Multiple domain integral equation method for 3D electromagnetic modeling in complex geoelectrical structures

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SUMMARY

We consider a new large-scale electromagnetic (EM) modeling technique based on the multiple-domain integral equation (IE) method. In geophysical applications, it is difficult to describe an earth structure using the horizontally layered background conductivity model, which is required for the efficient implementation of the conventional IE approach. The new method allows us to consider multiple inhomogeneous domains, where the conductivity distribution is different from that of the background, and to use independent discretizations for different domains. This reduces dramatically the computational resources required for large-scale modeling. In addition, using this method, we can analyze the response of each domain separately without an inappropriate use of the superposition principle for the EM field calculations. The method was carefully tested for the modeling the marine controlled-source electromagnetic (MCSEM) and the Magnetotelluric (MT) fields for complex geoelectric structures with multiple inhomogeneous domains. We have also used this technique to investigate the EM coupling effects between inhomogeneous domains, which can distort the EM response from the geophysical exploration target.

INTRODUCTION

The integral equation (IE) method represents an effective computational technique for electromagnetic modeling. In the framework of the IE method, the conductivity distribution is divided into two parts: 1) the background conductivity, σ_b, which is used for the Green's functions calculation, and 2) the anomalous conductivity, $\Delta\sigma_a$, within the domain of integration, D.

In many practical geological applications, however, it is difficult to describe an earth structure using the horizontally layered background conductivity model, which is required for the efficient implementation of the conventional IE approach. As a result, a large domain of interest with anomalous conductivity distribution needs to be discretized. Zhdanov et al. (2006) have recently developed a method to address this problem, the inhomogeneous background conductivity (IBC) IE method. In order to take into account the EM coupling between the anomalous domain and the inhomogeneous background domain, the IBC IE method can be applied iteratively (Zhdanov et al., 2006; Endo et al., 2008).

We have extended this iterative IBC IE method to the modeling of multiple inhomogeneous domains (Endo et al., 2009). In the framework of this method, we can construct a model with any number of inhomogeneous domains and take into account the return induction effects between any pairs of the inhomogeneous domains by using the iterative method. The important point is that by using this method we can evaluate the individual response from every domain, which includes the possible EM coupling effects between the different domains. A rigorous separate calculation of the EM fields produced by different anomalous domains representing different geological structures (e.g., a salt dome and a hydrocarbon (HC) reservoir) represents an important practical problem of EM exploration.

In summary, in this paper we not only demonstrate the effectiveness of the MD IE method but also examine the effects of the EM coupling between the different inhomogeneous domains which can distort a useful EM response and complicate the interpretation of the EM data.

INTEGRAL EQUATION FORMULATION FOR MULTIPLE INHOMOGENEOUS DOMAIN MODELING

In this seciton we summarize the principles of the IE method of EM modeling with multiple inhomogenesous domains. We assume that N inhomogeneous domains ($D_i, i = 1, \ldots, N$) are located within a horizontally layered earth (Figure 1). The conductivity of the horizontally layered earth (normal conductivity) is σ_n, while the inhomogeneous (anomalous) conductivity within each inhomogeneous domain is denoted as $\Delta\sigma_{D_i} (i = 1, \ldots, N)$. The total EM fields at any point $\mathbf{r}$, $\mathbf{E}^t(\mathbf{r})$, and $\mathbf{H}^t(\mathbf{r})$, can be expressed as a sum of normal fields $\mathbf{E}^n(\mathbf{r})$, $\mathbf{H}^n(\mathbf{r})$, and the EM fields induced by every inhomogeneous domain $\mathbf{E}^{\Delta\sigma_{D_i}}(\mathbf{r})$, $\mathbf{H}^{\Delta\sigma_{D_i}}(\mathbf{r})$ $(i = 1, \ldots, N)$:

$$\begin{aligned} \mathbf{E}^t(\mathbf{r}) &= \mathbf{E}^n(\mathbf{r}) + \sum_{i=1}^{N} \mathbf{E}^{\Delta\sigma_{D_i}}(\mathbf{r}) \\ &= \mathbf{E}^n(\mathbf{r}) + \sum_{i=1}^{N} G_E^{D_i}\left[\Delta\sigma_{D_i}\mathbf{E}^t\right], \qquad (1) \end{aligned}$$

$$\begin{aligned} \mathbf{H}^t(\mathbf{r}) &= \mathbf{H}^n(\mathbf{r}) + \sum_{i=1}^{N} \mathbf{H}^{\Delta\sigma_{D_i}}(\mathbf{r}) \\ &= \mathbf{H}^n(\mathbf{r}) + \sum_{i=1}^{N} G_H^{D_i}\left[\Delta\sigma_{D_i}\mathbf{E}^t\right], \qquad (2) \end{aligned}$$

where $G_E^{D_i}$ and $G_H^{D_i}$ are electric and magnetic Green's operators acting within domain D_i, respectively. Then the EM modeling problem is reduced to the calculation of the total electric fields inside each inhomogeneous domain.

Rearranging the equation (1) for the electric field induced in inhomogeneous domain D_N, we have:

$$\mathbf{E}^{\Delta\sigma_{D_N}}(\mathbf{r}) = \mathbf{E}^t(\mathbf{r}) - \mathbf{E}^n(\mathbf{r}) - \sum_{i=1}^{N-1} \mathbf{E}^{\Delta\sigma_{D_i}}(\mathbf{r}). \qquad (3)$$

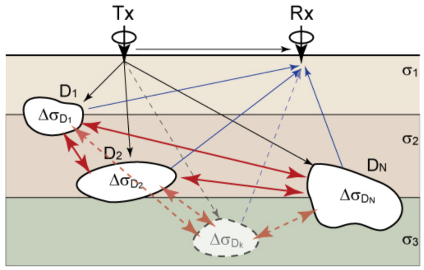

Figure 1: A sketch of a 3D geoelectrical model with horizontally layered (normal) conductivity and N inhomogeneous domains.

In practice, at the first step of the field calculation, we do not know the values of any electric fields in equation (3). We thus first calculate the electric field in domain D_1 without taking into account the induction effect from any other domains:

$$\mathbf{E}^{\Delta\sigma_{D_1}}(\mathbf{r}) = G_E^{D_1}\left[\Delta\sigma_{D_1}\mathbf{E}^t\right] = \mathbf{E}^t(\mathbf{r}) - \mathbf{E}^n(\mathbf{r}). \qquad (4)$$

Equation (4) can be written as an integral equation with respect to the field $\mathbf{E}^{\Delta\sigma_{D_1}}$:

$$\mathbf{E}^{\Delta\sigma_{D_1}}(\mathbf{r}) = G_E^{D_1}\left[\Delta\sigma_{D_1}\left(\mathbf{E}^t + \mathbf{E}^{\Delta\sigma_{D_1}}\right)\right]. \qquad (5)$$

This integral equation is solved using the contraction form of integral equations (Hursán and Zhdanov, 2002) and the complex generalized minimal residual (CGMRES) method (Zhdanov, 2002).

In the calculation of the field due to the currents induced in the next domain D_2, we take into account the electric field induced from the inhomogeneous domain D_1, $\mathbf{E}^{\Delta\sigma_{D_1}}(\mathbf{r})$:

$$\mathbf{E}^{\Delta\sigma_{D_2}}(\mathbf{r}) = G_E^{D_2}\left[\Delta\sigma_{D_2}\mathbf{E}^t\right] = \mathbf{E}^t(\mathbf{r}) - \mathbf{E}^n(\mathbf{r}) - \mathbf{E}^{\Delta\sigma_{D_1}}(\mathbf{r}). \quad (6)$$

The last equation is equivalent to the following integral equation:

$$\mathbf{E}^{\Delta\sigma_{D_2}}(\mathbf{r}) = G_E^{D_2}\left[\Delta\sigma_{D_2}\left(\mathbf{E}^t + \mathbf{E}^{\Delta\sigma_{D_2}} + \mathbf{E}^{\Delta\sigma_{D_1}}\right)\right], \qquad (7)$$

which is solved again by CGMRES method.

Finally, for the last inhomogeneous domain D_N, we already know the electric fields in all the other inhomogeneous domains and thus we can calculate the electric field $\mathbf{E}^{\Delta\sigma_{D_N}}(\mathbf{r})$ as described by equation (3).

To improve the accuracy, we can use this scheme iteratively. In the subsequent iterations, we use the fields obtained in the previous iteration to calculate the induced fields in the given domain. For example, in the second iteration, the calculation of the electric fields from the inhomogeneous domain D_1 will use the electric fields from other domains obtained in the first iteration as:

$$\mathbf{E}_{(2)}^{\Delta\sigma_{D_1}}(\mathbf{r}) = \mathbf{E}_{(1)}^t(\mathbf{r}) - \mathbf{E}^n(\mathbf{r}) - \sum_{i=2}^{N}\mathbf{E}_{(1)}^{\Delta\sigma_{D_i}}(\mathbf{r}), \qquad (8)$$

where the numerical field subscripts denote the iteration number. The electric fields from the other inhomogeneous domains are calculated similarly, always using the latest obtained electric fields for the given domain. For example, for the electric fields due to the domain D_2 at the second iteration:

$$\mathbf{E}_{(2)}^{\Delta\sigma_{D_2}}(\mathbf{r}) = \mathbf{E}_{(1)}^t(\mathbf{r}) - \mathbf{E}^n(\mathbf{r}) - \sum_{i=3}^{N}\mathbf{E}_{(1)}^{\Delta\sigma_{D_i}}(\mathbf{r}) - \mathbf{E}_{(2)}^{\Delta\sigma_{D_1}}(\mathbf{r}). \quad (9)$$

This process is repeated until the electric fields within all the inhomogeneous domains reach self consistency, i.e., the norm of difference between the electric fields in any domain at iterations i and $(i-1)$ is less than a certain threshold ε. In the kth inhomogeneous domain, for example, the electric fields satisfy the following inequality:

$$\frac{\left\|\mathbf{E}_{(i)}^{\Delta\sigma_{D_k}}(\mathbf{r}_j) - \mathbf{E}_{(i-1)}^{\Delta\sigma_{D_k}}(\mathbf{r}_j)\right\|_2}{\left\|\mathbf{E}_{(i)}^{\Delta\sigma_{D_k}}(\mathbf{r}_j)\right\|} < \varepsilon, \qquad \mathbf{r}_j \in D_k. \qquad (10)$$

The developed multiple-domain integral equation (MD IE) method is implemented in a new version of the parallel computer code PIE3D_MD. We will present the results of numerical modeling using this new algorithm and code in the following sections.

APPLICATION OF MD IE METHOD FOR 3D EM MODELING

We consider two numerical examples: a synthetic marine CSEM survey in the Nordkapp Basin, offshore Norway, and a synthetic MT survey in the area of the Radomiro Tomic mine in northern Chile.

Model 1: Nordkapp basin

The hydrocarbon (HC) explorations in the Nordkapp basin started in the 1980s. Apparently, three wells have been drilled to date, all on the flanks of the basin. The recent results of geological and geophysical explorations and discovery of hydrocarbons in wells outside the basin indicate that there is a potential for HC reservoirs discovery within the Nordkapp basin.

The complex salt diapirs represent the major geological structures known in this area. The 3D full-tensor gradiometry (FTG) survey has been carried out in this area in order to provide additional information for evaluation of these complex salt overhang geometries. The focusing inversion of FTG data has been successfully carried out by Wan and Zhdanov (2008).

The geoelectrical model is constructed based on the seismic cross section and the result of the focusing inversion of FTG data. The resistivity cross section of the geoelectrical model is shown in Figure 3. In this model, we construct a resistive HC reseroir (inhomogeneous domain 2) with the resistivity of 100

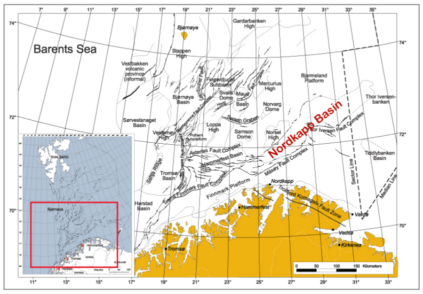

Figure 2: A map of the Nordkapp Basin location.

ohm-m within a complicated inhomogeneous background strucrure (inhomogeneous domain 1) which includes a salt structure (with the resistivity of 30 ohm-m) and folding sediments (with the resistivity varying from 1 to 5 ohm-m). The resistivity of the water layer is 0.3 ohm-m, and the depth of the seafloor is 300 m below the sea level.

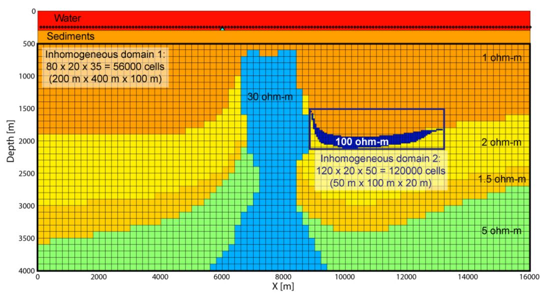

Figure 3: Geoelectrical model of Nordkapp basin. The vertical cross section of a synthetic resistivity model is constructed based on the typical resistivities of the salt diapir and the seabottom sediments.

The EM field in this model is excited by an x-directed electric horizontal bipole with a length of 1 m (the moment of the transmitter is normalized), which is located at the points with horizontal distances from -10 km to 10 km (every 100 m) in the x-direction from an electric receiver, which is located at $X = 6000$ m, as shown in Figure 3. The transmitter generates the frequency-domain EM field at a frequency of 0.25 Hz.

Following the main principles of the MD IE method for multiple inhomogeneous domains, the modeling area is divided into two modeling domains, outlined by the solid lines in Figure 3 (two domains are overlapped with each other). The inhomogeneous domain 1 covers the area with conductivity variations associated with complicated stratification of the seabottom sediments and a salt diapir, while the inhomogeneous domain 2 corresponds to the location of the HC reservoir.

Figure 4 shows the amplitude-versus-offset (AVO) plots of the total electric fields (calculated by the full geoelectrical model containing both inhomogeneous domains) normalized by normal (horizontally layered background) and "normal + inhomogeneous domain 1".

This figure demonstrates that we can evaluate the horizontal location of the reservoir (target) more easily by using the total field normalized by the field which is calculated as a sum of the currents induced in all domains except for the target domain (in this case, except for the field of the inhomongeneous domain 2). Otherwise, it is very difficult to detect the location of the reservoir (target in this model), because the field normalized by only normal field become larger at the location of the large salt diapir.

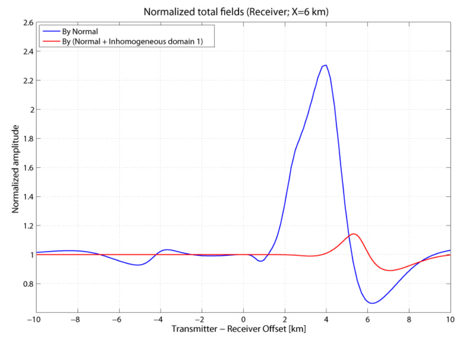

Figure 4: The amplitude-versus-offset (AVO) plots of the x (in-line) component of the electric fields at the receiver ($x = 6$ km) normalized by the normal field (blue line) and (normal + inhomogeneous domain 1) field (red line).

Model 2: Radomiro Tomic

Radomiro Tomic is the first mine to have been entirely developed by the Chilean state copper-mining company, Codelco. It is located at 3000 m above sea level in the Atacama Desert of northern Chile.

In a schematic vertical cross section (Figure 5), copper-bearing clay minerals tend to characterize the upper, very heterogeneous part of the oxidation zone, wheares the lower zone generally is dominated by atacamite mineralization (Brimhall et al., 2001). The copper-bearing minerals form veinlets and coat fractures.

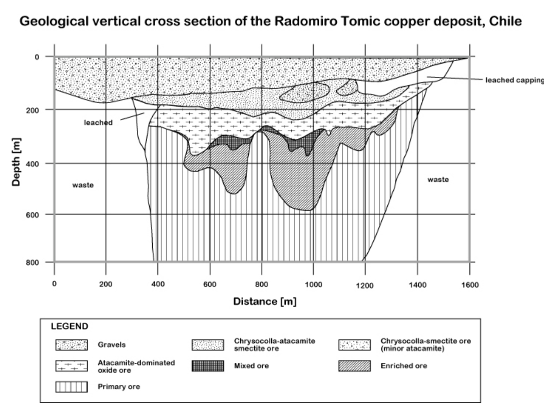

Figure 5: The geological cross section of the Radomiro Tomic copper deposit (after Brimhall et al., 2001).

The geoelectrical model is constructed based on the geological cross section and typical resistivities of rocks. The resistivity cross section of the geoelectrical model is shown in Figure 6. In this model, we construct a conductive copper deposit (inhomogeneous domain 2) with the resistivity of 1 ohm-m within a complicated inhomogeneous background strucrure (inhomogeneous domain 1) with the resistivity varying from 20 to 500 ohm-m.

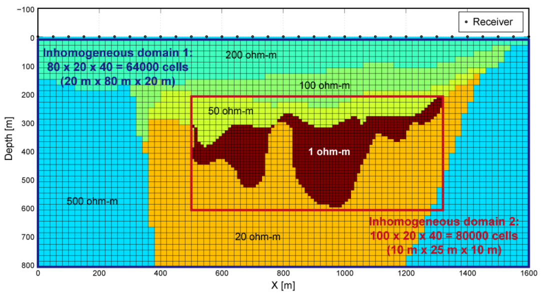

Figure 6: Geoelectrical model of Radomiro Tomic copper deposit. The vertical cross section of a synthetic resistivity model is constructed based on the typical resistivities of the rocks.

We consider the Magnetotelluric (MT) survey along a survey line. The receivers are located at points with horizontal distances from 0 m to 1600 m (every 50 m) in the x-direction, as shown in Figure 6. The frequencies used for the observation are from 0.001 to 1000 Hz (8 frequencies).

Following the main principles of the MD IE method for multiple inhomogeneous domains, the modeling area is divided into two modeling domains, outlined by the solid lines in Figure 6 (two domains are overlapped with each other). The inhomogeneous domain 1 covers the area with conductivity variations associated with complicated stratification of the sediments, while the inhomogeneous domain 2 corresponds to the location of the copper deposit.

Figure 7 shows the x-component of the electric field due to currents induced in the inhomogeneous domain 2 (conductive copper deposit) only. It is clearly shown that we can evaluate the field due to each inhomogeneous domain separately using our MD IE method.

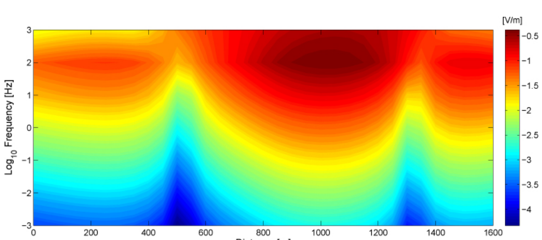

Figure 7: The cross section of the x -component of the electric field due to currents induced in the inhomogeneous domain 2 (conductive copper deposit) only.

Figure 8 shows the total field normalized by; (a) normal (horizontally layered background) field, and (b) (normal + inhomogeneous domain 1) fields. As same as the case of models we demonstrate above, we can evaluate the horizontal location of the target (conductive copper deposit, in this case) more easily by using the total field normalized by the field which is calculated as a sum of the currents induced in all domains except for the target domain.

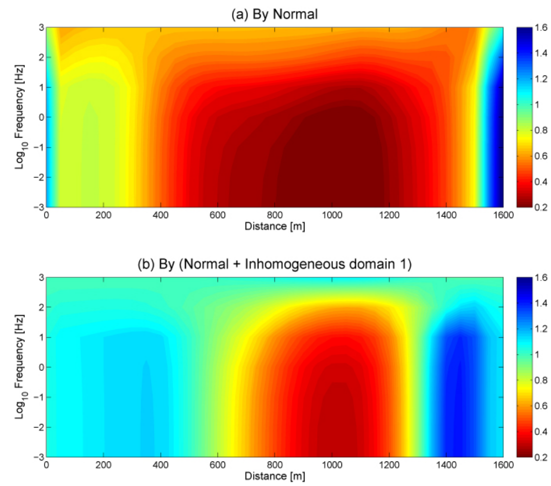

Figure 8: Cross sections of the total electric field (calculated for a full geoelectrical model containing complex stratification of the sediments and copper deposit) normalized by; (a) normal (layered background), and (b) (normal + inhomogeneous domain 1) fields.

CONCLUSIONS

In this paper we have examined the EM data for the models with complex geoelectrical structures using the MD IE method. This method can be used for the complex geoelectrical models with multiple inhomogeneous domains. Contrary to the conventional IE, finite-difference (FD), or finite-element (FE) techniques, the MD IE method requires discretization of the domains with the anomalous conductivity only. At the same time, this method provides a rigorous solution of the EM modeling problem by taking into account the EM coupling between the different domains. In addition, because the MD IE approach is based on the IE method, we can analyze the response of each domain, which can be discretized using its unique cell size, separately, without an inappropriate use of the superposition principle for the EM field calculations.

The numerical modeling results demonstrate that the MD IE method can be effectively used not only for studying the EM fields in complex geoelectrical models with multiple inhomogeneous domains, but also for evaluating the detectability of the EM survey to the complex geoelectrical targets.

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EDITED REFERENCES

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