3D lithological inversion of geophysical data

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Summary

We have developed a deterministic method for directly inverting geophysical data to 3D lithological models. This method is based upon a lithology-based model transform of the model parameters and their sensitivities from their physical property basis to one of a lithology basis. This method is general, as it can be applied to both linear and nonlinear geophysical methods, and that the physical properties defining a lithology may have a statistical distribution. We demonstrate the method with a case study for the 3D inversion of airborne electromagnetic (AEM) data for bathymetry mapping in the Backstairs Passage in South Australia, where the 3D earth model is characterized as discrete lithologies of seawater, sediment, and basement. Our results are shown to be in very good agreement with LiDAR bathymetry from the same area.

Introduction

Deterministic methods based upon least-squares model optimization are routinely applied to the regularized 3D inversion of geophysical data (e.g., Zhdanov, 2002). The 3D physical property models contain a continuum of values, and these are subsequently interpreted for geology by classifying their ranges as discrete lithologies. An alternative approach directly inverts geophysical data for lithological models of pre-determined physical properties (e.g., Bosch, 1999, 2004). The strength of this approach, the discrete physical properties, is also its major weakness. Since lithological models are defined by discrete physical properties, deterministic methods based on derivative-based minimization of the objective functional have not been applicable. As such, various stochastic methods have been implemented for potential fields since the modeling operators are linear (e.g., Guillen et al., 2004; Krahenbuhl and Li, 2009). Unfortunately, stochastic methods are more difficult to implement for 3D modeling operators that are nonlinear. Since we remain interested in solving the lithological inversion for all geophysical methods in a deterministic manner, we have introduced a transform of the model parameters and their sensitivities from their physical property basis to one of a lithology basis. In our case, a lithology is defined by a physical property value with some statistical variance. The method is general, and can be applied to any geophysical method. We demonstrate this with a model study. We also present a case study of 3D AEM inversion for bathymetry mapping in the Backstairs Passage in South Australia, where the 3D earth model was characterized by discrete lithologies of seawater, sediment, and basement.

Lithology-based model transform

We transform our physical properties into a model space defined by continuous range of model parameters. Yet, this model space maps the model parameters into discrete groups, or lithologies. The model space we have chosen is defined by a cumulative density function (cdf). Each model parameter is defined by its mean and standard deviation, which are chosen a priori. Figure 1 illustrates this for the simple case of seawater (4.7 S/m ± 10%) and basement (0.001 S/m ± 10%). Figure 1a shows the probability density function (pdf) of the log normal distribution of the model parameters. Figure 1b shows the cdf representing the model space. A small constant is added to the pdf, and this is incorporated into the cdf. This constant accounts for the non-vanishing probability that the physical property of a cell in the 3D earth model does not fall within any of the pre-defined lithologies. This also eliminates singularities in the transform between the cdf and physical properties.

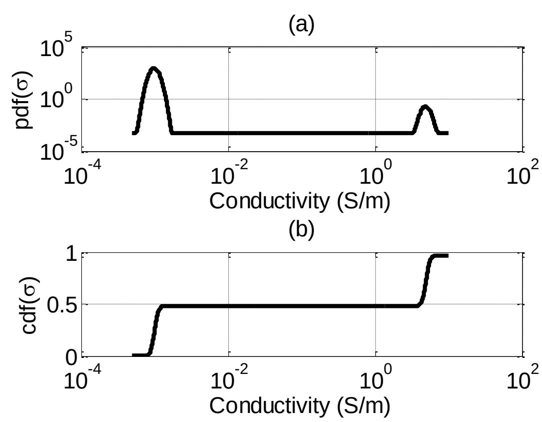

Figure 1. *(a) The probability density function (pdf) of two lithologies. (b) Cumulative density function (cdf) of the same two lithologies. The ordinate axis corresponds to the lithology, while the abscissa is the physical parameters (i.e., conductivity).*

Our inversion operates in the space of the cdf. However, we can transform the cdf back to physical properties. The transformation of the physical properties into model parameters is accomplished with a simple log normal cdf function. The sensitivities must also be transformed from data with respect to the physical properties into data with respect to the cdf. This is accomplished by simply

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multiplying the sensitivities of data with respect to conductivity (found through reciprocity) by the inverse of the pdf:

$$\frac{\partial d}{\partial \mathrm{cdf}(\sigma)} = \frac{\partial d}{\partial \sigma}\frac{\partial \sigma}{\partial \mathrm{cdf}(\sigma)} = \frac{\partial d}{\partial \sigma}\left(\frac{\partial \mathrm{cdf}(\sigma)}{\partial \sigma}\right)^{-1} = \frac{\partial d}{\partial \sigma}(\mathrm{pdf}(\sigma))^{-1}. \quad (1)$$

The model space has now been transformed into the pdf space. In the above example, cdf values between 0 and 0.5 will transform to the basement lithology, and values between 0.5 and 1 will transform to the seawater lithology. Note that, this can be extended to any number of lithologies, and that it can be applied to any deterministic inversion algorithm, implying that a priori models, constraints, and the variety of smooth and focusing stabilizers may still be used with only minor modification.

Model study: 3D AEM inversion

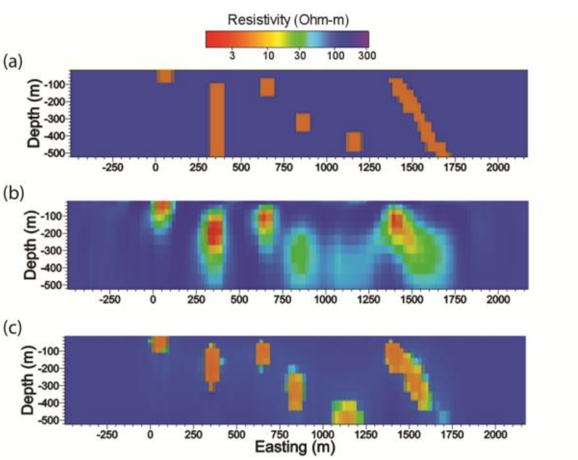

Figure 2. *(a) Vertical cross section of the 3D resistivity model for which MEGATEM data were simulated. (b) Vertical cross section of the 3D resistivity model from conventional 3D inversion of the MEGATEM data. (c) Vertical cross section of the 3D resistivity model from lithology-based inversion of the same MEGATEM data.*

To demonstrate our methodology, we present an AEM-based model study. Cox and Zhdanov (2007) and Cox et al. (2010) recently introduced the concept of a moving sensitivity domain for 3D AEM inversion. According to this concept, one only needs to calculate the responses and sensitivities for that part of the 3D earth model that is within the AEM system's sensitivity domain, and then superimpose the sensitivities for all sensitivity domains into a single, sparse sensitivity matrix for the entire 3D earth model. For each AEM system footprint, our 3D modeling is based on a frequency-domain contraction integral equation method (e.g., Hursán and Zhdanov, 2002) that solves for the total electric field from the background electric fields. For time-domain AEM responses and sensitivities, the frequency-domain responses and sensitivities are transformed to the time domain via a cosine transform and convolution with the system waveform (Raiche, 1998). The Tikhonov parametric functional is minimized using the reweighted regularized conjugate gradient (RRCG) method (Zhdanov, 2002).

The 3D earth model consisted of several discrete, conductive targets of 10 Ωm embedded at various depths and dips embedded in an otherwise homogeneous half-space of 100 Ωm. A MEGATEM fixed-wing time-domain AEM system with 90 Hz base frequency was simulated for inline and vertical component dB/dt data acquired at a 120 m flight height, with the bird towed 120 m behind and 35 m below the aircraft. As this was intended as a demonstration of the method, no noise was added to the synthetic data. We inverted this data where the inversion had to choose between a lithology of 0.1 S/m ±5% (corresponding to the targets) or a lithology of 0.01 S/m ±5% (corresponding to the host). For comparison, we also inverted the same data using standard 3D AEM inversion with a minimum norm stabilizer. As shown in Figure 2, we observe that the lithology-based inversion better recovered all bodies. In particular, we note the continuum of model parameters is more

Case study: 3D AEM inversion for bathymetry

Bathymetry is commonly estimated from manual soundings or acoustic echo reflection measured from vessels which may be endangered by reefs or shallow waters. In clear water, airborne LiDAR is preferred because of high spatial resolution. However, LiDAR is inaccurate in areas of high turbidity or turbulence because of suspended particles or air bubbles. AEM methods have been used for bathymetry mapping in shallow water (e.g., Vrbancich, 2011), but AEM-derived bathymetry has not yet established itself as a reliable technique. Recent studies in Australian waters have shown that AEM bathymetry can provide accurate water depths down to approximately 55 m using fixed-wing and helicopter time-domain systems. These previous studies focused on areas where the bathymetry was relatively featureless and 1D inversion or conductivity-depth imaging sufficient for interpretation (e.g., Vrbancich et al., 2000; Vrbancich and Fullagar, 2007).

The bathymetry of the Backstairs Passage (BSP), located between the Fleurieu Peninsula and Kangaroo Island approximately 100 km south-west of Adelaide, South Australia (Figure 3), contains two unique features that represent 3D bathymetric targets accessible from coastline

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using a helicopter AEM system. The principle bathymetric feature is a series of ridges resembling sandwave structures centered around Yatala Shoals (35.74^o S, 138.18^o E) which fan northwards and decrease in height as they spread over the seafloor (Figure 4). A HoisTEM helicopter central loop time-domain AEM survey was flown over the area in 2002, and a RepTEM helicopter central loop time-domain AEM survey was flown over the same area in 2010. In the latter survey, prevailing conditions did not permit the use of a vessel to obtain seawater conductivity soundings. This value of seawater conductivity is relatively high and has been observed in the nearby Port Lincoln area following the warmer summer months during which time the coastal waters have warmed.

Our 3D lithology-based AEM inversion discretized the earth model into cells of 10 m dimension in the inline direction, 25 m in the cross-line direction, and 2 m in the vertical direction through the area of interest. Two conductivities were assigned to the model: seawater (4.7 S/m ± 5%), sediment (1 S/m ± 20%) and basement (0.001 ± 10%). Profiles of the bathymetry as recovered from the 3D lithology-based inversion of the HoisTEM and RepTEM data are shown in Figures 5 and 6 (blue lines), with comparison to 1D inversion (red lines) and LiDAR acquired in 2000 (black lines). We note that the 3D and CDI results are similar. Further, there is a seemingly improved resolution of several bathymetric highs as recovered by the 3D inversion. This is reasonably expected. Given the time lapse between LiDAR and AEM surveys, some of the differences between the AEM-derived bathymetry and LiDAR bathymetry may be due to actual changes in the bathymetry. This is currently being investigated.

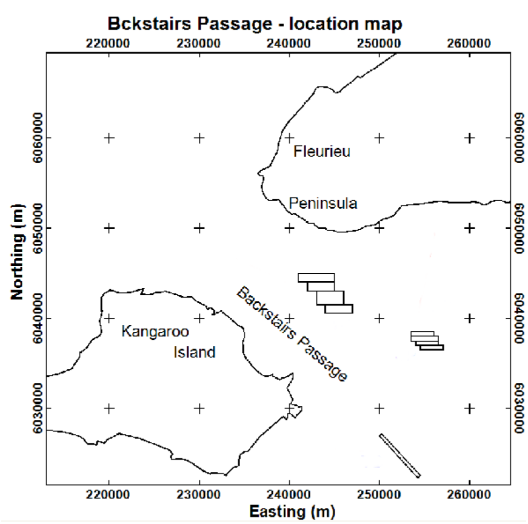

Figure 3. *Backstairs Passage location map.*

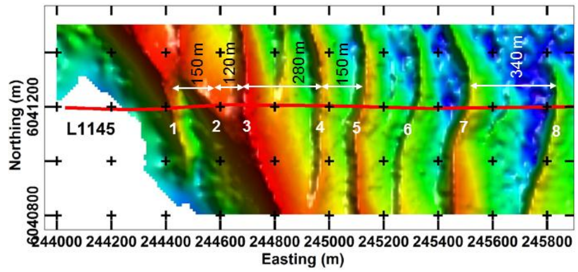

Figure 4. *Yatala Shaols bathymetry, with RepTEM line L1145 superimposed*

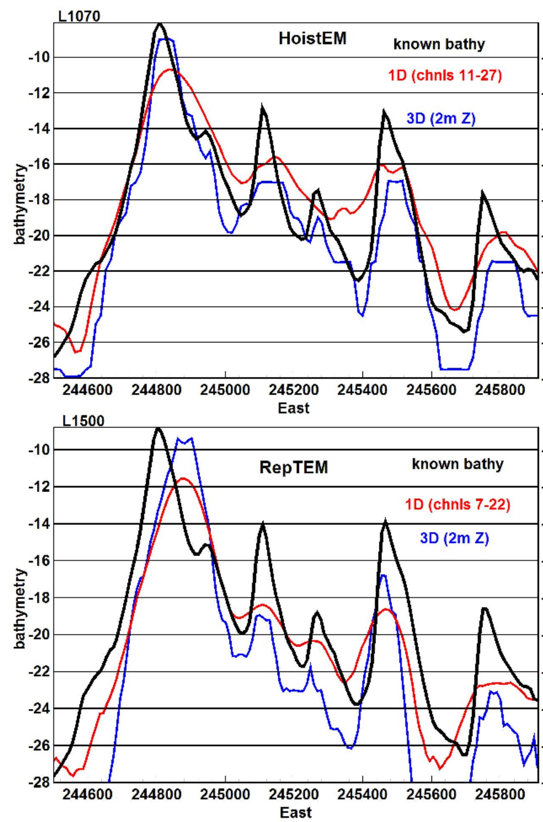

Figure 5. *Comparison of LiDAR-derived bathymetry (from 2000) with AEM-derived bathymetry using 1D inversion and 3D lithological inversion for collocated HoisTEM line L1070 and RepTEM line L1500.*

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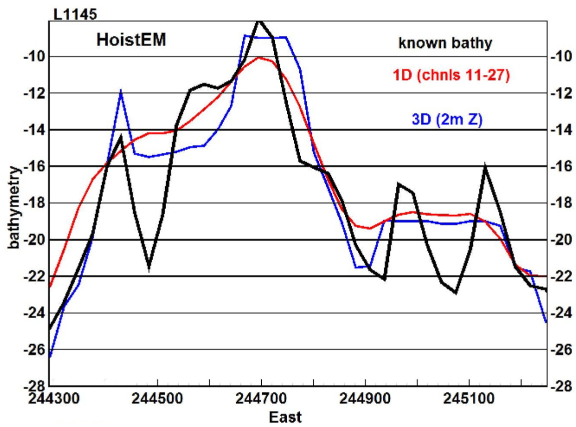

Figure 6. *Comparison of LiDAR-derived bathymetry (from 2000) with AEM-derived bathymetry using 1D inversion and 3D lithological inversion for HoistEM line L1145.*

Conclusions

In this paper, we have described a deterministic method for directly inverting geophysical data to 3D lithological models. This method is based upon a lithology-based transform of the model parameters and their sensitivities from their physical property basis to one of a lithology basis. This method is general, and can be applied to both linear and nonlinear geophysical methods. We have demonstrated the method as applied to 3D AEM inversion for bathymetry mapping where the 3D earth model is characterized as discrete lithologies of seawater, sediment, and basement. This method can be further generalized for the simultaneous joint inversion of multiple geophysical data, and this is the subject of our on-going research.

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EDITED REFERENCES

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