Anisotropy of induced polarization in the context of the generalized effective-medium theory

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SUMMARY

In this paper, we study the anisotropy effect in IP data in the context of the generalized effective-medium theory of induced polarization (GEMTIP). The effective-medium conductivity defined by the GEMTIP model, in a general case, is represented by a tensor function. This tensorial property of the effective-medium conductivity provides a new insight on the anisotropy phenomenon in the IP effect. As an example, we consider a multiphase composite polarized model of a rock formation with ellipsoidal inclusions. We demonstrate that the effective conductivity of this formation may be anisotropic, even if the host rock and all the grains are electrically isotropic.

INTRODUCTION

It is well known that the effective conductivity of rocks is not necessarily a constant and real number but may vary with frequency and be complex. There are several explanations for these properties of effective conductivity. Most often they are explained by the physical-chemical polarization effects of mineralized particles of the rock material and/or by the electrokinetic effects in the poroses of reservoirs (Wait, 1959; Marshall and Madden, 1959; Luo and Zhang, 1998). This phenomenon is called the induced polarization (IP) effect.

It was demonstrated in Zhdanov (2006a,b, 2008), that the IP phenomenon can be mathematically explained by a new composite geoelectrical model of rock formations. This model is based on the effective-medium approach, which takes into account both the volume polarization and the surface polarization of the grains. This new generalized effective-medium theory of the IP effect (GEMTIP) can be used for examining the IP effect in complex rock formations with different mineral structures and electrical properties.

The effective-medium conductivity defined by the GEMTIP model, in a general case, is represented by a tensor function. This tensorial property of the effective-medium conductivity shines a new light on the anisotropy phenomenon in the IP effect. The study of the IP anisotropy is the major subject of this paper. In order to develop a quantitative characterization of the IP anisotropy, we use as an example a medium with ellipsoidal inclusions, which allows us to calculate the analytical solutions for the effective-conductivity tensor of the model with the IP effect.

BASIC FORMULAS OF THE EFFECTIVE-MEDIUM THEORY OF INDUCED POLARIZATION

In the framework of the GEMTIP model, we represent a complex heterogeneous rock formation as a composite model formed by a homogeneous host medium of a volume V with a complex conductivity tensor $\widehat{\sigma}_0(\mathbf{r})$ (where $\mathbf{r}$ is an observation point) filled with grains of arbitrary shape and conductivity. In the present problem, the rock is composed of a set of N different types of grains, the lth grain type having a complex tensor conductivity, $\widehat{\sigma}_l$. The grains of the lth type have a volume fraction f_l in the medium and a particular shape and orientation.

Following Zhdanov (2006a,b, 2008), we can write the following expression for the effective conductivity of the polarized inhomogeneous medium:

$$\widehat{\sigma}_e = \widehat{\sigma}_0 + \sum_{l=1}^{N} \left[\widehat{\mathbf{I}} + \widehat{\mathbf{p}}_l\right]^{-1} \left[\widehat{\mathbf{I}} - \Delta\widehat{\sigma}_l^p \cdot \widehat{\Gamma}_l\right]^{-1} \cdot \left[\widehat{\mathbf{I}} + \widehat{\mathbf{p}}_l\right] \cdot \Delta\widehat{\sigma}_l f_l, \tag{1}$$

where $\widehat{\sigma}_e$ is an effective-medium conductivity tensor; $\Delta\widehat{\sigma}_l$ is an anomalous conductivity tensor; $\Delta\widehat{\sigma}_l^p = \left[\widehat{\mathbf{I}} + \widehat{\mathbf{p}}_l\right] \cdot \Delta\widehat{\sigma}_l$ is the polarized anomalous conductivity; $\widehat{\mathbf{p}}_l$ is a surface polarizability tensor; $\widehat{\Gamma}_l$ is a volume depolarization tensor; and index l corresponds to the grain of the lth type.

The last formula provides a general solution of the effective conductivity problem for an arbitrary multiphase composite polarized medium. This formula allows us to find the effective conductivity for inclusions with arbitrary shape and electrical properties. That is why the new composite geoelectrical model of the IP effect may be used to construct the effective conductivity for realistic rock formations typical for mineralization zones and/or petroleum reservoirs.

EFFECTIVE CONDUCTIVITY OF THE MULTIPHASE COMPOSITE POLARIZED MEDIUM WITH ELLIPSOIDAL INCLUSIONS

We assume now, that all grains have an ellipsoidal shape, and their axes are parallel to one another. It is clear that in this situation, similar to the case of the conventional effective-medium theory without IP effects (Sihvola, 2000), the effective medium becomes anisotropic.

We show in Figure 1, as an example, typical rotational ellipsoids for different values of the ellipticity $\varepsilon = c/a$ ranging from 0.125 to 8.

It can be demonstrated based on formula 1 that the effective resistivity of the multiphase composite polarized medium with ellipsoidal inclusions aligned along axis z, can be represented

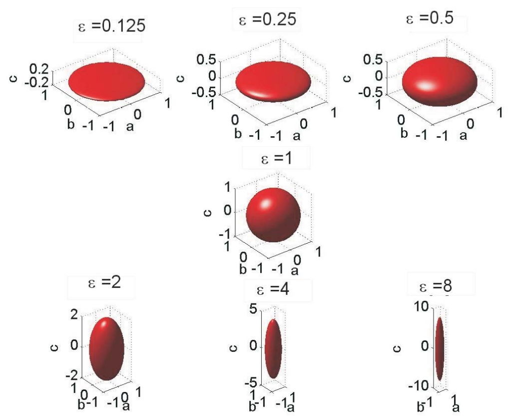

Figure 1: Typical rotational ellipsoids for different values of the ellipticity $\varepsilon = b/a$ ranging from 0.125 to 8.

as a diagonal resistivity tensor:

$$\widehat{\rho}_e = \begin{bmatrix} \rho_{lx} & 0 & 0 \\ 0 & \rho_{ly} & 0 \\ 0 & 0 & \rho_{lz} \end{bmatrix}. \tag{2}$$

where:

$$\rho_{e\alpha} = \rho_0 \{1 + \sum_{l=1}^{N} \left[f_l \frac{\rho_0 - \rho_l}{\rho_l + \gamma_{l\alpha}(\rho_0 - \rho_l) + k_l \lambda_{l\alpha}} \right] \}^{-1}, \alpha = x, y, z. \tag{3}$$

According to the experimental data, the surface polarizability factor is a complex function of frequency (Luo and Zang, 1998). Following Wait (1982) and Zhdanov (2008), we use the model

$$k_l = \alpha_l (i\omega)^{-C_l}, \tag{4}$$

which fits the experimental data, where α_l is some empirical surface polarizability coefficient, measured in the units $[\alpha_l] = (Ohm \times m^2) / \sec^{C_l}$, and C_l is the relaxation parameter of the lth grain.

Substituting formula 4 into expression 3, after some algebra, we have

$$\rho_{e\alpha} = \rho_0 \{1 + \sum_{l=1}^{N} \left[f_l \frac{\rho_0 - \rho_l}{\rho_l + \gamma_{l\alpha}(\rho_0 - \rho_l) + \bar{a}_l (\rho_l + \rho_0/2)(i\omega\tau_l)^{-C_l} \lambda_{l\alpha}} \right] \}^{-1},$$
$$\alpha = x, y, z \tag{5}$$

where $\bar{a}_l$ is an average value of the equatorial (a_{lx} and a_{ly}) and polar (a_{lz}) radii of the ellipsoidal grains, i.e.:

$$\bar{a}_l = \frac{(a_{lx} + a_{ly} + a_{lz})}{3}. \tag{6}$$

The last formula provides a general solution of the effective resistivity problem for a multiphase composite anisotropic medium filled with ellipsoidal inclusions. We will call the conductivity relaxation model described by formula (5) a *GEMTIP-AE model*.

TWO-PHASE COMPOSITE POLARIZED MEDIUM WITH ELLIPSOIDAL INCLUSIONS

In the case of a two-phase composite model, we have a homogeneous host medium of a volume V with a complex resistivity ρ_0 and ellipsoidal inclusions with resistivity ρ_1. Formula 5 is simplified:

$$\rho_{e\alpha} = \rho_0 \{1 - \frac{f_l(\rho_0 - \rho_l)}{\rho_l + \gamma_{l\alpha}(\rho_0 - \rho_l) + \bar{a}_l (\rho_l + \rho_0/2)(i\omega\tau_l)^{-C_l} \lambda_{l\alpha} + f_l(\rho_0 - \rho_l)} \}^{-1}. \tag{7}$$

Formula 7 provides an analytical expression for the components of the effective conductivity tensor of a two-phase composite polarized medium with ellipsoidal inclusions oriented along the axis of the Cartesian coordinates x, y, *and* z.

We present below some typical GEMTIP resistivity relaxation models for two-phase composite polarized media. The models are formed by a homogeneous host rock with a resistivity ρ_0 filled with ellipsoidal grains. The resistivities of the host rock and of the grains are $\rho_0 = 2$ Ohm-m and $\rho_1 = 0.2$ Ohm-m, respectively. The volume fraction of grain f_1 is equal to 0.05; the time parameter, τ_1, and the relaxation parameter, C_1, are equal to:

$$\tau_1 = 0.4 \text{ sec}; \; C_1 = 0.8.$$

We select the horizontal (equatorial) radius of the rotational ellipsoid, a, equal to 1.25 cm, while the vertical (polar) axis, b, is variable, depending on the ellipticity ε. The effective resistivity was calculated for the following values of the ellipticity, $\varepsilon = b/a = 5$; 0.25; 0.5; 1; 2; 4; 8, as shown in Figure 2. The plots in the top parts of panels a), b), and c) represent the real parts of the effective resistivity, while the bottom parts of the same panels show the imaginary parts. For comparison, the bold curves in these plots show the effective resistivity of the isotropic two-phase medium filled with spherical grains. One can clearly see that the presence of the ellipsoidal grains results in an anisotropy effect in the effective resisitivity.

ANISOTROPY COEFFICIENT FOR TWO-PHASE COMPOSITE POLARIZED MEDIUM WITH ELLIPSOIDAL INCLUSIONS

The results presented in the previous section demonstrate that a composite polarized medium with ellipsoidal inclusions may be characterized by a strong conductivity anisotropy. It is interesting to evaluate the anisotropy coefficient for this model. The

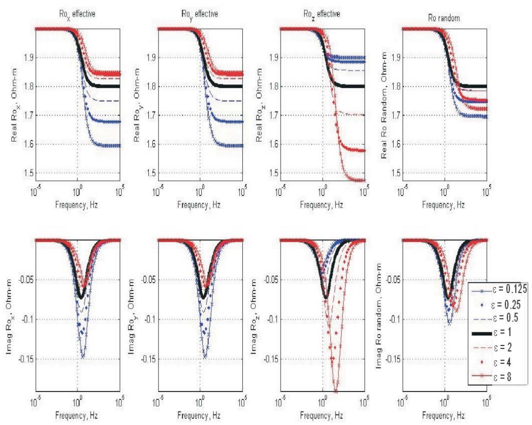

Figure 2: Resistivity relaxation models of two-phase heterogeneous rock filled by ellipsoidal grains. Panels a), b), and c) correspond to a model with the grains aligned along the Cartesian coordinates. Panel d) presents the resistivity model for a case with random orientation of the grains. The upper panels show the real part of the complex effective resistivity, while the bottom panels present the imaginary part of the complex effective resistivity. The different curves in each panel correspond to the different ellipticities of the grains: $\varepsilon = b/a = 5$; 0.25; 0.5; 1; 2; 4; 8, respectively.

coefficient of anisotropy κ of the effective anisotropic medium is defined as

$$\kappa = \sqrt{\frac{\rho_{ez}}{\rho_{ex}}}. \tag{8}$$

Note that in our case both the horizontal, ρ_{ex}, and the vertical, ρ_{ez}, effective resistivities are complex and frequency-dependent functions. Therefore, the anisotropy coefficient should also be described by a complex function of the frequency:

$$\kappa = \sqrt{\frac{\rho_{ez}}{\rho_{ex}}} = \sqrt{\left|\frac{\rho_{ez}}{\rho_{ex}}\right|} \exp\left[\frac{i}{2}(\varphi_z - \varphi_x)\right]. \tag{9}$$

Thus the amplitude and phase of the complex anisotropy coefficient can be calculated as:

$$|\kappa| = \sqrt{\left|\frac{\rho_{ez}}{\rho_{ex}}\right|}$$

and:

$$\varphi_\kappa = \frac{1}{2}(\varphi_z - \varphi_x) = \frac{1}{2}\left[\tan^{-1}\left(\frac{\Im\rho_{ez}}{\Re\rho_{ez}}\right) - \tan^{-1}\left(\frac{\Im\rho_{ex}}{\Re\rho_{ex}}\right)\right], \tag{10}$$

respectively.

To emphasize the close connection between the ellipticity, ε, of the ellipsoidal grains and the resistivity anisotropy, we present in Figure 3 plots of the amplitude and phase of the anisotropy coefficient versus frequency for the different ellipticity $\varepsilon = b/a$, where b is a polar radius and a is an equatorial radius of the rotational ellipsoid. These plots indicate that the anisotropy in the induced polarization effect is relatively strong at a frequency above 1 Hz. For the prolate (cigar-shaped) ellipsoid ($b > a$ and $\varepsilon > 1$), the real and imaginary parts of the anisotropy coefficient are smaller than 1. This means that the absolute value of the resistivity in the z direction (the direction of the long axis of the ellipsoid) is smaller than the resistivity in the x and/or y directions. In the case of the oblate (disk-shaped) ellipsoid ($b < a$ and $\varepsilon < 1$), we have quite an opposite situation. The real and imaginary parts of the anisotropy coefficient becomes greater than 1, and the resistivity in the z direction is greater than in the x and/or y directions. These behaviors of the complex resistivity have a very clear physical explanation. In the case of the prolate ellipsoidal grains, it is easier for the electric current to flow along the long axes of the ellipsoids than in the perpendicular direction. In the case of the oblate ellipsoidal grains, the preferred direction of the current flow is along the horizontal axis, while the ellipsoids that are relatively thin in the z direction provide a stronger resistance to the current flow in the z direction.

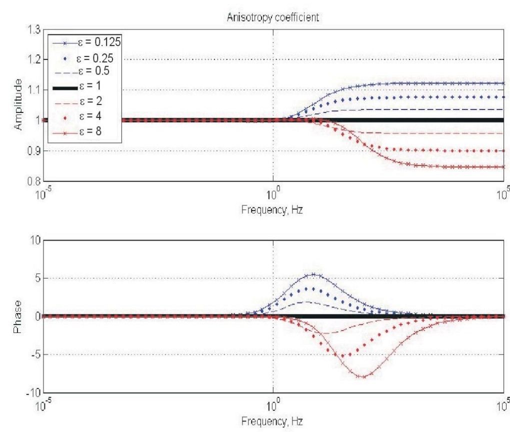

Figure 3: Frequency dependence of the amplitude and phase of the anisotropy coefficient for the different ellipticity $\varepsilon = b/a$, where b is a polar radius and a is an equatorial radius of the rotational ellipsoid.

Figure 4 shows a zoom in the low frequency range of Figure 3 for the same set of input model parameters. It is evident from Figures 2 and 3 that, for the given parameters of the composite medium, the ellipticity of the grains dramatically changes both the amplitude and phase of the effective resistivity, especially in the proximity of the frequencies, where the IP effect occurs. While at the low frequencies the coefficient of anisotropy also exhibits a relatively weak and monotonic ellipticity dependence, the big changes occur at the IP frequencies. It is also notable that the position of the peak of the phase of the anisotropy coefficient depends on the ellipticity. In addition, the position of the phase peak corresponds to the area of intensive amplitude growth. Thus, the relative phase shift of the peak of the IP response should contain information on both the

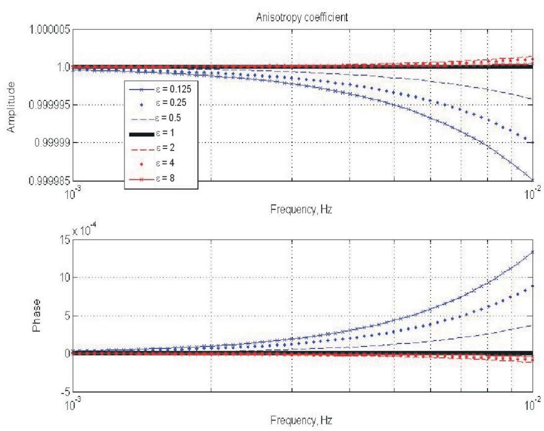

Figure 4: Low-frequency dependence of the amplitude and phase of the anisotropy coefficient for the different ellipticity $\varepsilon = b/a$, where b is a polar radius and a is an equatorial radius of the rotational ellipsoid.

anisotropy in the effective resistivity media and the ellipticity of the grains in the original composite medium.

RANDOMLY ORIENTED ELLIPSOIDS

We consider now a medium with completely randomly oriented ellipsoids. The effective conductivity of this medium can be calculated by taking an average over the orientation in the correspondent formulas for conductivities, $\sigma_{e\alpha}$, based on equation 3. As a result we obtain

$$\sigma_e = \sigma_0 + \sum_{l=1}^{N} \frac{1}{3} f_l D_1 \tag{11}$$

where:

$$D_l = \sum_{\alpha=x,y,z} \frac{\rho_0 - \rho_l}{\rho_l + \gamma_{l\alpha}(\rho_0 - \rho_1) + k_l \lambda_{l\alpha}}.$$

The expression for the effective resistivity, $\rho_e = 1/\sigma_e$, of a medium filled with completely randomly oriented ellipsoidal grains can be written as:

$$\rho_e = \rho_0 \left[1 + \frac{1}{3} \sum_{l=1}^{N} f_l D_l \right]^{-1} =$$

$$\rho_0 \left\{ 1 + \frac{1}{3} \sum_{l=1}^{N} f_l (\rho_0 - \rho_l) \sum_{\alpha=x,y,z} \left[\rho_l + \gamma_{l\alpha}(\rho_0 - \rho_l) + k_l \lambda_{l\alpha} \right]^{-1} \right\}^{-1}, \tag{12}$$

where, according to expression 4, the surface polarizability factor k_l of the lth grain is given by the formula

$$k_l = \bar{a_l}\,(\rho_l + \rho_0/2)\,(i\omega\tau_l)^{-C_l}.$$

In particular, for a simplified two-phase composite model, we have:

$$\sigma_e = \sigma_0 \left[1 + \frac{1}{3} f_l D_1 \right], \tag{13}$$

and

$$\rho_e = \rho_0 \left\{ 1 + \frac{1}{3} f_1 (\rho_0 - \rho_1) \sum_{\alpha=x,y,z} \left[\rho_1 + \gamma_{1\alpha}(\rho_0 - \rho_1) + k_1 \lambda_{1\alpha} \right]^{-1} \right\}^{-1}. \tag{14}$$

As an example, we show in Figure 2 (panel d) the resistivity relaxation curve for the two-phase composite model with a random orientation of the grains. It is interesting to notice that the isotropic effective resistivity of this model differs from the isotropic resistivity of a similar model with spherical grains. As one would expect, resistivity in isotropic media is less sensitive to grain eccentricity than in anisotropic media, as shown in Figure 2.

CONCLUSIONS

In this paper we have examined the tensorial properties of the effective-medium conductivity defined by the GEMTIP model. As a basic model of our research, we have used a multiphase composite polarized model of a rock formation filled with ellipsoidal inclusions. We considered two types of the composite media formed by ellipsoidal grains: 1) a medium with the grains oriented all in one direction and 2) a medium with randomly oriented grains. We have found that the effective conductivity of the composite media may be anisotropic even if the host rock and all the grains are electrically isotropic. As a result, the medium of the first type has anisotropic effective conductivity, while the medium of the second type has isotropic effective conductivity.

The discovered anisotropy phenomenon in the IP effect is manifested in the different conductivities for the different directions. This anisotropy should be taken into account in the quantitative interpretation of the field IP data.

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EDITED REFERENCES

Note: This reference list is a copy-edited version of the reference list submitted by the author. Reference lists for the 2008 SEG Technical Program Expanded Abstracts have been copy edited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

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