Joint inversion of multimodal data using focusing stabilizers and Gramian constraints

Michael S. Zhdanov and Martin Čuma
University of Utah and TechnoImaging

SUMMARY

This paper demonstrates that focusing stabilizers can be used for joint inversion of gravity and magnetic data. We consider joint minimum support and joint minimum gradient support stabilizers in the context of re-weighted regularized conjugate gradient inversion, which is implemented in our GPU enabled potential fields inversion program. The joint focusing stabilizers force the anomalies of different physical properties either overlap or experience a rapid change in the same areas, thus enforcing the structural correlation. We compare this joint inversion approach to independent and joint inversions using Gramian stabilizers on models and real gravity gradient and magnetic data from the Ring of Fire area, Ontario. Joint focusing stabilizers converge faster than the joint Gramian stabilizers, although, the latter allow for a better tailored mix of smooth and focusing as joint focusing tends to overfocus the anomalies. The Ring of Fire inversion result correlate well with known mineral discoveries.

INTRODUCTION

As gravity and magnetic inversion is an ill-posed problem, a regularization must be introduced to recover the most geologically plausible solutions from the infinite number of mathematically equivalent models. The regularization effectively selects the class of models from which a unique solution is sought. Over the years, a variety of methods have been developed for 3D inversion of potential field data with both smooth (e.g., Li and Oldenburg, 1996, 1998) and focusing (e.g., Zhdanov, 2002, 2009, 2015; Kirkendall et. al., 2007) regularizations. Nonuniqueness can also be reduced by incorporating additional information derived from available geological and/ or geophysical data in the survey area to reduce the searching space for the solution. This additional information can be incorporated in the form of a joint inversion. Different geophysical fields provide information about the different physical properties of rock formations. In many cases this information is mutually complementary, which makes it a natural for consideration in a joint inversion of the different geophysical data.

There are different approaches to joint inversion. In a case where the corresponding model parameters are identical or mutually correlated, the joint inversion can explore the existence of this known correlation (e.g., Jupp and Vozoff, 1975; Hoversten et al., 2003, 2006). In a case, where the model parameters are not correlated, but nevertheless have similar geometrical features, the joint inversion can be based on structure-coupled constraints. It is often based on minimizing a value of the cross gradients between different model parameters. This has now been widely used in the joint inversion of geophysical data (e.g., Colombo and De Stefano, 2007). Zhdanov et al. (2012) developed a generalized approach to joint inversion based on Gramian constraints which correlate and/or impose structural similarities between different physical properties without a priori knowledge about a specific form of these cross-model relationships (Zhdanov, 2015).

Another approach can be used based on a joint total variation functional (Molodtsov and Troyan, 2016), or on joint focusing stabilizers (Molodtsov, 2017), e.g., minimum support and minimum gradient support constraints (Zhdanov, 2015). It is well known that the focusing stabilizers minimize the areas with anomalous physical properties (in the case of the minimum support stabilizer) or the areas where major changes in physical properties occurs (in the case of the minimum gradient support stabilizer). The joint focusing stabilizers force the anomalies of different physical properties to either overlap or experience a rapid change in the same areas, thus enforcing the structural correlation. We will demonstrate these properties in the model study, presented in our paper.

JOINT INVERSION USING FOCUSING STABILIZERS

Considering forward geophysical problems for multiple geophysical data sets, we can describe these problems by the operator relationships

$$\mathbf{d}^{(i)} = \mathbf{A}^{(i)}(\mathbf{m}^{(i)}), \quad i = 1, 2, 3, ..., N; \tag{1}$$

where, in a general case, $\mathbf{A}^{(i)}$ is a nonlinear operator, $\mathbf{d}^{(i)}$ ($i = 1, 2, 3, ..., N$) are different observed data sets (which may have different physical natures and/or parameters), and $\mathbf{m}^{(i)}$ ($i = 1, 2, 3, ..., N$) are the unknown sets of model parameters.

Note that, in a general case, different model parameters may have different physical dimensions (e.g., density is measured in g/cm^3, resistivity is measured in Ohm-m, etc.). It is convenient to introduce the dimensionless weighted model parameters, $\widetilde{\mathbf{m}}^{(i)}$, defined as follows:

$$\widetilde{\mathbf{m}}^{(i)} = \mathbf{W}_m^{(i)} \mathbf{m}^{(i)}, \tag{2}$$

where $\mathbf{W}_m^{(i)}$ is the corresponding linear operator of the model weighting.

For the solution of a nonlinear inverse problem (1), we introduce the following parametric functional with the focusing stabilizers,

$$P^{\alpha}(\widetilde{\mathbf{m}}^{(1)}, \widetilde{\mathbf{m}}^{(2)}, ..., \widetilde{\mathbf{m}}^{(n)}) = \sum_{i=1}^{N} \widetilde{\varphi}^{(i)} + \alpha S_{JMS,\ JMGS} \tag{3}$$

where $\widetilde{\varphi}^{(i)} = \left\| \widetilde{\mathbf{A}}^{(i)}(\widetilde{\mathbf{m}}^{(i)}) - \widetilde{\mathbf{d}}^{(i)} \right\|_D^2$ is the misfit and α is the regularization parameter.

The terms S_{JMS}, and S_{JMGS} are the joint stabilizing functionals, based on minimum support and minimum gradient support constraints, respectively (Molodtsov, 2017).

For example, a joint minimum support stabilizer can be introduced as follows:

$$S_{JMS} = \int_V \frac{\sum_{i=1}^{N} \left(\widetilde{m}^{(i)} - \widetilde{m}_{apr}^{(i)}\right)^2}{\sum_{i=1}^{N} \left(\widetilde{m}^{(i)} - \widetilde{m}_{apr}^{(i)}\right)^2 + e^2} dv. \qquad (4)$$

In a similar way, we can introduce a joint minimum gradient support functional (JMGS):

$$S_{JMGS} = \int_V \frac{\sum_{i=1}^{N} \left(\nabla\widetilde{m}^{(i)} \cdot \nabla\widetilde{m}^{(i)}\right)}{\sum_{i=1}^{N} \left(\nabla\widetilde{m}^{(i)} \cdot \nabla\widetilde{m}^{(i)}\right) + e^2} dv. \qquad (5)$$

According to the basic principles of the regularization method, we have to find the models $\widetilde{\mathbf{m}}_\alpha^{(1)}$,.... $\widetilde{\mathbf{m}}_\alpha^{(2)}$, ...$\widetilde{\mathbf{m}}_\alpha^{(N)}$, a quasi-solution of the inverse problem, which minimizes the parametric functional:

$$P^\alpha(\widetilde{\mathbf{m}}_\alpha^{(1)}, \widetilde{\mathbf{m}}_\alpha^{(2)}, ...\widetilde{\mathbf{m}}_\alpha^{(N)}) = \min. \qquad (6)$$

The minimization of the parametric functional P^α is based on the re-weighted regularized conjugate gradient method (RRCG, Zhdanov, 2002), which iteratively updates the model parameters, $\mathbf{m}^{(i)}$; to minimize the misfit, $\varphi^{(i)}$, between the observed and predicted data. The inversion iterates until the misfit, $\varphi^{(i)}$, reaches a given threshold, or until a maximum number of iterations is reached.

JOINT INVERSION USING THE GRAMIAN STABILIZER

Consider two different geophysical data sets, $\mathbf{d}^{(i)}$ $(i = 1, 2)$, and the related two physical properties, $\mathbf{m}^{(i)}$ $(i = 1, 2)$. The joint inversion recovers two physical properties simultaneously using a single parametric functional according to the following formula:

$$P^\alpha(\mathbf{m}^{(1)}, \mathbf{m}^{(2)}) = \sum_{i=1}^{2} \mathbf{r}^{(i)}$$

$$+\sum_{i=1}^{2} \alpha^{(i)} s_M(\mathbf{m}^{(i)}) + \beta s_G(L^{(1)}\mathbf{m}^{(1)}, L^{(2)}\mathbf{m}^{(2)}). \qquad (7)$$

The coefficients $\alpha^{(i)}$ are the independent property regularization parameters, and β is the joint stabilizer regularization parameter. Both $\alpha^{(i)}$ and β decrease as the inversion progresses. The term s_M is a stabilizing functional and $s_G(L^{(1)}\mathbf{m}^{(1)}, L^{(2)}\mathbf{m}^{(2)})$ is the Gramian constraint (Zhdanov et al., 2012, 2015), which in a case of two physical properties can be written, using matrix notations, as follows:

$$s_G(L^{(1)}\mathbf{m}^{(1)}, L^{(2)}\mathbf{m}^{(2)}) =$$

$$\begin{vmatrix} (L^{(1)}\mathbf{m}^{(1)}, L^{(1)}\mathbf{m}^{(1)}) & \left(L^{(1)}\mathbf{m}^{(1)}, L^{(2)}\mathbf{m}^{(2)}\right) \\ \left(L^{(2)}\mathbf{m}^{(2)}, L^{(1)}\mathbf{m}^{(1)}\right) & (L^{(2)}\mathbf{m}^{(2)}, L^{(2)}\mathbf{m}^{(2)}) \end{vmatrix}, \qquad (8)$$

where operators $L^{(i)}$ $(i = 1, 2)$ represent some linear transformation of the model parameters; and operation $(\cdot, \cdot)$ stands for the inner product of two vectors in the corresponding Gramian space (Zhdanov, 2015). By minimizing a parametric functional with the Gramian constraint, we enforce some linear correlation between the model parameters. Generalizing to arbitrary $L^{(i)}$, we can use the Gramian constraint to enhance linear correlation between the transformed model parameters, which provides the way for complex nonlinear correlation between the model parameters.

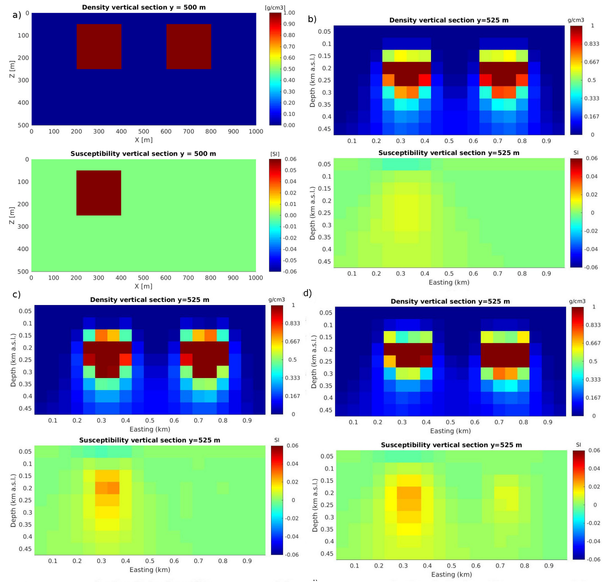

Figure 1: Model vertical cross-section, a) true model, b) independent inversion with minimum support focusing, c) joint minimum support inversion,d) joint Gramian inversion with gradient property coupling and minimum support focusing.

If the model parameter gradients are correlated, $L\mathbf{m} = \nabla m$, this imposes structural similarities between the different physical properties, similar to using the cross-gradient functional. For nontrivial $\nabla m^{(i)}$, this functional is minimized, when the two gradients are parallel to each other, resulting in the following linear relationship:

$$\nabla m^{(1)}(x, y, z) = k_g \nabla m^{(2)}(x, y, z). \qquad (9)$$

Expression (8) is minimized when the two gradients, $\nabla m^{(1)}$ and $\nabla m^{(2)}$, are parallel to each other, which is equivalent to condition (9).

We apply the Gramian and joint focusing inversion to gravity and magnetic data using our parallel GPU accelerated inversion program (Čuma and Zhdanov, 2014).

MODEL STUDY

In a representative model we test on how well the joint inversion does with two different anomalies. We have two cubic 200x200x200 m^3 anomalies 50 m below surface, both having 1 g/m^3 density contrast, and only the left one having 0.06 SI magnetic susceptibility (Figure 1 a)). The observed data consist of vertical gravity g_z, six gravity gradient components and

10.1190/segam2018-2998495.1

total magnetic intensity (TMI) collected over 1 km^2 area over the target with data spacing at 50 m.

The inversion was run for 50 iterations turning focusing on at iteration 5, except for the Gramian joint inversions which we run for 100 iterations starting focusing at iteration 20. Slower convergence of the Gramian joint inversion is common as the Gramian stabilizer couples the material properties strongly at the start of the inversion, which reduces the speed of the misfit decrease. The adaptive regularization decay coefficients were set to 0.9 for the individual model stabilizers and 0.95 for the Gramian joint stabilizer. In the Gramian joint inversion, it is important to adjust the weights of the model parameters to be roughly equal. We use sensitivity based model weights. Data weights based on the norm of each data component were also used to equalize the data influence on the inversion.

Figure 1 shows the vertical cross sections of the model obtained with the different inversion approaches. The gravity anomalies are resolved fairly well, albeit at a slightly deeper location. The magnetic anomaly is quite weak in the independent inversion, and stronger in the joint inversions. The joint focusing inversions also correctly identify no magnetic anomaly for the right body.

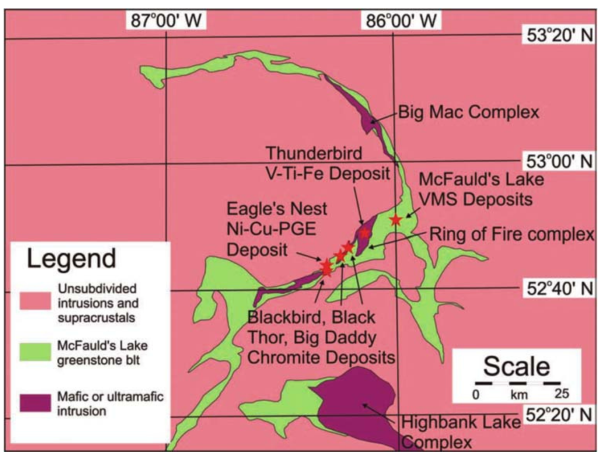

Figure 2: Geological map of the Ring of Fire area with marked known deposits (from Mungall et al., 2010).

CASE STUDY: MCFAULDS LAKE, ONTARIO

McFaulds Lake covers the Ring of Fire intrusive complex located in the James Bay lowlands of northwestern Ontario. Ring of Fire is a roughly north-south trending Archean green belt (Figure 2). It is composed of mafic metavolcanic flows, felsic metavolcanic flows and pyroclastic rocks and a suit of layered mafic to ultramafic intrusions that trend subparallel with and obliquely cut the westernmost part of the belt, close to a large granitoid batholith lying west of the belt. The major layered intrusion at its base, hosts Ni-Cu-PGE deposits of exceptional grade as well as overlying stratiform chromite deposits further east and higher in the layered intrusion stratigraphy (Ontario Geological Survey and Geological Survey of Canada, 2011).

In order to map regional geology and locate further potential mineral resources, an airborne geophysical survey was carried out in the McFaulds Lake region between 2010 and 2011. Both airborne gravity gradiometer (AGG) and magnetic data were collected. This project was collaboratively operated between the Ontario Geological Survey (OGS) and the Geological Survey of Canada (GSC). The survey was flown with the Fugro Airborne Surveys gravity gradiometer and magnetic system.

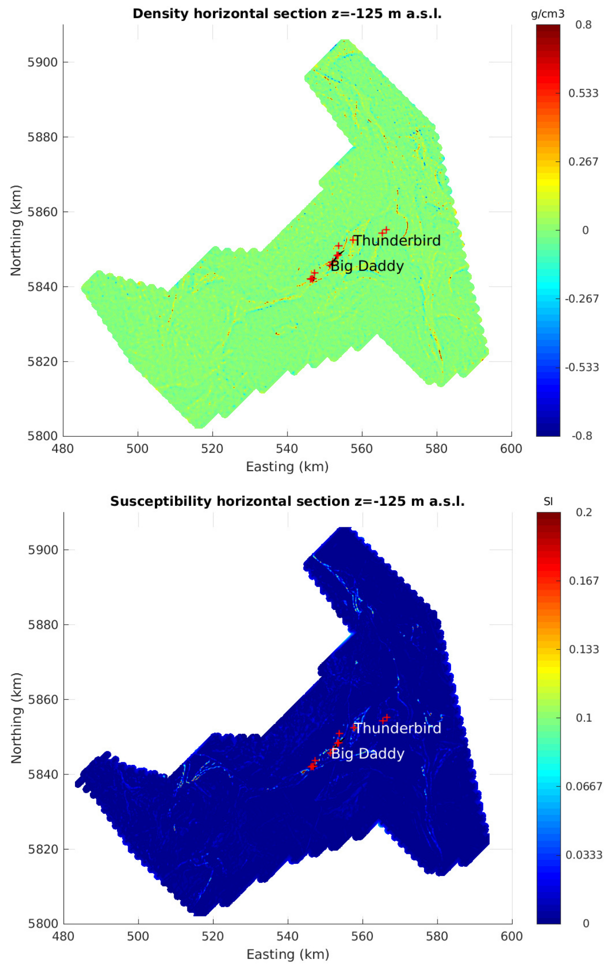

Figure 3: Horizontal cross section of the a) density and b) susceptibility obtained from the inversion of the entire McFaulds Lake survey.

We have inverted the entire survey area jointly with the Gramian stabilizer of the model gradients on a 50x50x50 m^3 grid up to a depth of 2 km. This resulted in ca. 200 million inversion cells. As the data points are located roughly every 6 m along the survey lines, we decimated the data by using every eighth, keeping ~2.5 million AGG and 470,000 TMI data values. We ran 85 iterations, 10 with smooth minimum norm stabilizer, followed by joint minimum support focusing stabilizer reweighting at every 10 iterations. This took 3 days on three cluster nodes, each with three Nvidia Volta V100 GPUs (nine GPUs total) and the final misfit of the gravity data was 18 % and of the magnetic data was 10 %. We also ran independent inversion and Gramian joint inversion coupling gradients of the model pa-

10.1190/segam2018-2998495.1

rameters which achieved similar misfit, although the Gramian inversion took 150 iterations to converge.

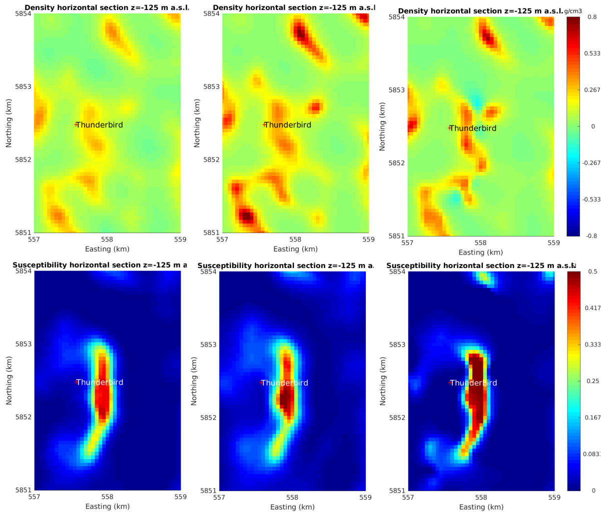

Figure 4: The horizontal cross sections of the a) density, and b) susceptibility around the Thunderbird V-Ti deposit obtained by the regional inversion.

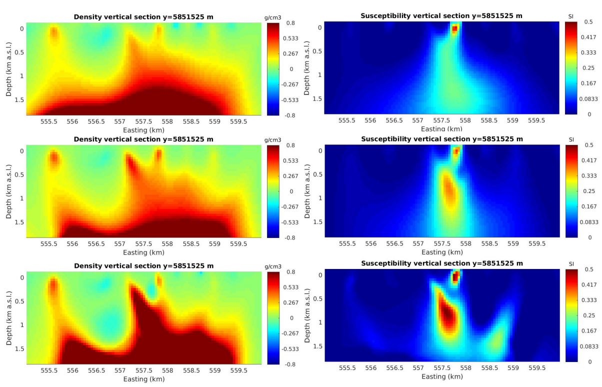

Figure 5: The vertical cross sections of the a) density, and b) susceptibility through the Thunderbird V-Ti deposit obtained by the regional inversion.

Figure 3 shows the horizontal cross sections of the density and susceptibility at a depth of 125 m below sea level (300 m below the surface) obtained by the joint inversion of the entire survey area. The locations of the known mineral deposits are shown as well. The correlation of the density and susceptibility anomalies with the surface geology (Figure 2) is quite obvious, depicting the concave greenstone belt along with several arms to the south-east.

The Thunderbird deposit consists of semi-massive vanadium and titanium enriched magnetite, which corresponds to a strong gravity and magnetic anomaly. Noront Resources, owner of the claim, estimates the ore body at 1.6 km long, 400 m wide and 500 m deep, based on gravity and magnetic data and limited core drilling. This deposit was not being developed yet and a more detailed analysis was lacking.

We use the regional 3D inversion result to focus on the area of the Thunderbird deposit. Figure 4 shows the density and susceptibility horizontal cross sections at 125 m below the surface. Especially, the magnetic body is well defined, albeit near the surface shifted a few hundred meters from the perceived deposit location. An east-west vertical cross section going through the middle of the body is shown in Figure 5. We see a strong magnetic body near the surface and another deeeper body roughly in the horizontal location of the deposit. The dense body is less defined near the surface at the deposit site, presumably due to weathering, but gets denser and wider at depth.

CONCLUSIONS

We have developed a computationally effective algorithm of joint inversion of the gravity and magnetic data based on a joint focusing stabilizer. This approach is included in our GPU enabled parallel inversion code, which allows us to invert large data sets. We have evaluated the performance of this approach on model studies, comparing them to independent inversions and to Gramian stabilized joint inversion. Joint focusing stabilizers emphasize the coupling between model parameters, where it exists, and reduce it, where it does not, while only gradient coupled joint Gramian stabilizers do this also. Joint focusing stabilizers tend to converge faster, although they may result in overfocusing of the anomaly.

The Gramian joint stabilizers converge considerably slower, but allow for more tailoring of smooth and focusing stabilizers to reduce the overfocusing problem. The Gramian stabilizers are also sensitive to the model weighting of the different model parameters, due to the single minimization scheme for all model parameters, while the joint focusing stabilizers avoid this problem by having independent minimization (aside from the stabilizer coupling) for each model parameter. Examining the model studies we believe we have come up with a good set of inversion parameters to produce realistic inversion results.

We applied the developed approach to the inversion of the regional scale airborne potential field data acquired over the Ring of Fire greenbelt in the northwestern Ontario. We used the regional 3D model to zoom into the Thunderbird V-Ti-Fe deposit, and we observed a good correlation with the little known characteristics of the ore body. The Thunderbird deposit is formed by magnetite rocks, which are both dense and magnetic.

ACKNOWLEDGEMENTS

The authors acknowledge the support of the University of Utah Consortium for Electromagnetic Modeling and Inversion (CEMI). We also acknowledge support and allocation of computer time provided by the University of Utah's Center for High Performance Computing (CHPC). We also acknowledge the Ontario Geological Survey for providing the survey data and supporting materials.

10.1190/segam2018-2998495.1

REFERENCES

Colombo, D., and M. De Stefano, 2007, Geophysical modeling via simultaneous joint inversion of seismic, gravity, and electromagnetic data: Application to prestack depth imaging: The Leading Edge, **26**, 326–331, https://doi.org/10.1190/1.2715057.

Cuma, M., and M. S. Zhdanov, 2014, Massively parallel regularized 3D inversion of potential fields on CPUs and GPUs: Computers and Geosciences, **62**, 80–87, https://doi.org/10.1016/j.cageo.2013.10.004.

Kirkendall, B., Y. Li, and D. W. Oldenburg, 2007, Imaging cargo containers using gravity gradiometry: IEEE Transactions on Geoscience and Remote Sensing, **45**, 1786–1797, https://doi.org/10.1109/TGRS.2007.895427.

Li, Y., and D. W. Oldenburg, 1996, 3-D inversion of magnetic data: Geophysics, **61**, 394–408, https://doi.org/10.1190/1.1443968.

Li, Y., and D. W. Oldenburg, 1998, 3-D inversion of gravity data: Geophysics, **63**, 109–119, https://doi.org/10.1190/1.1444302.

Mungall, J. E., J. D. Harvey, S. J. Balch, B. Azar, J. Atkinson, and M. A. Hamilton, 2010, Eagle's Nest: A magmatic Ni- Sulfide deposit in the James Bay Lowlands, Ontario, Canada: Society of Economic Geologists, Special Publication 15, 539–557.

Zhdanov, M. S., 2002, Geophysical inverse theory and regularization problems: Elsevier.

Zhdanov, M. S., 2009, New advances in 3D regularized inversion of gravity and electromagnetic data: Geophysical Prospecting, **57**, 463–478, https://doi.org/10.1111/j.1365-2478.2008.00763.x.

Zhdanov, M. S., 2015, Inverse theory and applications in geophysics: Elsevier.

Zhdanov, M. S., A. V. Gribenko, and G. Wilson, 2012, Generalized joint inversion of multimodal geophysical data using Gramian constraints: Geophysical Research Letters, **39**, L09301, https://doi.org/10.1029/2012gl051233.