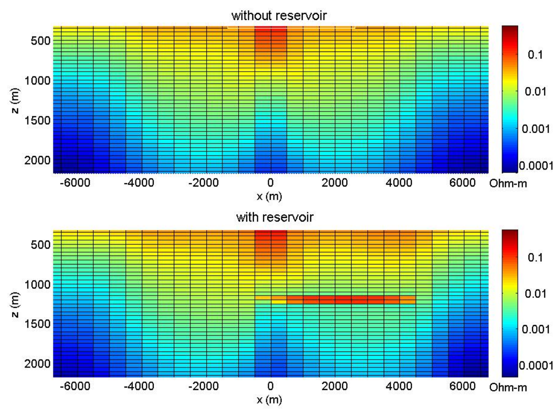

Figure 7: The integrated sensitivity distributions in the vertical section along the *x* axis. The top panel presents the sensitivity without the resistive body, while the bottom panel shows the sensitivity with the resistive body.

The sensitivity of the MCSEM data to the depth and thickness of the sea-bottom resistive target (petroleum reservoir)

The final numerical experiment is aimed at the analysis of the sensitivity of the MCSEM data with respect to the depth and thickness of the sea-bottom resistive target (e.g., a petroleum reservoir). In this experiment we consider a reservoir of resistivity 50 Ohm-m with the fixed length of 5 km and width of 1.5 km. The thickness of the reservoir varies from 50 m to 300 m, and the depth of the top of the reservoir varies from 350 m to 1850 m.

We have computed the integrated sensitivity for this model using the numerical technique developed in this paper. These calculations were done for the different depths and thicknesses of the reservoir. An example of the integrated sensitivity distribution in the vertical section for the reservoir with thickness 100 m and the depth 1150 m is shown in Figure 7.

We use these sensitivity plots to calculate the integrated sensitivity for different thicknesses and depths of the resistive body, as shown in Figure 8. The closer the body is to the sea floor, the higher the sensitivity is. The dependence of the sensitivity on the reservoir thickness is significantly weaker. However, the larger thickness corresponds to the higher sensitivity.

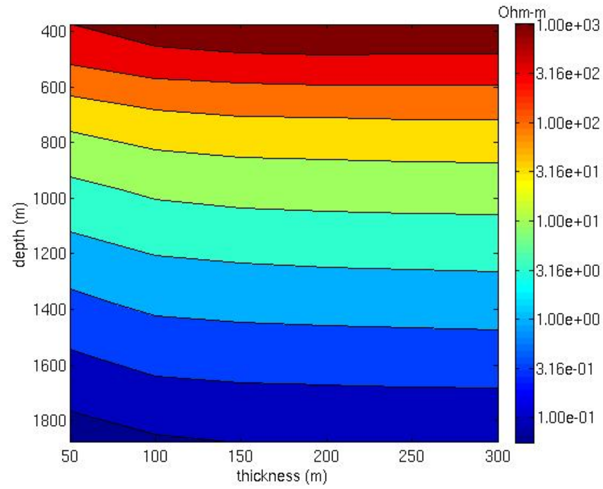

Figure 8: The integrated sensitivity for different thicknesses and depths of the resistive body.

CONCLUSIONS

In this paper, we introduce a simple technique of EM data sensitivity analysis based on the reciprocity principle and integrated sensitivity calculations. We have applied this technique to evaluate the sensitivities of the various survey configurations used in MCSEM geophysical methods. We have showed the effects of the frequency of the transmitted EM signal, the different components of EM field, and the number and orientation of the lines in the survey. All these parameters play an important role in determining the overall sensitivity of the observed MCSEM data.

In summary, we conclude that the developed methodology of sensitivity analysis can provide useful information for planning and designing MCSEM surveys for offshore petroleum exploration.

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