Foundations of the method of EM field separation into upgoing and downgoing parts and its application to MCSEM data

Michael S. Zhdanov and Shuming Wang, University of Utah*

Summary

The renewed interest in the methods of electromagnetic field decomposition into upgoing and downgoing parts is generated by the practical problem of removing the effect of EM airwaves on marine controlled source electromagnetic data collected in shallow water. In this paper we consider the principles of solving this problem using the classical methods of EM field separation into external and internal parts. We demonstrate that the most general approach to upgoing/downgoing field decomposition is based on the theory of Stratton-Chu type integrals. This approach allows us to separate the field observed on an arbitrary surface. In the case of flat observational surface, the Stratton-Chu method is equivalent to the decomposition technique based on the Fourier transform in the spatial frequency domain. We present also a novel method of EM field separation using the method of horizontal gradients of the EM field. The new technique is tested on synthetic MCSEM data. We demonstrate also that the method represents a useful tool for a rapid qualitative interpretation of MCSEM data.

Introduction

The problem of electromagnetic field separation into upgoing and downgoing parts is one of the oldest problems of geophysics (e.g. , Chapman and Bartels, 1940). The most general approach to the solution of this problem is based on the formalism of the Stratton-Chu type integrals, introduced by Zhdanov (1980, 1988). Later on this technique was also extended to the separation of EM fields measured at the sea bottom (Zhdanova and Zhdanov, 1999).

Recently, we have observed a renewed interest in the problem of upgoing/downgoing EM field separation, generated by increased research and development of marine controlled source electromagnetic (MCSEM) methods. A problem that arises in the MCSEM method is that EM energy may travel from the source to the receiver along many paths. For example, seafloor receivers of EM data measure not only a response from the sea-bottom geoelectrical formations (the "upgoing field") but they also measure a direct part of the primary field from the source, the field travelling from the source to the sea surface and reflected back to the sea bottom (a so-called airwave), and the natural magnetotelluric field. The latter parts of the total EM field form the "downgoing field" because the sources of these fields are located above the receivers (above the seafloor). The methods of EM field decomposition into upgoing and downgoing components in application to MCSEM data are discussed in the recent paper by Amundsen et al., 2006. They presented a constructive method of the decomposition of the EM field given on a horizontal plane, and provided practical examples of its application for 1D and 2D models.

In the current paper we demonstrate how the theory of the Stratton-Chu type integrals can be used for decomposition of the EM field measured on an arbitrary surface, e.g., on a seafloor with a variable bathymetry.

In a case where the data are measured on a horizontal plane, the most effective technique for field separation can be developed using a spatial Fourier transform in the (k, ω) domain (Berdichevsky and Zhdanov, 1984; Zhdanova and Zhdanov, 1999). In this paper, we introduce a new method of the field separation based on using horizontal gradients of the observed EM fields. This method allows us to develop a fast and accurate method for separation of 3D EM fields into upgoing and downgoing fields, which can be effectively used for interpretation of the MCSEM field data.

Application of the Stratton-Chu Type Integrals for EM Field Separation

A general integral method of EM field decomposition into upgoing and downgoing components has been developed by Zhdanov (1980, 1988). We will demonstrate below that this theory provides the foundations for the solution of the problem of EM field separation into upgoing and down-going parts.

Let us consider a typical MCSEM survey conducted in an area with rough bathymetry. The conductivity of seawater is known and it is equal to σ_w. The frequency domain EM field is generated by an electric bipole transmitter T_x located at some depth within the sea-water layer. This field is measured by a system of receivers located at the sea-bottom.

We can represent the electromagnetic field **E**, **H** as a sum of the upgoing E^u, H^u and downgoing E^d, H^d components:

$$E = E^u + E^d, \quad H = H^u + H^d. \qquad (1)$$

Foundations of EM field separation

The problem of field decomposition can be formulated as follows: find the upgoing and downgoing components at the surface of observation S from the total EM field E, H, observed also at S.

We apply the theory of the Stratton-Chu type integrals to solve this problem. Let us take a point at the surface of observation, $r_0 \in S$, and draw a sphere O_R of radius R with the center at r_0 (see Figure 1). We denote by Γ_R^+ a piece-wise smooth close surface formed by the semisphere O_R^+ and S_R, $\Gamma_R^+ = O_R^+ \cup S_R$. This surface Γ_R^+ is a close boundary of the upper part, D_R^+, of the ball bounded by a sphere O_R, located above surface S.

We can introduce now the Stratton-Chu integrals over a piece-wise smooth close surface Γ_R^+:

$$C^E_{\Gamma_R^+}(\mathrm{r}') = \iint_{\Gamma_R^+} [(\mathrm{n}\cdot\mathrm{E})\nabla G_w + (\mathrm{n}\times\mathrm{E})\times\nabla G_w + i\omega\mu(\mathrm{n}\times\mathrm{H})G_w]ds, \tag{2}$$

$$C^H_{\Gamma_R^+}(\mathrm{r}') = \iint_{\Gamma_R^+} [(\mathrm{n}\cdot\mathrm{H})\nabla G_w + (\mathrm{n}\times\mathrm{H})\times\nabla G_w + \sigma_w(\mathrm{n}\times\mathrm{E})G_w]ds, \tag{3}$$

where $\mathrm{r}' \in D_R^+$, n is the unit vector of an inward pointing normal to D_R^+, and G_w is the fundamental Green's function for the Helmholtz equation in a homogeneous full space with the seawater conductivity σ_w.

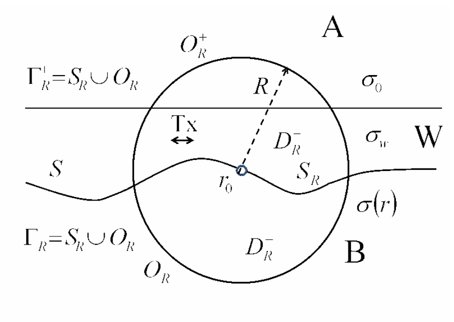

Figure 1: A scheme illustrating the applications of the Stratton-Chu type integrals to the separation of the upgoing and downgoing fields.

Using the properties of Stratton-Chu type integrals (Zhdanov 1988), one can demonstrate that

$$\mathrm{E}^u(\mathrm{r}_0) = \frac{1}{2}\mathrm{E}(\mathrm{r}_0) + C_S^E(\mathrm{r}_0)\ , \tag{4}$$

and

$$\mathrm{E}^d(\mathrm{r}_0) = \frac{1}{2}\mathrm{E}(\mathrm{r}_0) - C_S^E(\mathrm{r}_0)\ . \tag{5}$$

Similar expressions can be derived for magnetic field components as well:

$$\mathrm{H}^u(\mathrm{r}_0) = \frac{1}{2}\mathrm{H}(\mathrm{r}_0) + C_S^H(\mathrm{r}_0)\ , \tag{6}$$

and

$$\mathrm{H}^d(\mathrm{r}_0) = \frac{1}{2}\mathrm{H}(\mathrm{r}_0) - C_S^H(\mathrm{r}_0)\ . \tag{7}$$

In formulas (4) through (7), expressions $C^E_S(\mathrm{r}_0)$ and $C^H_S(\mathrm{r}_0)$ stand for the Stratton-Chu type integrals over the entire observation surface S, determined at a singular point $\mathrm{r}_0 \in S$ in terms of the Cauchy principal value.

Equations (4) through (7) describe a general integral transformation of the total EM field, observed on the arbitrary surface S within the water layer, into upgoing and downgoing parts. These equations serve as a theoretical foundation of the method of up/down decomposition of the EM field.

Electromagnetic Field Separation into Upgoing and Downgoing Parts Using Horizontal Gradients

We should note that the method of EM field decomposition based on Stratton-Chu type transforms requires measurements of the total EM field on the entire surface of observation S. In practice, however, we measure these fields at a discrete number of observation points located within a limited part of S. That is why, it is important to develop simplified methods of field decomposition which would require measurements at just a few points. The simplest way to solve this problem is based on an assumption that the observed data can be approximated by a vertically propagated plane EM wave. This approach, for example, was discussed in Amundsen et al. (2006) and it is widely used for decomposition of the field MCSEM data.

However, one can produce a more accurate but still rapid decomposition method assuming just slow horizontal variations of the EM field observed within the area of observation. In this case, the EM field within the seawater can be expressed approximately as (Zhdanov et al., 1996):

$$\mathrm{E}(x,y,z) = \mathrm{Q}_E^u(x,y,z)\exp[-ik(z-z_0)] + \mathrm{Q}_E^d(x,y,z)\exp[ik(z-z_0)]\ ,$$
$$\mathrm{H}(x,y,z) = \mathrm{Q}_H^u(x,y,z)\exp[-ik(z-z_0)] + \mathrm{Q}_H^d(x,y,z)\exp[ik(z-z_0)]\ , \tag{8}$$

where $z_b \geq z \geq 0$, z_b is the depth of the sea bottom, z_0 is the depth of the location of the receivers, and $k = k_w = \sqrt{i\omega\mu\sigma_w}$.

We assume that the vector functions Q^u_E, Q^d_E, Q^u_H, Q^d_H vary relatively slowly with the depth. The goal is to find the upgoing and downgoing components of the EM field. This problem can be solved by differentiation of both sides of equations (8) with respect to the vertical coordinate z:

$$\partial \mathrm{E}(x,y,z)/\partial z = ik\mathrm{Q}_E^d \exp[ik(z-z_0)] - ik\mathrm{Q}_E^u \exp[-ik(z-z_0)],$$
$$\partial \mathrm{H}(x,y,z)/\partial z = ik\mathrm{Q}_H^d \exp[ik(z-z_0)] - ik\mathrm{Q}_H^u \exp[-ik(z-z_0)]. \tag{9}$$

Multiplying both sides of equation (8) by ik and adding (9), we have:

$$ik\mathrm{E}(x,y,z) - \partial \mathrm{E}(x,y,z)/\partial z = 2ik\mathrm{Q}_E^u \exp[-ik(z-z_0)],$$
$$ik\mathrm{H}(x,y,z) - \partial \mathrm{H}(x,y,z)/\partial z = 2ik\mathrm{Q}_H^u \exp[-ik(z-z_0)]. \tag{10}$$

From the last expression we find immediately that:

$$\mathrm{Q}_E^u(x,y,z) = \tfrac{1}{2}[\mathrm{E}(x,y,z) - \tfrac{1}{ik}\partial \mathrm{E}(x,y,z)/\partial z]\exp[ik(z-z_0)],$$
$$\mathrm{Q}_H^u(x,y,z) = \tfrac{1}{2}[\mathrm{H}(x,y,z) - \tfrac{1}{ik}\partial \mathrm{H}(x,y,z)/\partial z]\exp[ik(z-z_0)]. \tag{11}$$

Thus the problem of field separation is reduced to computations of the vertical derivatives of the electric and magnetic fields. The last problem can be solved using Maxwell's equations. It can be shown that the vertical derivatives of electric and magnetic fields, $\partial E/\partial z$ and $\partial H/\partial z$, can be expressed using horizontal gradients of these fields. For example, we have for electric field:

$$Q_{E_x}^u = \tfrac{1}{2}E_x - \tfrac{1}{2ik}\tfrac{\partial}{\partial x}E_z - \tfrac{i\omega\mu}{2ik}H_y,$$
$$Q_{E_y}^u = \tfrac{1}{2}E_y - \tfrac{1}{2ik}\tfrac{\partial}{\partial y}E_z + \tfrac{i\omega\mu}{2ik}H_x, \tag{12}$$
$$Q_{E_z}^u = \tfrac{1}{2}E_z + \tfrac{1}{2ik}\tfrac{\partial}{\partial x}E_x + \tfrac{1}{2ik}\tfrac{\partial}{\partial y}E_y.$$

Note that the expressions $\frac{\partial}{\partial x}E_x, \frac{\partial}{\partial y}E_y, \frac{\partial}{\partial x}E_z, \frac{\partial}{\partial y}E_z$, etc. represent the horizontal gradients of the electric and magnetic field components.

After we determine the vector coefficients Q_E^u, Q_E^d, and Q_H^u, Q_H^d, we can find the upgoing and downgoing fields themselves, e.g.:

$$\mathrm{E}^u(x,y,z) = \mathrm{Q}_E^u \exp[-ik(z-z_0)],$$
$$\mathrm{H}^u(x,y,z) = \mathrm{Q}_H^u \exp[-ik(z-z_0)]. \tag{13}$$

Numerical Examples of Marine EM Data Decomposition

In order to check the effectiveness of the separation technique developed above based on the horizontal gradients, we have conducted a number of numerical experiments with synthetic EM data. We present below just one of these results.

For a model shown in Figure 2, the geoelectrical section of this model is formed by a seawater layer with a resistivity of 0.4 ohm-m and a thickness of 340 m, and conductive sea-bottom sediments with a resistivity of 1 ohm-m, respectively. A 3D reservoir with 100 ohm-m resistivity is embeded in the sea bottom at a depth of 500 m below the seafloor. The EM field in the model is generated by a horizontal electric bipole transmitter towing behind the ship at an elevation of 50 m above the seafloor. The receivers measuring all six components of the EM field (or four horizontal components only) are located at the seafloor along several survey lines with a separation of 500 meters in both x and y directions.

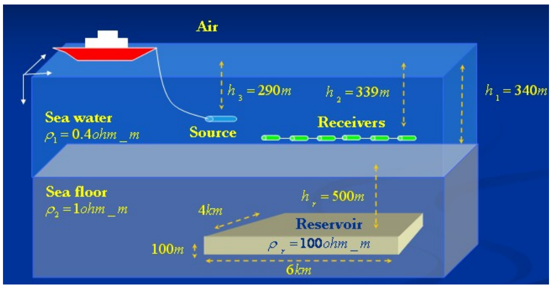

Figure 2: The geoelectrical section of the model is formed by a seawater layer and conductive sea-bottom sediments. A 3D is embeded in the sea bottom at a depth of 500 m below the seafloor.

We should note that the problem of upgoing/downgoing field separation can be solved using the data along one survey line only. However, the most profound result can be seen in the maps of the upgoing and downgoing parts of the field (see Figures 3 and 4). One can clearly see in these figures that the separation of the upgoing part of the total electric field results in effective imaging of the horizontal location of the reservoir. A similar effect is observed in the maps of the horizontal component of the magnetic field H_y after a decomposition transformation.

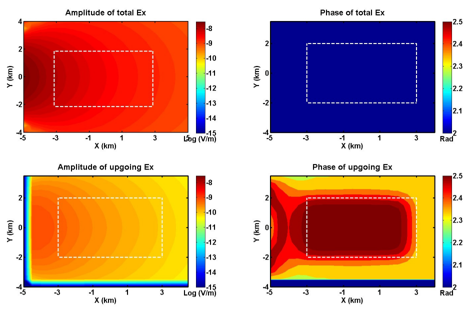

Figure 3: The top panels show maps of the amplitude (left top panel) and phase (right top panel) of the total in-line electric field E_x at a frequency of 0.5 Hz. The bottom panels present maps of the amplitude (left bottom panel)

and phase (right bottom panel) of upgoing in-line electric field E_x^u separated using the method of horizontal gradients.

Note that in Figures 3 and 4 a dashed white rectangle outlines the location of a resistive reservoir. From these results, it is clear, the upgoing field (real and/or imaginary parts, amplitude and/or phase) separated using the developed decomposition technique, reflects the location of the target much better than the total EM field data. Numerical data, shown above, were generated for a frequency of 0.5 Hz. We also used other frequencies from 0.1 Hz to 10 Hz and other locations of the transmitting bipole source to test our technique. We have obtained similar results for all these frequencies and different transmitter locations.

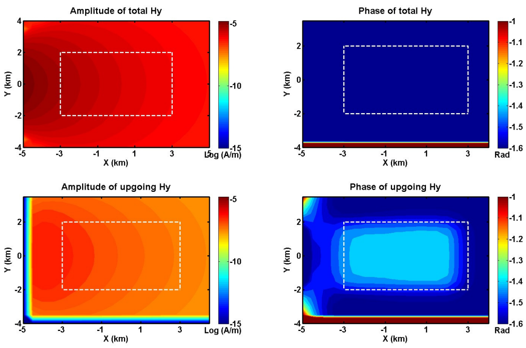

Figure 4: Model 1: The top panels show maps of the amplitude (left top panel) and phase (right top panel) of the total magnetic field H_y at a frequency of 0.5 Hz. The bottom panels present maps of the amplitude (left bottom panel) and phase (right bottom panel) of upgoing magnetic field H_y^u separated using the method of horizontal gradients.

Conclusions

The modern technique of EM field decomposition into upgoing and downgoing parts has its roots in the classical methods of geomagnetic field separation into external and internal parts. The most general approach to the solution of this problem is based on the theory of the Stratton-Chu type integrals. This approach makes it possible to separate the MCSEM data observed on the seafloor in areas with rough bathymetry. However, Stratton-Chu type transformation, in a general case, requires measurement of the data in the relatively large area of observation.

We have developed a method of solving this problem which is free of these limitations. It is based on the calculations of the horizontal gradients of the field, and therefore it can be used for a decomposition of MCSEM data measured at a few points only. The numerical study shows that this novel method can be used as a rapid transformation of MCSEM data for a qualitative evaluation of the location of the typical exploration targets, e.g., an HC reservoir.

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EDITED REFERENCES

Note: This reference list is a copy-edited version of the reference list submitted by the author. Reference lists for the 2009 SEG Technical Program Expanded Abstracts have been copy edited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

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