Magnetotelluric inversion for depth-to-basement estimation

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Summary

The magnetotelluric (MT) method can be effectively applied for depth-to-basement estimation, because there exists a strong contrast in resistivity between a conductive sedimentary basin and a resistive crystalline basement. Conventional inversions of MT data are usually aimed at determining the volumetric distribution of the conductivity within the inversion domain. By the nature of the MT method, the recovered distribution of the subsurface conductivity is typically diffusive, which makes it difficult to select the sediment-basement interface. This paper develops a novel approach to 3D MT inversion for the depth-to-basement estimate. The key to this approach is selection of the model parameterization with the depth to basement being the major unknown parameter. In order to estimate the depth to the basement, the inversion algorithm recovers both the thickness and the conductivities of the sedimentary basin. The forward modeling is based on the contraction integral equation approach. The inverse problem is solved using a regularized conjugate gradient method. The Fréchet derivative matrix is calculated based on quasi-Born approximation. The developed method and the algorithm for MT inversion for the depth-to-basement estimate are illustrated on several realistic geoelectrical models.

Introduction

There is a strong interest in developing effective geophysical methods for depth-to-basement estimation. It is well known that seismic imaging is characterized by the highest resolution of the subsurface structures. However, in the case of complex near-surface heterogeneity (e.g., shallow, high-velocity, highly heterogeneous basalt sills), typical for many frontier exploration regions, interpretation of seismic data represents a significant challenge, while using 3D seismic surveys is very expensive. These circumstances stimulated growing interest in using nonseismic geophysical methods, which could provide reasonable resolution but with lower cost (Tournerie and Chouteau, 2005).

Among the passive-source geophysical methods, potential field surveys have been widely used to estimate the depth to basement for decades (e.g., Barbosa et al., 1997; Gallardo-Delgado et al., 2003; Martins et al., 2010; Silva et al., 2001; Cai and Zhdanov, 2015a, b). Modern approaches to solving this problem are mostly based on the 3D inversion of gravity and magnetic data to recover the thickness of the columns, which are used to discretize the sedimentary basin. In the inversion, the horizontal dimensions of the columns are fixed and the column thickness is updated to fit the observed data. The low resolution of potential field inversion in this application can be compensated by joint inversion with seismic refraction data collected at some sparsely distributed receivers, with minor extra cost.

It is well known that electromagnetic (EM) data can provide higher resolution for subsurface formation than the gravity and magnetic data due to the frequency dependence of the EM field and the depth of investigation (Zhdanov, 2009). The MT method also provides an effective approach for sedimentary basin analysis such as depth-to-basement estimation based on the conductivity contrast between the sediments and bedrocks (Zevallos et al., 2004; Tournerie and Chouteau, 2005). Conventional inversions of the MT data are usually aimed at determining the volumetric distribution of the conductivity within the inversion domain (Berdichevsky and Dmitriev, 2008; Zhdanov, 2002, 2009). By the nature of the MT method, the recovered distribution of the subsurface conductivity is typically diffusive, although it can be focused by adopting more advanced regularization schemes such as focusing stabilizers (Zhdanov, 2002).

In the problem of depth-to-basement estimation using geophysical data, the goal is to recover a sharp boundary between a sedimentary basin and a crystalline basement. Therefore, we need to adopt a sharp boundary parametrization of the subsurface for the inversion.

In this paper, we suggest using a column parameterization for the MT inversion, similar to the discretization used in Gallardo-Delgado et al. (2003) for potential field inversion. For simplicity, it is assumed that the subsurface comprises a conductive layer of sediments and a resistive bedrock foundation. The interface between the sediments and the bedrock has an arbitrary shape. The sediment packs are discretized into a grid of columns with known horizontal dimensions. The MT response of the geoelectrical model is computed using the integral equation (IE) method. We demonstrate that in the inversion one can calculate the Fréchet derivatives of the data with respect to the columns' thickness and the sediment's conductivity using the quasi-Born approximation. A realistic model study shows that the developed method can be used for fast and accurate estimation of the depth to basement using MT data.

Principles of inversion of MT data for the depth-to-basement estimate using the integral equation method

In this paper, 3D modeling of MT data is based on the integral equation (IE) method. We use the parallelized contraction IE algorithm (Zhdanov et al., 2006), which is capable of modeling large geoelectrical structures. In the framework of the IE method, the anomalous EM field can be expressed as an integral of the excess currents within the anomalous domain as follows:

$$\mathbf{E}^a(\mathbf{r}_j) = \iiint_D \widehat{\mathbf{G}}_E(\mathbf{r}_j|\mathbf{r})\Delta\sigma(\mathbf{r}) \cdot [\mathbf{E}^b(\mathbf{r}) + \mathbf{E}^a(\mathbf{r})]\, dv, \quad (1)$$

$$\mathbf{H}^a(\mathbf{r}_j) = \iiint_D \widehat{\mathbf{G}}_H(\mathbf{r}_j|\mathbf{r})\Delta\sigma(\mathbf{r}) \cdot [\mathbf{E}^b(\mathbf{r}) + \mathbf{E}^a(\mathbf{r})]\, dv, \quad (2)$$

where $\widehat{\mathbf{G}}_E$ and $\widehat{\mathbf{G}}_H$ are the electric and magnetic Green's tensors defined for a medium with the background conductivity, σ_b. The components of the MT impedance tensor are then computed using the known linear relationships between the horizontal components of the electric and magnetic fields (Zhdanov and Keller, 1994; Berdichevsky and Dmitriev, 2008).

In order to formulate a corresponding inverse MT problem, we consider a model of the sedimentary basin shown in Figure 1. The basement has the background conductivity σ_b, and domain D represents the conductive sediments. We assume for simplicity that the sediments have a uniform conductivity of σ_s; however, in a general case, the method can be extended to the case of an arbitrary distribution of the conductivity,

$$\sigma_s(\mathbf{r}) = \sigma_b(\mathbf{r}) + \Delta\sigma(\mathbf{r}).$$

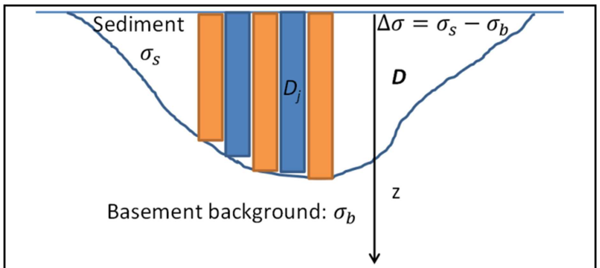

Figure 1: A sketch of a conductivity model of the sedimentary basin. Domain D represents the conductive sediments with conductivity σ_s, which is discretized into a grid of vertical columns.

In the inversion, domain D is discretized into N columns, denoted as subdomains D_j, with conductivity σ_s. The horizontal dimension of each subdomain is known and fixed. Contrary to the conventional MT inversion, which recovers a volumetric distribution of the subsurface conductivities, the goal is to find the depth of each column. If the conductivity of the sediments is unknown, the inversion can also recover $\sigma_s(\mathbf{r})$ jointly with the depth-to-basement estimate. We should note that, for IE forward modeling, the columns should be further discretized in the vertical direction.

The inversion of MT data is an ill-posed problem. In order to obtain a stable and geologically reasonable result, one has to apply regularization to impose some restrictions on the solution. The regularized inversion is based on the minimization of the Tikhonov parametric functional (Tikhonov and Arsenin, 1977):

$$P^a(\mathbf{m}, \mathbf{d}) = (\mathbf{W}_d\mathbf{A}(\mathbf{m}) - \mathbf{W}_d\mathbf{d})^*(\mathbf{W}_d\mathbf{A}(\mathbf{m}) - \mathbf{W}_d\mathbf{d}) + (\mathbf{W}_m\mathbf{m} - \mathbf{W}_m\mathbf{m}_{apr})^T(\mathbf{W}_m\mathbf{m} - \mathbf{W}_m\mathbf{m}_{apr}) \quad (3)$$

where A is the forward modeling operator for the MT impedance data; $\mathbf{d}$ is the vector of the observed data; $\mathbf{W}_d$ is a diagonal data weighting matrix; $\mathbf{m}$ is the vector of the model parameters; and "*" is the symbol of complex conjugate transposition.

A diagonal matrix of the model parameters weights, $\mathbf{W}_m$, is calculated based on the integrated sensitivity as follows (Zhdanov, 2009):

$$\mathbf{W}_m = diag(\boldsymbol{F}^T\boldsymbol{F})^{1/2}, \quad (4)$$

where $\mathbf{F}$ is the Fréchet derivative matrix.

One of the most expensive part of the inversion is the computation of the Fréchet derivative of the observed data with respect to the thickness of the sediments The simplest way of solving this problem is using the quasi-Born approximation (Zhdanov, 2009), which provides a reasonable estimation of the Fréchet derivative for the depth-to-basement inversion. Indeed, the anomalous field in the receiver's positions, $\mathbf{r}_i$, can be calculated according to formula (1) as follows:

$$\mathbf{E}^a(\mathbf{r}_i) = \sum_{j=1}^{N} \mathbf{E}_j^a(\mathbf{r}_i), \quad (5)$$

where $\mathbf{E}_j^a(\mathbf{r}_i)$ represents the anomalous field at receiver $\mathbf{r}_i$ contributed from the j^{th} column, D_j, of the sediments' pack, which can be written explicitly as a combination of the surface integral over the horizontal section of the j^{th} column, S_j, and a linear integral along the vertical coordinate from the surface, z=0, down to the bottom of the j^{th} column, $z = z_j$:

$$\mathbf{E}_j^a(\mathbf{r}_i) = \int_0^{z_j} \left\{ \iint_{S_j} \widehat{\mathbf{G}}_E(\mathbf{r}_i|(x,y,z)) \cdot [\Delta\sigma(\mathbf{r})\mathbf{E}(x,y,z)]\, dxdy \right\} dz. \quad (6)$$

The Fréchet derivative of the anomalous field to the thickness, z_j, of the j^{th} column can be calculated considering the variation, $\delta\mathbf{E}^a(\mathbf{r}_i)$, of the anomalous electric field with respect to variations, δz_j, of the depth of the j^{th} column, as follows:

$$F_{ij} = \frac{\delta\mathbf{E}^a(\mathbf{r}_i)}{\delta z_j} = \frac{\delta \sum_{j=1}^{N} \mathbf{E}_j^a(\mathbf{r}_i)}{\delta z_j} = \frac{\delta\mathbf{E}_j^a(\mathbf{r}_i)}{\delta z_j}. \quad (7)$$

Taking into account equation (6) and using the concept of quasi-Born approximation (Zhdanov, 2009), the Fréchet derivative in equation (7) can be reduced to the following expression:

$$F_{ij} = \iint_{S_j} \widehat{\mathbf{G}}_E(\mathbf{r}_i|(x,y,z)) \cdot [\Delta\sigma\mathbf{E}(x,y,z)]\, dxdy. \quad (8)$$

Expression (8) requires knowledge of the total electric field, $\mathbf{E}(x,y,z)$ within the anomalous domain. On the first iteration of the inversion, we may substitute the background electric field, $\mathbf{E}^b(x,y,z_j)$, for the total electric field, just arriving at the the conventional Born approximation for the Fréchet derivative calculation. However, on iteration number n, following Zhdanov (2009), one can use a quasi-Born approximation, which is based on substituting the total electric field, $\mathbf{E}^{(n)}$, computed for the current iteration, for the unknown total electric field, $\mathbf{E}$, in formula (8), as follows:

$$F_{ij}^{(n)} = \iint_{S_j} \widehat{\mathbf{G}}_E\left(\mathbf{r}_i \middle| (x,y,z_j)\right) \cdot \left[\Delta\sigma \mathbf{E}^{(n)}(x,y,z_j)\right] dxdy. \quad (9)$$

The advantage of using the quasi-Born approximation is that it provides an explicit expression for the Fréchet derivative, which is at the same time very accurate due to the presence of the total electric field, $\mathbf{E}^{(n)}$, estimated at the current iteration, in formula (9). The surface integrals in formula (9) are computed numerically with high accuracy using a fine discretization of the column in the x and y directions.

The developed theory and method have been implemented in the computer code that was tested on several synthetic models, discussed below.

Model studies

In this section, we will illustrate our inversion algorithm using a realistic synthetic model of the sediment-basement interface. In the model study, we consider the inversion for both the depth to the basement and the conductivity of the sediments. We should note, however, that in practical applications, one should apply a conventional 3D inversion of the MT data first in order to determine the volumetric distribution of the conductivity in the subsurface. The inverse model produced by the conventional MT inversion can be used to create the initial model for the depth-to basement estimate using the developed novel algorithm.

Model 1 represents a sediment-basement interface (Figure 2) with a maximum depth of 600 m. The conductivity of the basement is 0.001 S/m, while the conductivity of the sediments is 0.05 S/m. Figure 3 shows a vertical cross section of the conductivity distribution for this model.

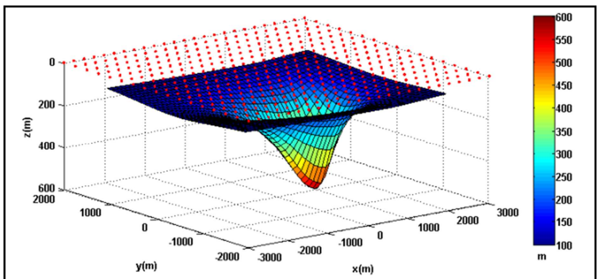

Figure 2: Model 1 of the sediment-basement interface with asymmetric shape. The MT stations' locations are shown by red dots.

We used 9 frequencies uniformly distributed from 0.01 Hz to 100 Hz in logarithmic space, and the data were contaminated by 5% random noise.

We have first applied the conventional MT inversion to recover a volumetric conductivity distribution based on integral equation method. Figure 4 shows the vertical section of the conventional MT inversion result at y=0. From this figure, we can see that the shape of the sedimentary basin and the sediment conductivity is roughly recovered. However, it is hard to determine the sediment-basement interface since the conductivity distribution recovered from conventional MT inversion is very diffusive. We have approximately determined the sediment-basement interface from the conventional MT inversion. We use this interface and the estimated sediment conductivity (0.03 S/m) from conventional MT inversion as the initial model for the depth to basement inversion.

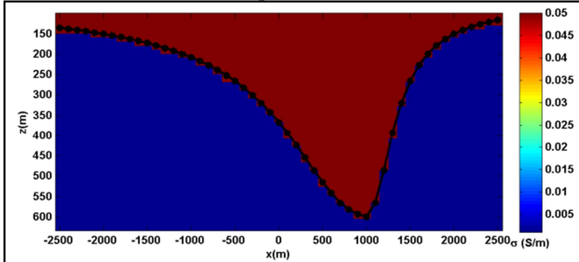

Figure 3: A vertical cross section at y=0 of Model 1 of the sediment-basement interface with asymmetric shape. The black line indicates the actual sediment-basement interface, while the prismatic approximation of the interface is shown by the dark red color, reflecting the conductivity of the sediments of 0.05 S/m on the corresponding color scale.

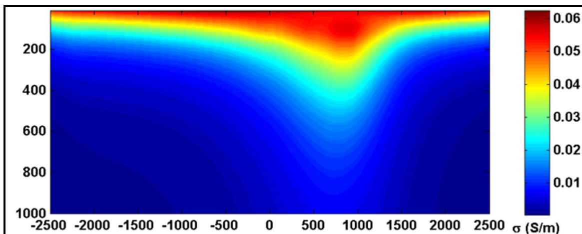

Figure 4: Model 1. Convential MT inversion result at y=0.

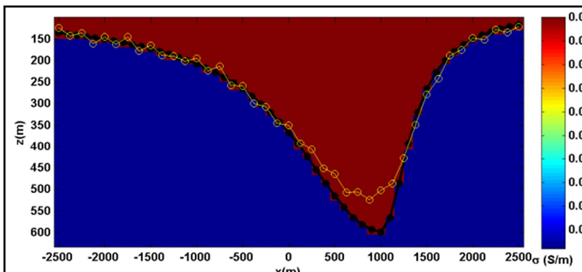

Figure 5: Model 1. Inversion with unknown conductivity of the sediments and with conventional 3D MT inversion as initial model: a vertical section of the inversion result at y=0 with yellow circles representing the recovered model, and the black stars show the true sediment-basement interface.

Figure 5 shows a vertical section of the inversion result with yellow circles representing the recovered model and black stars indicating the true sediment-basement interface. One can see that recovered sediment-basement interface is very close to the true model. The inverted sediment conductivity converged to the value of 0.0454 S/m which was very close to the true value of 0.05 S/m.

Inversion of the MT data for a USGS basin model

In this section we will consider the inversion of MT data computer simulated for the USGS basin model (Big Bear Lake basin). The Big Bear Lake area is located in the southeast part of California. It is characterized by deep sediment basins surrounded by uplifted bedrocks. The basin was well-studied by using collected gravity anomaly data (Roberts et al., 2002). Cai and Zhdanov (2015b) also inverted the depth to basement in this area using the released gravity data. However, the MT data were not available in this area.

We computer simulated the synthetic MT data at 441 MT stations located on a rectangular grid at 9 frequencies ranging from 0.01 Hz to 100 Hz using the basin model that was produced by the gravity inversion (Cai and Zhdanov, 2015b). A 5% random noise was added to the synthetic data as well. In the USGS basin model the conductivities of the basement and sediments were selected as 0.001 S/m and 0.05 S/m, respectively. The inversion was done for the depth-to-basement estimate only, considering that the conductivities of sediment and basement were well known based on other geophysical data (e.g., resistivity logging). The inversion process was terminated after 23 iterations, when the misfit between the observed and predicted data reached the noise level.

Figure 6 shows a comparison of the maps of the true model and the inversion result. One can see that the geometry of the USGS basin model was reconstructed very well. The recovered maximum depth of the basin was 862 m, which was very close to the actual maximum depth of 850 m. Figure 7 presents the vertical sections of the inversion results along two profiles, y=-1300 m and y=200 m, shown by the dashed white lines in Figure 7. One can see that the inversion did a good job in determining the correct interface between the sedimentary basin and basement.

Conclusions

We have developed a novel approach to the inversion of the MT data for the depth to the basement. The key component of this approach is selection of the model parameterization with the depth to the basement being the major unknown parameter. An effective and accurate method of computing the Fréchet derivatives with respect to the depth to the basement has been introduced based on the quasi-Born approximation of the anomalous EM fields.

The developed method and computer code were tested using several typical sedimentary basin models. The numerical studies have also demonstrated that the MT inversion can simultaneously recover both the thickness of the sedimentary basin and its conductivity.

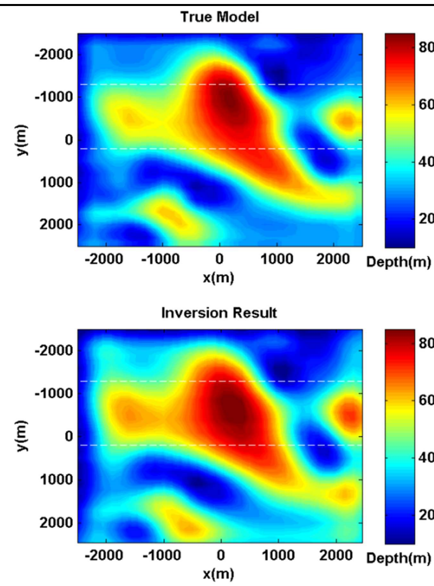

Figure 6: Maps of the true model of the sediment-basin interface (top panel) and of the inversion result (bottom panel) for the USGS model. The dashed white lines show two selected profiles at y=-1300 m and y=200 m, respectively.

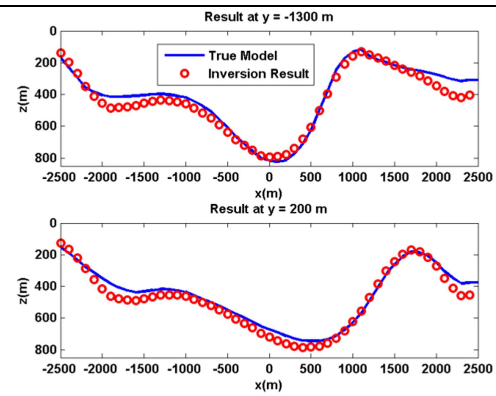

Figure 7: A comparison of the true interface and inversion result for the USGS model at the profiles y=-1300 m and y=200 m. The blue curve shows the true model, while the red circles represent the inversion result.

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