Finite element time domain electromagnetic modeling with IP effects using adaptive Padé series

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Summary

The induced-polarization (IP) method has been widely used in geophysical exploration. The correct interpretation of IP data requires techniques which can simulate IP responses caused by fully 3D dispersive conductivity structures. We developed an edge-based finite element time domain (FETD) method to simulate the electromagnetic fields in 3D dispersive medium. The vector Helmholtz equation for total electric field is solved using the edge based finite element method with unstructured tetrahedral mesh and the backward Euler method with adaptive time stepping. We use the direct solver based on LU decomposition to solve the system of equations. The Cole-Cole conductivity relaxation model in frequency domain is expanded using truncated Padé series. The Ohm's law with Cole-Cole model is transformed into time domain. By using Padé expansion, the fractional differential equation problem can be avoided. During time stepping, we select the center point and orders for Padé series expansion adaptively. The developed method was tested for several synthetic dispersive conductivity models to validate our algorithm.

Introduction

The time domain electromagnetic (TEM) methods has been widely used to delineate the subsurface conductivity (Ward and Hohmann, 1988). Comparing to the frequency domain method, the TEM method usually has better resolution to the deep target for typical survey configurations and broad time scales (Zaslavsky et al., 2011). Correct interpretation of field data requires arcuate tools to model the TEM response (Um et al., 2012). The common approach is based on the Fourier transform of the frequency domain response (Mulder et al., 2007; Ralph-Uwe et al., 2008). However, the accuracy of such transformation is affected by the frequency sampling and the transformation methods (Li, 2016).

One can also directly discretize the Maxwell's equation in time domain (Um et al., 2012). The finite difference time domain (FDTD) methods has been adopted for advancing the electromagnetic response in time domain (Yee, 1966; Wang and Hohmann, 1993; Commer and Newman, 2004). The TEM simulation requires a large computation domain to address the Boundary condition and the mesh needs to be refined nearby transmitters, receivers and the domain with abrupt conductivity variation (Zaslavsky et al., 2011). The size of problem for FDTD can be very large (Um et al., 2012) and the complex geometries can only be approximated by a stair-cased model (Um et al., 2012).

To overcome these problems, the FETD method has been introduced (Jin, 2014) and applied to large scale CSEM modeling (Um, 2011). The FETD method, with unstructured spatial discretization, can reduce the size of the problem dramatically (Um, 2011; Jin, 2014). We adopt the FETD scheme proposed by Um (2011) for solving TEM modeling problem. We also update the time step size adaptively, to reduce the computational cost (Um, 2011).

The conventional time domain electromagnetic (TEM) modeling considers non-dispersive conductivity. Pelton et al. (1978) studied the frequency-dependent conductivity which is manifested as induced polarization (IP) effect. Such effect has been well studied in frequency domain using Cole-Cole model (Luo and Zhang, 1998), which needs to be represented by the convolution of the electric field in time domain which is introduced into Maxwell's equations through the fractional time derivative (Zaslavsky et al., 2011). Solving such equations with fractional derivative term requires the electric field at all previous stage since either the convolution or the fractional derivative corresponds to a global operator. Due to this problem, the TEM data with IP effect are rarely modeled directly in time domain.

The Padé series (Baker, 1996) can be used to avoid the fractional derivative problem for modeling dispersive medium (Weedon and Rappaport, 1997). The fractional differential equation can be transformed to differential equation with integer order and further to be solved using numerical methods (Rekanos, 2010). Based on the work of Weedon and Rappaport (1997) and Rekanos (2010), Marchant et al. (2014) proposed a finite volume time domain method for simulating IP effect with Cole-Cole model. However, these methods use the Taylor expansion in the vicinity of one point to calculate the Padé coefficient during the time domain modeling. The corresponding Padé approximation is only accurate near the selected center point in the situation of broad dispersion.

We implemented the FETD modeling with IP effect using the Padé approximation. Instead of using a fixed-point Taylor expansion to calculate the Padé coefficient, we update the Padé coefficient adaptively. We keep the same Padé coefficient for each *n* steps and update the Padé series after these *n* time steps. For a given model, we use a halfspace model with the same dispersion behavior to calculate the time domain response at the receivers. We also approximate this halfspace Cole-Cole model using Padé series with different choice of expansion point and order. The optimized expansion point and order is adjusted until the Padé model produce closer result to this halfspace Cole-Cole model. We repeat this process for each time segment.

FETD discretization of electric field equation

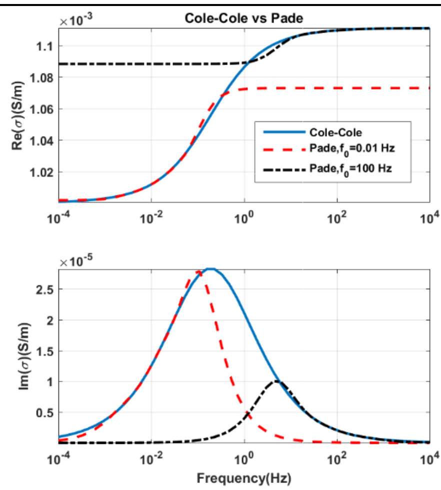

Figure 1: A comparison between the Cole-Cole conductivity spectrum and the Padé approximation with two different center frequencies of 0.01 Hz and 100 Hz.

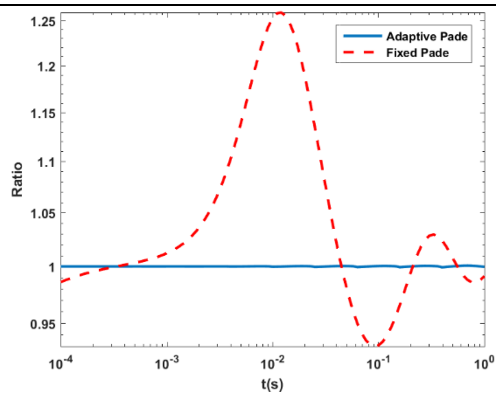

Figure 2: A comparison between the normlaized error for the Padé model with both fixed and adaptive center frequency

In a general 3D dispersive medium, the electric field $\mathbf{E}$ satisfies the following diffusion equation (Um et al., 2012):

$$\nabla \times \nabla \times \mathbf{E}(t) + \mu \frac{\partial \mathbf{j_e}(t)}{\partial t} = -\mu \frac{\partial \mathbf{J_s}(t)}{\partial t}, \quad (1)$$

where $\mathbf{J_s}$ is the current density of the source, and $\mathbf{j_e}$ is the induction current which can be related to the electric field by Ohm's law as follows (Ward and Hohmann, 1988; Zhdanov, 2009):

$$\mathbf{j_e} = \hat{\sigma}\mathbf{E}, \quad (2)$$

and $\hat{\sigma}$ is the electric conductivity tensor (for a general anisotropic medium). In the nondisperssive medium, the electric conductivity $\hat{\sigma}$ is time invariant.

We use $\mathbf{E}(t)$, $\mathbf{j_e}(t)$, and $\mathbf{J_s}(t)$ to emphasize that the electric field and current density are functions of time. We first consider a non-dispersive conductivity and (2) can be directly substituted into (1) to get the following equation:

$$\nabla \times \nabla \times \mathbf{E}(t) + \mu\hat{\sigma} \frac{\partial \mathbf{E}(t)}{\partial t} = -\mu \frac{\partial \mathbf{J_s}(t)}{\partial t}. \quad (3)$$

We adopt the Nédélec basis function (Nédélec, 1980; Jin, 2014) for unstructured tetrahedral mesh. The electric field inside the tetrahedral at any time t can be represented as a linear combination of the fields along the edge:

$$\mathbf{E}^e(t) = \sum_{i=1}^{6} \mathbf{N}_i^e E_i^e(t) \quad (4)$$

After applying edge-based finite element analysis to (3), we can get a system of equation as follows (Jin, 2002, 2014):

$$KE(t) + \mu\hat{\sigma}M \frac{\partial \mathbf{E}(t)}{\partial t} = -\mu M \frac{\partial \mathbf{J_s}(t)}{\partial t} \quad (5)$$

where the stiffness matrix K and M are defined as:

$$K_{ij}^e = \int_{\Omega_e} (\nabla \times \mathbf{N}_i^e) \cdot (\nabla \times \mathbf{N}_j^e)\, dv \quad (6)$$

$$M_{ij}^e = \int_{\Omega_e} \mathbf{N}_i^e \cdot \mathbf{N}_j^e\, dv, \quad (7)$$

We use the implicit backward Euler approximation, which is unconditionally stable for time stepping:

$$\frac{\partial \mathbf{E}(t)}{\partial t} \approx \frac{\mathbf{E}(t) - \mathbf{E}(t - \Delta t)}{\Delta t}. \quad (8)$$

After assembling, we can get a system of equation:

$$A\mathbf{E}(t) = \boldsymbol{b} \quad (9)$$

For the FETD modeling, we consider an impulse source waveform approximated by a Gaussian function. As a result, we can use a zero initial condition for (9). With this initial condition and the Dirichlet boundary condition which assumes the electric field vanishes on the boundary, equation (9) is ready to be solved using sparse LU decomposition. We use an adaptive time stepping method in such a manner that a fixed time step size is used for n steps and then it will be doubled if the accuracy can be guaranteed (Um, 2011). The finite element matrix only needs to be factorized once for the same time step size.

Modeling IP effects with adaptive Padé series

We consider the Cole-Cole model in frequency domain:

$$\rho(\omega) = \rho_0 \left(1 - \eta \left(1 - \frac{1}{1 + (i\omega\tau)^c}\right)\right) \quad (10)$$

where ρ_0 is the DC resistivity, η is the chargeability, τ is the time constant and c is the frequency dependence term with the value ranges from 0 to 1 (Pelton, 1977). The relaxation model is named as Debye model when c=1 (Marchant et al., 2014).

By subsisting (10) into (2), we can obtain the following equation in frequency domain:

$$\sigma_0 \mathbf{E}(\omega) + (i\omega)^c \tau^c \sigma_0 \mathbf{E}(\omega) = \mathbf{j_e}(\omega) + (i\omega)^c (1 - \eta)\tau^c \mathbf{j_e}(\omega) \quad (11)$$

here $\mathbf{E}(\omega)$ and $\mathbf{j_e}(\omega)$ are the electric field and induction current density in frequency domain, σ_0 represents the DC conductivity.

Following Marchant et al. (2014), we apply the inverse Fourier transform to (13) and get the following fractional differential equation (Miller and Ross, 1993; Meerschaert and Tadjernan, 2004):

$$\sigma_0 \mathbf{E}(t) + \tau\sigma_0 \frac{\partial^c \mathbf{E}(t)}{\partial t^c} = \mathbf{j_e}(t) + \tau(1-\eta)\frac{\partial^c \mathbf{j_e}(t)}{\partial t^c} \quad (12)$$

The Caputo fractional derivative with real order c is defined as follows for $n-1 \le c < n$ (Miller and Ross, 1993; Meerschaert and Tadjernan, 2004):

$$\frac{\partial^c f(t)}{\partial t^c} = \frac{1}{\Gamma(n-c)}\int_c^t \frac{f^{(n)}(s)ds}{(t-s)^{c-n+1}} \quad (13)$$

where Γ is the Gamma function.

Note that for the Debye dispersion with $c = 1$, (12) becomes an first order differential equation and can be solved together with (1) in a coupled manner using the described FETD method. For a general case with $c \neq 1$, we approximate the term $(i\omega)^c$ by the Padé series of order M (Marchant et al., 2014):

$$(i\omega)^c = \frac{P_0 + \sum_{m=1}^{M} P_m (i\omega)^m}{1 + \sum_{n=1}^{N} Q_n (i\omega)^m} \quad (14)$$

By subsisting (14) into (11), we can obtain the following ordinary differential equation for Ohm's law:

$$a_0\sigma_0\mathbf{E}(\omega) + \left[\sum_{m=1}^{M} a_m (i\omega)^m\right]\sigma_0\mathbf{E}(\omega) = b_0\mathbf{j_e}(\omega) + \left[\sum_{m=1}^{M} b_m (i\omega)^m\right]\mathbf{j_e}(\omega) \quad (15)$$

where we define:

$$a_0 = 1 + P_0\tau^c \,,\; a_m = Q_m + P_m\tau^c$$
$$b_0 = 1 + P_0(1-\eta)\tau^c \,,\; b_m = Q_m + P_m(1-\eta)\tau^c$$

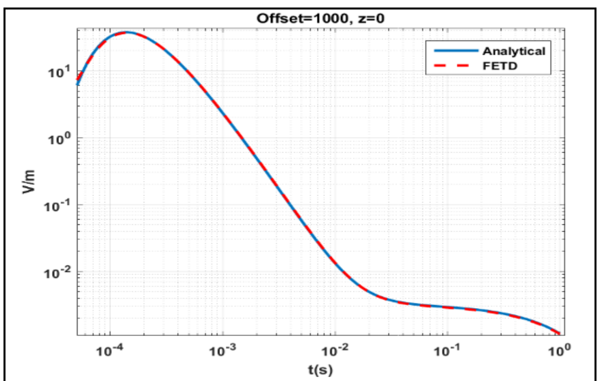

Figure 3: A comparison between the analytical solution and FETD solution for the halfspace model with Debye dispersion.

By applying inverse Fourier transform to (15) we can get the following differential equation with integer order in time domain (Marchant et al., 2014):

$$a_0\sigma_0\mathbf{E}(t) + \sum_{m=1}^{M}\left(a_m\sigma_0\frac{\partial^m \mathbf{E}(t)}{\partial t^m}\right) = b_0\mathbf{j_e}(t) + \sum_{m=1}^{M}\left(b_m\frac{\partial^m \mathbf{j_e}(t)}{\partial t^m}\right) \quad (16)$$

We use the higher order backward Euler method to approximate the time derivative term in (16).

As a result, the coupling between (3) and (16) can be solved using the FETD method to take into account the IP effects in TEM data for a general dispersion of $c \neq 1$.

However, the accuracy of the proposed method depends on how we can accurately approximate the Cole-Cole model with Padé series which is only guaranteed to be accurate in the vicinity of the center frequency of Padé expansion.

To illustrate this problem, we consider a Cole-Cole model with $\sigma_0 = 0.001\ S/m, \tau = 1\ s, \eta = 0.1, c = 0.6$. We use two center frequencies for the Padé expansion to approximate the Cole-Cole model. From Figure 1, we can see that low frequency part of the spectrum can be well fitted, with center frequency f_0=0.01 Hz, but the high frequency part shows a clear discrepancy, and vice versa.

We proposed a method to adaptively select the center frequency for Padé series. We gradually decrease the value of center frequency with time increase. The total observation time period is divided in to a series of time segment. For a model (can be 3D) with given Cole-Cole dispersion, the true time domain response of a halfspace model with the same/equivalent dispersion parameter will be calculated by cosine transform for each time segment. For each time segment, the response for the Padé model will be calculated for a series of trial center frequency. The optimal center frequency will be selected, for each time segment, based on the misfit between the Padé model and the halfspace response.

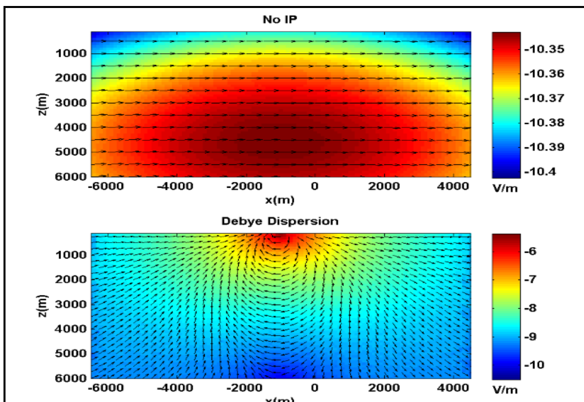

Figure 4: A comparison between TEM response for the non-dispersive halfspace model and halfspace Debye model at y=300m and t=0.22s. The arrow represents the direction of the total electric field on this vertical plane. The color scale is in logarithmic space.

To illustrate this approach, we consider a halfspace model with the model parameters described before for Figure 1. The EM field is excited by a horizontal electric ground wire with the moment of 10^5 Am. We first calculate the time domain response of the in-line E_x field at the offset of 1000 m using cosine transform for the actual Cole-Cole model. Then, we calculate the TEM response for the Padé model with both optimized fixed and adaptive center frequency. We normalize the solution for the Padé model (with both fixed and adaptive center frequency) by the actual Cole-Cole model solution. Figure 2 shows a comparison between the normalized error (value of 1 indicates perfect fitting) for the Padé model with both fixed and adaptive center frequency. We can clearly see that the modeling accuracy is improved significantly by the proposed adaptive method.

Model studies

We now consider a halfspace model with earth conductivity of 10^{-3} S/m. The EM field is excited by a *x* oriented ground wire with the center located at (-1000,0,0) m and a length of 10 m. We assume an impulse current, with the maximum value of 10^4 A, is injected into the ground through the wire. The E_x component will be recorded. The modeling domain is selected to be 80km×80km×80km based on the experience. The corresponding unstructured tetrahedral mesh contains 162,394 elements and 193,341 edges. For the Cole-Cole parameters, we set $\tau = 1\ s, \eta = 0.1$. Figure 3 shows the comparisons between FETD modeling results and the analytical solutions for the halfspace model with Debye dispersion (*c*=1) at the offset of 1000 m, on the earth's surface. We can see that the FETD results compare well to the analytical solutions. Figure 4 shows comparison between the FETD solution for the halfspace model with no IP and Debye dispersion at the vertical plane of y=300 m. We can clearly see that the time domain response is distorted significantly by the IP effect.

Finally, we consider a model with non-dispersive halfspace background and localized dispersive 3D anomaly with a dyke shape. The conductivity of the background is 0.01 S/m. For this model, we consider several different scenarios with and without IP (both Debye and general dispersion). For IP case, we set τ=0.1s, η=0.5.The dipping angle of the dyke is 45 degree. The dimension of the dyke in y direction is 200 m. Figure 5 shows the tetrahedral mesh of this model at the plane of y=0. The magenta color represents the location of the dyke. We consider a grounded wire source with the length of 10 m, located at (-1000, 0, 0) m. The wire carries the electric current of 10000 Amper with Gaussian impulse waveform. Figure 6 shows the decay of electric field E_x on the earth's surface, directly above the center of the dyke. For the general dispersion ($c \neq 1$), the optimized Padé series with third order and adaptive center frequency is selected using the proposed method.

Conclusions

We have developed an edge-based finite element time domain method for simulating electromagnetic fields in conductive and general dispersive mediums. We used the Padé series to approximate the Cole-Cole model for the conductivity dispersion. Using this method, the differential equation in the time domain with fractional derivative can be approximated by the differential equation with integer order. We proposed a method with the adaptive Padé series in such a manner that the center frequency for the Padé series expansion was updated with time stepping. Our algorithm automatically calculate the optimized center frequency at different time stages. We validated the accuracy and effectiveness of the developed algorithm based on several model studies.

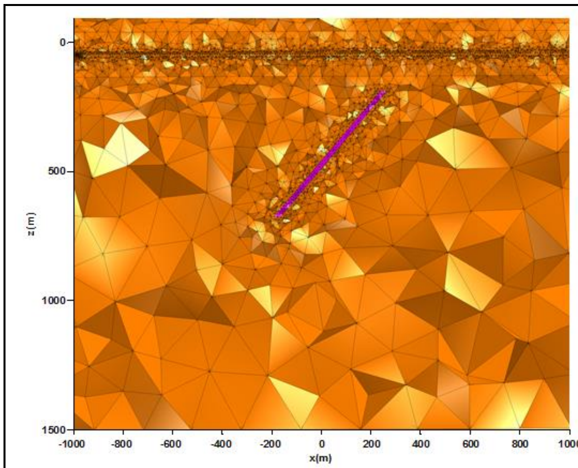

Figure 5: Tetrahedral mesh of the dyke model at y=0.

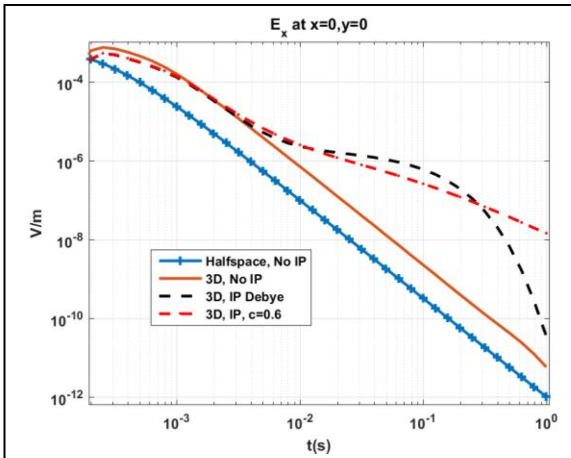

Figure 6: A comparing between the time domain responses above the center of the dyke for different scenario.

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EDITED REFERENCES

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REFERENCES

Baker, G. A., and P. R. Graves-Morris, 1996, Padé approximants: Cambridge University Press.

Commer, M., and G. Newman, 2004, A parallel finite-difference approach for 3D transient electromagnetic modeling with galvanic sources: Geophysics, **69**, 1192–1202, http://dx.doi.org/10.1190/1.1801936.

Jin, J., 2002, Finite element method in electromagnetics (2nd ed.): Wiley-IEEE Press.

Jin, J., 2014, Finite element method in electromagnetics (3rd ed.): Wiley-IEEE Press.

Li, J., C. G. Farquharson, and X. Hu, 2016. Three effective inverse Laplace transform algorithms for computing time-domain electromagnetic responses: Geophysics, **81**, no. 2, E113–E128, http://dx.doi.org/10.1190/geo2015-0174.1.

Luo, Y., and G. Zhang, 1998, Theory and application of spectral induced polarization: Society of Exploration Geophysicists, http://dx.doi.org/10.1190/1.9781560801856.

Marchant, D., E. Haber, and D. W. Oldenburg, 2014, Three-dimensional modeling of IP effects in time-domain electromagnetic data: Geophysics, **79**, no. 6, E303–E314, http://dx.doi.org/10.1190/geo2014-0060.1.

Meerschaert, M. M., and C. Tadjeran, 2004, Finite difference approximations for fractional advection--dispersion flow equations: Journal of Computational and Applied Mathematics, **172**, 65–77, http://dx.doi.org/10.1016/j.cam.2004.01.033.

Miller, K. S., and B. Ross, 1993, An introduction to the fractional calculus and fractional differential equations: Wiley-Blackwell.

Mulder, W. A., M. Wirianto, and E. C. Slob, 2007, Time-domain modeling of electromagnetic diffusion with a frequency-domain code: Geophysics, **73**, no. 1, F1–F8, http://dx.doi.org/10.1190/1.2799093.

Nédélec, J. C., 1980, Mixed finite elements in R^3: Numerische Mathematik, **35**, 315–341, http://dx.doi.org/10.1007/BF01396415.

Pelton, W. H., 1977, Interpretation of induced polarization and resistivity data: Ph.D dissertation, University of Utah.

Pelton, W. H., S. H. Ward, P. G. Hallof, W. R. Sill, and P. H. Nelson, 1978, Mineral discrimination and removal of inductive coupling with multifrequency IP: Geophysics, **43**, no. 3, 588–609, http://dx.doi.org/10.1190/1.1440839.

Ralph-Uwe, B., O. G. Ernst, and K. Spitzer, 2008, Fast 3-D simulation of transient electromagnetic fields by model reduction in the frequency domain using Krylov subspace projection: Geophysical Journal International, **173**, 766–780, http://dx.doi.org/10.1111/j.1365-246X.2008.03750.x.

Rekanos, I. T., and T. G. Papadopoulos, 2010, An auxiliary differential equation method for FDTD modeling of wave propagation in Cole-Cole dispersive media: IEEE Transactions on Antennas and Propagation, **58**, no. 11, 3666–3674, http://dx.doi.org/10.1109/TAP.2010.2071365.

Um, E. S., 2011, Three-dimensional finite-element time-domain modeling of the marine controlled-source electromagnetic method: Ph.D. dissertation, Stanford University.

Um, E. S., J. M. Harris, and D. L. Alumbaugh, 2012, An iterative finite element time-domain method for simulating three-dimensional electromagnetic diffusion in earth: Geophysical Journal International, **190**, 871–886, http://dx.doi.org/10.1111/j.1365-246X.2012.05540.x.

Wang, T., and G. W. Hohmann, 1993, A finite-difference, time-domain solution for three-dimensional electromagnetic modeling: Geophysics, **58**, no. 6, 797–809, http://dx.doi.org/10.1190/1.1443465.

Ward, S. H., and G. W. Hohmann, 1988, Electromagnetic theory for geophysical applications: Society of Exploration Geophysicists, 130–311, http://dx.doi.org/10.1190/1.9781560802631.ch4.

Weedon, W. H., and C. M. Rappaport, 1997, A general method for FDTD modeling of wave propagation in arbitrary frequency-dispersive media: IEEE Transactions on Antennas and Propagation, **45**, no. 3, 401–410, http://dx.doi.org/10.1109/8.558655.

Yee, K. S., 1966, Numerical solution of initial value problems involving Maxwell's equations in isotropic media: IEEE Transactions on Antennas and Propagation, **14**, no. 3, 302–307, http://dx.doi.org/10.1109/TAP.1966.1138693.

Zaslavsky, M., V. Druskin, and L. Knizhnerman, 2011, Solution of 3D time-domain electromagnetic problems using optimal subspace projection: Geophysics, **76**, no. 6, F339–F351, http://dx.doi.org/10.1190/geo2011-0088.1.

Zhdanov, M. S., 2009, Geophysical electromagnetic theory and methods: Elsevier.