# Rigorous 3D inversion of marine magnetotelluric data in the area with complex bathymetry

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## Summary

Three-dimensional (3D) magnetotelluric (MT) inversion is an emerging technique for offshore hydrocarbon (HC) exploration. In this paper we introduce a new approach to 3D inversion of MT data for offshore HC exploration based on the integral equation method. The method is implemented in a fully parallel computer code. We have applied the developed method and software for the inversion of marine MT data collected by the Scripps Institution of Oceanography (SIO) in the Gemini Prospect, Gulf of Mexico. The inversion domain was discretized into 1.7 M cells. It took 9 hours to complete 51 iterations on the 832 processor cluster with a final misfit between the observed and predicted data of 6.2%. The inversion results reveal a resistive salt structure which is confirmed by a comparison with the seismic data. These inversion results demonstrate that we can map resistive geoelectrical structures like salt domes or HC reservoirs with reasonable accuracy using 3D inversion of marine MT data.

#### Introduction

Controlled source electromagnetic (CSEM) and magnetotelluric (MT) techniques have become widely used in oil and gas exploration offshore and in the deep sea environment.

There were several publications presenting the results of marine magnetotelluric (MT) surveys (Constable et al., 1998; Hoversten et al., 1998, 2000; Ellingsrud et al., 2002; Key, 2003; Key et al., 2006). In all these publications, however, the interpretation of the sea-bottom MT data was based, as a rule, on 1D or 2D modeling, which limited the practical effectiveness of the MT method.

In this paper we introduce a method of rigorous 3D inversion of MT data, based on the integral equation (IE) method. We use the re-weighted regularized conjugate gradient method (RRCG) for nonlinear MT inversion. The main distinguishing feature of the RRCG algorithm is application of the special stabilization functionals which allow construction of both smooth images of the underground geoelectrical structures and models with sharp geoelectrical boundaries (Zhdanov, 2002).

The method of regularized focusing inversion of the MT data is implemented in a new fully parallelized version of the computer code, which can be run on a PC cluster. One

distinguished feature of the new method and computer code is the possibility of taking into account the effect of seabottom bathymetry in the inversion of MT data. This is a very important problem in marine EM geophysics, because the effect of sea-bottom bathymetry can significantly distort the useful MT response from sub sea-bottom geoelectrical structures, which are the main target of offshore MT surveys.

We apply the developed method to the interpretation of MT data collected by the Scripps Institution of Oceanography in Gemini Prospect, Gulf of Mexico. The main objective of this paper is to demonstrate the capability of imaging a seabottom resistivity structure based on large-scale 3D inversion of marine MT data.

### Principles of The Regularized MT Data Inversion

In the MT method the earth's natural electromagnetic field is used as a source field. The observed MT data are represented in the form of the impedance tensor in a Cartesian coordinate system (Berdichevsky and Dmitriev, 2002):

$$Z = \begin{bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{bmatrix}.$$
 (1)

The observed impedances are independent on the strength of the source. They depend only on the frequency of the signal and the electrical conductivity of the subsurface earth. The MT inversion is carried out most commonly for the principal impedances,  $Z_{xy}$  and  $Z_{yx}$ .

We can describe the discrete magnetotelluric inverse problem by an operator equation:

$$\mathbf{d} = \mathbf{A}(\mathbf{m}) , \qquad (2)$$

where **d** stands for a *data vector* formed by the components of the principal impedances, **A** is the *nonlinear forward operator* symbolizing the governing equations of the MT modeling, and  $\mathbf{m} = \Delta \boldsymbol{\sigma}$  is a vector formed by an unknown set of *anomalous conductivity* (*model parameters*) within the targeted domain.

We use the integral equation (IE) method (Hursán and Zhdanov, 2002; Zhdanov, 2002) for numerical calculation of the forward modeling operator in equation (2).

We apply the Tikhonov regularization to solve the inverse problem (2). It is based on minimization of the Tikhonov parametric functional:

$$P(\Delta\sigma) = \varphi(\Delta\sigma) + \alpha \, s(\Delta\sigma) = \min \,, \tag{3}$$

where  $\varphi(\Delta \sigma)$  is the misfit functional between the predicted data  $\mathbf{A}(\Delta \sigma)$  and the observed data  $\mathbf{d}$ , and  $s(\Delta \sigma)$  is a stabilizing functional,  $\alpha$  is a regularization parameter.

The main role of the stabilizing functional in the inversion process is selecting the appropriate solution of the inverse problem from a class of models with assigned properties. There are several possible choices for the stabilizer. We use three different types of stabilizing functionals  $s(\Delta\sigma)$  -- a minimum-norm (MN), minimum-support (MS), and minimum vertical --support (MVS) stabilizers:

$$s(\Delta\sigma) = \begin{cases} s_{MN}(\Delta\sigma) = \beta_{MN} \left\| \mathbf{W}_{m}(\Delta\sigma - \Delta\sigma_{apr}) \right\|_{L_{2}}^{2} \\ s_{MS}(\Delta\sigma) = \beta_{MS} \left\| \mathbf{W}_{e}^{MS} \mathbf{W}_{m} \Delta\sigma \right\|_{L_{2}}^{2} \\ s_{MVS}(\Delta\sigma) = \beta_{MVS} \left\| \mathbf{W}_{e}^{MVS} \mathbf{W}_{m} \Delta\sigma \right\|_{L_{2}}^{2} \end{cases}$$
(4)

In equations (4),  $\mathbf{W}_{m}$  is the model parameter weighting matrix;  $\mathbf{W}_{e}^{MS}$  and  $\mathbf{W}_{e}^{MVS}$  are focusing matrices (for definition see Zhdanov, 2002); the  $\Delta\sigma_{apr}$  term is a priori information about the anomalous conductivity model. The coefficients  $\beta_{MN}$ ,  $\beta_{MS}$ , and  $\beta_{MVS}$  are called the minimum-norm, minimum-support, and minimum vertical-support coefficients. These coefficients can be selected in the inversion process by the user based on the nature of the problem and its required solution.

In summary, the minimum-norm stabilizer selects the inverse model from the class of models with the leastsquare norm. The minimum-support stabilizer insures that the solution belongs to the class of models with the smallest domain of anomalous conductivity. The minimum verticalsupport stabilizer provides solutions for problems having the smallest vertical dimensions of the domain of anomalous conductivity.

The minimization problem (3) can be solved using any gradient-type technique. We use the regularized conjugate gradient (RCG) method. The implementation details of this algorithm are specified in Zhdanov (2002). The stability and uniqueness of the regularized inverse solution depend on the selection of optimal value of regularization parameter. The optimal value  $\alpha_{opt}$  of the regularization parameter is determined from the misfit condition,

$$\varphi(\mathbf{m}_{apr}) = \delta^2, \qquad (5)$$

where  $\delta$  is the noise level of the data.

### Inversion of Marine MT Data Collected in The Gemini Prospect, Gulf of Mexico

Gemini Prospect is located about 200 km southeast of New Orleans in water about 1km deep in the northern Gulf of Mexico, which is shown in Figure 1. The salt body at Gemini Prospect has been determined by 3D seismic reflection survey, revealing a complex 3D salt structure at depths 1 to 5 km beneath the seafloor in 1km deep water which has a high electrical resistivity compared with surrounding sediments (Key, 2003). The reflection imaging technique, widely available and ever popular in exploration, can provide detailed images of the top and base of the salt surface, sedimentary layer, and basement formations. However, seismic imaging may not always provide sufficient details to interpret salt and nearby sedimentary structures. The salt in the Gulf of Mexico has a high acoustic contrast with surrounding sediments, which makes seismic section difficult to interpret.



Figure 1. Location of Gemini Prospect in the northern Gulf of Mexico. Topography and bathymetry from Smith and Sandwell (1997).

It is generally well known that rocks with high seismic velocity and impedance contrast are also higher in electrical resistance than surrounding sediments. The high contrast in electrical conductivity between the salt and the surrounding sediments makes Gemini Prospect an attractive target for marine MT method.

Figure 2 shows the bathymetry and MT site locations at Gemini Prospect. One can see that the depth of the sea bottom varies from about 900 m in the NW part of the survey up to 1500 m in the SE corner of the survey area. There is a significant conductivity contrast between the seawater and the sea-bottom sediments. That is why we should take into account the bathymetry effect on the observed MT data in 3D inversion.



Figure 2. Bathymetry and MT site locations at Gemini Prospect. MT sites are shown as circles with the color indicating the year the site was acquired.



Figure 3. Location of MT profiles with the observation sites in Gemini Prospect, which were used in the 3D inversion (after Key, 2003).

MT data were collected at 42 sites (Figure 3) in a twodimensional (2D) grid over the Gemini salt body using broadband MT sensors developed by the Scripps Institution of Oceanography (Constable et al., 1998).

Three-dimensional inversion of the Gemini MT data was performed for all the transect lines shown in Figure 3. The inversion domain was selected from -3 km to 13 km and from -15 km to 10 km in the horizontal *x* and *y* directions, and from 1 km to 12 km in the vertical direction. The background geoelectrical model was obtained by a one-dimensional (1D) inversion. The inversion domain was divided into 128 x 200 x 64 = 1,638,400 cells with a cell size of 125m x 125m in the *x* and *y* directions and 50~500 m in the vertical direction (the vertical size of the cells increases with the depth, varying from 50 to 500 m). The bathymetry domain was also extended from -3 km to13 km in the *x* direction, from -15 km to 10 km in the *y* direction, and from 600 m to 1000 m in the *z* direction.

To accomplish the inversion, we have used our newly developed parallel MT code that is capable of running on massively parallel supercomputers. For increased efficiency, the forward modeling part of the code uses two levels of parallelization. On the coarser level, we parallelize over the frequencies of the MT signal, on the finer level, we parallelize over the vertical dimension of the inversion domain. The two-level parallelization was employed over all 13 frequencies and over all 64 Z layers, thus requiring 13 x 64 = 832 CPUs. We ran the inversion on the Updraft cluster at the Center for High Performance Computing at the University of Utah. Updraft has 256 nodes connected with the Qlogic InfiniBand network. Each node includes 8 Intel CPU cores running at 2.8 GHz and 16 GB of RAM. The inversion took 9 hours to complete 51 iterations on the 832 processors and required 130 GB of disk space for intermediate files.

Figures 4 and 5 present the vertical geoelectrical cross sections over Lines A and I combined with seismic reflection sections.



Figure 4. Line A: the combined 3D MT inversion results overlap with a seismic section. The green lines show depthmigrated reflections from the 3D seismic survey.

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Figute 5. Line I: the combined 3D MT inversion results overlap with a seismic section. The green lines show depth-migrated reflections from the 3D seismic survey.

Figure 6 shows 3D image of the inversion result for the Gemini prospect MT data and the bathymetry in the area of the survey. One can clearly see the location and shape of the salt dome structure in this image.



Figure 6. 3D image of the inversion result for the Gemini prospect MT data in the presence of the sea-bottom bathymetry.

Figure 7 presents a comparison between the observed and predicted data (as an example in the form of apparent resistivity and phase, respectively) for the XY polarization at a frequency of 0.064091 Hz. From the observed and predicted maps, one can see a good match between the observed and predicted data.

### Conclusions

We have developed and analyzed a new version of the 3D inversion algorithm for interpretation of MT data. This new algorithm is based on the IE method. It utilizes different focusing stabilizing functionals, which allows us to produce stable and focused structures of a geoelectrical target in an offshore environment. The new algorithm can take into

account the effect of the sea-bottom bathymetry on the observed MT data. The method is fully parallelized and can be run on a PC cluster.



Figure 7. Maps of observed and predicted apparent resistivity (upper panels) and observed and predicted phase (lower panel) for XY polarization at a frequency of 0.064091 Hz.

We have applied this inversion method for interpretation of the field MT data collected in Gemini Prospect, Gulf of Mexico. The inversion for almost 1.8 M discretization cells took only 9 hours on the 832 processors and required 130 GB of disk space for intermediate files. The obtained 3D inverse model correlates well with the shape and location of the salt-dome structure that was determined using 3D seismic prestack depth migration. The vertical cross sections of the inverse image demonstrate reasonable recovery of the true geological features.

### Acknowledgments

The authors acknowledge the support of the University of Utah Consortium for Electromagnetic Modeling and Inversion (CEMI).

We are thankful to the Scripps Institution of Oceanography (SIO) for providing the real sea-bottom MT data and permission to publish the results.

We also acknowledge allocation of computer time provided by the University of Utahs Center for High Performance Computing (CHPC).

## EDITED REFERENCES

Note: This reference list is a copy-edited version of the reference list submitted by the author. Reference lists for the 2009 SEG Technical Program Expanded Abstracts have been copy edited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

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