JOINT INVERSION USING FOCUSING STABILIZERS

Considering forward geophysical problems for multiple geophysical data sets, we can describe these problems by the operator relationships

\[ \mathbf{d}^{(i)} = \mathbf{A}^{(i)}(\mathbf{m}^{(i)}), \quad i = 1, 2, 3, ..., N; \]

where, in a general case, \( \mathbf{A}^{(i)} \) is a nonlinear operator, \( \mathbf{d}^{(i)} \) \((i = 1, 2, 3, ..., N)\) are different observed data sets (which may have different physical natures and/or parameters), and \( \mathbf{m}^{(i)} \) \((i = 1, 2, 3, ..., N)\) are the unknown sets of model parameters.

Note that, in a general case, different model parameters may have different physical dimensions (e.g., density is measured in g/cm\(^3\), resistivity is measured in Ohm-m, etc.). It is convenient to introduce the dimensionless weighted model parameters, \( \tilde{\mathbf{m}}^{(i)} \), defined as follows:

\[ \tilde{\mathbf{m}}^{(i)} = \mathbf{W}^{(i)}_{m} \mathbf{m}^{(i)}, \]

where \( \mathbf{W}^{(i)}_{m} \) is the corresponding linear operator of the model weighting.

For the solution of a nonlinear inverse problem (1), we introduce the following parametric functional with the focusing stabilizers,

\[ \Phi^{(m)}(\tilde{\mathbf{m}}^{(1)}, \tilde{\mathbf{m}}^{(2)}, ..., \tilde{\mathbf{m}}^{(N)}) = \sum_{i=1}^{N} \tilde{\varphi}^{(i)} + \alpha S_{JMS, JMGS} \]

where \( \tilde{\varphi}^{(i)} = \left\| \tilde{\mathbf{A}}^{(i)}(\tilde{\mathbf{m}}^{(i)}) - \mathbf{d}^{(i)} \right\|_{D}^{2} \) is the misfit and \( \alpha \) is the regularization parameter.

The terms \( S_{JMS} \) and \( S_{JMGS} \) are the joint stabilizing functionals, based on minimum support and minimum gradient support constraints, respectively (Molodtsov, 2017).
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For example, a joint minimum support stabilizer can be introduced as follows:

$$S_{JM} = \sqrt{\sum_{i=1}^{N} \frac{(\tilde{m}^{(i)} - \tilde{m}_{\text{ap}r}^{(i)})^2}{\sum_{i=1}^{N} (\tilde{m}^{(i)} - \tilde{m}_{\text{ap}r}^{(i)})^2 + \epsilon^2} \, dv.}$$  \hspace{1cm} (4)

In a similar way, we can introduce a joint minimum gradient support functional (JMGS):

$$S_{JMS} = \sqrt{\sum_{i=1}^{N} \frac{(\nabla \tilde{m}^{(i)}) \cdot (\nabla \tilde{m}^{(i)})}{\sum_{i=1}^{N} (\nabla \tilde{m}^{(i)}) \cdot (\nabla \tilde{m}^{(i)}) + \epsilon^2} \, dv.}$$  \hspace{1cm} (5)

According to the basic principles of the regularization method, we have to find the models $\tilde{m}_{\text{ap}r}^{(1)},...,\tilde{m}_{\text{ap}r}^{(2)},...,\tilde{m}_{\text{ap}r}^{(N)}$, a quasi-solution of the inverse problem, which minimizes the parametric functional:

$$P^\alpha(\tilde{m}_{\text{ap}r}^{(1)},\tilde{m}_{\text{ap}r}^{(2)},...,\tilde{m}_{\text{ap}r}^{(N)}) = \min.$$  \hspace{1cm} (6)

The minimization of the parametric functional $P^\alpha$ is based on the re-weighted regularized conjugate gradient method (RRCG, Zhdanov, 2002), which iteratively updates the model parameters, $\tilde{m}^{(i)}$, to minimize the misfit, $\varphi^{(i)}$, between the observed and predicted data. The inversion iterates until the misfit, $\varphi^{(i)}$, reaches a given threshold, or until a maximum number of iterations is reached.

JOINT INVERSION USING THE GRAMIAN STABILIZER

Consider two different geophysical data sets, $d^{(i)}$ ($i = 1, 2$), and the related two physical properties, $\tilde{m}^{(i)}$ ($i = 1, 2$). The joint inversion recovers two physical properties simultaneously using a single parametric functional according to the following formula:

$$P^\alpha(\tilde{m}^{(1)},\tilde{m}^{(2)}) = \sum_{i=1}^{2} \alpha^{(i)} s_M(\tilde{m}^{(i)}) + \beta s_G(\tilde{L}^{(1)}\tilde{m}^{(1)},\tilde{L}^{(2)}\tilde{m}^{(2)}).$$  \hspace{1cm} (7)

The coefficients $\alpha^{(i)}$ are the independent property regularization parameters, and $\beta$ is the joint stabilizer regularization parameter. Both $\alpha^{(i)}$ and $\beta$ decrease as the inversion progresses. The term $s_M$ is a stabilizing functional and $s_G(\tilde{L}^{(1)}\tilde{m}^{(1)},\tilde{L}^{(2)}\tilde{m}^{(2)})$ is the Gramian constraint (Zhdanov et al., 2012, 2015), which in a case of two physical properties can be written, using matrix notations, as follows:

$$s_G(\tilde{L}^{(1)}\tilde{m}^{(1)},\tilde{L}^{(2)}\tilde{m}^{(2)}) = \begin{vmatrix} (\tilde{L}^{(1)}\tilde{m}^{(1)},\tilde{L}^{(1)}\tilde{m}^{(1)}) & (\tilde{L}^{(1)}\tilde{m}^{(1)},\tilde{L}^{(2)}\tilde{m}^{(2)}) \\ (\tilde{L}^{(2)}\tilde{m}^{(2)},\tilde{L}^{(1)}\tilde{m}^{(1)}) & (\tilde{L}^{(2)}\tilde{m}^{(2)},\tilde{L}^{(2)}\tilde{m}^{(2)}) \end{vmatrix},$$  \hspace{1cm} (8)

where operators $\tilde{L}^{(i)}$ ($i = 1, 2$) represent some linear transformation of the model parameters; and operation $(\cdot, \cdot)$ stands for the inner product of two vectors in the corresponding Gramian space (Zhdanov, 2015). By minimizing a parametric functional with the Gramian constraint, we enforce some linear correlation between the model parameters. Generalizing to arbitrary $\tilde{L}^{(i)}$, we can use the Gramian constraint to enhance linear correlation between the transformed model parameters, which provides the way for complex nonlinear correlation between the model parameters.

If the model parameter gradients are correlated, $\tilde{L}\tilde{m} = \nabla \tilde{m}$, this imposes structural similarities between the different physical properties, similar to using the cross-gradient functional. For nontrivial $\nabla \tilde{m}^{(i)}$, this functional is minimized, when the two gradients are parallel to each other, resulting in the following linear relationship:

$$\nabla \tilde{m}^{(1)}(x, y, z) = k_g \nabla \tilde{m}^{(2)}(x, y, z).$$  \hspace{1cm} (9)

Expression (9) is minimized when the two gradients, $\nabla \tilde{m}^{(1)}$ and $\nabla \tilde{m}^{(2)}$, are parallel to each other, which is equivalent to condition (9).

We apply the Gramian and joint focusing inversion to gravity and magnetic data using our parallel GPU accelerated inversion program (Čuma and Zhdanov, 2014).

MODEL STUDY

In a representative model we test on how well the joint inversion does with two different anomalies. We have two cubic 200×200×200 m³ anomalies 50 m below surface, both having 1 g/m³ density contrast, and only the left one having 0.06 SI magnetic susceptibility (Figure 1 a)). The observed data consist of vertical gravity $g_z$, six gravity gradient components and...
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The inversion was run for 50 iterations turning focusing on at iteration 5, except for the Gramian joint inversions which we run for 100 iterations starting focusing at iteration 20. Slower convergence of the Gramian joint inversion is common as the Gramian stabilizer couples the material properties strongly at the start of the inversion, which reduces the speed of the misfit decrease. The adaptive regularization decay coefficients were set to 0.9 for the individual model stabilizers and 0.95 for the Gramian joint stabilizer. In the Gramian joint inversion, it is important to adjust the weights of the model parameters to be roughly equal. We use sensitivity based model weights. Data weights based on the norm of each data component were also used to equalize the data influence on the inversion.

Figure 1 shows the vertical cross sections of the model obtained with the different inversion approaches. The gravity anomalies are resolved fairly well, albeit at a slightly deeper location. The magnetic anomaly is quite weak in the independent inversion, and stronger in the joint inversions. The joint focusing inversions also correctly identify no magnetic anomaly for the right body.

CASE STUDY: MCFAULDS LAKE, ONTARIO

McFaulds Lake covers the Ring of Fire intrusive complex located in the James Bay lowlands of northwestern Ontario. Ring of Fire is a roughly north-south trending Archean green belt (Figure 2). It is composed of mafic metavolcanic flows, felsic metavolcanic flows and pyroclastic rocks and a suit of layered mafic to ultramafic intrusions that trend subparallel with and obliquely cut the westernmost part of the belt, close to a large granitoid batholith lying west of the belt. The major layered intrusion at its base, hosts Ni-Cu-PGE deposits of exceptional grade as well as overlying stratiform chromite deposits further east and higher in the layered intrusion stratigraphy (Ontario Geological Survey and Geological Survey of Canada, 2011).

In order to map regional geology and locate further potential mineral resources, an airborne geophysical survey was carried out in the McFaulds Lake region between 2010 and 2011. Both airborne gravity gradiometer (AGG) and magnetic data were collected. This project was collaboratively operated between the Ontario Geological Survey (OGS) and the Geological Survey of Canada (GSC). The survey was flown with the Fugro Airborne Surveys gravity gradiometer and magnetic system.

We have inverted the entire survey area jointly with the Gramian stabilizer of the model gradients on a 50x50x50 m³ grid up to a depth of 2 km. This resulted in ca. 200 million inversion cells. As the data points are located roughly every 6 m along the survey lines, we decimated the data by using every eighth, keeping ~2.5 million AGG and 470,000 TMI data values. We ran 85 iterations, 10 with smooth minimum norm stabilizer, followed by joint minimum support focusing stabilizer reweighting at every 10 iterations. This took 3 days on three cluster nodes, each with three Nvidia Volta V100 GPUs (nine GPUs total) and the final misfit of the gravity data was 18 % and of the magnetic data was 10 %. We also ran independent inversion and Gramian joint inversion coupling gradients of the model pa...
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rameters which achieved similar misfit, although the Gramian inversion took 150 iterations to converge.

Figure 4: The horizontal cross sections of the a) density, and b) susceptibility around the Thunderbird V-Ti deposit obtained by the regional inversion.

Figure 5: The vertical cross sections of the a) density, and b) susceptibility through the Thunderbird V-Ti deposit obtained by the regional inversion.

Figure 3 shows the horizontal cross sections of the density and susceptibility at a depth of 125 m below sea level (300 m below the surface) obtained by the joint inversion of the entire survey area. The locations of the known mineral deposits are shown as well. The correlation of the density and susceptibility anomalies with the surface geology (Figure 2) is quite obvious, depicting the concave greenstone belt along with several arms to the south-east.

The Thunderbird deposit consists of semi-massive vanadium and titanium enriched magnetite, which corresponds to a strong gravity and magnetic anomaly. Noront Resources, owner of the claim, estimates the ore body at 1.6 km long, 400 m wide and 500 m deep, based on gravity and magnetic data and limited core drilling. This deposit was not being developed yet and a more detailed analysis was lacking.

CONCLUSIONS

We use the regional 3D inversion result to focus on the area of the Thunderbird deposit. Figure 4 shows the density and susceptibility horizontal cross sections at 125 m below the surface. Especially, the magnetic body is well defined, albeit near the surface shifted a few hundred meters from the perceived deposit location. An east-west vertical cross section going through the middle of the body is shown in Figure 5. We see a strong magnetic body near the surface and another deeper body roughly in the horizontal location of the deposit. The dense body is less defined near the surface at the deposit site, presumably due to weathering, but gets denser and wider at depth.

We have developed a computationally effective algorithm of joint inversion of the gravity and magnetic data based on a joint focusing stabilizer. This approach is included in our GPU enabled parallel inversion code, which allows us to invert large data sets. We have evaluated the performance of this approach on model studies, comparing them to independent inversions and to Gramian stabilized joint inversion. Joint focusing stabilizers emphasize the coupling between model parameters, where it exists, and reduce it, where it does not, while only gradient coupled joint Gramian stabilizers do this also. Joint focusing stabilizers tend to converge faster, although they may result in overfocusing of the anomaly.

The Gramian joint stabilizers converge considerably slower, but allow for more tailoring of smooth and focusing stabilizers to reduce the overfocusing problem. The Gramian stabilizers are also sensitive to the model weighting of the different model parameters, due to the single minimization scheme for all model parameters, while the joint focusing stabilizers avoid this problem by having independent minimization (aside from the stabilizer coupling) for each model parameter. Examining the model studies we believe we have come up with a good set of inversion parameters to produce realistic inversion results.

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