Effective-medium modeling of the induced polarization effect in multiphase artificial mineral rocks

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Summary

This paper demonstrates that the generalized effective-medium theory of induced polarization (GEMTIP) can correctly represent the induced polarization (IP) phenomenon in the artificial rock samples. These samples were manufactured using pyrite and magnetite particles. The results of our study show that the conventional Cole-Cole model cannot adequately describe the IP effect in artificial rocks containing both the pyrite and magnetite. However, the GEMTIP model not only predicted the IP response correctly, but it also opens the possibility of discriminating between rock samples containing pyrite and magnetite, based on complex resistivity (CR) data. Based on the GEMTIP inversion results for a total of 35 artificial rock samples, we demonstrate that the GEMTIP model best represents the CR response of the artificial rock samples.

Introduction

The induced polarization (IP) method has been widely used in the exploration of sulfide minerals such as porphyry copper, and Carlin-style gold deposits because these minerals are characterized by strong IP effects. However, in the presence of magnetite, which also has an IP effect, it is difficult to distinguish the sulfide minerals from the magnetite in the target rocks. In order to address the problem of discriminating between the sulfide minerals and magnetite using the IP method, Takakura et al. (2014) studied the complex resistivity (CR) response of the artificial samples containing both the pyrite and magnetite particles using the Cole-Cole model (Cole and Cole, 1941). The authors of the cited paper demonstrated that, for two-phase artificial rocks, containing separately pyrite or magnetite particles mixed with glass beads and a 0.01M KCl solution, the Cole-Cole model provided a reasonable representation of the observed CR spectra. However, for a three-phase artificial rock sample, containing particles of both minerals mixed with glass beads, they were unable to find the Cole-Cole model, which would represent the observed SIP data.

In this paper we demonstrate that the generalized effective-medium theory of induced polarization (GEMTIP) can correctly represent the induced polarization (IP) phenomenon in multiphase artificial rock samples. The GEMTIP model is a rigorously formulated CR model, which was developed to characterize the complex resistivity of multiphase heterogeneous rocks and their petrophysical and structural properties, including grain size, grain shape, porosity, anisotropy, polarizability, volume fraction, and conductivity of the inclusions in the pore space (Zhdanov, 2008; Zhdanov et al., 2009). This paper uses the GEMTIP model as a basis for determining the intrinsic characteristics of the two-phase (pyrite or magnetite) and three-phase (pyrite and magnetite) artificial rock samples from the observed CR data. With the GEMTIP model, we analyzed the SIP responses of 35 artificial rock samples manufactured by Takakura et al. (2014) using pyrite and magnetite particles mixed with glass beads and a 0.01M KCl solution.

In order to invert the CR data for the GEMTIP model parameters, we have applied the hybrid method based on a genetic algorithm with simulated annealing and the regularized conjugate gradient method (SAAGA-CG). The results of this study demonstrate that the generalized effective-medium theory of induced polarization (GEMTIP) can correctly represent the IP phenomenon in the artificial rock samples, both for two-phase and three-phase artificial rocks. We show also that, using the GEMTIP model, it is possible to discriminate between the rock samples containing sulfide minerals and magnetite using the CR data. This result opens the possibility of applying the field IP method to discriminate between economic and noneconomic rocks.

Cole-Cole vs. GEMTIP models

The Cole-Cole model (Cole and Cole, 1941) is an empirical model, which is widely used to represent the complex resistivity of the polarized rock formations (Pelton et al., 1978). The frequency-dependent complex resistivity can be described by the following expression:

$$\rho(\omega) = \rho_0\{1 - m\left[1 - \frac{1}{1 + (i\omega\tau)^C}\right]\}, \qquad (1)$$

where ρ_0 is the matrix resistivity (Ohm-m); ω is the angular frequency; τ is the time constant; m is the intrinsic chargeability; and C is the relaxation parameter.

The three-phased GEMTIP model, developed by Zhdanov (2008), in a case of ellipsoidal inclusions representing different minerals, can be described by the following formula:

$$\rho_e = \rho_0\{1 + \\ + \sum_{l=1}^{2} \sum_{\alpha=x,y,z} \frac{f_l}{3\gamma_{l\alpha}}\left[1 - \frac{1}{1 + (i\omega\tau_l)^{C_l} \frac{\gamma_{l\alpha}}{2\overline{a_l}\lambda_{l\alpha}}}\right]\}^{-1}, \qquad (2)$$

where ρ_0 is the matrix resistivity, e is the ellipticity of the grains, and f is their volume fraction. Parameters τ and C are similar to the Cole-Cole model and represent the time constant and relaxation parameter, respectively. The constants $\bar{a}_l$ are the average values of the equatorial (a_{lx} and a_{ly}) and polar (a_{lz}) radii of the ellipsoidal grains. The coefficients $\gamma_{l\alpha}$ and $\lambda_{l\alpha}$ are the structural parameters defined by the geometrical characteristics of the ellipsoidal inclusions.

Artificial rock samples

The artificial rock samples assembled by Takakura et al. (2014) were composed of mineral grains (pyrite or magnetite particles), glass beads, and a 0.01 mol/L KCL solution. Six different two-phase sample sets were prepared. Each sample set contained six different weighted concentrations of minerals, either pyrite or magnetite. The weighted concentrations for the first five sample sets were 1%, 3%, 5%, 10%, 15% and 20%, respectively. In sample set #4 the weighted concentrations were 1%, 3%, 5% and 10%. The different sample sets varied by (1) mineral, either pyrite or magnetite; (2) the size of the particles; and (3) the size of the glass beads. For example, the first sample set contained 6 different weighted concentrations of pyrite (1%, 3%, 5%, 10%, 15% and 20%) in glass beads. The size of pyrite particles was 1.4-2 mm and the size of glass beads was 1 mm. In the second sample set the concentration range and the mineral type and the size of the glass beads were the same as in the first sample set, but the size of the pyrite particles was 0.7-1 mm. The third sample set contained samples with the same characteristics as the first sample set, with the exception of the size of the glass beads, which was 0.05 mm. The fourth sample set contained samples with the same characteristics as the third sample set, with the exception of the size of the pyrite particles, which was in the 0.5-0.7 mm range. This set contained only four concentrations of pyrite (1%, 3%, 5% and 10%) instead of six, as in the all other samples. Overall, 34 individual two-phase samples were prepared, and their CR spectra were measured and modeled by the three-phase GEMTIP model, as will be explained below.

In addition to the two-phase samples, we tested one three-phase sample. This sample contained 20 weighted % of magnetite and 10 weighted % of pyrite mixed with glass beads. All mineral particles and glass beads had the same size, between 1.4 mm and 2 mm, and this mixture was saturated by a 0.01M solution of KCl, similar to all of the two-phase samples.

According to Takakura et al. (2014), 1% content weight corresponded to 4 g in mass, and the volume of the artificial rock was 192 cm^3. Considering that the densities of the pyrite and magnetite are 5 g/cm^3 and 5.15 g/cm^3, respectively, we calculated that the volume fractions (content volume) for 1% (content weight) pyrite and magnetite in the rock samples are 0.42% and 0.40%, respectively.

The 1260 Impedance/Gain-Phase Analyzer with ZPlot® Impedance Spectroscopy software was employed for the CR measurement in the frequency range of 0.01 to 1 MHz, five frequencies per decade, and Vpp=100 mV. The methodology of the CR measurement, used in this project, was originally developed to study the IP effect in clay minerals and described in Takakura et al., (2014).

GEMTIP analysis of CR spectra

The observed CR data were inverted for the GEMTIP model parameters using a hybrid method based on a genetic algorithm with simulated annealing and the regularized conjugate gradient method (SAAGA-CG) (Lin et al., 2015). The GEMTIP inverse problem can be formulated as the following operator equation:

$$\mathbf{d} = \mathbf{A}(\mathbf{m}), \tag{3}$$

where **A** is a forward modeling operator described by equation (2), and **d** is the vector of the observed data (the values of the complex resistivity as a function of the frequency):

$$\mathbf{d} = [\rho_e(\omega_1), \rho_e(\omega_2), \ldots, \rho_e(\omega_n)]. \tag{4}$$

Vector **m** represents the unknown model parameters, defined above in equation (3):

$$\mathbf{m} = [\rho_0, e_1, \tau_1, C_1, f_1, e_2, \tau_2, C_2, f_2], \tag{5}$$

In order to find the parameters of the GEMTIP model, we need to solve equation (3) with respect to **m**. A conventional way of solving this problem is based on substituting the following minimization problem for inverse problem (3):

$$P^{\alpha}(\mathbf{m}) = \|\mathbf{d} - A(\mathbf{m})\|^2 + \alpha\left\|\mathbf{W}_m\mathbf{m} - \mathbf{W}_m\mathbf{m}_{apr}\right\|^2 = \min, \tag{6}$$

where α is a regularization parameter, $\mathbf{W}_m$ is the weighting matrix of the model parameters, and $\mathbf{m}_{apr}$ is some a priori model selected based on all available rock physics data for the rock sample under consideration (Zhdanov, 2002).

There are different methods available for solving minimization problem (6). In our study, we have used a new hybrid method based on a genetic algorithm with simulated annealing and the regularized conjugate gradient method (SAAGA-CG), introduced by Lin et al (2015).

Complex resistivity of two-phase artificial mineral rocks

We ran the inversion using the two-phase GEMTIP model, where one phase was represented by either pyrite or magnetite, and another phase was represented by the glass beads. Figure 1 shows a comparison of the complex resistivity spectra (dots) of the artificial mineral rocks composed of pyrite particles, (panels (a) and (b)), and magnetite particles (panels (c) and (d)). The pyrite and magnetite particles have are of the same size, 1.4 - 2 mm, and the glass beads are 1 mm in size. The six CR spectra correspond to the six mixing concentrations, 1%, 3%, 5%, 10%, 15% and 20%. The theoretical CR curves based on the GEMTIP model are shown by the solid lines for all experimental CR data. Panels (a) and (c) show the real parts of the complex resistivity; panels (b) and (d) present the imaginary parts of the complex resistivity.

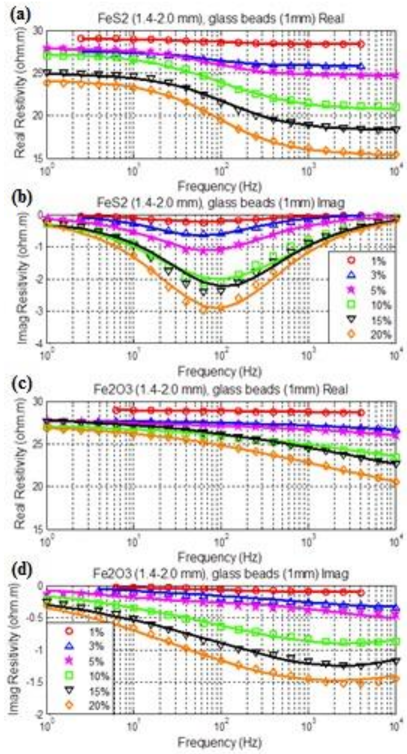

Figure 1: Observed and predicted CR spectra for artificial mineral rocks with pyrite (panels a and b) and magnetite (panels c and d) particles. The pyrite and magnetite particles are the same size, 1.4 - 2 mm. The plots present the real (panels a and c) and imaginary (panels b and d) resistivities for six different mixing concentrations, 1%, 3%, 5%, 10%, 15% and 20% , as functions of the frequency. The solid lines show the theoretical predicted CR curves based on the GEMTIP models.

Figure 2 also compares the complex resistivity spectra (dots) of the artificial mineral rocks composed of pyrite particles, (panels (a) and (b)) and magnetite particles (panels (c) and (d)). However, the pyrite and magnetite particles are smaller, 0.7-1 mm, than in the previous case, and the glass beads have the same size of 1 mm. Similar to Figure 3, the six datasets of each spectrum corresponds to the six mixing concentrations, 1%, 3%, 5%, 10%, 15% and 20%. The theoretical CR curves based on the GEMTIP model are shown by the solid lines for all experimental CR data. Panels (a) and (c) show the real parts of the complex resistivity; panels (b) and (d) present the imaginary parts of the complex resistivity.

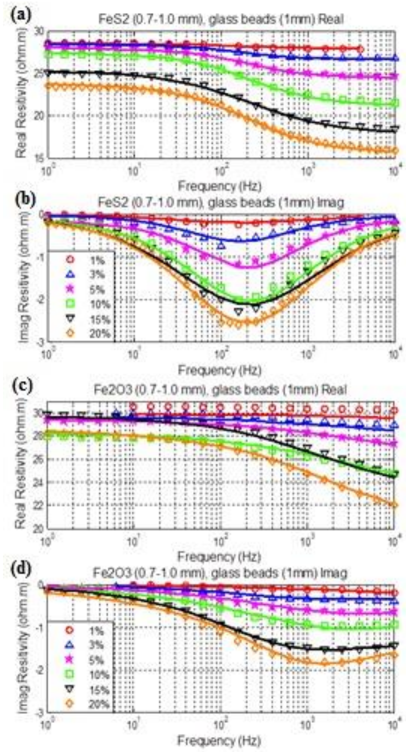

Figure 2: Observed and predicted CR spectra for artificial mineral rocks with pyrite (panels a and b) and magnetite (panels c and d) particles. The pyrite and magnetite particles are the same size, 0.7 - 1 mm. The plots present the real (panels a and c) and imaginary (panels b and d) resistivities for six different mixing concentrations, 1%, 3%, 5%, 10%, 15% and 20% , as functions of the frequency. The solid lines show the theoretical predicted CR curves based on the GEMTIP models.

The inversion results indicate that the predicted curves fit the observed data very well for all cases. In order to find the difference between the pertrophysical properties of pyrite and magnetite, we analyzed the content dependencies of the time constant (τ), the relaxation parameter (C), and the matrix resistivity (ρ_0) for the different types of artificial rock samples. We have found that, for the pyrite particles, the time constants are in the range of 4×10^{-4} to 5×10^{-3}s, while for magnetite the range is of 5×10^{-6} to 1×10^{-4}s. We have also found that, the size of the particles, and the content of the minerals are the major factors which affect the value of time constant. For the same size particle, the time constant for the pyrite sample is about 100 times larger than that for the magnetite sample.

In the case of the different sizes of pyrite, the artificial rock samples with the bigger particles tend to have a larger time constant. The time constant of the pyrite samples decreases with the content of the particles increases in the range of 0-15%, and for larger content the curve tends to be steady.

In the case of the magnetite samples, the time constant does not vary with the size. The difference in the time constant curves between the pyrite and magnetite samples may be due to the physical fact that pyrite is conductive, while magnetite behaves as a dielectric, so that the conductor (pyrite) requires more time to release electrons after the current cut-off during the measurement of the IP effect. Since the electrons do not attract magnetite, the time constant of the artificial rock sample is not affected by the content of the magnetite.

The analysis of recovered relaxation parameters (C) has demonstrated that, for the pyrite particles, the relaxation parameters are within the range of 0.7 to 0.9, while for magnetite the range is of 0.3 to 0.5. From the inversion results we have also found that the relaxation parameter of the pyrite samples is affected by the size of the mineral particles and the glass beads. The artificial rock samples with the bigger pyrite particles and smaller glass beads tend to have larger C. In the case of the magnetite samples, the recovered relaxation parameters of the smaller magnetite particles are larger than those of the bigger particles.

Complex resistivity of three-phase artificial mineral rocks

The last artificial rock sample (Set 7) contains 10% pyrite and 20% magnetite (in weight), mixed with glass beads, which represents a three-phase medium. Takakura et al. (2014) failed to recover the Cole-Cole parameters for this sample, because the conventional Cole-Cole model can represent two-phase rocks only.

At the same time, by using the GEMTIP model, we were able to successfully invert the CR data for this sample and determined the GEMTIP parameters for both the pyrite and magnetite. Figure 3 demonstrates that the predicted curves for both the real and imaginary resistivities represent the observed CR data well. One can see in Table 1 that the recovered time constants (τ_1, τ_2) and relaxation parameters (C_1, C_2) of the pyrite and magnetite are within the ranges listed above, which also shows that the GEMTIP model can be used to distinguish between pyrite and magnetite in rock samples by analyzing the CR data using the GEMTIP parameters.

Misfit(%)		0.3	
$\rho_0(\Omega \cdot m)$		22.98	
Mineral 1: Pyrite		Mineral 2: Magnetite	
e_1	3.64	e_2	4.22
$\tau_1(10^{-3}s)$	1.40	$\tau_2(10^{-6}s)$	8.01
C_1	0.80	C_2	0.46
$f_1(\%)$	4.16	$f_2(\%)$	8.08

Table 1: Inversion result for rock sample (Set 7) using the hybrid SAAGA and RCG method.

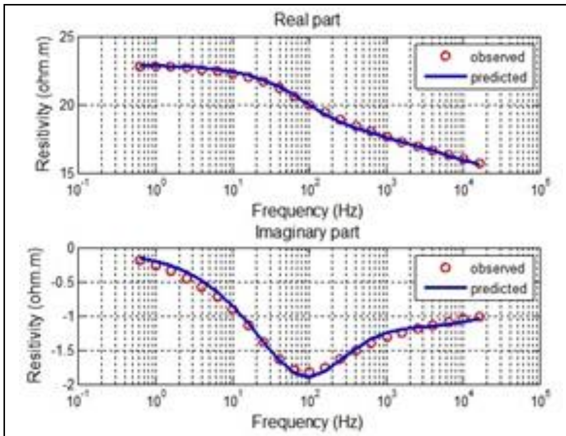

Figure 3: Plots of the observed and predicted CR spectrum for Set 7 (1.4-2.0 mm pyrite (10%) and magnetite (15%) with 1 mm glass beads).

Conclusion

We successfully applied the three-phase GEMTIP model to the artificial rock sample with pyrite and magnetite and inverted the CR data for the GEMTIP parameters. The inversion results indicate that the mineral type, the size of the particles, and the content of the minerals are the major factors that affected the time constant and relaxation parameters. Based on the inversion results of the CR data measured for the artificial rock samples, we determined that the approximate range of the time constant for magnetite is 5×10^{-6}, 1×10^{-4}, while the time constant of pyrite decreases from $5\times10^{-3}$to 4×10^{-4} with the increasing weight content of pyrite, and the ranges of the relaxation parameter for pyrite and magnetite are 0.7 - 0.9 and 0.3 - 0.5, respectively. Thus, it is possible to distinguish pyrite and magnetite from the observed CR data using the GEMTIP model.

In summary, based on the GEMTIP inversion results for a total of 35 artificial rock samples, we have demonstrated that the GEMTIP model best represents the CR response of the artificial rock samples. This model not only predicts the IP response correctly, but it also opens the possibility of discriminating between the rock samples, containing pyrite and magnetite, based on the complex resistivity (CR) data.

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EDITED REFERENCES

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