# Inversion of gravity data in the Big Bear Lake Area to recover depth to basement using Cauchy-type integrals

*Hongzhu Cai*, University of Utah, Michael Zhdanov, University of Utah and Technoimaging*

**Summary**

One of the important applications of the gravity method is evaluation of the depth to the basement, which is characterized by a significant density contrast with the sedimental layeres. We have introduced recently a new method of modeling and inversion of potential field data generated by a density contrast surface based on 3D Cauchy-type integral representation of the potential fields (Zhdanov and Cai, 2013). The technique of the Cauchy-type integrals makes it possible to represent the gravity field and its gradients as surface integrals. In the previous work, it was assumed that the density contrast between sediment and basement was a constant value. However, in some practical situations, the density of sediments may change with the depth. As a result, the density contrast between sediment and basement is also a function of depth. In this paper, we develop a method for modeling the gravity response caused by sediment-basement interface with variable density in depth. We have also developed the inversion of gravity data to recover the depth to basement given the density profile with depth.

**Introduction**

It was demonstrated in the papers by Zhdanov and Liu (2013) and Zhdanov and Cai (2013) that 3D Cauchy-type integrals can be effectively used for modeling and inversion of the gravity and gravity gradient data in models where the anomalous field is generated by a density-contrast surface.
This paper extends the approach based on 3D Cauchy-type integrals to modeling and inversion of the gravity response caused by sediment-basement interface with variable density contrast. In our study, we consider a model formed by two quasi-horizontal layers, the first layer representing the sediments and the second layer describing the basement.

We have developed an inversion scheme to determine both the density contrast surface and the density contrast function. Gravity field and/or full tensor gravity gradiometry data can be used for the inversion. The inversion scheme is based on the re-weighted regularized conjugate gradient method (Zhdanov, 2002). Note that the method based on the Cauchy-type integrals requires the discretization of the contrast surface only, which reduces dramatically the computing resources in comparison with the conventional methods based on the discretization into prismatic cells.

**Representation of a gravity field by Cauchy-type integrals for a variable density contrast model**

The 3D analog of the Cauchy-type integral and its derivation was presented in Zhdanov (1988) as follows:

$$\mathbf{C}^S(\mathbf{r}',\boldsymbol{\varphi}) = -\frac{1}{4\pi}\iint_S \left[(\boldsymbol{n}\cdot\boldsymbol{\varphi})\nabla\frac{1}{|\mathbf{r}-\mathbf{r}'|} + (\boldsymbol{n}\times\boldsymbol{\varphi})\times\nabla\frac{1}{|\mathbf{r}-\mathbf{r}'|}\right]ds, \qquad (1)$$

where $S$ is some closed surface bounding a domain $D$, $\boldsymbol{\varphi}=\boldsymbol{\varphi}(\mathbf{r})$ is some vector function defined on the closed surface $S$, and $\mathbf{n}$ is the normal vector to the surface $S$ pointing outside $D$. The vector function $\boldsymbol{\varphi}$ is called the vector density of the Cauchy-type integral.

It was shown by Zhdanov (1988) and Zhdanov and Cai (2013) that, the gravity field caused by a three dimensional body with constant density can be expressed by a Cauchy-type integral as follows:

$$\mathbf{g}(\mathbf{r}') = \frac{4\pi}{3}\gamma_g\rho_0\mathbf{C}^{\Gamma_R}(\mathbf{r}',\mathbf{r}-\mathbf{r}'). \qquad (2)$$

We consider a model of the sediment-basement interface with a constant density contrast at some surface Γ shown in Figure 1. The surface Γ is described by equation $z = h(x,y) - H_0$, and a horizontal plane $P$ is given by equation $z = -H_0$, where $H_0 \geq -h(x,y) \geq 0$, and: $h(x,y) - H_0 \to 0$ for $\sqrt{x^2+y^2} \to \infty$, where $H_0$ is a constant. Let us draw a sphere $O_R$ of radius $R$ with the center in the origin of the Cartesian system of coordinates. We denote by $\Gamma_R$ and $P_R$ the parts of the surfaces Γ and P, respectively, located within the sphere $O_R$. The gravity anomaly is caused by the density volume $D_R$: which is bounded by a closed surface, formed by $\Gamma_R$ and $P_R$ and the parts of the sphere $O_R$ between these two surfaces as shown in Figure 1.

It is demonstrated in Zhdanov (1988) that the gravity field caused by volume $D_R$ is expressed by:

$$\mathbf{g}(\mathbf{r}') = 4\pi\gamma_g\rho_0\mathbf{C}^{\Gamma_R}(\mathbf{r}',(z+H_0)\mathbf{d}_z). \qquad (3)$$

As a result, in the case where $\Gamma_{R\to\infty} \to P$ at infinity, the Cauchy-type integral in equation 5 is calculated along an infinitely extended surface Γ only. In a general case the density contrast value between sediment and basement is a function of depth:

$$\Delta\rho = f(z). \qquad (4)$$


DOI http://dx.doi.org/10.1190/segam2014-0251.1

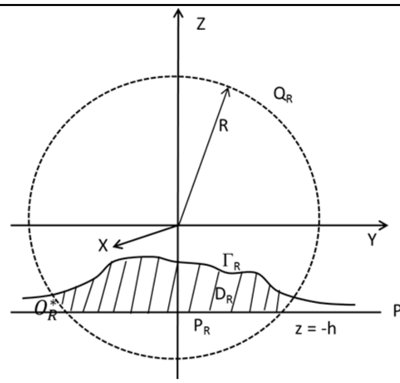


Figure 1: An illustration of the contrast density model for a sediment-basement interface. The plane P is the average depth of the sediment-basement interface, and $\Gamma_R$ is the actual sediment-basement interface.

In this case, the representation of gravity field caused by the sediment-basement interface takes the following form (Zhdanov, 1988):

$$\mathbf{g}(\mathbf{r}') = 4\pi\gamma_g\rho_0\mathbf{C}^{\Gamma_R}(\mathbf{r}', R(\mathbf{z})\mathbf{d}_z), \qquad (5)$$

where: $R(\mathbf{z}) = \int_{-H_0}^{z} f(z)dz.$

Similar equations can be derived for the gravity gradient component by taking the spatial derivative of the forward operator for gravity field.

Note that, function $f(z)$ describing the density variations with the depth can be approximated by a polinomial or exponencial function, depending on a number of parameters, $p_n$.

**Inversion for a density contrast surface based on 3D Cauchy-type integrals**

The main difference between the traditional approach to the inversion, based on volume integral representation of the gravity field, and this new approach, based on the surface Cauchy-type integrals, is that in the latter case the model parameters are the elevations, $h^{(k)} = h(x_k, y_k)$ of the density contrast surface with respect to the reference horizontal plane. In adition, parameters $p_n$ of the density contrast function, $f(z)$, consitutte the unknown model parameters as well. This inverse problem is a nonlinear one, and its solution requires the calculation of the corresponding Fréchet derivative operator. The advantage of representing the forward modeling operator (5) for the gravity field using Cauchy-type integrals is that this Fréchet derivative operator has an analytical form by taking the differential of the forward operator with respect to the model parameter.

As usual, the inversion of gravity and gravity gradient data is an ill-posed problem. In order to obtain a stable and geologically reasonable result, we use the Tikhonov regularization based on the minimization of the following parametric functional (Tikhonov and Arsenin, 1977).

The minimization of the Tikhonov parametric functional is based on the reweighted regularized conjugate gradient method (Zhdanov, 2002).

The developed theory and method have been implemented in the computer code which was tested on synthetic gravity data and USGS field data.

**Model studies**

In this section, we present a model study for the modeling and inversion of gravity data caused by density-contrast surface with a density contrast that is variable with the depth. We consider a model with the density contrast between sediment and basement changing exponentially with the depth:

$$\Delta\rho = ae^{-bz} + ce^{-ez}. \qquad (6)$$

Figure 2 shows a plot of the density contrast variation with the depth for this model. Figure 3 shows a prism approximation for this model in a vertical section.

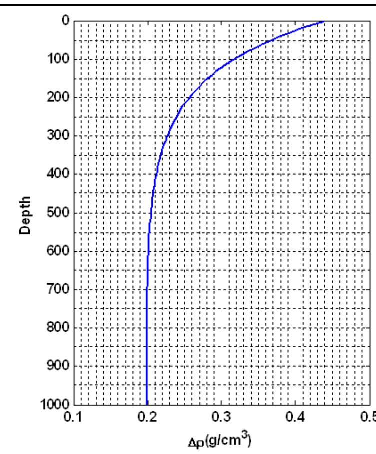


Figure 2: A plot of the exponential density contrast variation with depth.

Forward modeling based on the Cauchy-type integral method is compared with that based on the traditional method. We have also applied the same inversion to the data simulated for the second model since a similar exponential density profile will be used for the inversion of the field data in the following section.



DOI http://dx.doi.org/10.1190/segam2014-0251.1

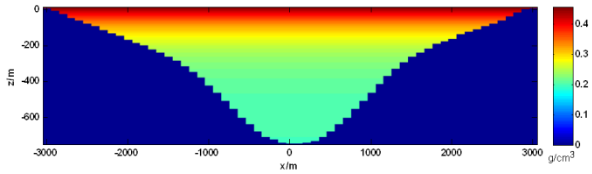

Figure 3: A prism approximation of the density contrast surface whith the density contrast changing exponentially with the depth.

We present the gravity responses computed using Cauchy-type integral and the traditional volume integral methods in Figure 4. One can see that the result produced by the new method practically coincides with that of the traditional method.

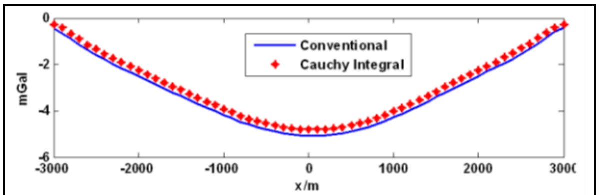

Figure 4: Comparison of forward modeling results obtained using Cauchy-type integral (dotted red line) and traditional volume integral (solid blue line) methods.

We applied the inversion algorithm introduced in the previous sections to the inversion of the synthetic data computer simulated for the model with exponential density variation. Figure 5 shows the inversion result for the synthetic model with exponential density variation with depth. One can see that both the depth and shape of the basement is well recovered.

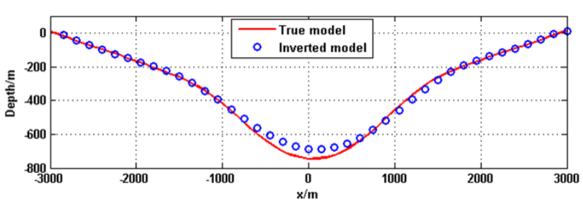

Figure 5: The inversion result for the model with exponential density contrast varying with depth.

**Inversion of gravity data at the Big Bear Lake Area**

The Big Bear Lake area is located in the southeast part of California. The area is characterized by a deep sedimental basin surrounded by uplifted bedrocks. The USGS produced a basin model from the surface geology, well-logs, and potential field data. Figure 6 shows that the whole basin area can be divided into three parts from the northeast to the southwest: Deadman Lake Basin, Surprise Spring Basin, and Joshua Tree Basin. The average depth and density variations between sediment and bedrocks may be slightly different.

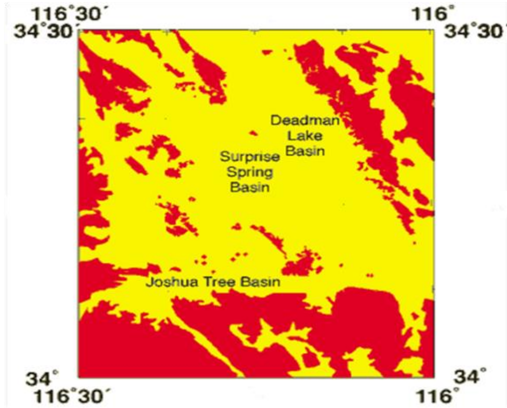

Figure 6: A USGS model of the basin for the Big Bear Lake area. The red zones indicate the location of bedrocks.

The released USGS isostatic Bouguer gravity data is gridded using an equivalent layer method. The gravity stations on the bedrocks are used to get the bedrock componnent of the gravity anomaly. The bedrock component of the gravity anomaly will be removed to obtain the isostatic Bouguer gravity gravity caused by the varaition of sediment basin.

One needs to know that the density variation with the depth in order to get an accurate model of the depth to the basement. As we mentioned above, this information can be obtained from well-log data. The density models of the DeadMan Lake and Surprise Spring Basins are slightly different from that of Joshua Tree Basin.

In the inversion, we used a grid size of 300 m by 300 m in the x and y directions, which is much finer than the USGS model grid for prismatic inversion (2 km by 2 km).

*Inversion of the Deadman Lake and Surprise Spring Basins' gravity data*

In order to take the variable density contrast into account, we needed to use some analytical function of depth to approximate the density contrast. For the USGS model, we found that it was better to use equation (6) to approximate the true density contrast. Figure 7 presents plots of the USGS staircase density variation model and our approximation by the exponential function. The results of the inversion are shown in Figure 8 overlapped with the DEM (digital elevation model) and the fault structure. One





DOI http://dx.doi.org/10.1190/segam2014-0251.1

can see that the northwest-southeast trending faults correspond well to the edge of the Surprise Spring and Deadman Lake Basins. The east edge of the recovered Deadman Lake Basin fits well with the mountain belt.

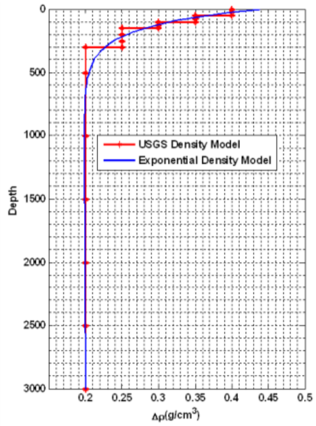


Figure 7: Approximation of the USGS stairwise density variation model by exponential function.

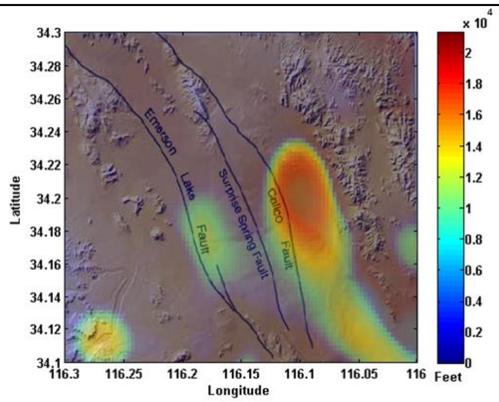


Figure 8: Results of the inversion of the gravity data for the Deadman Lake Basin and Surprise Spring Basin overlapped with the surface geology.

We should note that the recovered basin geometry obtained by our method correlates well with the USGS model. However, the locations of the maximum depth are different. USGS' report mentioned that the recovered depth of the basement for the Deadman Lake basin in their inversion may be underestimated due to the absence of the well's data.

*Inversion of the Joshua Tree Basin*

For the inversion of the gravity data in Joshua Tree Basin, the USGS used several different density models. They found that a constant density contrast of 0.55g/cm3 was a good approximation of the true density distribution. We use the same value in our inversion.

Figure 9 shows our inversion results overlapped with the DEM and fault structure. One can see that the edges of the inverse gravity model of the basin correspond well to the Pinto Mountain belt. The recovered depth is close to zero on the bedrocks. We should note that, the recovered location of the basin is very similar to the USGS model.
The maximum depth determined by our inversion is also in a good agreement with the USGS model.

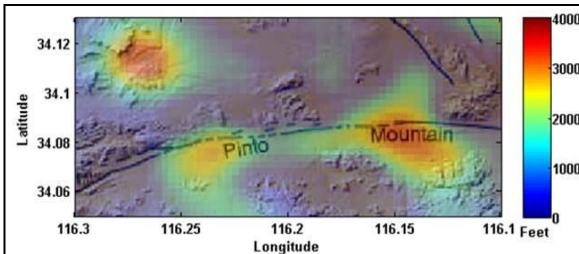


Figure 9: Results of the inversion of the gravity data for the Joshua Tree Basin overlapped with the DEM map.

## Conclusions

We have developed a new method for modeling gravity data caused by a sediment-basement interface with a variable density contrast distribution in the vertical direction. Our method is based on the Cauchy-type integral approach, which reduces the volume integration to the surface integration. We validated our forward modeling algorithm for linear and exponential density contrast distributions with depth by comparing our result with conventional prism-based modeling. We have also developed an inversion algorithm to recover the depth to basement for the models with variable density contrast with depth. The method was used for inversion of the field data collected by the USGS in the Big Bear Lake area.

The results show that, using surface Cauchy-type integrals reduces the computational expenses significantly in comparison with the conventional volume integral methods. The developed approach to interpretation of gravity data makes it practical to invert gravity data on a large scale while using very fine discretization of the sediment-basement interface.

## Acknowledgements

The authors acknowledge the support of the University of Utah's Consortium for Electromagnetic Modeling and Inversion (CEMI) and TechnoImaging. We are thankful to the USGS for releasing the field gravity data.



DOI http://dx.doi.org/10.1190/segam2014-0251.1

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**EDITED REFERENCES**

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