Large-scale electromagnetic modeling for multiple inhomogeneous domains

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SUMMARY

We present an integral equation (IE) method for three-dimensional (3D) electromagnetic (EM) field computations in large-scale models with multiple inhomogeneous domains. This method can take into account the EM coupling between the different inhomogeneous domains by making iterative calculations. The method was tested for modeling the marine CSEM field for complex geoelectrical structures with multiple inhomogeneous domains, such as bathymetry, salt domes, and reservoirs. Because this method is based on the IE approach, it can calculate the response of each inhomogeneous domain separately. We have also investigated the return induction effects from regional geoelectrical structures, e.g., bathymetry and salt domes, which can distort the EM response from the geological exploration target.

INTRODUCTION

In the framework of the IE method, the conductivity distribution is divided into two parts: 1) the background conductivity, σ_b, which is used for the Green's functions calculation, and 2) the anomalous conductivity, $\Delta\sigma_a$, within the domain of integration, D. One principal advantage of the IE method over the other numerical techniques is that the IE method requires discretization of the anomalous domain D only. It is very well known, however, that the main limitation of the IE method is that the background conductivity model must have a simple structure to allow for an efficient Green's function calculation.

The most widely used background models in EM exploration are those formed by horizontally homogeneous layers. Any deviation from this 1D background model must be treated as an anomalous conductivity. In some practical geological applications, however, it is difficult to describe an earth structure using a horizontally layered background conductivity model, which is required for the efficient implementation of the conventional IE approach. As a result, a large domain of interest with anomalous conductivity distribution needs to be discretized. This discretization may become too large, however, for a feasible calculation of the fields generated by the geoelectrical structures.

Zhdanov et al. (2006) have recently developed a method to address this problem, the inhomogeneous background conductivity (IBC) IE method. In the current paper we have extended this iterative IBC IE method to the modeling of multiple inhomogeneous domains. In the framework of this method, we can construct a model with any number of inhomogeneous domains and take into account the return induction effects between any pairs of the inhomogeneous domains by using the iterative method. The important point is that by using this method we can evaluate the individual response from every domain, which includes the possible EM coupling effects between the different domains. A rigorous separate calculation of the EM fields produced by different anomalous domains representing different geological structures (e.g., a salt dome and a hydrocarbon [HC] reservoir) represents an important practical problem of EM exploration. In summary, in this paper we demonstrate the effectiveness of the new forward-modeling method and also examine the effects of the EM coupling between the different inhomogeneous domains which can distort a useful EM anomaly and complicate the interpretation of the marine CSEM data.

INTEGRAL EQUATION FORMULATION FOR MULTIPLE INHOMOGENEOUS-DOMAIN MODELING

We assume that N inhomogeneous domains (D_i, $i = 1,\ldots,N$) are located within a horizontally layered earth (Figure 1). The conductivity of the horizontally layered earth (normal conductivity) is σ_n, while the inhomogeneous (anomalous) conductivity within each inhomogeneous domain is denoted as $\Delta\sigma_{D_i}$ ($i = 1,\ldots,N$). The total EM fields at any point $\mathbf{r}$, $\mathbf{E}^t(\mathbf{r})$, and $\mathbf{H}^t(\mathbf{r})$, can be expressed as a sum of the normal fields $\mathbf{E}^n(\mathbf{r})$, $\mathbf{H}^n(\mathbf{r})$, and the EM fields induced by every inhomogeneous domain $\mathbf{E}^{\Delta\sigma_{D_i}}(\mathbf{r})$, $\mathbf{H}^{\Delta\sigma_{D_i}}(\mathbf{r})$ ($i = 1,\ldots,N$):

$$\mathbf{E}^t(\mathbf{r}) = \mathbf{E}^n(\mathbf{r}) + \sum_{i=1}^{N} \mathbf{E}^{\Delta\sigma_{D_i}}(\mathbf{r}) = \mathbf{E}^n(\mathbf{r}) + \sum_{i=1}^{N} G_E^{D_i}\left[\Delta\sigma_{D_i}\mathbf{E}^t\right], \tag{1}$$

$$\mathbf{H}^t(\mathbf{r}) = \mathbf{H}^n(\mathbf{r}) + \sum_{i=1}^{N} \mathbf{H}^{\Delta\sigma_{D_i}}(\mathbf{r}) = \mathbf{H}^n(\mathbf{r}) + \sum_{i=1}^{N} G_H^{D_i}\left[\Delta\sigma_{D_i}\mathbf{E}^t\right], \tag{2}$$

where $G_E^{D_i}$ and $G_H^{D_i}$ are electric and magnetic Green's operators acting within domain D_i, respectively. Then the EM modeling problem is reduced to the calculation of the total electric fields inside each inhomogeneous domain.

In practice, at the first step of the field calculation, we do not know the values of any electric fields in equation (1). We thus first calculate the electric field in domain D_1 without taking into account the induction effect from any other domains:

$$\mathbf{E}^{\Delta\sigma_{D_1}}(\mathbf{r}) = G_E^{D_1}\left[\Delta\sigma_{D_1}\left(\mathbf{E}^n + \mathbf{E}^{\Delta\sigma_{D_1}}\right)\right]. \tag{3}$$

This integral equation is solved using the contraction form of integral equations (Hursán and Zhdanov, 2002) and the complex generalized minimal residual (CGMRES) method (Zhdanov, 2002). In the calculation of the field due to the currents induced in the next domain (2), we take into account the electric field induced from the inhomogeneous domain N,

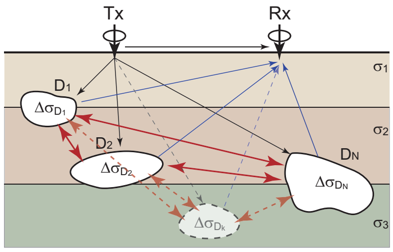

Figure 1: A sketch of a 3D geoelectrical model with horizontally layered (normal) conductivity and N inhomogeneous conductivities.

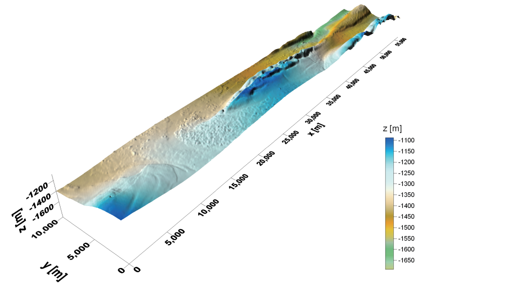

Figure 2: A 3D relief of the bathymetry for the Sabah model.

$\mathbf{E}^{\Delta\sigma_{D_1}}(\mathbf{r})$. Finally, for the last inhomogeneous domain, D_N, we already know the electric fields in all the other inhomogeneous domains and thus we can calculate the electric field $\mathbf{E}^{\Delta\sigma_{D_N}}(\mathbf{r})$ as described by equation (1). To improve the accuracy, we can use this scheme iteratively. In the subsequent iterations, we use the fields obtained in the previous iteration to calculate the induced fields in the given domain. This process is repeated until the electric fields within all the inhomogeneous domains reach self-consistency, i.e., the norm of difference between the electric fields in any domain at iterations i and $(i-1)$ is less than a certain threshold ε.

APPLICATION OF THE MD IE METHOD FOR STUDYING THE EM COUPLING EFFECTS IN MARINE CSEM DATA

In this section we will present the application of the developed MD IE method to investigate the EM coupling effects in marine CSEM data collected over areas with a rough sea-bottom bathymetry. This is a very important problem in marine EM geophysics, because the effect of the sea-bottom bathymetry can significantly distort the useful EM response from a hydrocarbon (HC) reservoir, which is the main target of offshore geophysical exploration. As a prototype of the bathymetry structure in all our models, we use the known bathymetry of the Sabah area, Malaysia. We have used a simplified model of the bathymetry data provided by Shell in constructing the geoelectrical models considered in this paper. A 3D relief of the true bathymetry of the Sabah area, Malaysia, is plotted in Figure 2.

Model 1: three-domain model (bathymetry, plus a salt dome, plus an HC reservoir)

A vertical section of the geoelectrical structure of Model 1 is shown in Figure 3. This figure shows a resistive HC reservoir with a resistivity of 100 Ohm-m and a salt dome with a resistivity of 30 Ohm-m located within conductive sea-bottom sediments whose resistivity is 1 Ohm-m. The EM field in this model is excited by an x-directed electric horizontal bipole. The transmitter generates the frequency-domain EM field at a frequency of 0.25 Hz. The electric field receivers are located along the y axis, as shown in Figure 3. Following the main principles of the MD IE method for multiple inhomogeneous domains, the modeling area is divided into three modeling domains, D_1, D_2, and D_3, outlined by the dashed lines in Figure 3. Modeling domain D_1 covers the area with conductivity variations associated with the bathymetry of the sea bottom, while modeling domains D_2 and D_3 correspond to the location of the salt dome and the HC reservoir, respectively. We use 120,000 cells with a cell size of $100 \times 100 \times 25$ m^3 for a discretization of the bathymetry structure. Domain D_2, the salt dome area, is discretized into 36960 cells with a cell size of $100 \times 100 \times 25$ m^3, and domain D_3, the reservoir area, is discretized into 12800 cells with a cell size of $100 \times 100 \times 25$ m^3.

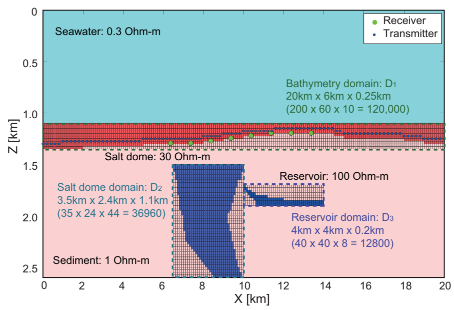

Figure 3: A vertical geoelectrical section of Model 1.

We used four CPUs (Opteron 2.0 GHz) for this calculation. The calculation time is around 43 min, and the required memory and the disc space are around 1.0 GB and 7.3 GB, respectively. It took just six iterations of the MD IE method to converge to the given level of the threshold $\varepsilon = 10^{-4}$ (Figure 4).

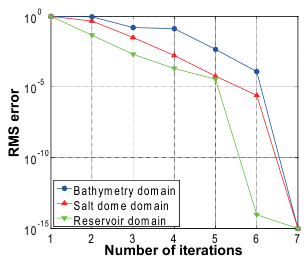

Figure 4: Convergence plots for the calculation of the EM field for Model 1.

Analysis of the maps of the amplitude of the electric fields observed at sea bottom

Figure 5 shows a map of the absolute value of the x (in-line) component of the anomalous electric field generated by the currents induced in the reservoir domain only, calculated by the MD IE method, which includes the return induction effects from all the other domains (bathymetry and salt dome). It is clear from this figure that the anomalous field associated with the reservoir is concentrated in the horizontal location of the reservoir.

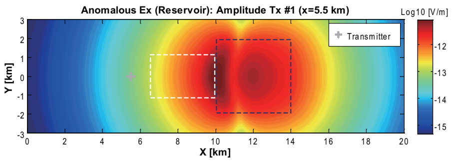

Figure 5: Model 1: A map of the absolute values of the x (in-line) component of the anomalous electric field generated by the currents induced in the reservoir domain only.

Figure 6 shows a map of the absolute value of the x (in-line) component of the anomalous electric field generated by the currents induced in the salt dome domain only, calculated by the MD IE method. As in the case of the response from the reservoir, the anomalous field is concentrated in the horizontal location of the salt dome.

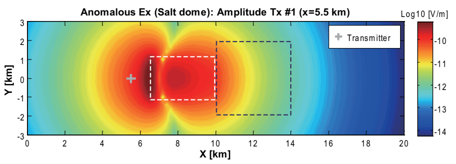

Figure 6: Model 1: A map of the absolute values of the x (in-line) component of the anomalous electric field generated by the currents induced in the salt dome domain only.

Analysis of the amplitude-versus-offset (AVO) plots of the electric fields

Figure 7 shows amplitude-versus-offset (AVO) plots of the electric fields calculated by the MD IE method for the x (in-line) component. One can see that, due to the EM coupling between the different inhomogeneous domains, the normalized AVO plots become very complicated. In this situation it is difficult to evaluate the horizontal location of the reservoir (or a salt dome) from these plots.

Comparison of the normalized AVO plots

Next, we have investigated the normalized AVO plots. Usually the amplitude of the EM field is normalized by the normal (layered background) field. Because we can calculate the response from the EM currents induced in each domain by our MD IE method, we can use any combination of these fields (not only the normal field) to normalize the AVO plots. Figure 8 shows the AVO plots of the total electric fields (calculated for the full geoelectrical model containing bathymetry, salt dome,

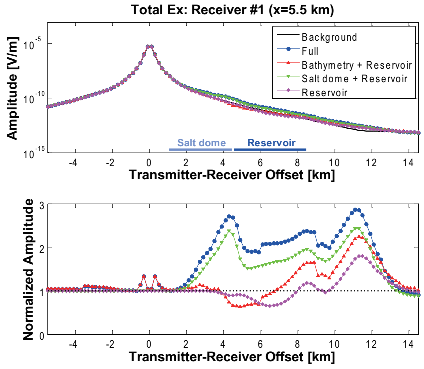

Figure 7: The top panel shows amplitude-versus-offset (AVO) plots of the x (in-line) component of the electric fields at receiver #1 ($x = 5.5$ km). The bottom panel presents the AVO plots of the same fields normalized by the absolute values of the normal electric fields.

and reservoir domains) normalized by "normal" (layered background), "normal + bathymetry," "normal + salt dome," and "full" (normal + bathymetry + salt dome) fields. This figure demonstrates that we can evaluate the horizontal location of the reservoir more easily by using the total field normalized by the field which is calculated as a sum of the currents induced in all domains except for the target domain (in this case, except for the field of the reservoir domain). In the practical data we observed the total field, which includes the EM coupling effects from all the inhomogeneities. Therefore, in the numerical modeling one should calculate the response of all the known inhomogeneous domains in order to be able to detect the location of the target effectively.

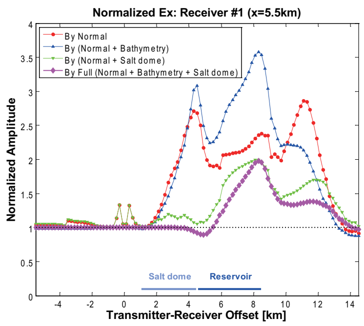

Figure 8: AVO plots of the total in-line electric field (calculated for a full geoelectrical model containing bathymetry, salt dome, and reservoir domains) normalized by "normal" (layered background), "normal + bathymetry," "normal + salt dome," and "full" (normal + bathymetry + salt dome) fields.

Model 2: four-domain model (bathymetry, plus two salt domes, plus a reservoir)

We have calculated an EM field for a model which includes four domains (a bathymetry, two salt domes, and a reservoir) to investigate the code performance for a model with many domains and to study the multiple-domain effect in the data.

IE method for multiple domains

A vertical section of the geoelectrical structure of Model 2 is shown in Figure 9. The MCSEM survey configuration in this model is the same as in Model 1. Following the main principles of the MD IE method for multiple inhomogeneous domains, the modeling area was represented by four modeling domains, D_1, D_2, D_3, and D_4, outlined by the dashed lines in Figure 9. Modeling domain D_1 covers the area with conductivity variations associated with the bathymetry of the sea bottom, while modeling domains D_2 and D_3 correspond to the location of the salt domes. Modeling domain D_4 corresponds to the location of the HC reservoir.

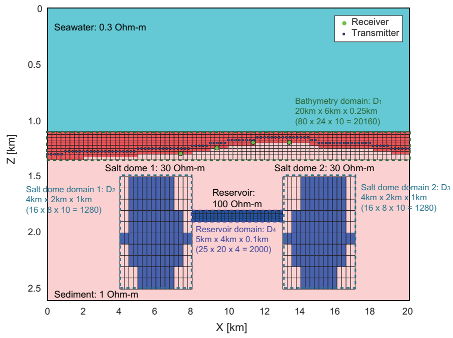

Figure 9: A vertical section of the geoelectrical structure of Model 2.

We used four CPUs (Opteron 2.0 GHz) for this calculation. The calculation time is around 12 min, and the required memory and the disc space are around 100 MB and 750 MB, respectively. It took just six iterations of the MD IE method to converge to the given level of the threshold $\varepsilon = 10^{-4}$ (Figure 10).

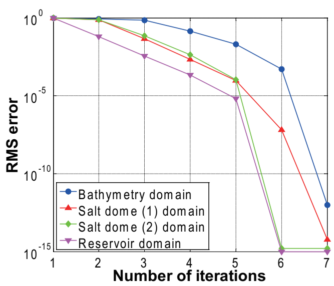

Figure 10: Convergence plots for the calculation of the EM field for Model 2.

Figure 11 shows a map of the absolute value of the *x* (in-line) component of the anomalous electric field generated by the currents induced in the reservoir domain only, calculated by the MD IE method, which includes the return induction effects from all the other domains (bathymetry and salt domes). It is clear from these figures that both of the components of the anomalous field associated with the reservoir are concentrated in the horizontal location of the reservoir.

Figure 12 shows a map of the absolute value of the *x* (in-line) component of the anomalous electric field generated by the currents induced in the salt dome domains only, calculated by the MD IE method. One can see that the anomalous field is concentrated in the horizontal location of the salt dome now.

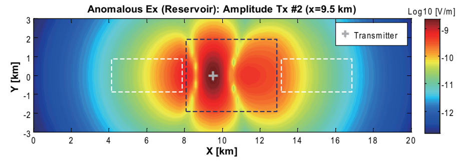

Figure 11: Model 2: A map of the absolute values of the *x* (in-line) component of the anomalous electric field generated by the currents induced in the reservoir domain only.

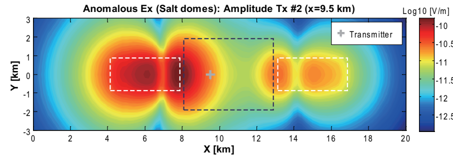

Figure 12: Model 2: A map of the absolute values of the *x* (in-line) component of the anomalous electric field generated by the currents induced in the salt dome domains only.

CONCLUSIONS

In this paper we have introduced a new MD IE method which can be used for complex geoelectrical models with multiple inhomogeneous domains. This method is based on the extension of the CEMI original inhomogeneous background conductivity integral equation (IBC IE) method. Contrary to the conventional IE, finite-difference (FD), or finite-element (FE) techniques, the new MD IE method requires discretization of the domains with the anomalous conductivity only. At the same time, this method provides a rigorous solution of the EM modeling problem by taking into account the EM coupling between the different domains. In addition, because the MD IE approach is based on the IE method, we can analyze the response of each domain separately, without an inappropriate use of the superposition principle for the EM field calculations. Using the new modeling facility, we have examined the MCSEM data for the models with multiple inhomogeneous domains, including bathymetry, salt dome, and reservoir structures. The numerical modeling results demonstrate that the new modeling method can be effectively used for studying the EM fields in complex geoelectrical models with multiple inhomogeneous domains.

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EDITED REFERENCES

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