Two-Dimensional Time Domain Electromagnetic Migration Using Integral Transformation

Toshiko Furukawa and Michael S. Zhdanov, University of Utah

SUMMARY

The method of time domain electromagnetic (TDEM) migration is used for fast imaging of subsurface geoelectrical structures. The computation of the migration field is based on downward extrapolation of the observed field in reverse time. In this paper we examine an effective method of EM migration based on the operator of an integral transformation in the spatial-temporal domain. The migration conductivity image is constructed using the convolution of the background and migration fields. We demonstrate in this paper that the method of EM migration can be effectively used for interpretation of multi-transient electromagnetic (MTEM) data.

INTRODUCTION

Time domain (transient) electromagnetic (TEM) methods are widely used in exploration geophysics both in mineral and petroleum applications (Zhdanov and Keller, 1994). Until recently, the interpretation of these methods was based on a simple 1D inversion of the data at any given observation point. This approach was reasonable for TEM surveys with relatively large separation between the observation points. A new modification of this technique has emerged recently, a so-called multi-transient electromagnetic (MTEM) method (Hobbs et al., 2005). In the framework of this technique the survey layout is similar to that of seismics, with multiple receivers and multiple transmitters. In this case, one still can use a conventional 1D inversion for interpretation of MTEM data by collating the 1D inversion results to form a 2D resistivity model of the geological formation. However, this 2D result may provide a very distorted image of the true complex 3D subsurface structure.

The rigorous 2D and/or 3D inversion of the MTEM data is a very challenging problem because of the enormous computations required for forward modeling the multi-transmitter and multi-receiver MTEM data. In this paper we discuss an alternative approach to MTEM data interpretation based on the principles of time domain electromagnetic (TDEM) migration. The basic principles of TDEM migration were formulated in publications by Zhdanov (1981, 1988, 1999, 2001, and 2002). In the current paper we apply these principles to fast imaging of MTEM data.

For the sake of simplicity, in this paper we will consider a 2D migration method only. We use the simple but effective technique of integral transformation to compute the migration EM field. The migration images are produced by a convolution between the migration and background EM fields computed inside the subsurface domain.

The migration imaging is tested on synthetic models of the MTEM survey over a resistive target (e.g., a hydrocarbon reservoir) and over a conductive target (e.g., an ore body). The numerical study demonstrates that this technique produces quite adequate images of subsurface geoelectrical structures from the MTEM data.

MIGRATION OF A 2D EM FIELD

Migration field equations

Consider a two-dimensional (2D) geoelectrical model in which the horizontal plane, $z=0$ separates the conductive earth $(z>0)$ from the insulating atmosphere $(z<0)$. In order to simplify our discussion of two-dimensional model, we will use coordinate systems in which the long extent of the two-dimensional structure lies along the y axis, and the axis z is directed downward. We will also assume that the current density in the transmitter does not vary along the y axis. Therefore, the electromagnetic field in a given model will also be two-dimensional; that is, it will not vary along the y axis.

The conductivity in the earth, $\sigma(x,z)$, is an arbitrary function of the coordinates. This conductivity function can be represented as the superposition of a constant background conductivity, σ_b, and an anomalous conductivity function, $\Delta\sigma(x,z)$:

$$\sigma(x,z)=\sigma_b+\Delta\sigma(x,z). \tag{1}$$

Everywhere outside the anomalous region, the electromagnetic field will satisfy a diffusion equation:

$$\widetilde{\nabla}^2\mathbf{H}-\mu_0\sigma_b\frac{\partial\mathbf{H}}{\partial t}=0, \tag{2}$$

$$\widetilde{\nabla}^2\mathbf{E}-\mu_0\sigma_b\frac{\partial\mathbf{E}}{\partial t}=0, \tag{3}$$

where $\widetilde{\nabla}$ is a two-dimensional Hamiltonian operator: $\widetilde{\nabla}=(\partial/\partial x,\ \partial/\partial z)$, and $\mu_0=4\pi\times10^{-7}H/m$ is the free-space magnetic permeability.

According to the general principles of EM migration, we introduce a reverse time:

$$\tau=T-t,$$

where T is the interval of the transient EM field observation.

Let $P^0(x,z,t)$ represent any of the components H_x^0,H_y^0,H_z^0,E_x^0 or E_y^0 that can be observed at the earth's surface. We will define the *migration field,* P^m, obtained from a specific scalar component, P^0, of the electromagnetic field as being the field that satisfies the following conditions:

$$P^m(x,z,\tau)\mid_{z=0}=\begin{cases}P^0(x,z,T-\tau)_{z=0} & for\,0\le\tau\le T\\ 0 & for\,\tau<0,\,\tau>T\end{cases}, \tag{4}$$

$$\widetilde{\nabla}^2P^m(x,z,\tau)-\mu_0\sigma_b\frac{\partial P^m(x,z,\tau)}{\partial\tau}=0\,for\,z>0, \tag{5}$$

$$P^m(x,z,\tau)\to0,\,for\sqrt{x^2+z^2}\to\infty,\,\dot{z}>0. \tag{6}$$

Note that if we exchange reverse time, τ, for ordinary time, t, in equation (5), we have an equation which is the conjugate to the diffusion equation:

$$\widetilde{\nabla}^2P^m(x,z,T-t)+\mu_0\sigma_b\frac{\partial P^m(x,z,T-t)}{\partial t}=0. \tag{7}$$

If the ordinary diffusion equation describes field propagation from the sources to receivers, then equation (7) describes the inverse process of propagation from receivers to sources.

The problem of establishing the migration field reduces to the continuation of the field P^0 from the earth's surface into the lower half-space in reverse time, τ. This procedure is called *electromagnetic field migration*.

It can be seen from these considerations that the calculation of a migrated field is reduced to the boundary value problem described by equations (4) through (6).

Migration based on integral transformation of the electromagnetic field

For a solution of the migration problem, we will specify an arbitrary point $\mathbf{r}' = (x', z')$ in the lower half-plane and draw a circle about this point with radius R. The part of the circle located in the lower half-plane will be denoted as O_R and the part of the line $z = 0$ located inside O_R will be denoted as L_R. The domain S is bounded by O_R and L_R. Let us consider a Green's function for the 2D diffusion equation, G^{2d}, which satisfies the equation:

$$\widetilde{\nabla}^2 G^{2d} + \mu_0 \sigma_b \frac{\partial G^{2d}}{\partial t} = \delta(\mathbf{r} - \mathbf{r}')\delta(t - t'), \quad (8)$$

where G^{2d} has the form (Morse and Feshbach, 1953):

$$G^{2d}(\mathbf{r}', t' \mid \mathbf{r}, t) = -\frac{1}{4\pi(t' - t)} \exp\left(-\mu_0 \sigma_b \mid \mathbf{r}' - \mathbf{r} \mid^2 / 4(t' - t)\right) H(t' - t), \quad (9)$$

written for a Heaviside excitation function, $H(t' - t)$:

$$H(t - t') = \begin{cases} 0; & t' - t < 0 \\ 1; & t' - t > 0 \end{cases} .$$

It can be demonstrated that the solution of the migration field equation with the corresponding boundary conditions is given by the following expression:

$$P^m(\mathbf{r}', T - t') =$$

$$\frac{(\mu_0 \sigma_b)(z')}{4\pi} \int_{t'}^{T} \int_{-\infty}^{\infty} P^0(\mathbf{r}, t) \frac{1}{(t - t')^2} e^{-\mu_0 \sigma_b \frac{(x' - x)^2 + (z')^2}{4(t - t')}} \, dx dt. \quad (10)$$

Formula (10) gives us an explicit form of the integral transformation for numerical 2D EM migration.

IMAGING SUBSURFACE RESISTIVITY USING ELECTROMAGNETIC MIGRATION

Migration imaging condition

It was demonstrated in Zhdanov (2002), that the corresponding imaging condition can be introduced by comparing the migration with the conventional Newton method of geophysical inversion. Indeed, the Newton method is based on the idea that one can find a solution of the inverse problem in one iteration as well.
We describe the corresponding inverse problem as a solution of the operator equation:

$$\mathbf{d} = A(\sigma), \quad (11)$$

where A is a nonlinear forward operator determined by the survey configuration, and σ is a model parameter vector formed by the anomalous conductivity distribution in a model with the given background conductivity σ_b:

$$\sigma = [\Delta\sigma_1, \Delta\sigma_2, ...\Delta\sigma_L]^T,$$

where L is the number of cells in the inversion domain, and $\mathbf{d}$ is a data vector, $\mathbf{E}^a$, formed by the anomalous electric fields observed in the receiver positions:

$$\mathbf{d} = \mathbf{E}^a = \{\mathbf{E}^a(\mathbf{r}_i), \, i = 1, 2, ...I\}. \quad (12)$$

We use Maxwell's equations to describe the forward modeling operator $A(\sigma)$. In the framework of the regularization theory, the solution of the inverse problem (11) is reduced to the minimization of the Tikhonov parametric functional:

$$P^{\alpha}(\sigma) = \varphi(\sigma) + \alpha s(\sigma) = \min, \quad (13)$$

where $\varphi(\sigma)$ is a misfit functional between the theoretical predicted $A(\sigma)$ and the observed data $\mathbf{d}$, $s(\sigma)$ is a stabilizing functional. In particular, we can write the parametric functional (13) as :

$$P^{\alpha}(\sigma) = \|\mathbf{W}_d (A(\sigma) - \mathbf{d})\|^2 + \alpha \left\|\mathbf{W}_m (\sigma - \sigma_{apr})\right\|^2 = \min, \quad (14)$$

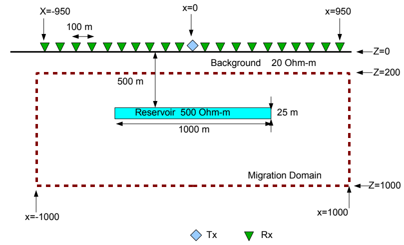

Figure 1: The detailed configuration of Model 1a used in the migration experiment.

where $\mathbf{W}_d$ and $\mathbf{W}_m$ are data and model parameter weighting matrices, α is the regularization parameter, and $\sigma_{apr} = \sigma_b$ is an a priori model equal in this case to the background conductivity.

In the framework of the Newton method one finds a solution of minimization problem (14) in one step (Zhdanov, 2002):

$$\begin{aligned} \sigma_1 &= \sigma_b + \delta\sigma \\ &\approx \sigma_b - k(\mathbf{W}_m^{\star}\mathbf{W}_m)^{-1} \mathbf{l}_0, \end{aligned} \quad (15)$$

where k is a scaling factor, $\mathbf{W}_m$ is model parameters weighting matrix, and $\mathbf{l}_0$ is a gradient direction, equal to a convolution of the background electric field, $\mathbf{E}^b$, and of the corresponding migration field:

$$\mathbf{l}_0 = \{l_0(\mathbf{r}_l), \, l = 1, 2, ...L\} =$$

$$\left\{ \int_T \mathbf{E}^b(\mathbf{r}_l, t) \cdot \mathbf{E}^m(\mathbf{r}_l, -t) \, dt, \, l = 1, 2, ...L \right\}. \quad (16)$$

Substituting formula (16) into equation (15), we arrive at the corresponding imaging condition:

$$\sigma_1 \approx \sigma_\mathbf{b} - k(\mathbf{W}_m^{\star}\mathbf{W}_m)^{-1} \left\{ \int_T \mathbf{E}^b(\mathbf{r}_l, t) \cdot \mathbf{E}^m(\mathbf{r}_l, -t) \, dt, \, l = 1, 2, ...L \right\}. \quad (17)$$

We will call the conductivity distribution obtained by formula (17) a *migration apparent conductivity.*

NUMERICAL RESULTS

Model 1: a resistive reservoir in a homogeneous half-space

Model 1a: a single dipole transmitter

The model is formed by a resistive reservoir located within a homogeneous half-space of 20 Ohm-m resistivity. The rectangular reservoir has a size of 1000 m by 25 m in the x and z directions, respectively, and its resistivity is of 500 Ohm-m. An electric dipole transmitter is located on the surface of the earth at $x = 0$. The current in the dipole source is 1 A. The electric field receivers are located on the ground every 100 meters along the x axis from $x = -950$ m up to $x = 950$ m. The migration domain is extended from 200 m down to 1000 m in the vertical direction and from $x = -1000$ m up to $x = 1000$ m in the horizontal direction. The detailed configuration of Model 1a is shown in Figure 1.

Migration field

We have developed a working version of the migration code based on equation (10). The migration field is computed in the nodes of the rectangular grid Σ covering the migration domain with a cell size of

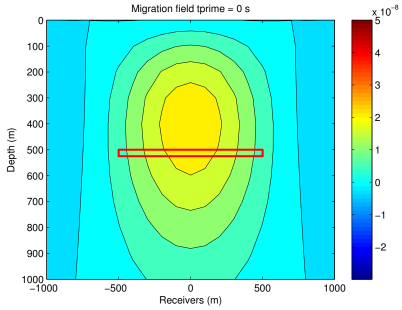

Figure 2: A snapshot of the migration field at migration time equals zero.

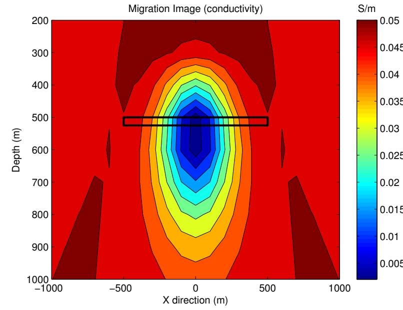

Figure 3: The final migration conductivity image for Model 1a. The rectangular box in the middle of this Figure shows the location of the reservoir.

100 m by 100 m from -1000 m to 1000 m in the x direction, and from 200 m to 1000 m in the z direction. We produce the migration field at zero migration time, $t' = 0$, and at logarithmically equally spaced time moments from 1 millisecond to 1 second with 10 time moments per one decade. Figure 2 shows a snapshot of the migration field at zero migration time, $t' = 0$. We can see that the maximum of the migration field is located above the center of the reservoir shown by the solid contour. Note that, the migration requires specification of the background conductivity, which can be found by 1D inversion of the data observed outside the anomalous domain.

Migration image

According to the imaging condition (17), the conductivity model can be obtained by convolution of the migration field with the background electric field. Note that the migration field depends on three parameters: spatial coordinates x and z, and time, t'. The background electric field is computed using the forward modeling code for the nodes of the same rectangular grid Σ covering the migration domain as the migration field and for the same time moments. The gradient direction $\mathbf{l}_0$ is obtained in the nodes of the grid Σ using a numerical analog of the convolution formula (16).

The coefficient k in equation (17) is a scaling factor. In practical applications this coefficient can be selected by minimizing the misfit between the observed and predicted data for the migration conductivity model:

$$\varphi(k) = \|A(\sigma_1) - \mathbf{d}\|^2 = \left\| A(\sigma_b - k(\mathbf{W}_m^{\star}\mathbf{W}_m)^{-1}\mathbf{l}_0) - \mathbf{d} \right\|^2 = \min. \tag{18}$$

In our model study we have used an empirical value for k, assuming that we know the true anomalous conductivity of the reservoir equal to-0.048 S/m.

The final conductivity model is constructed following equation (17). The corresponding migration conductivity image is shown in Figure 3.

The migration image in Figure 3 seems to be diffused around the center of the reservoir location. We can estimate the depth of the reservoir, but the horizontal resolution is not good enough to characterize the horizontal extension of the reservoir. In order to improve the horizontal resolution, we need to migrate simultaneously the electric field data recorded for multiple transmitters distributed along the x-axis on the surface of the earth.

Model 1b: multiple dipole transmitters

We consider the same Model 1 of a resistive reservoir within the conductive host rock. However, we now computer-simulate a MTEM survey with the multiple transmitters, Tx_j $(j = 0, 1, 2, ...N)$ located every 200 m along the x axis from $x = -1000$ m to $x = 1000$ m. The delta-pulse signal generated by every transmitter, Tx_j, is recorded by a corresponding set of receivers distributed every 100 meters along the profile with the maximum offset of 950 m from the transmitter. A schematic configuration of the MTEM survey for Model 1b is shown in Figure 4. We use the same migration domain for this model as for Model 1a. The electric field observed in the corresponding receivers for every transmitter position is migrated in the lower half-space using the integral migration transformation (10) to produce the corresponding migration field $\mathbf{E}_j^m(\mathbf{r},t)$, where j is the index of the transmitter Tx_j. We also compute the corresponding background electric fields, $\mathbf{E}_j^b(\mathbf{r},t)$, for every transmitter position. After that, the total migration field $\widetilde{\mathbf{E}}^m(\mathbf{r},t)$ and the total background field $\widetilde{\mathbf{E}}^b(\mathbf{r},t)$ are determined by summations:

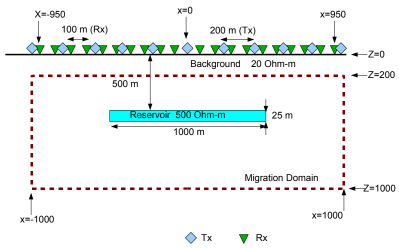

Figure 4: A schematic configuration of the MTEM survey with multiple transmitters and receivers for Model 1b.

$$\widetilde{\mathbf{E}}^m(\mathbf{r},t) = \sum_{j=1}^{N} \mathbf{E}_j^m(\mathbf{r},t), \; \widetilde{\mathbf{E}}^b(\mathbf{r},t) = \sum_{j=1}^{N} \mathbf{E}_j^b(\mathbf{r},t). \tag{19}$$

These fields are used in the migration imaging conditions (17) to produce the final conductivity model for the MTEM survey. This final conductivity image is shown in Figure 5. One can see that the horizontal extent of the reservoir is reconstructed in this image quite well.

Model 2: conductive body in a homogeneous half-space

Model 2a: a single dipole transmitter

Model 2a is formed by a rectangular conductive body located within a homogeneous half-space of 20 Ohm-m resistivity (Figure 6). The conductor has a size of 400 m by 200 m in the x and z directions, respectively, and its resistivity is 1 Ohm-m. The configuration of the electrical survey is the same as in Model 1a. An electric dipole transmitter is located on the surface of the earth at $x = 0$. The current in the dipole source is 1 A. The electric field receivers are located on the ground every 100 meters along the x axis from $x = -950$ m up to

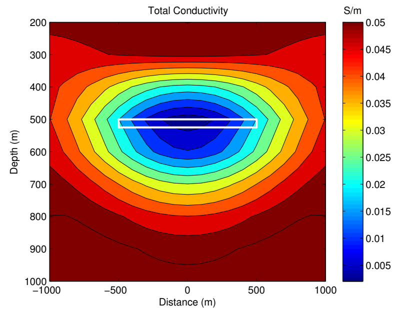

Figure 5: The final migration conductivity image for Model 1b. The rectangular box in the middle of this Figure shows the location of the reservoir.

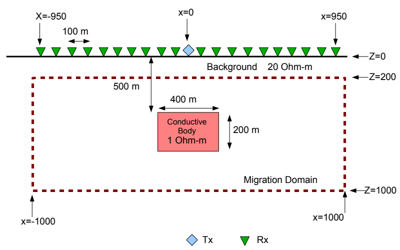

Figure 6: The detailed configuration of Model 2a used in the migration experiment. Tx denotes the transmitter position, while Rx corresponds to the receiver positions.

$x = 950$ m. The migration domain is extended from 200 m down to 1000 m in the vertical direction and from $x = -1000$ m up to $x = 1000$ m in the horizontal direction.

We have applied the migration transformation to the synthetic time domain EM data computed for this model. Figure 7 shows the final migration conductivity image for this model. We can clearly see the conductive target in this image.

Model 2b: multiple dipole transmitters

Finally, we consider the MTEM survey for a model with a conductive body. We use the same survey configuration as for a model of a resistive target. The corresponding migration conductivity image is shown in Figure 8. We can see reasonably well the conductive body in this image. Note that the computation of the migration image is an extremely fast operation. It takes just a few seconds on a PC to

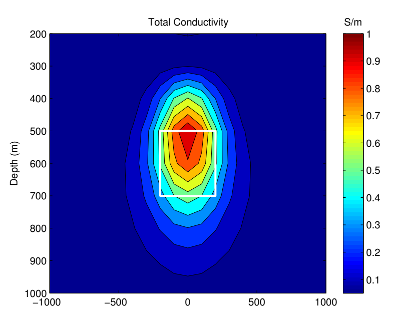

Figure 7: The final migration conductivity image for Model 2a. The rectangular box in the middle of this figure shows the location of the conductive body.

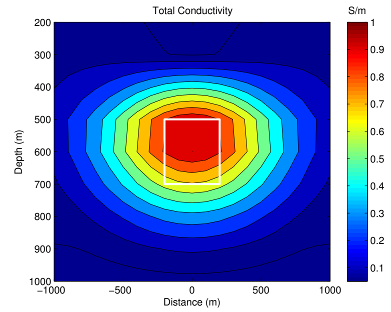

Figure 8: he final migration conductivity image for Model 2b. The rectangular box in the middle of this figure shows the location of the conductive body.

compute the migration field, and a few more minutes to generate the conductivity image.

CONCLUSIONS

In this paper we have introduced a new imaging technique for fast interpretation of multi-transient time domain EM data. This technique is based on the principles of time domain EM holography/migration. The migration field is produced by an integral transformation operator acting on the spatial-temporal distribution of the observed electric field in the receivers. This transformation produces a migration field as a function of spatial coordinates and time. The time domain convolution of the migration and the background electric fields generates a conductivity distribution in the vertical section.

Thus, the TDEM migration allows us to quickly generate a conductivity image of the subsurface structures.

In the current paper we have implemented a 2D version of this technique. Future research will be focused on extending the migration imaging for full 3D interpretation of the MTEM data.

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